A study of Rainfall-Runoff Process considering two uncertainties in Basin with multiple dams

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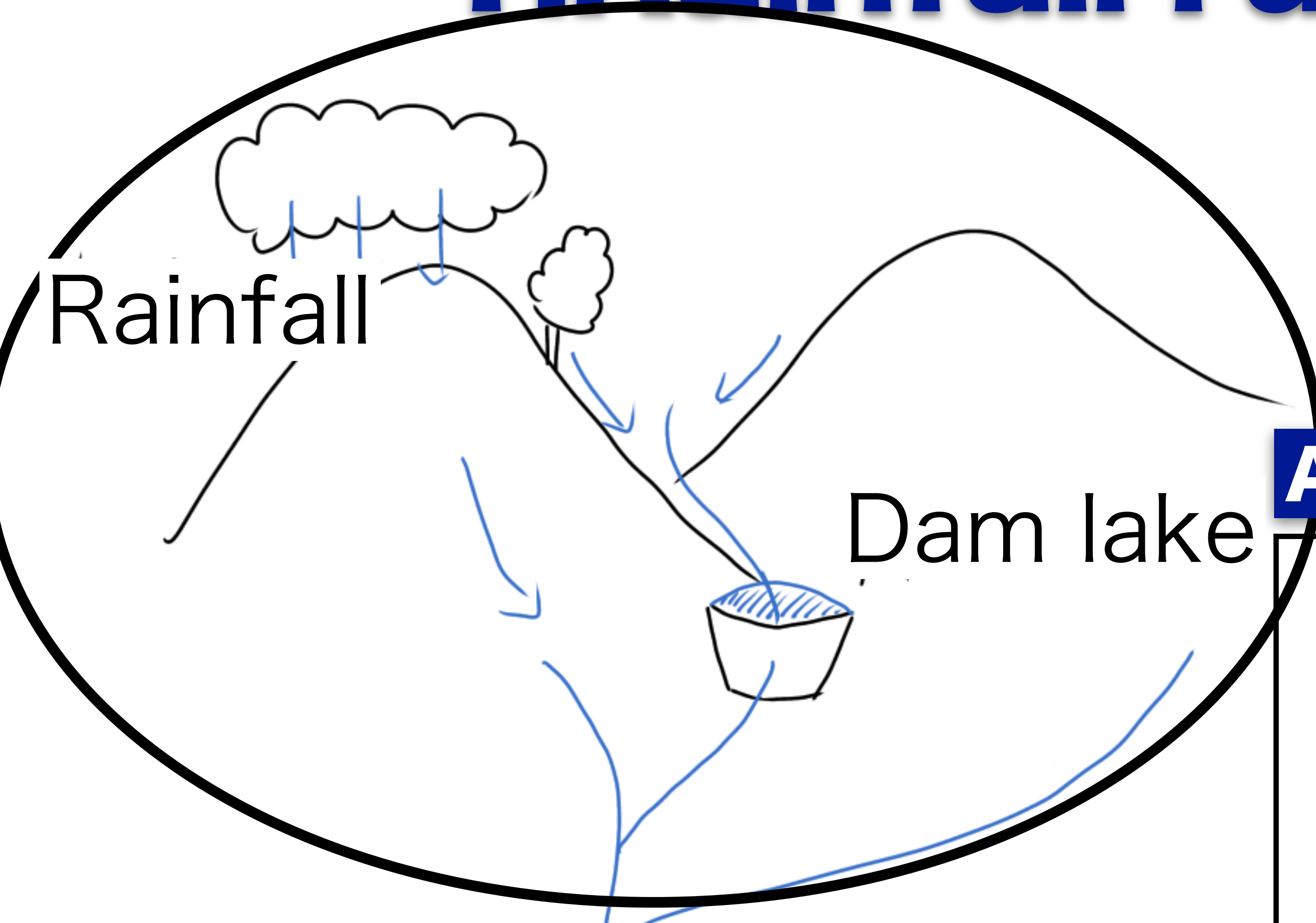
1.Rainfall runoff process



Runoff
- **Direct runoff**
- Base flow

Analysis of Runoff, Physical Model

Basic equations are derived from the Kinematic wave equation.

$$\begin{cases} \dfrac{\partial q}{\partial x} + \dfrac{\partial h}{\partial t} = r_e & \text{- ①} \\ q = \alpha h^m & \text{- ②} \end{cases}$$

q :discharge per unit width、
h:depth of flooding
θ:gradient of steep、τ_b:frictional stress

The observation results show that runoff coming from the vicinity of the river contribute direct runoff .

Yamada(2003)

$$q(x,t) \simeq l q_*(t) \quad \text{-③}$$

q_* :runoff height、l:length of the contributing regions

$$\frac{dq_*}{dt} = \alpha q_*^{\beta}(r_e - q_*) \quad \text{- ④}$$

q_*:runoff height r_e:effective rainfall

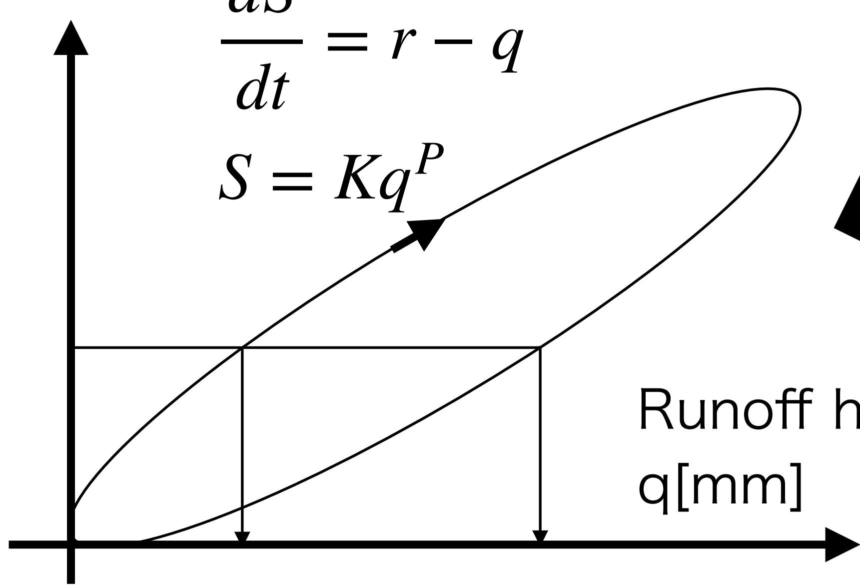
α, β:parameters

貯留関数法

Storage Height s[mm]

$$\frac{dS}{dt} = r - q$$

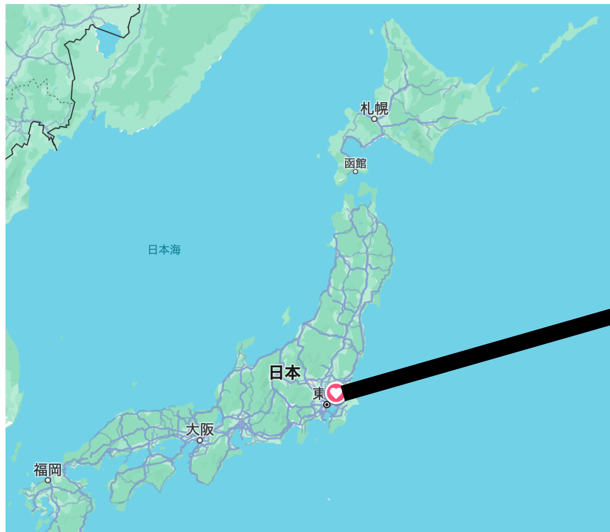
$$S = Kq^P$$



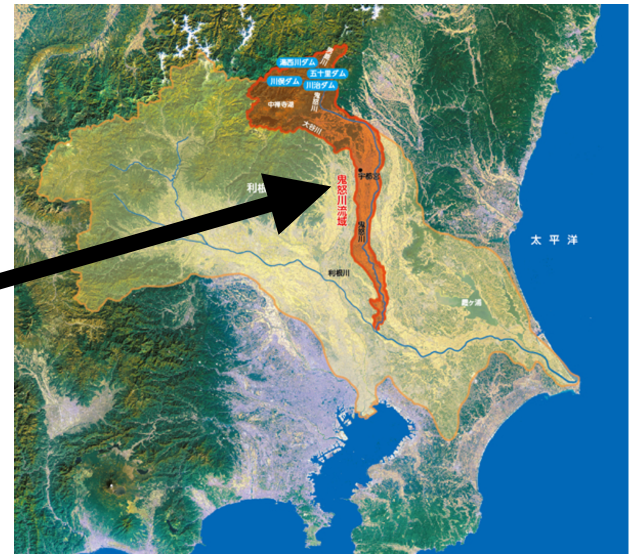
Accumulation of observation data of heavy rainfall cases in recent decades

