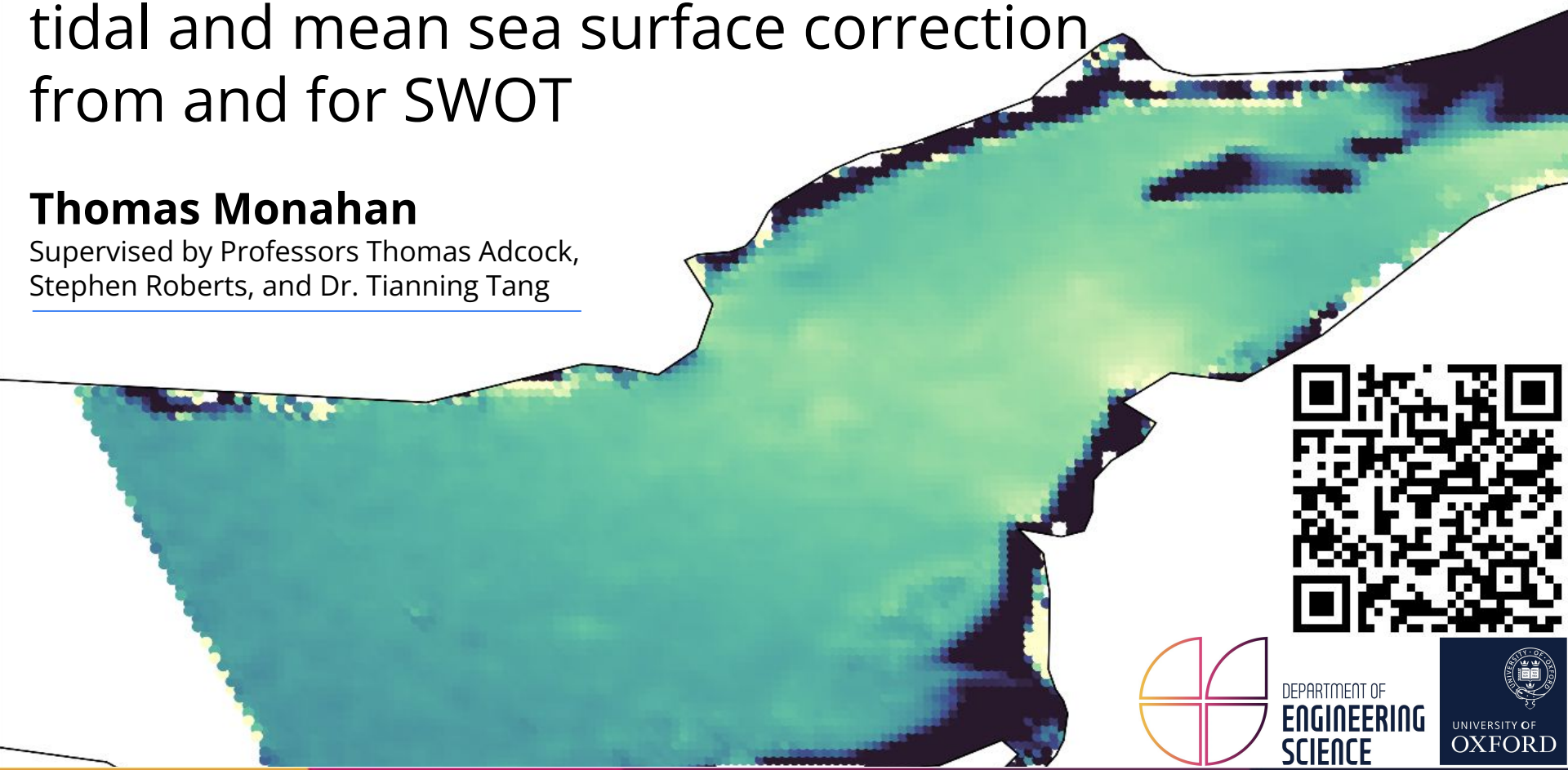
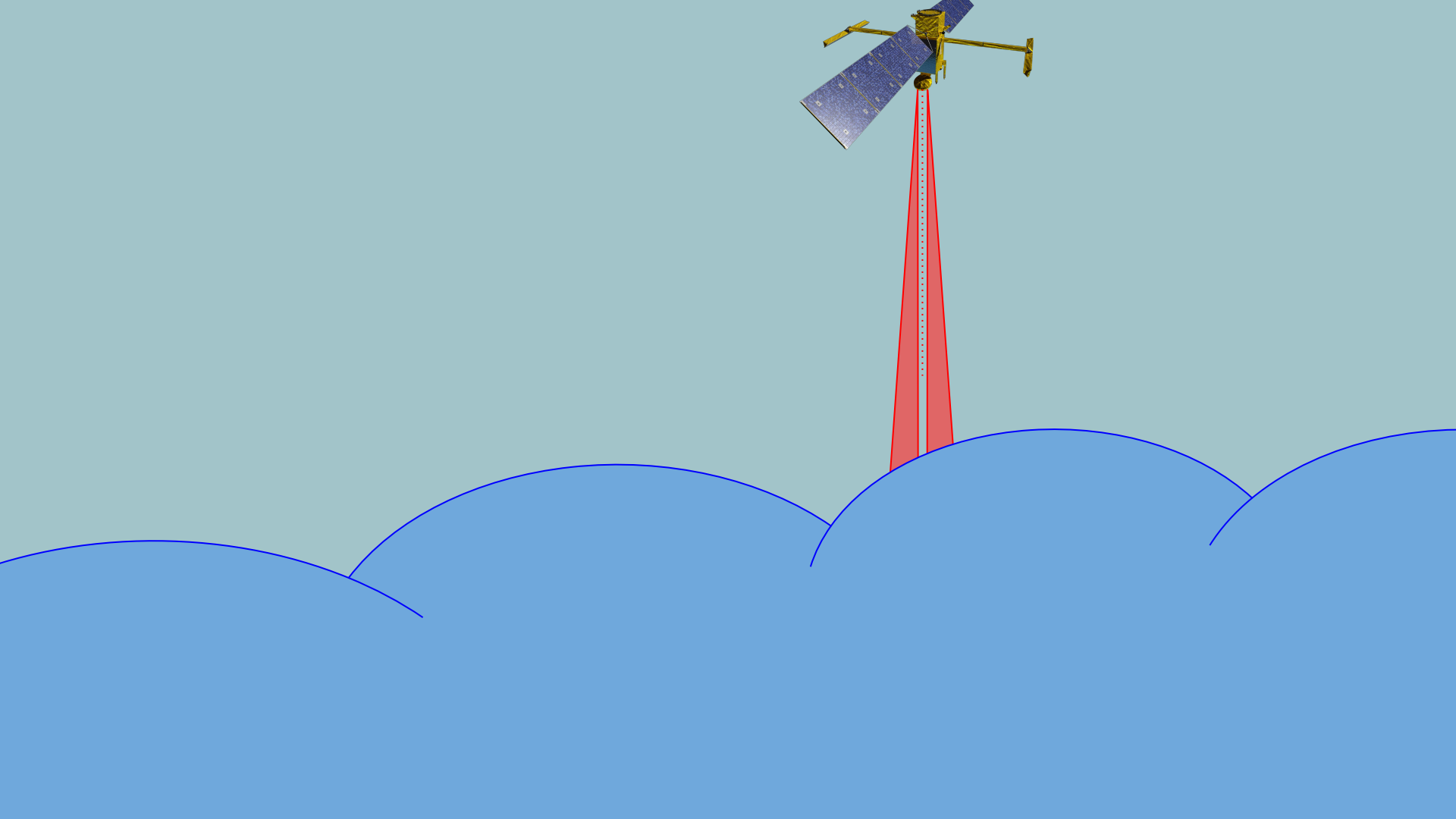
A framework for coastal and estuarine tidal and mean sea surface correction from and for SWOT

