

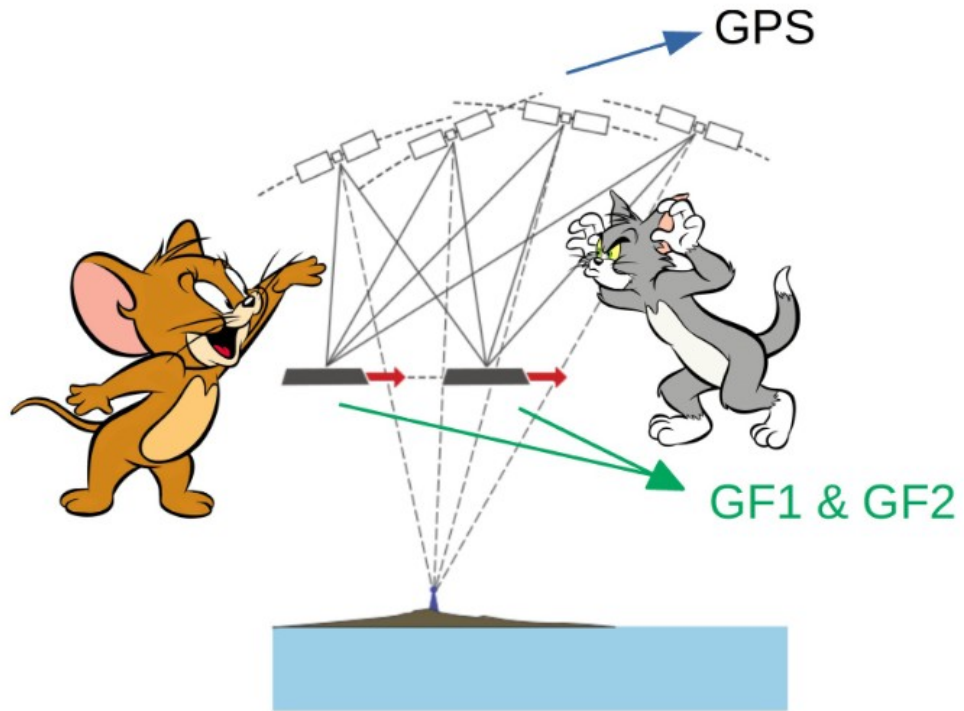
*u*<sup>b</sup>

# Integration of Laser Ranging Range-Rate Observations into the GRACE Follow-On Processing at the AIUB

**Martin Lasser, Ulrich Meyer, Daniel Arnold and Adrian Jäggi**  
EGU General Assembly 2024, 14 – 19 April 2024, Vienna, Austria

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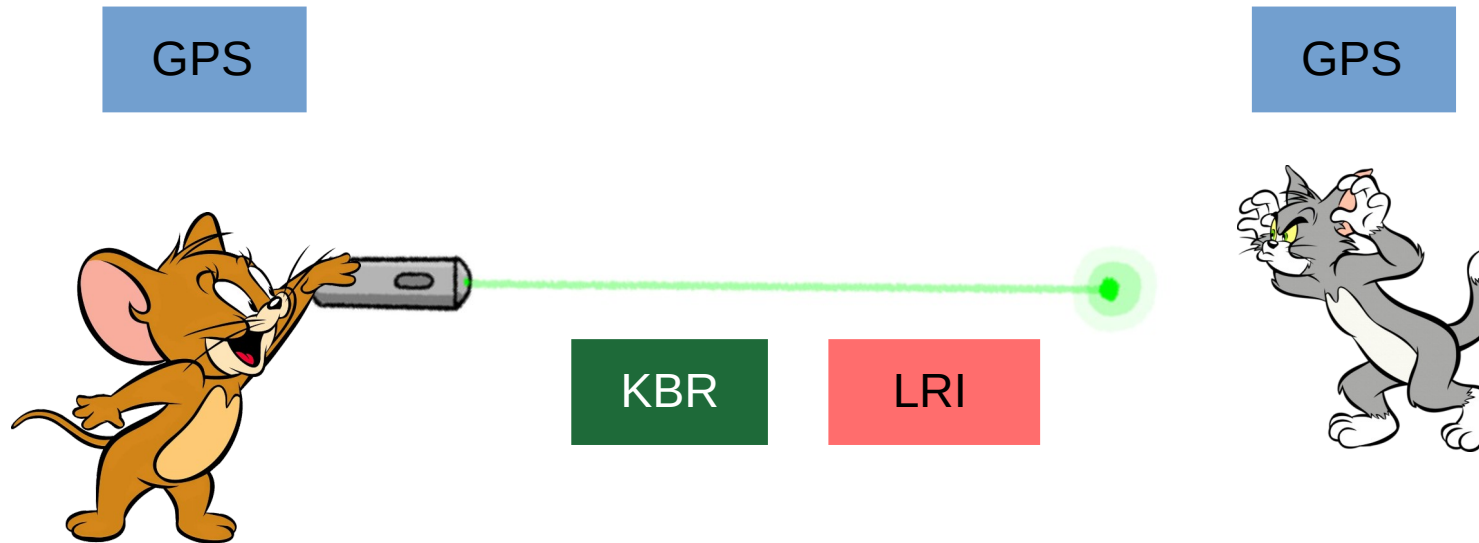
# GRACE/GRACE Follow-On Observation concept