Object of the study: To propose an equation form that takes into account the uncertainty of the variables (rainfall information)

2.Uncertainty in rainfall observations

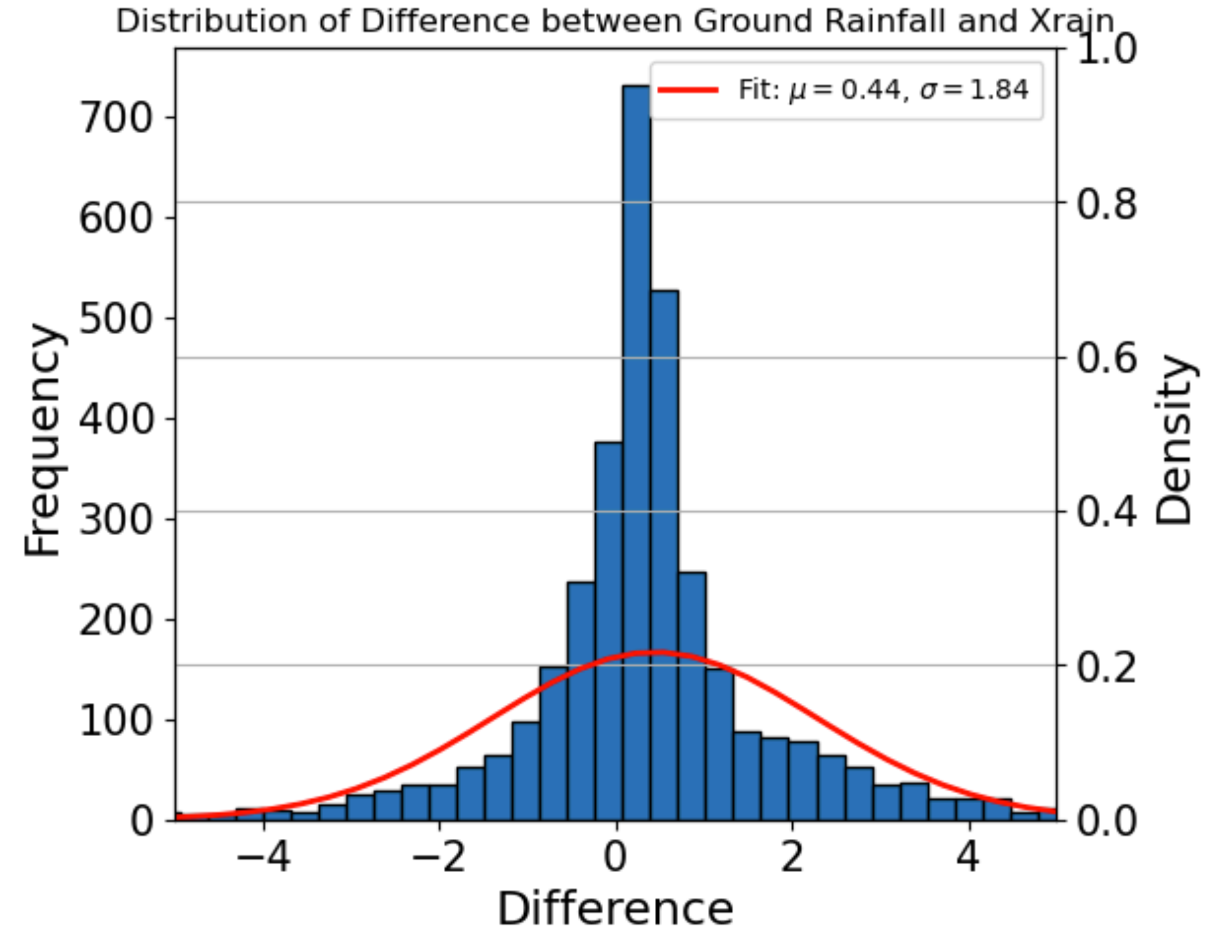


Japan,Google map



Japan,Kinugawa basin,MLIT

the 2015 Kanto Tohoku Heavy Rainfall,Nikkei x tech



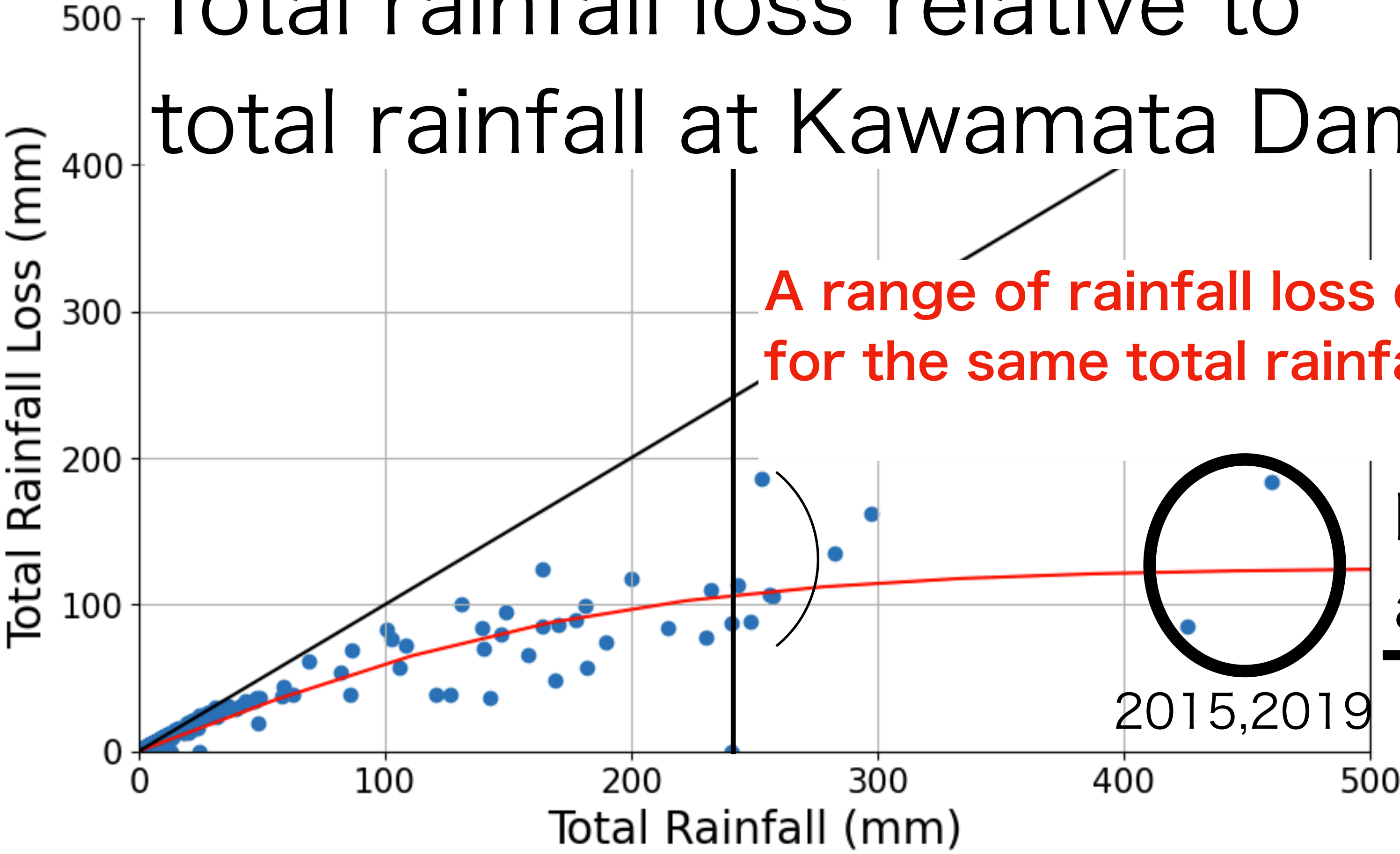
Histograms of Differences between XRAIN Rain Gauge and AMeDAS Rain Gauge in the Kinugawa River Basin of the 2015 Kanto Tohoku Heavy Rainfall and the 2019 East Japan Typhoon