Thomas Monahan
Supervised by Professors Thomas Adcock, Stephen Roberts, and Dr. Tianning Tang

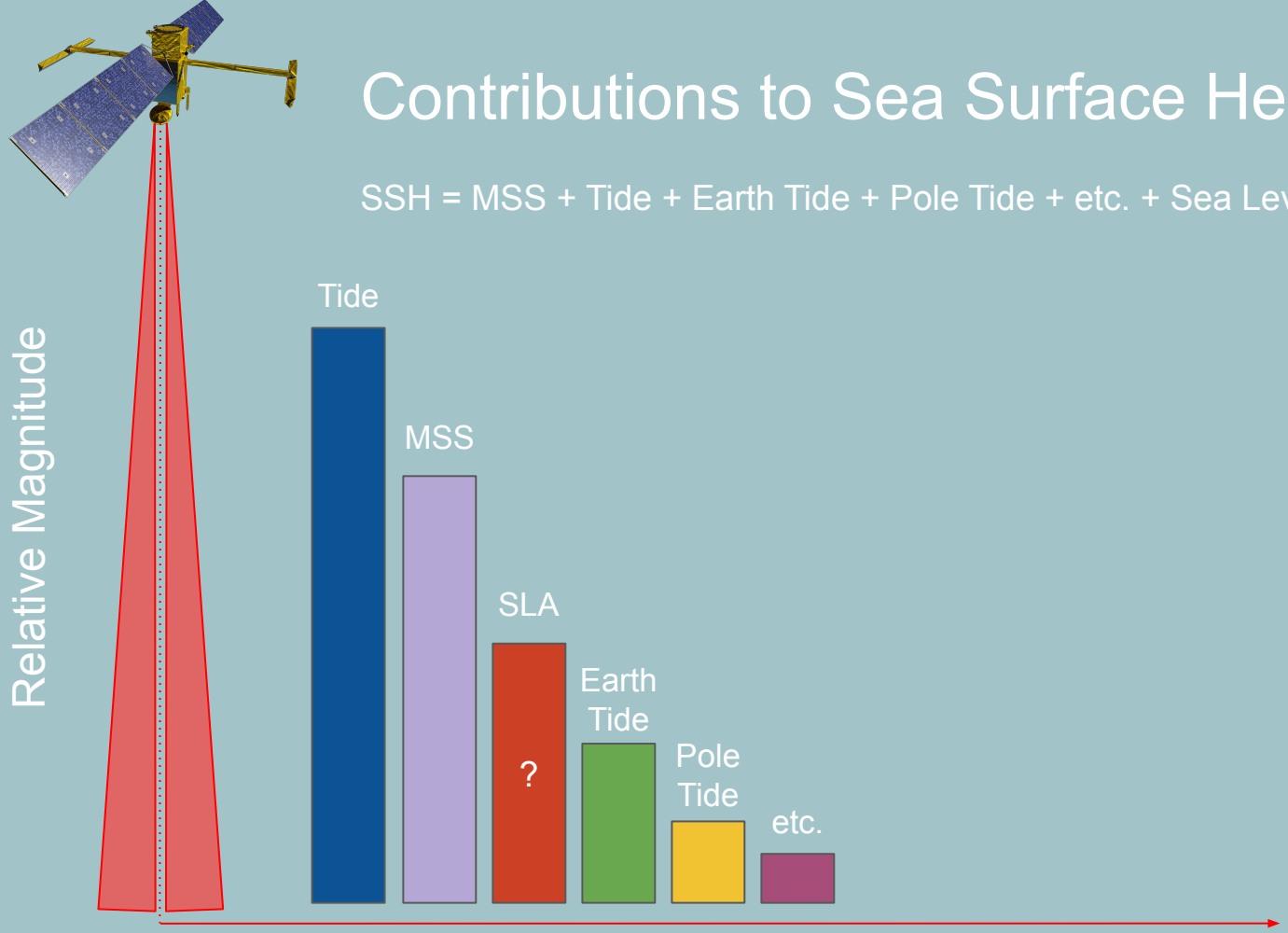


DEPARTMENT OF ENGINEERING SCIENCE
UNIVERSITY OF OXFORD

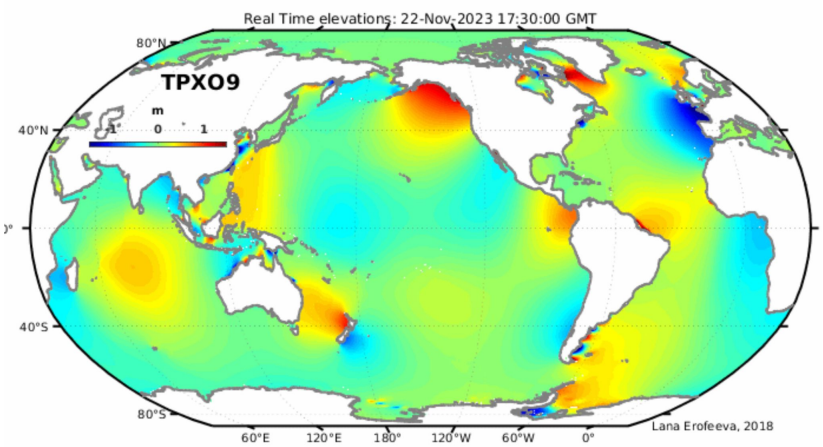


Contributions to Sea Surface Height

SSH = MSS + Tide + Earth Tide + Pole Tide + etc. + Sea Level Anomaly



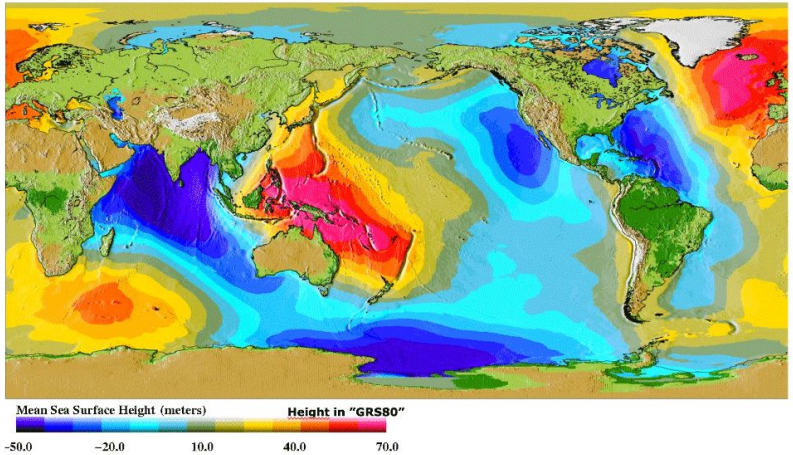
Tidal Corrections



Shelf Error	**6-10%**
Coastal Error	**7-27%**

1.Egbert, Gary D., and Svetlana Y. Erofeeva. "Efficient inverse modeling of barotropic ocean tides." Journal of Atmospheric and Oceanic Technology 19.2 (2002): 183-204.