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# GRACE Follow-On

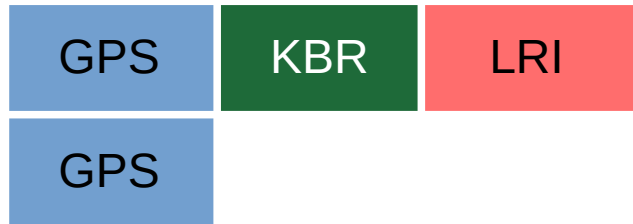
## Observables in L2 processing



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# GRACE Follow-On

## Observables in L2 processing

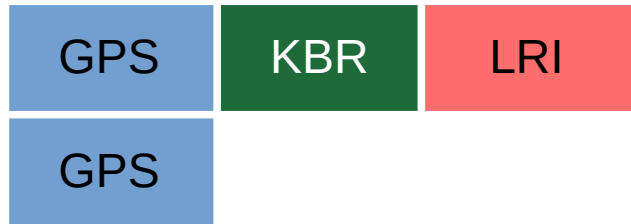


$$l = \begin{bmatrix} \text{GPS} \\ \text{KBR} \\ \text{LRI} \end{bmatrix} \begin{matrix} \rightarrow \mathbf{A}_1 \mathbf{P}_1 \\ \rightarrow \mathbf{A}_2 \mathbf{P}_2 \\ \rightarrow \mathbf{A}_3 \mathbf{P}_3 \end{matrix} \left. \vphantom{\begin{matrix} \rightarrow \mathbf{A}_1 \mathbf{P}_1 \\ \rightarrow \mathbf{A}_2 \mathbf{P}_2 \\ \rightarrow \mathbf{A}_3 \mathbf{P}_3 \end{matrix}} \right\} \begin{matrix} \mathbf{N} = (\mathbf{A}^T \mathbf{P} \mathbf{A}) \\ \mathbf{b} = \mathbf{A}^T \mathbf{P} l \end{matrix} \rightarrow \hat{\mathbf{x}} = \left( \sum_{k=1}^{K=3} \mathbf{N}_k \right)^{-1} \sum_{k=1}^{K=3} \mathbf{b}_k$$

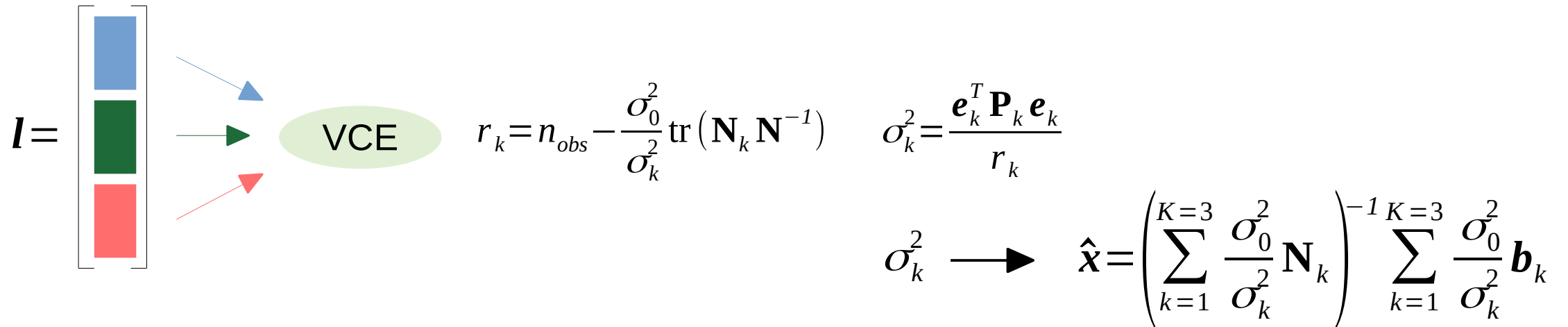
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# GRACE Follow-On

## Observables



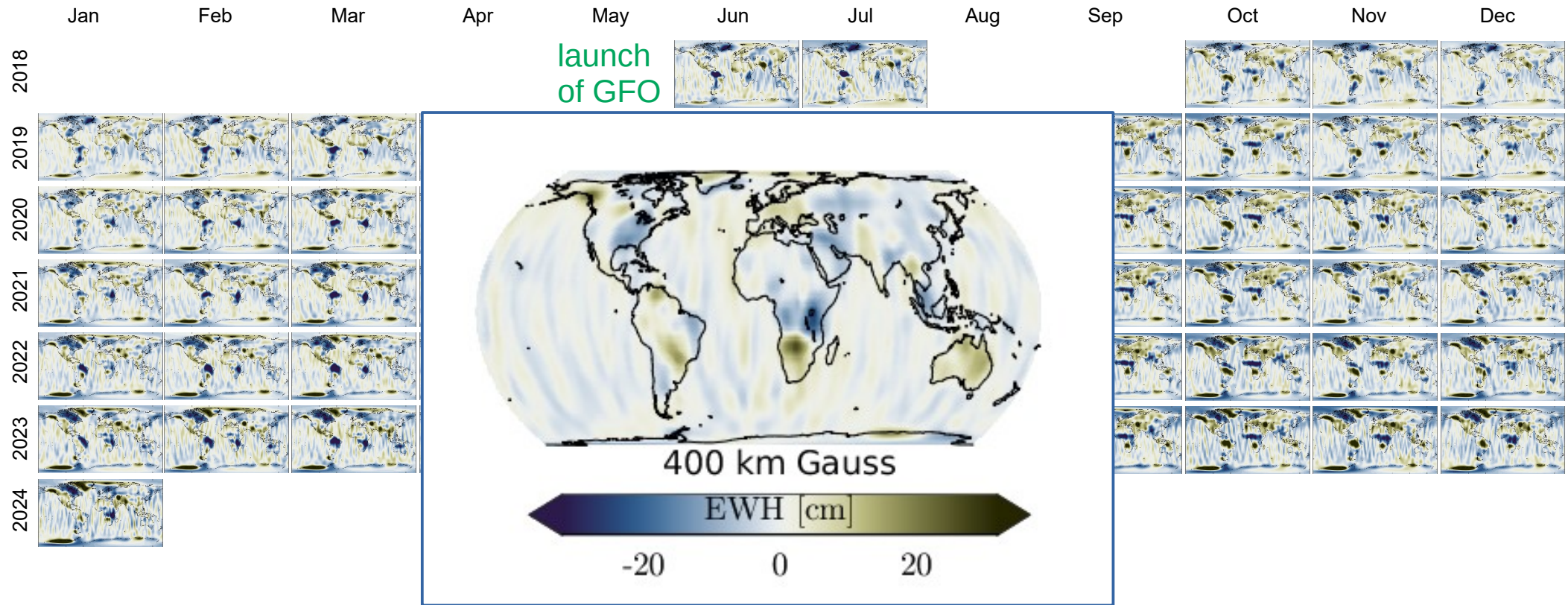
VCE: Each group of observations gets a weight based on its contribution to the final solution