The accumulation of observational data has made it possible to quantify the range of uncertainty.

Consider uncertainties due to limitations of observations in the rainfall information to be input

3.Uncertainty in soil wetness

Total rainfall loss relative to total rainfall at Kawamata Dam

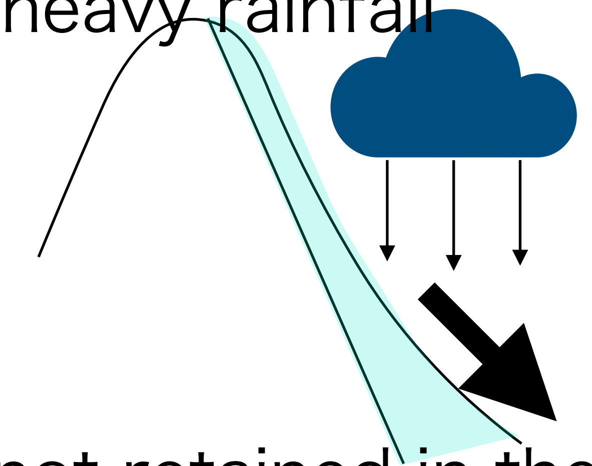


Total rainfall loss converges to a constant value as total precipitation increases.

Accumulation of observation data of heavy rainfall cases in recent decades

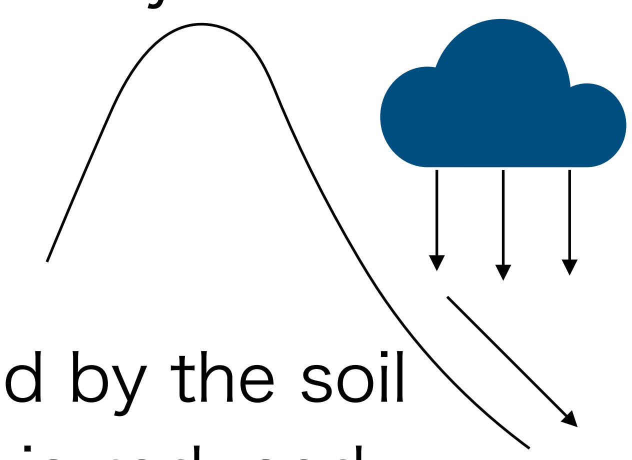
The range of uncertainty in converging values can now be quantified.

①If the soil is saturated just before a heavy rainfall



Rainfall is not retained in the soil and direct runoff is greater.

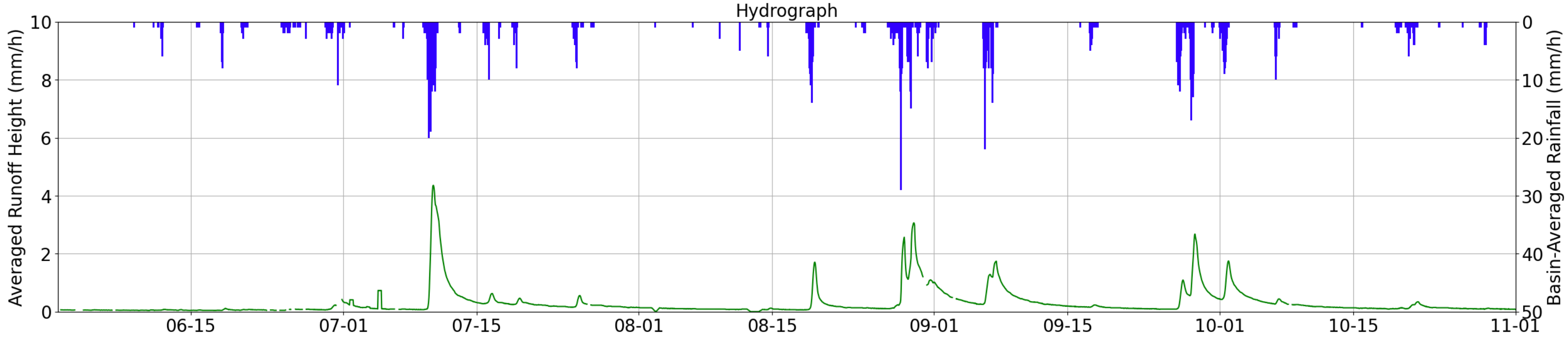
②If the soil was dry just before a heavy rainfall