2.Stammer, Detlef, et al. "Accuracy assessment of global barotropic ocean tide models." *Reviews of Geophysics* 52.3 (2014): 243-282.

MSS Corrections



- For wavelengths shorter than 100 km, the residual errors of recent models represent nearly 30% of the SLA variance.
- The error can be 2.5 times higher for uncharted tracks over rough bathymetry.

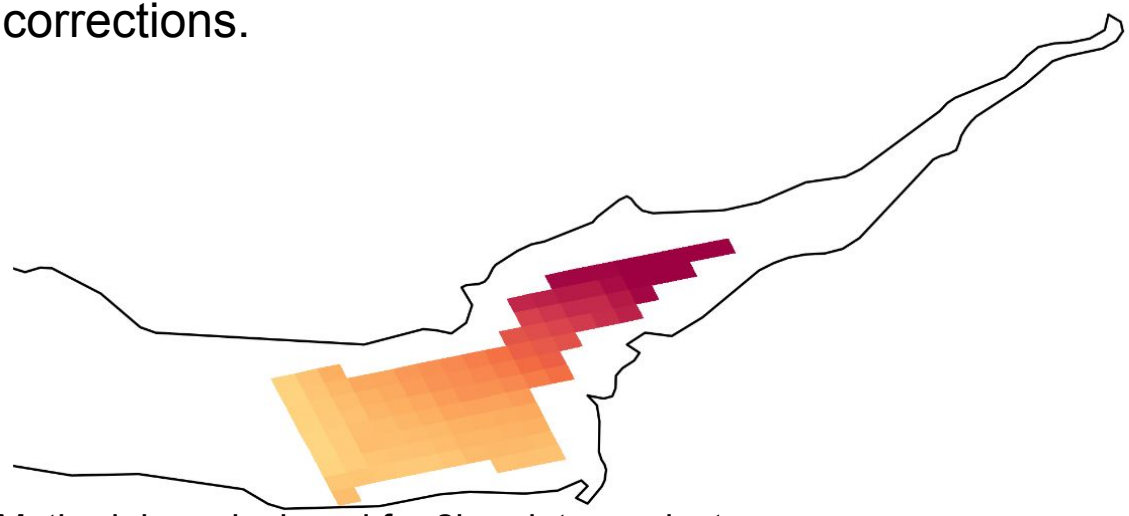
1. Andersen, Ole Baltazar, et al. "The DTU21 global mean sea surface and first evaluation." *Earth System Science Data Discussions* 2023 (2023): 1-19.
2.Pujol, Marie, Isabelle, et al. "Gauging the improvement of recent mean sea surface models: A new approach for identifying and quantifying their errors."

DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD

Objectives

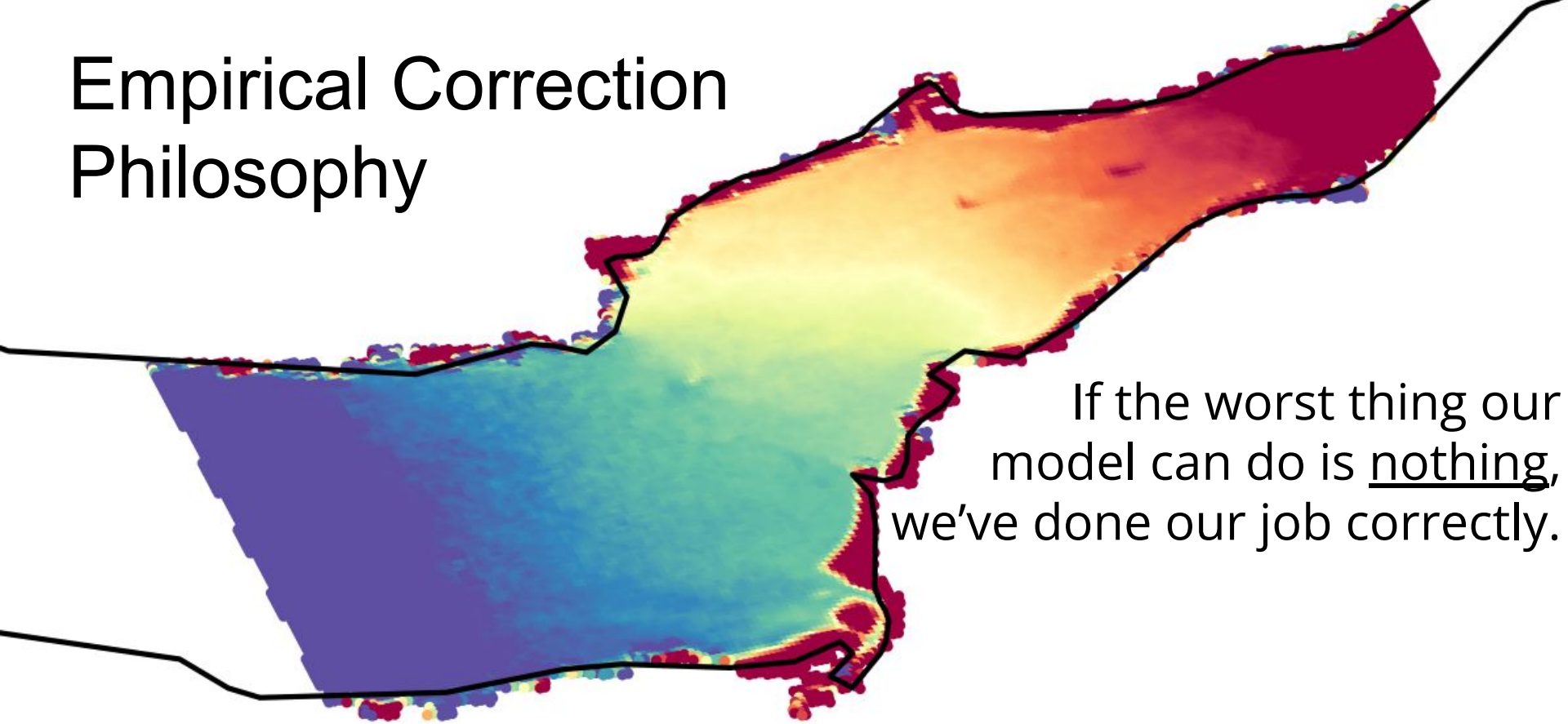
1. Provide accurate uncertainty estimation for present geophysical corrections.
2. Improve current coastal and estuarine tidal and mean sea surface corrections.