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# Operational GRACE Follow-On Solution

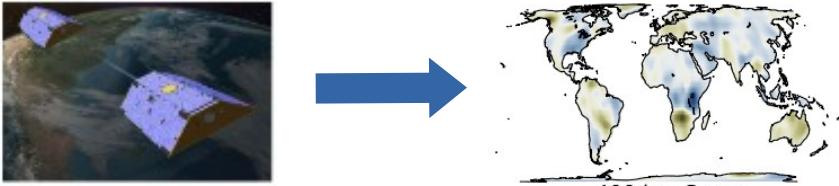
## Mosaic Jun 2018 – Jan 2024



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# Operational GRACE Follow-On Solution

## Monthly gravity fields – parametrisation



### Basic parametrisation

- Initial conditions 2x[6]
- Accelerometer bias 2x[3] | [6]
- Accelerometer scaling 2x[3] | [9]

Parameters per arc 24 | 42

since 2023 a full scale matrix estimated

### Additional parameters

- 15 min PCA per satellite in
  - radial 2x[96]
  - along-track 2x[96]
  - cross-track 2x[96]

Parameters per arc 576

in daily arcs (30 days):

~ 18000 <orbit> parameters

+ 9405 gravity field d/o=2..96

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# Stochastic Noise Modelling

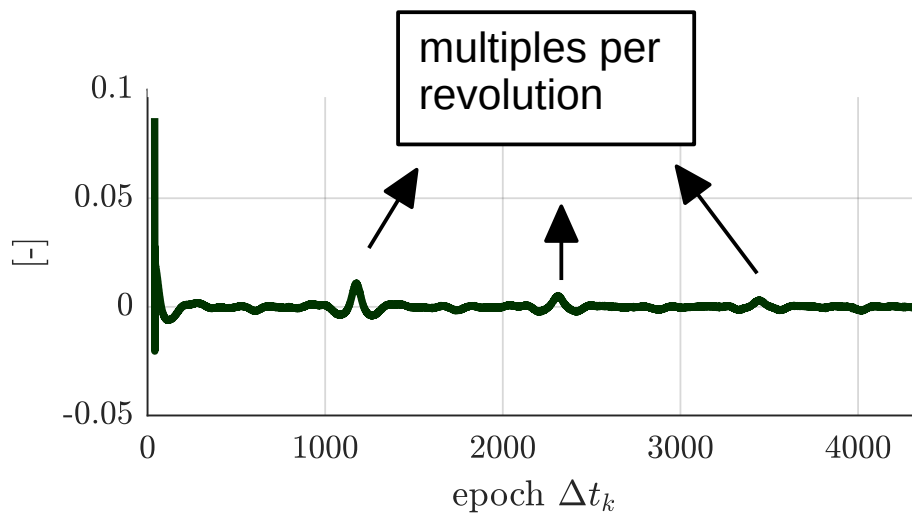
## Empirical model from post-fit residuals

### Serial correlation of post-fit residuals

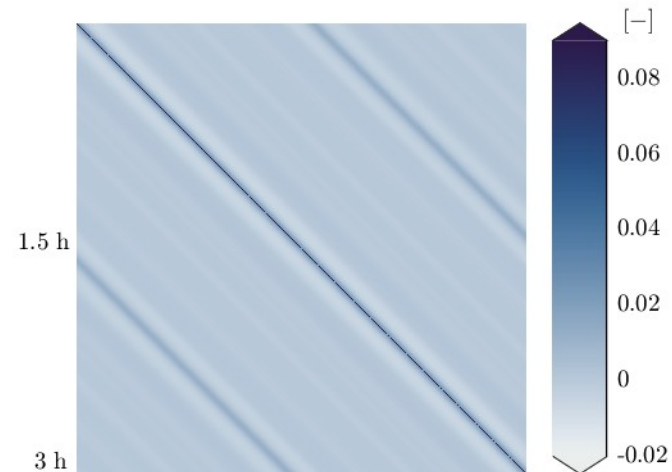
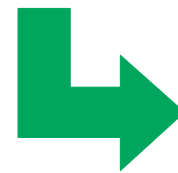
$$\hat{e} = l - A \hat{x} \quad (\text{post-fit residuals})$$

$$\text{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^N \hat{e}(t_i) \hat{e}(t_i + \Delta t_k)$$

- stationarity assumed
- biased estimation of auto-covariance  
→ covariance matrix nondegenerate