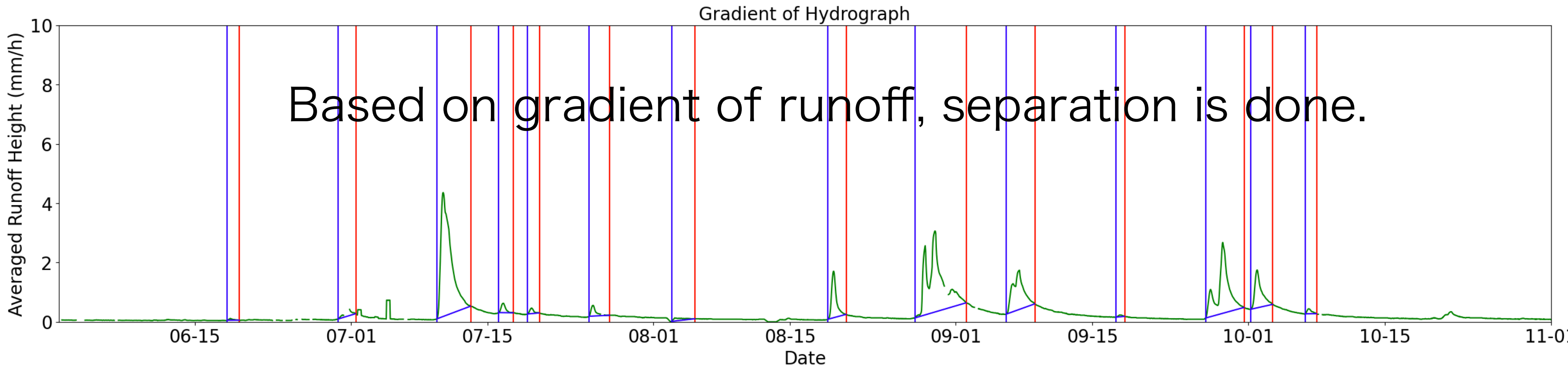


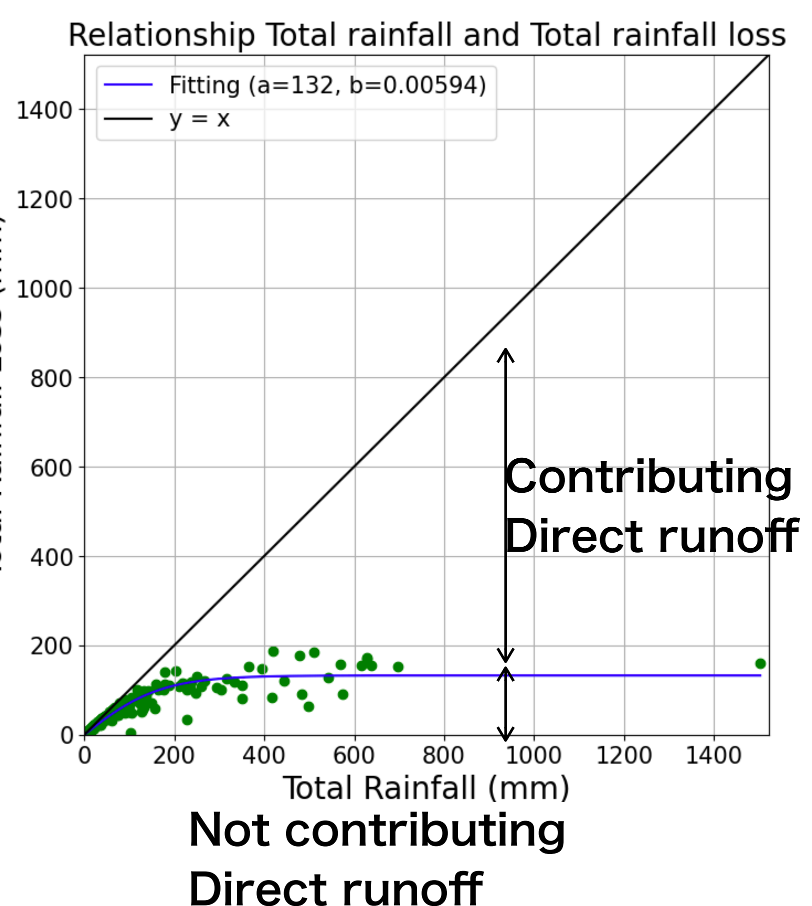
Rainfall is retained by the soil and direct runoff is reduced.

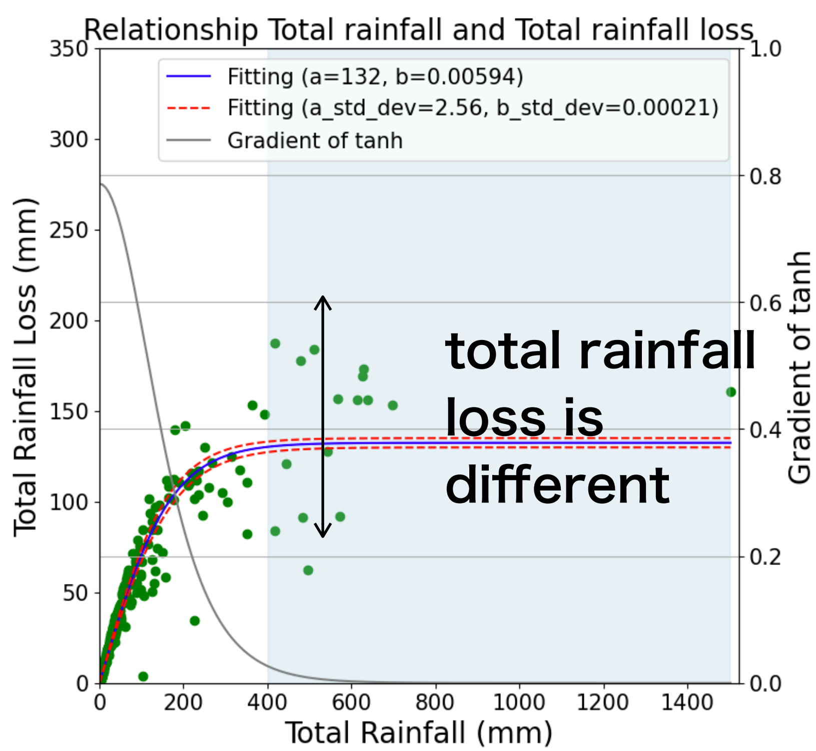
There is uncertainty in direct runoff not only from observational uncertainty, but also from the wetness of the soil immediately prior to heavy rainfall

Hachisu dam (Japan, Mie prefecture) **June to November of 2002**









What do I want to do here?

I want to analyze why total rainfall loss is different even if total rainfall is the same amount.

↓

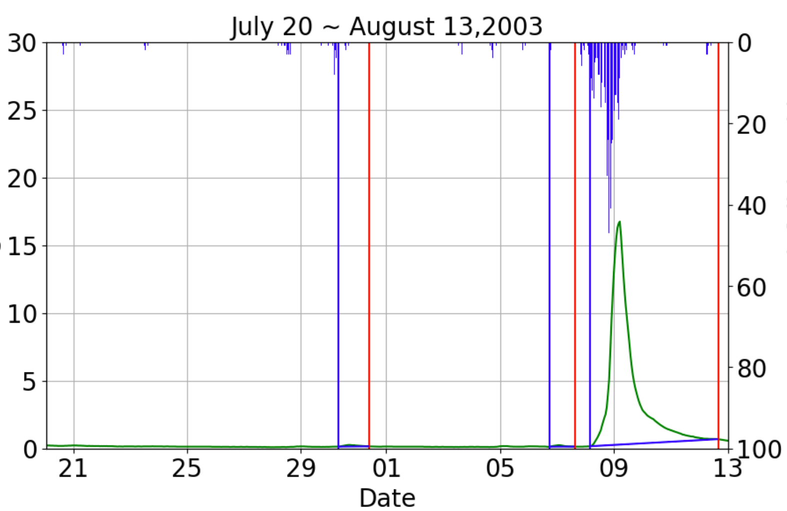
I extract rainfall event with a total rainfall amount of 400 or more and find out what happened to the previous rainfall event.

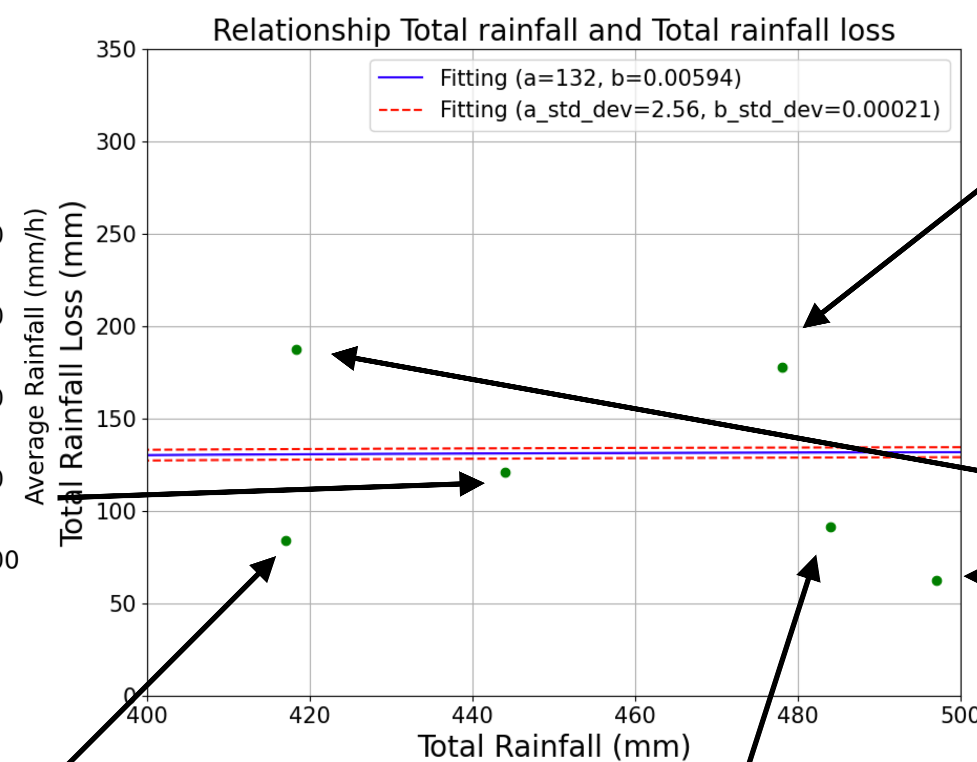
Hachisu dam (Japan, Mie prefecture)