** Methodology designed for 2km data product

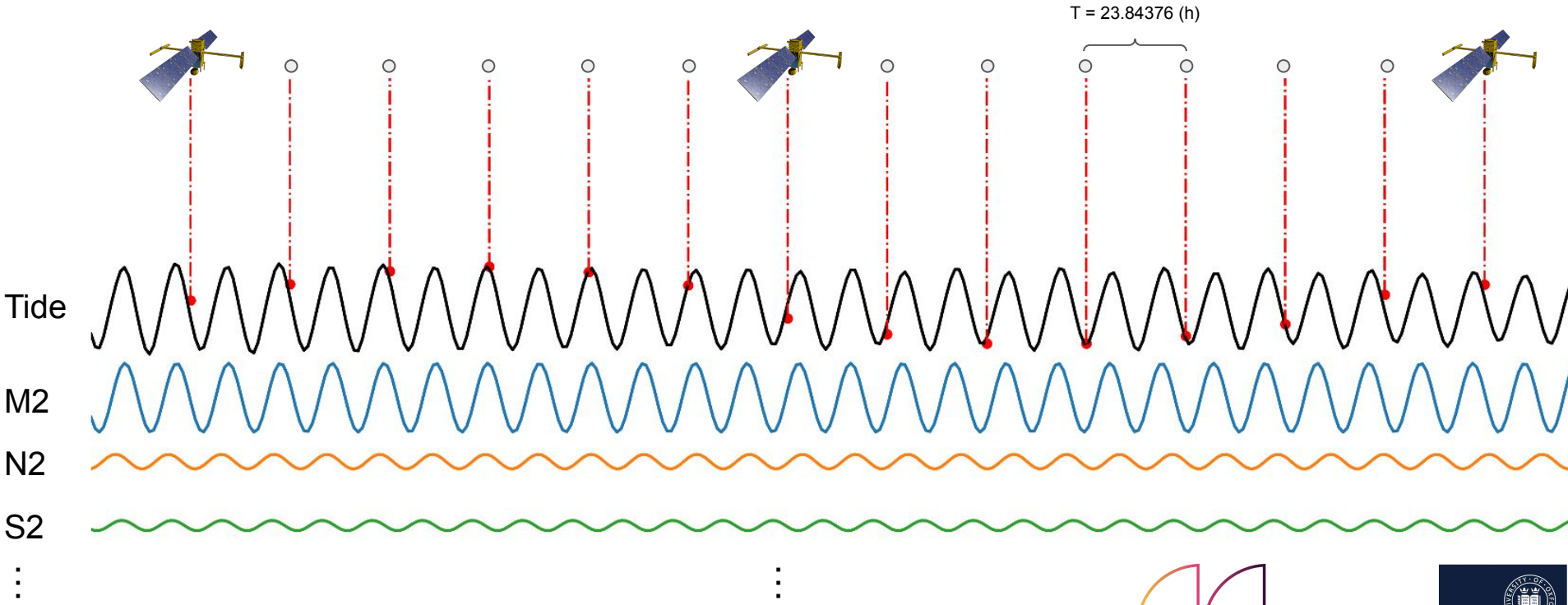
DEPARTMENT OF ENGINEERING SCIENCE
UNIVERSITY OF OXFORD

Empirical Correction Philosophy



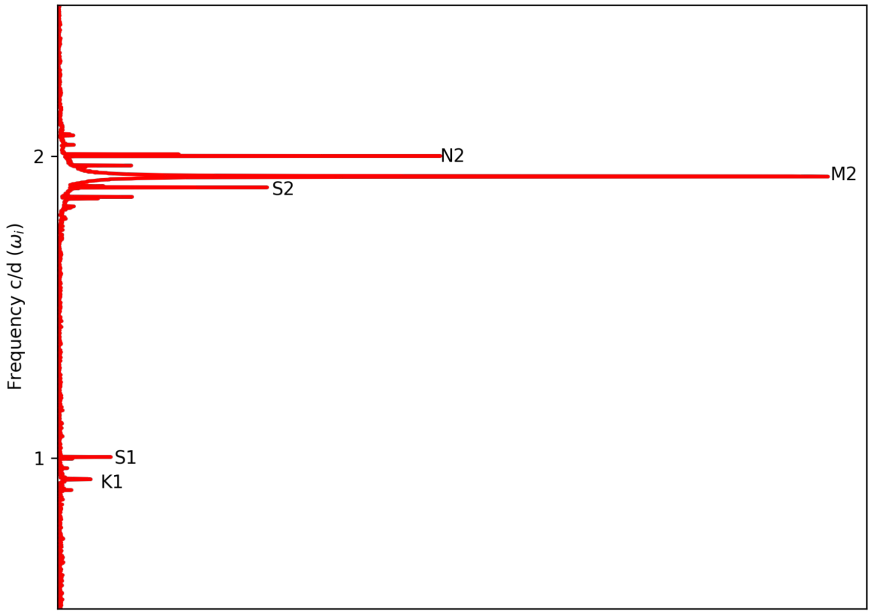
If the worst thing our model can do is nothing, we've done our job correctly.

Tides From and For SWOT



Conventional Tidal Analysis

Harmonic Analysis (HA): $\zeta(t) = x_0 + x_1 t + \sum_{k=1,\ldots,N} c_k \sin(\omega_i t + \phi_k)$



$$y_t = x_t w + c$$

OLS: $\hat{\beta} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$

Bayesian Tidal Analysis