block  
Toeplitz  
matrix

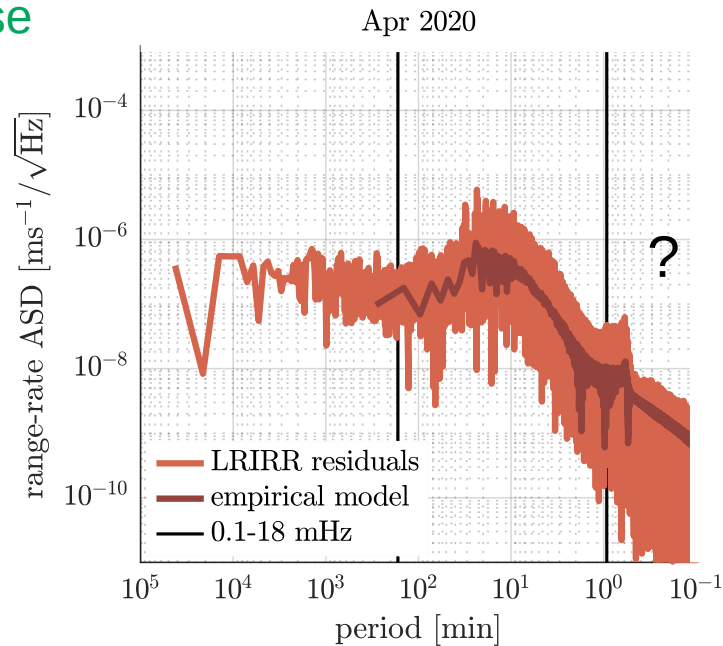
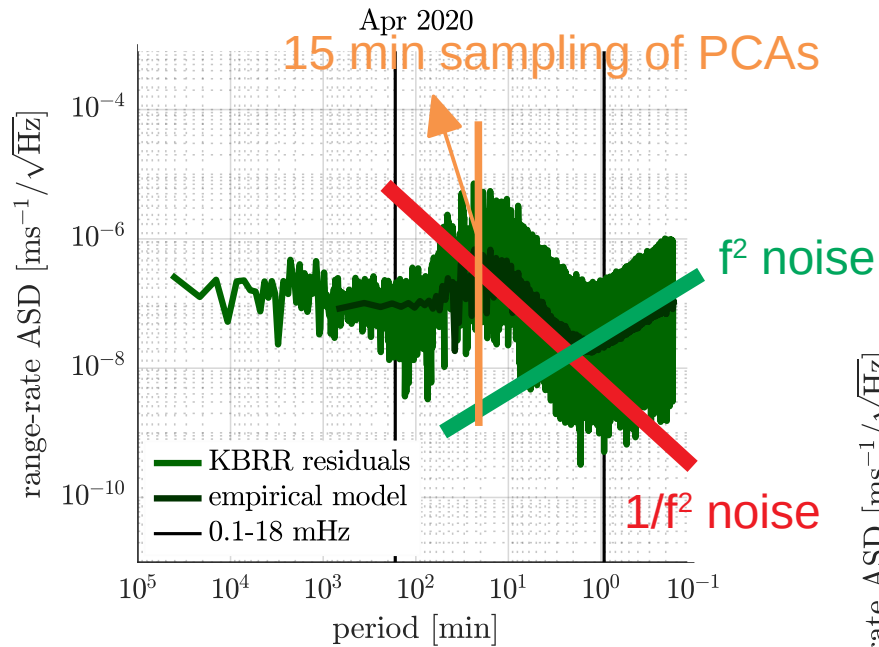
**P**



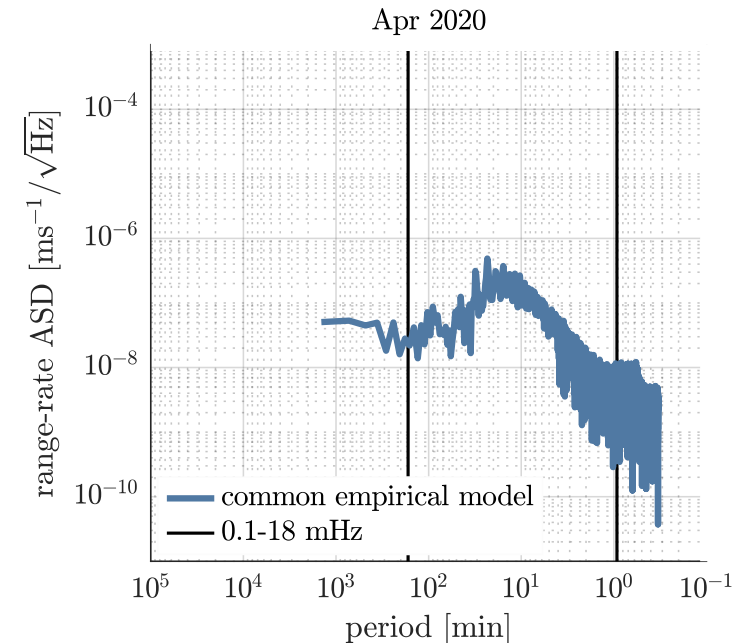
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# Post-fit Residuals

## Spectral domain



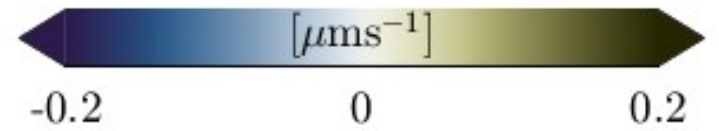
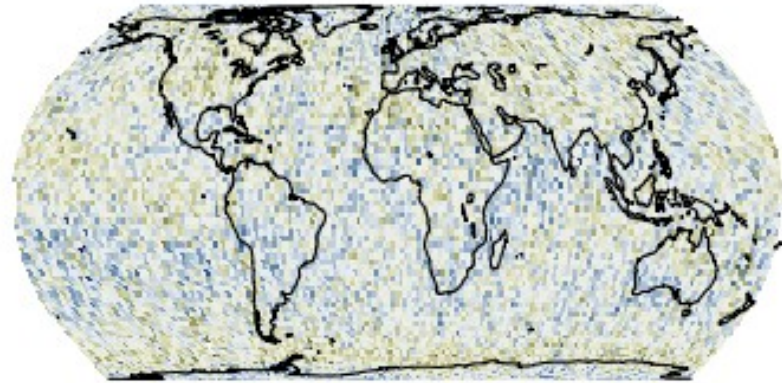
obtained by cross-correlation  
not used