400 <= Total rainfall <= 500

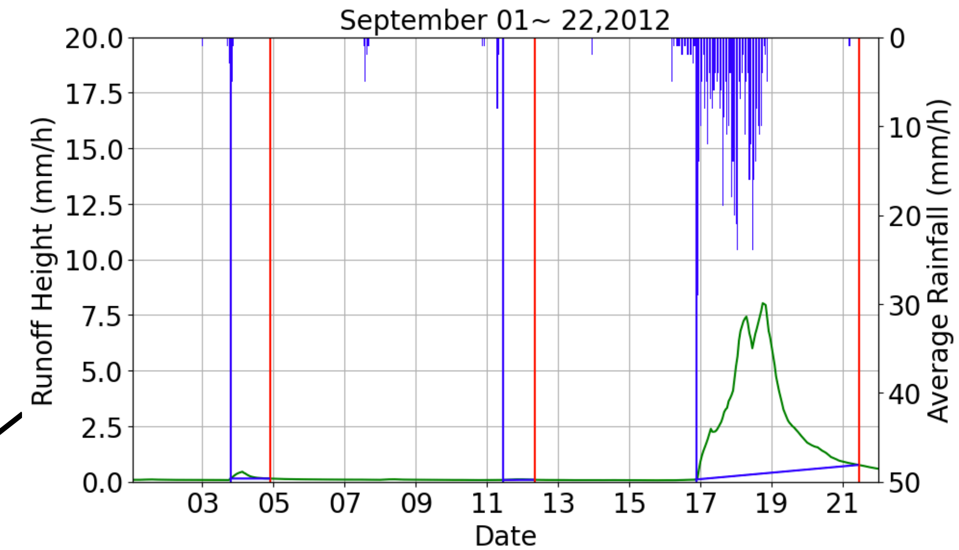
Runoff characteristics below the fitting curve have rained in previous rainfall events