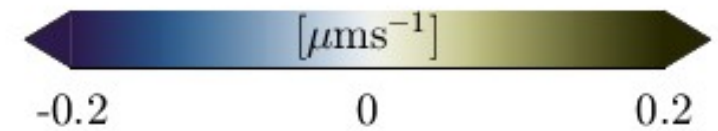
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# Post-fit Residuals Geographic domain

Apr 2020

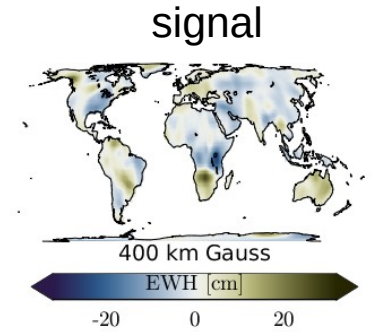
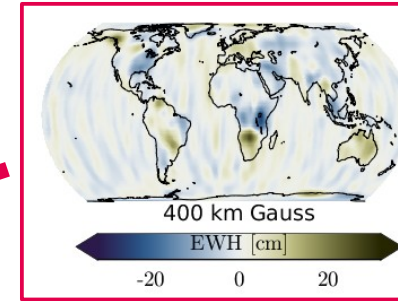
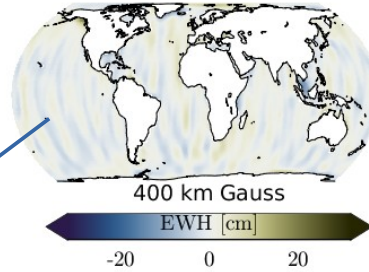


Apr 2020

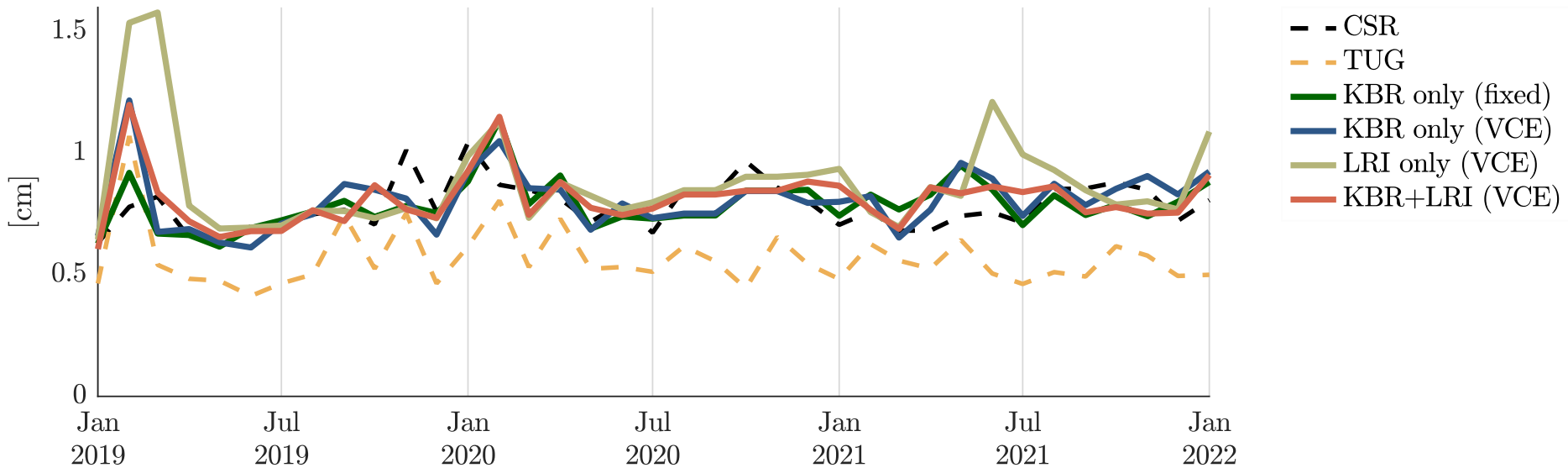


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# Noise evaluation RMS over the oceans



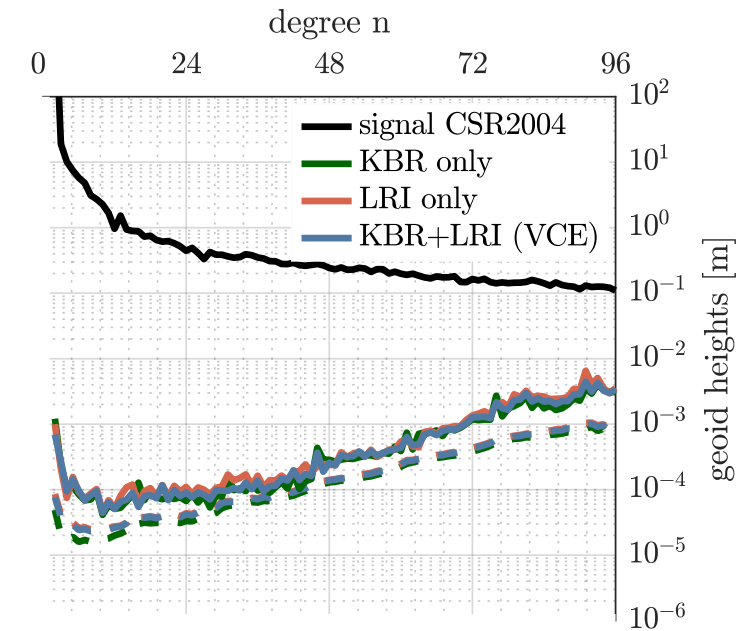
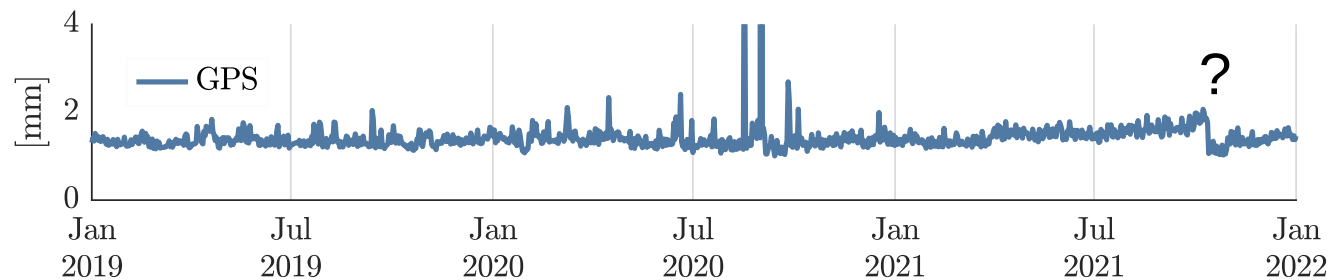
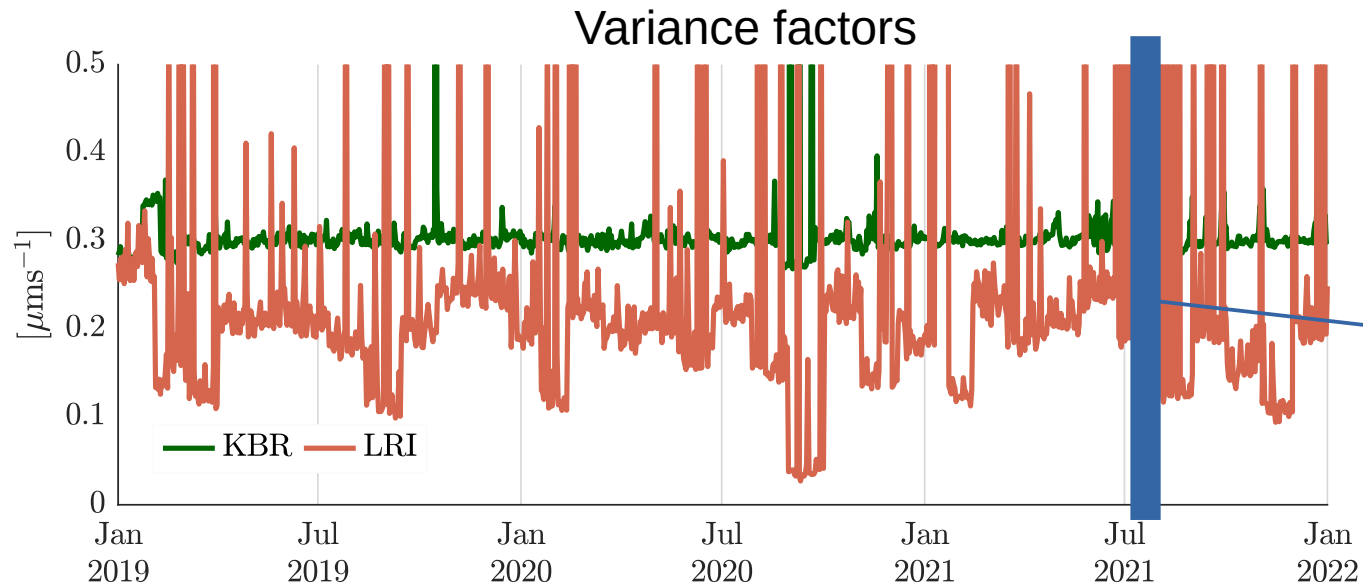
RMS over the oceans



$u^b$ 

# Variance Component Estimation

## Arc-wise results



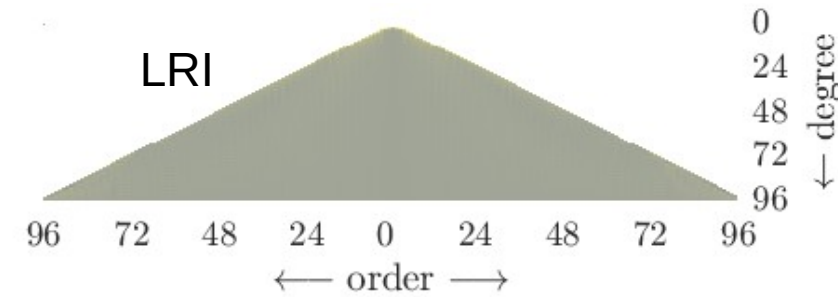
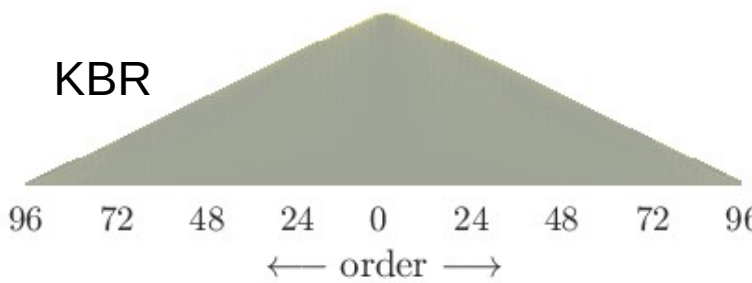
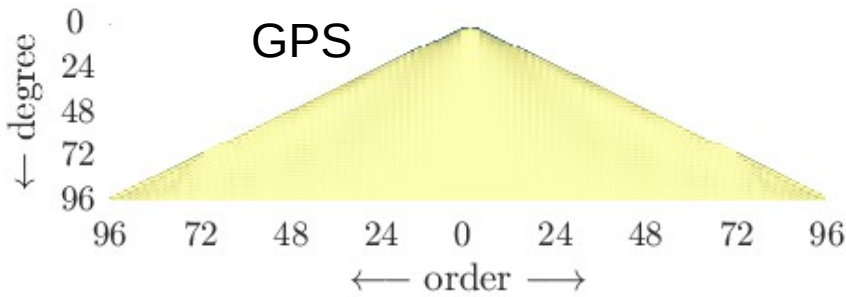
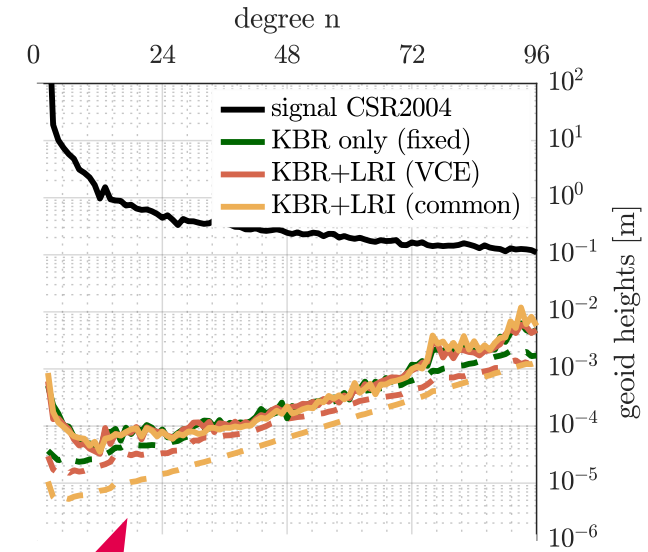
# $u^b$ Contribution Analysis