August
8~12,2003



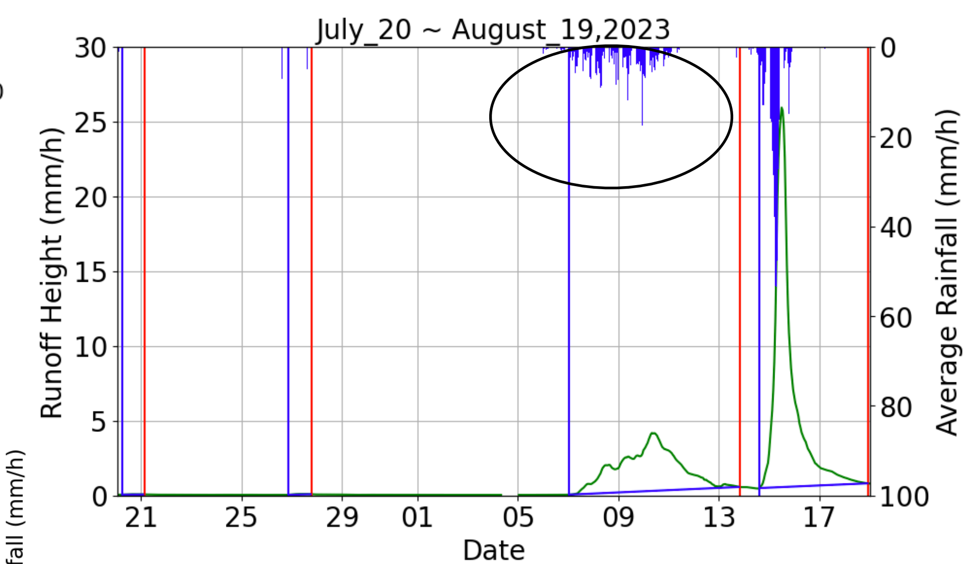


September
16~21,2012

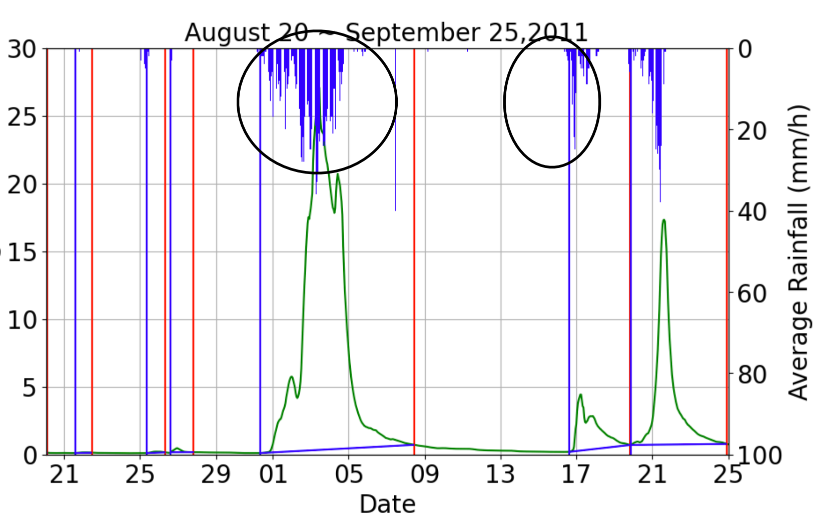


August
7~13,2023

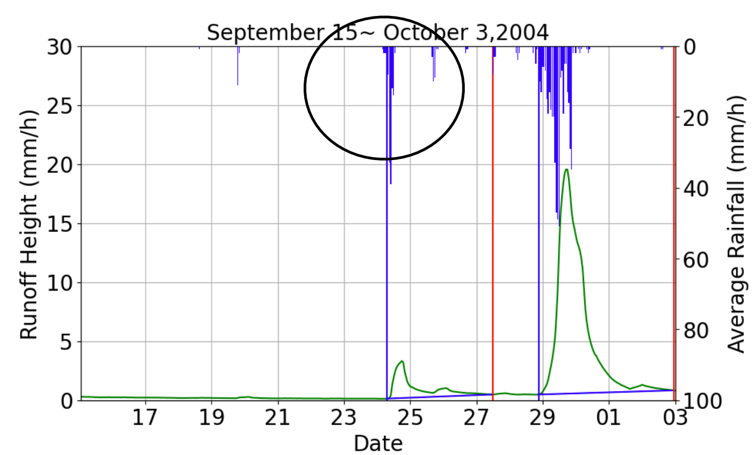
August
14~18,202



September
19~24,2011



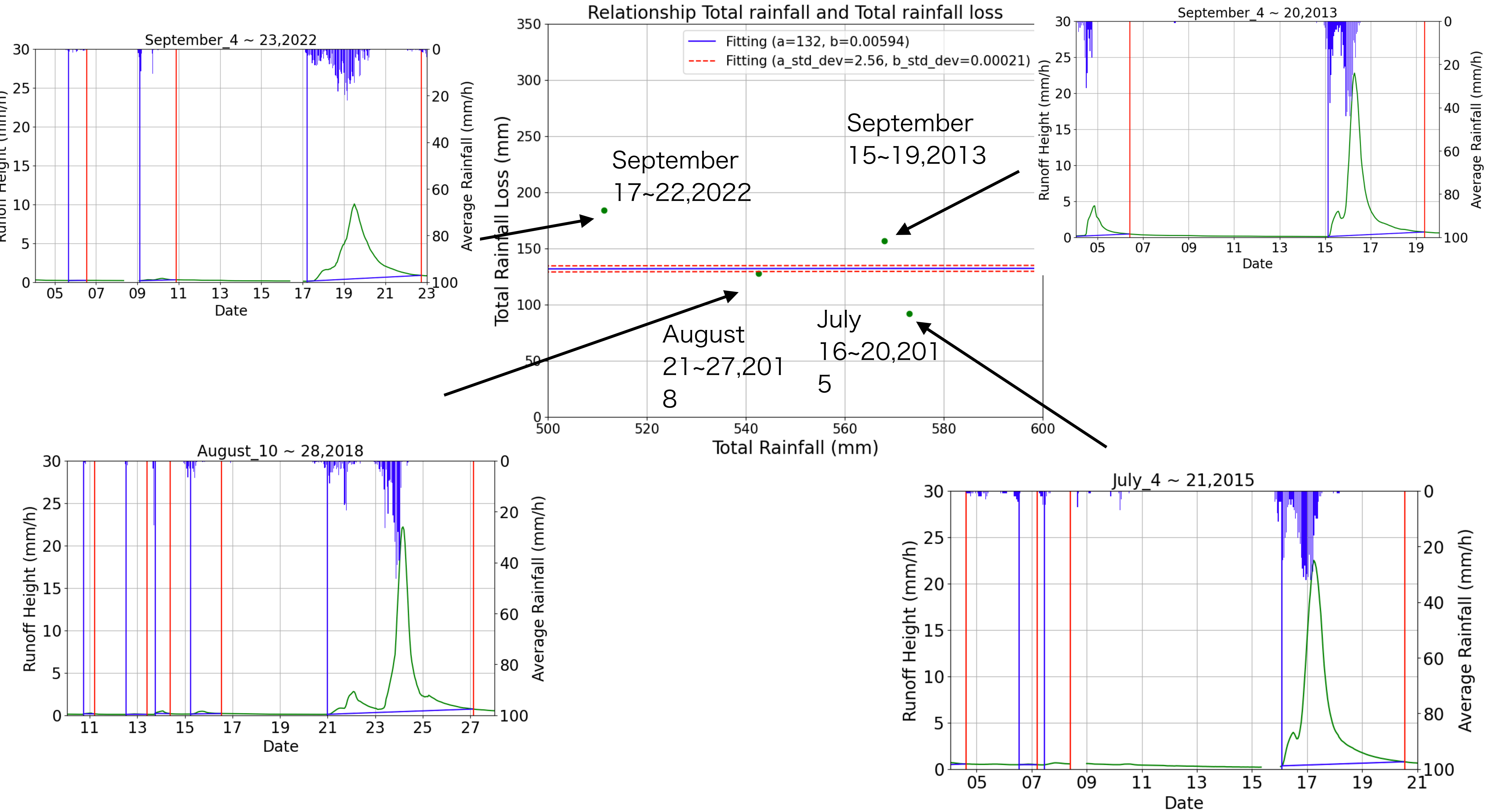
September
28~October02,2004



Hachisu dam (Japan, Mie prefecture)

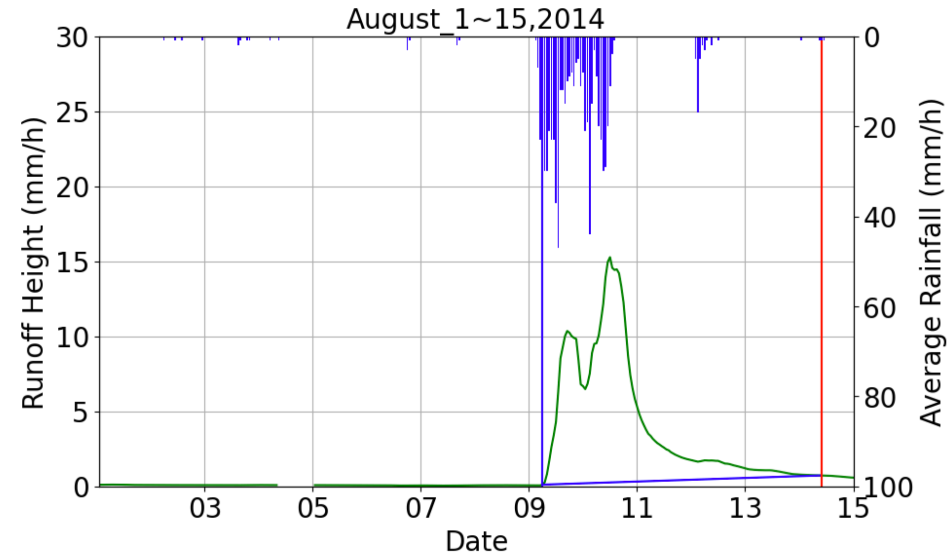
500 <= Total rainfall <= 600

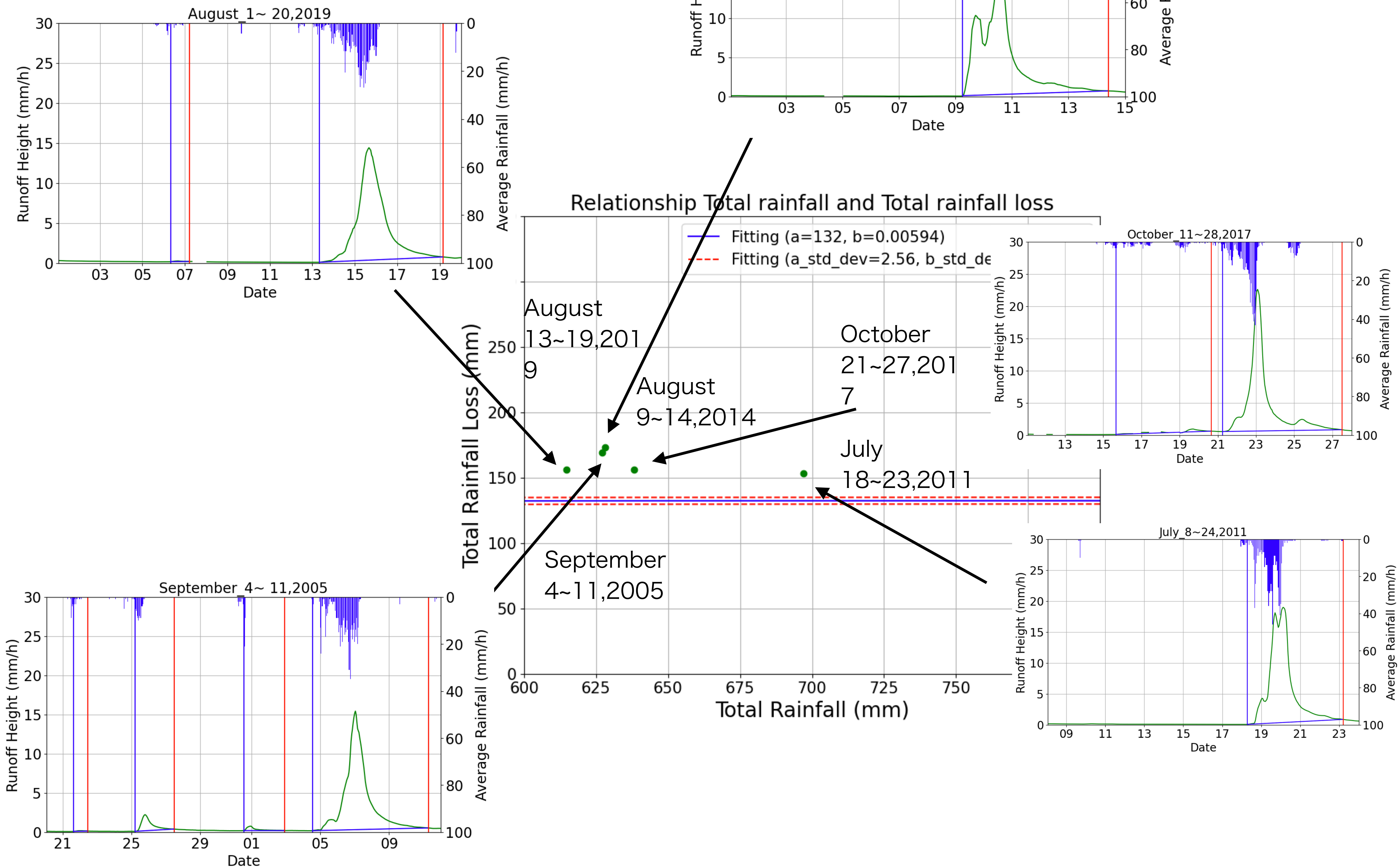
Runoff from July 4th to 21st, 2015 shows smaller values of lost rainfall than that in the fitting curve, but there was no significant rainfall in the previous rainfall event