## GPS vs. KBR vs. LRI

$$\hat{\mathbf{x}} = \left( \sum_{k=1}^{K=3} \mathbf{N}_k \right)^{-1} \sum_{k=1}^{K=3} \mathbf{b}_k$$

■  $\mathbf{N}_k$   
■  $\mathbf{b}_k$   
■  $c = \text{diag}(\mathbf{N}_k \mathbf{N}_k^{-1})$

- No noise modelling
- No VCE (all weights fixed)
- Equal weight for KBR and LRI
- $\sigma_{\text{KBR}} = \sigma_{\text{LRI}} = 0.3 \mu\text{m/s}$ ;  $\sigma_{\text{GPS}} = 12 \text{ mm}$
- Only common epochs (10 s sampling)




0 0.5 1

0 0.5 1

0 0.5 1

# $u^b$ Contribution Analysis

## GPS vs. KBR vs. LRI



$$\hat{\mathbf{x}} = \left( \sum_{k=1}^{K=3} \mathbf{N}_k \right)^{-1} \sum_{k=1}^{K=3} \mathbf{b}_k$$

$$\mathbf{c} = \text{diag}(\mathbf{N}_k \mathbf{N}_k^{-1})$$

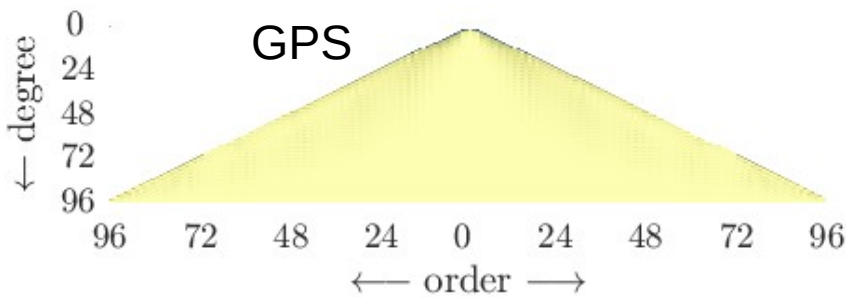
With noise modelling

With VCE

$\sigma_{\text{KBR}} \sim 0.3 \mu\text{m/s}$ ;  $\sigma_{\text{LRI}} \sim 0.2 \mu\text{m/s}$

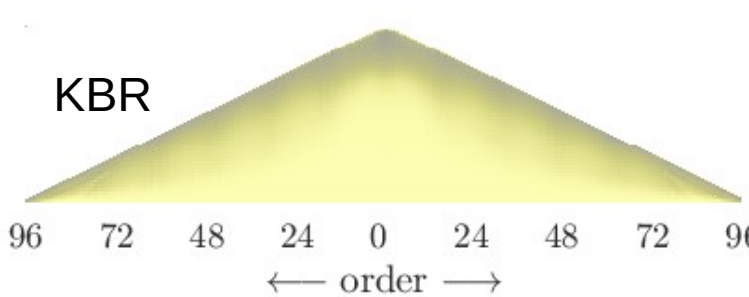
$\sigma_{\text{GPS}} \sim 1.6 \text{ mm}$

Only common epochs (10 s sampling)



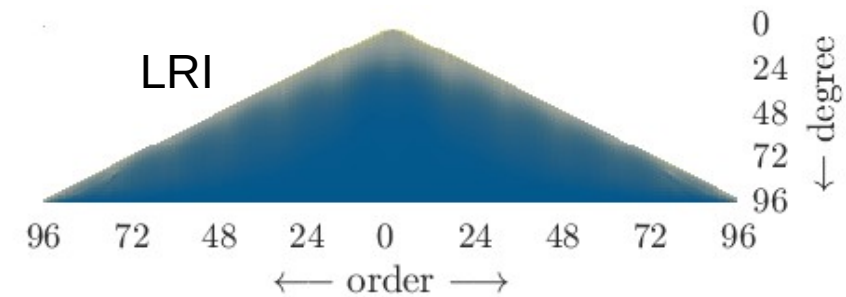
0 0.5 1

14/19 18 Apr 2024, Vienna, Austria



0 0.5 1

Astronomical Institute University of Bern



0 0.5 1



# $u^b$ Contribution Analysis

## GPS vs. KBR vs. LRI

$$\hat{\mathbf{x}} = \left( \sum_{k=1}^{K=3} \mathbf{N}_k \right)^{-1} \sum_{k=1}^{K=3} \mathbf{b}_k$$

■  $\mathbf{b}_k$   
■  $\mathbf{N}_k$   
■  $\mathbf{c} = \text{diag}(\mathbf{N}_k \mathbf{N}_k^{-1})$

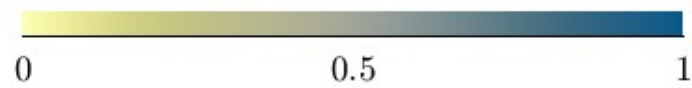
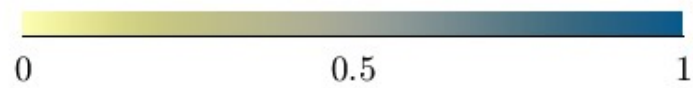
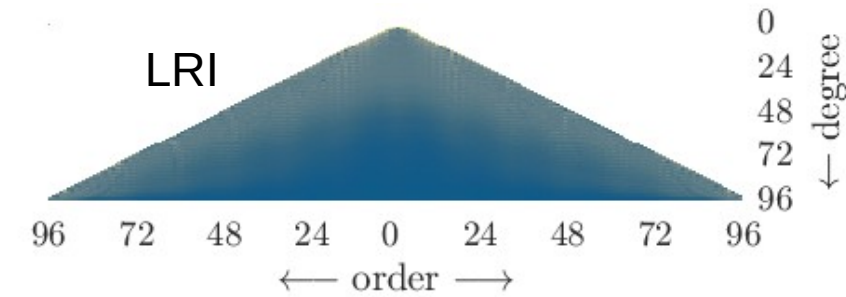
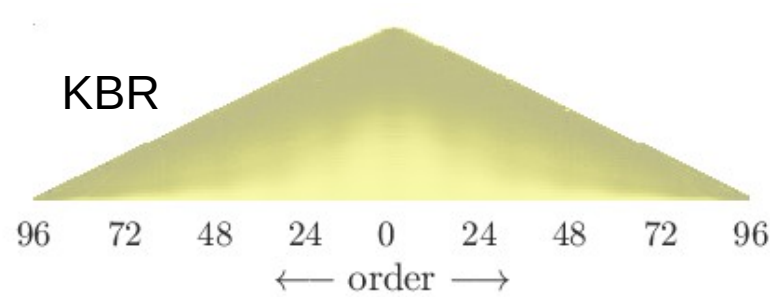
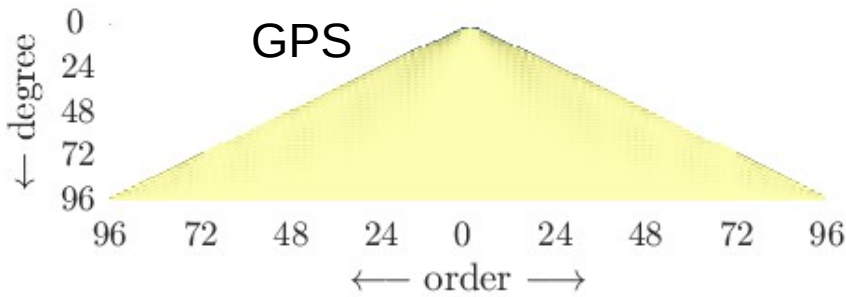
With noise modelling

With VCE

$\sigma_{\text{KBR}} \sim 0.3 \mu\text{m/s}$ ;  $\sigma_{\text{LRI}} \sim 0.2 \mu\text{m/s}$

$\sigma_{\text{GPS}} \sim 1.6 \text{ mm}$

5 s KBR and 2 s LRI sampling



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# Benefits of adding the LRI

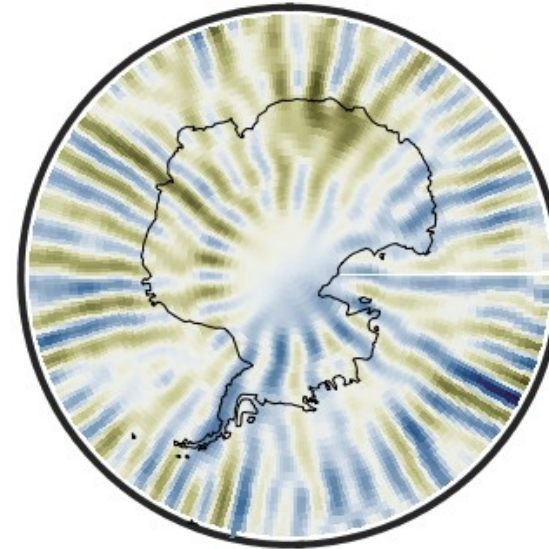
## Antarctica

KBR only  
Nov 2020



GOCO06s + trend  
+ annual variations & 100 km Gauss  
[cm]  
-1 0 1

KBR + LRI  
Nov 2020



GOCO06s + trend  
+ annual variations & 100 km Gauss  
[cm]  
-1 0 1



$u^b$

# Thank you for your attention

## Contact

Martin Lasser

[martin.lasser@unibe.ch](mailto:martin.lasser@unibe.ch)

# References

## A

Beutler, G., Jäggi, A., Mervart, L. and Meyer, U. [2010]: The celestial mechanics approach: theoretical foundations. *Journal of Geodesy*, vol. 84(10), pp. 605-624. <https://doi.org/10.1007/s00190-010-0401-7>

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Kvas, A., Behzadpour, S., Ellmer, M., Klinger, B., Strasser, S., Zehentner, N. and Mayer-Gürr, T. [2019]: Overview and evaluation of a new GRACE-only gravity field time series. *Journal of Geophysical Research: Solid Earth*. ISSN 2169-9313. <https://doi.org/10.1029/2019JB017415>

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## B

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Lasser, M., Meyer, U., Arnold, D. and Jäggi, A. [2020]: AIUB-GRACE-FO-operational - Operational GRACE Follow-On monthly gravity field solutions. <https://doi.org/10.5880/ICGEM.2020.001>

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