Hachisu dam (Japan, Mie prefecture)

600 <= Total rainfall <= 800

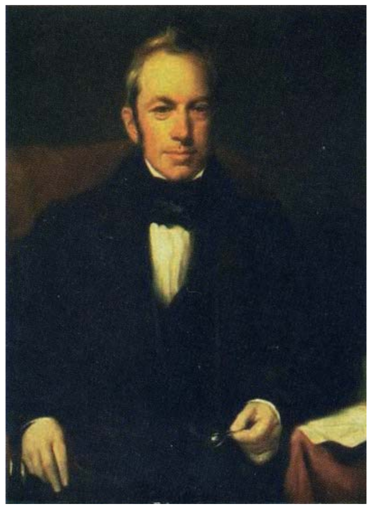


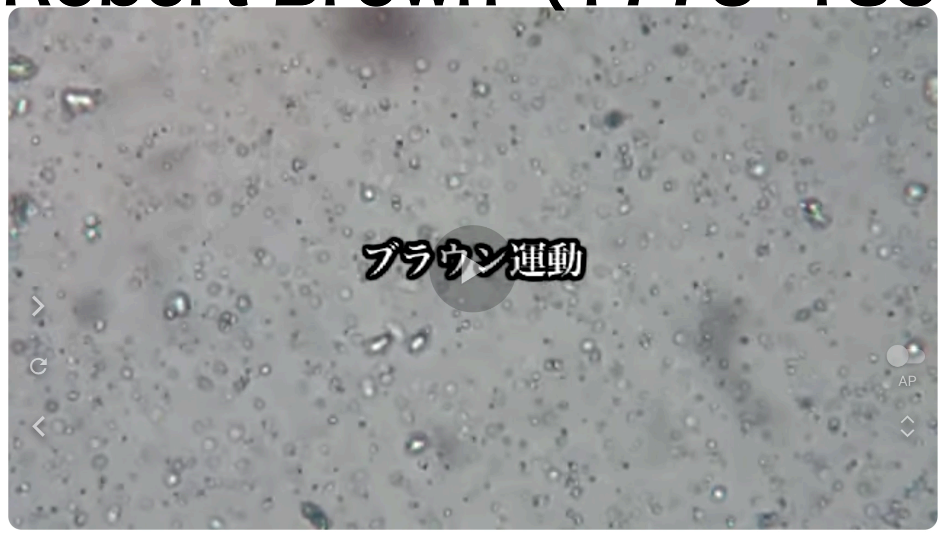


4.A method for mathematically evaluating the range of uncertainty

Wikipedia

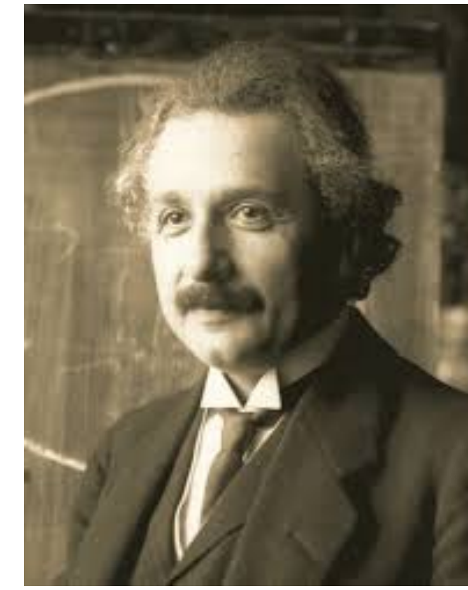
Robert Brown（1773~1858)





The phenomenon of Brownian motion was theorized, and it was shown that molecules exist.

Wikipedia



He discovered that the movement of microparticles in water, which was thought to be biological, is a physical phenomenon.

Einstein（1773~1858)

Langevin equation

Average force

$$m\frac{dv}{dt} = -\mu v(t) + f(t)$$

Randam force

m :mass、v :velocity、μ :coefficient

↓

Ito stochastic differential equation

$$dx(t) = \mu(x,t)dt + \sum_{j=1}^{n} \sigma_j(x,t)dw_j$$

↓

Fokker-Planck equation

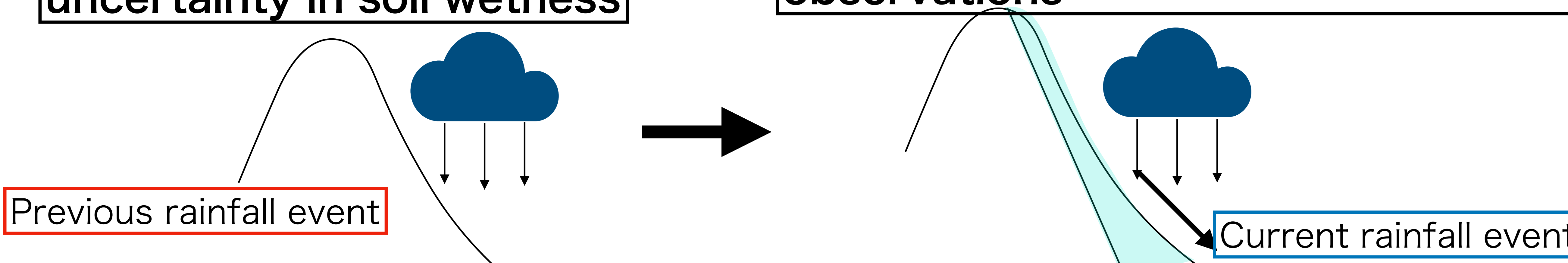
$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial\left(\mu(x,t)p(x,t)\right)}{\partial t} + \frac{1}{2}\frac{\partial^2\left(\sum_{j=1}^{n}\sigma_j^2(x,t)p(x,t)\right)}{\partial x^2}$$

5.Relationship between two uncertainties

Uncertainty due to limitations of observations and uncertainty in soil wetness are independent

uncertainty in soil wetness

Uncertainty due to limitations of observations



The effective rainfall term can be decomposed as a linear sum of the effective rainfall from observations, the uncertainty in soil wetness, and the uncertainty in observations

$$r_e = \bar{r}_e + \boxed{r'} + \boxed{r''}$$

(r':the uncertainty in observations, r'':the uncertainty in soil wetness)

$$\frac{dq_*}{dt} = \alpha q_*^{\beta}(r_e - q_*) \longrightarrow dq_* = \alpha q_*^{\beta}(\bar{r}_e - q_*)dt + \alpha q_*^{\beta}\boxed{\sigma_1}\sqrt{T_w}dw + \alpha q_*^{\beta}\boxed{\sigma_2}\sqrt{T_w}dw$$

(σ_i, T_i, dw is standard devitation, time constant (0,dt)normal distribution)

Two uncertainties are introduced into the traditional deterministic basic equation

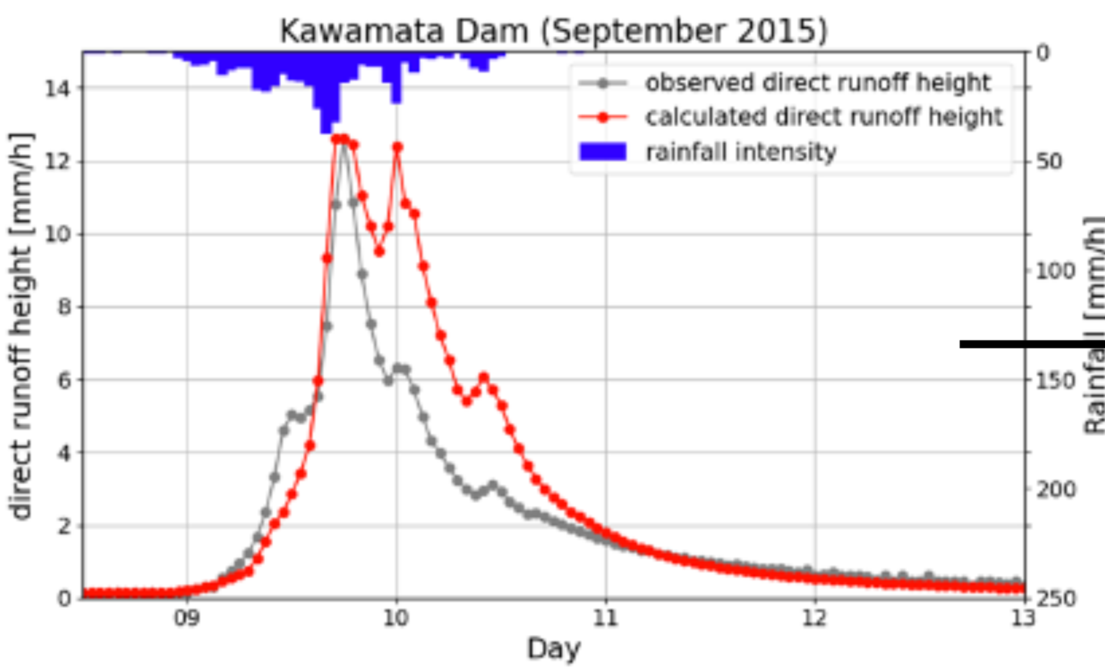
6.Stochastic rainfall runoff process

Deterministic method → **Method considering uncertainty**

$$\frac{dq_*}{dt} = \alpha q_*^{\beta}(r_e - q_*)$$

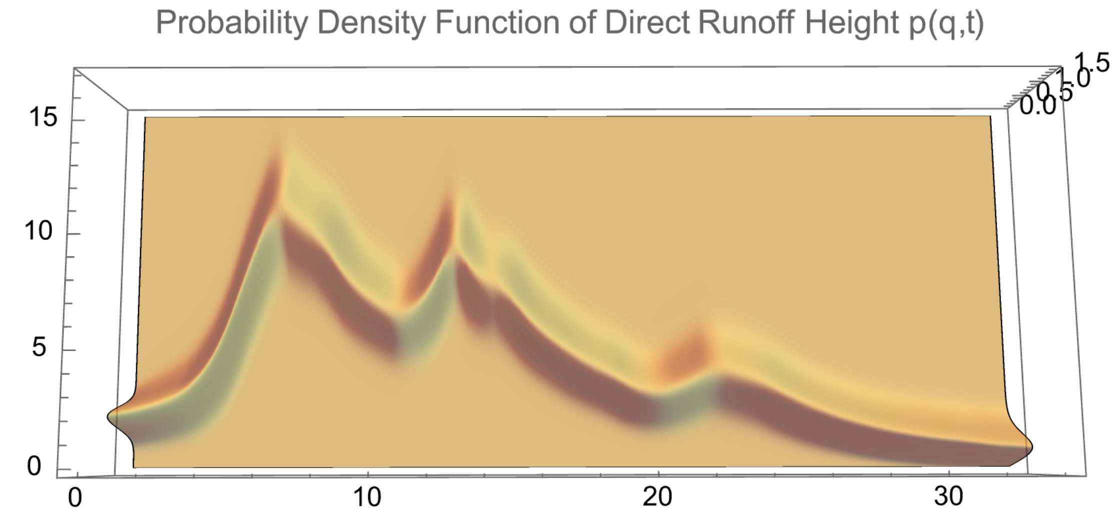
α, β:parameter q_*:runoff

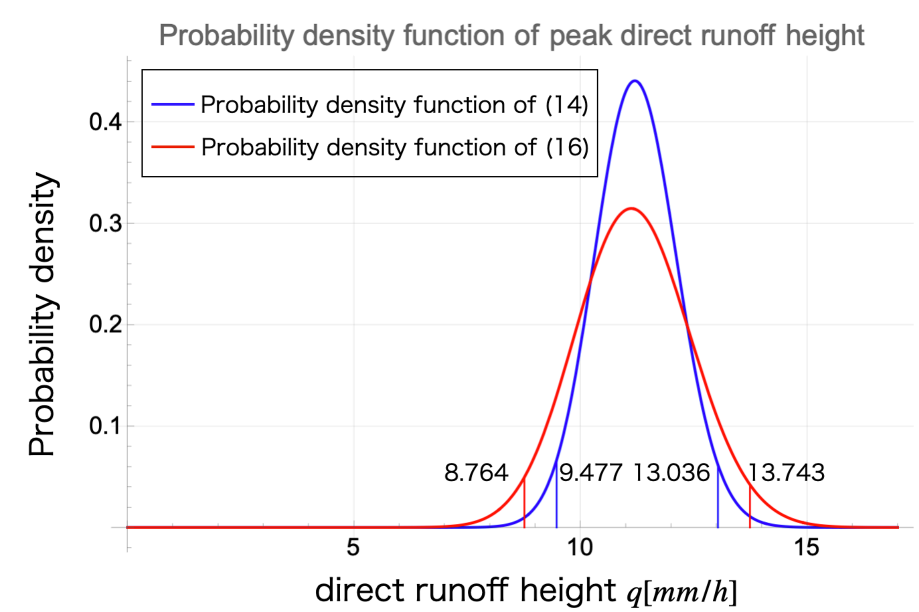
r_e:effective rainfall



Conventional deterministic models settle on one outflow

$$\frac{\partial p(q_*,t)}{\partial t} = -\frac{\partial\left(a_0 q_*^{\beta}(r_e - q_*)p(q_*,t)\right)}{\partial t} + \frac{1}{2}\frac{\partial^2\left((a_0 q_*^{\beta}\sigma_1(q_*,t))^2 p(q_*,t)\right)}{\partial q_*^2} + \frac{1}{2}\frac{\partial^2\left((a_0 q_*^{\beta}\sigma_2(q_*,t))^2 p(q_*,t)\right)}{\partial q_*^2}$$





Now that the uncertainty of rainfall information become known, it is possible to evaluate runoff heights taking the uncertainty into account.