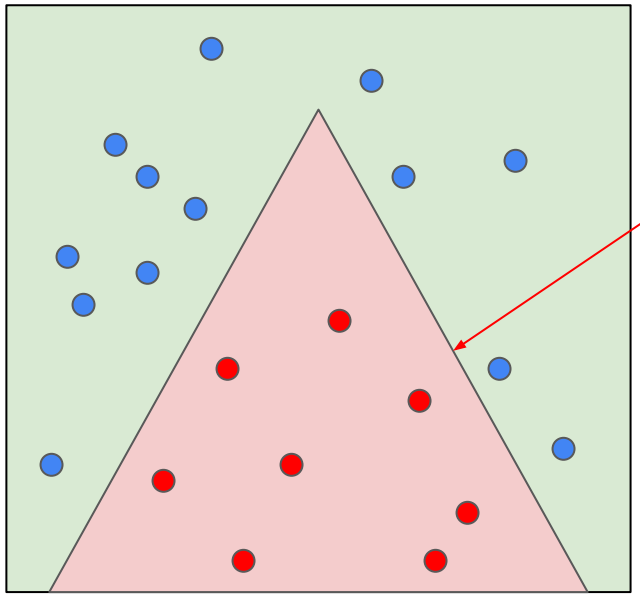
Harmonic Analysis (HA): $\zeta(t) = x_0 + x_1 t + \sum_{k=1,\ldots,N} c_k \sin(\omega_i t + \phi_k)$

$$\zeta(t) = X_0 + X_1 t + \sum_{k=1,\ldots,N} A_k \sin \omega_i t + B_k \cos \omega_i t$$

$$X_0, X_1, A_k, B_K \sim \mathcal{N}(\mu, \sigma^2)$$

The problem…

$$\zeta(t) = X_0 + X_1 t + \sum_{k=1,\ldots,N} A_k \sin \omega_i t + B_k \cos \omega_i t$$



Posterior **Likelihood** **Prior**

$$P(Model|\ New\ Data) = \frac{P(New\ Data\ |\ Model)\ P(Model)}{P(New\ Data)}$$

Evidence

Bayesian Harmonic Analysis

Pros

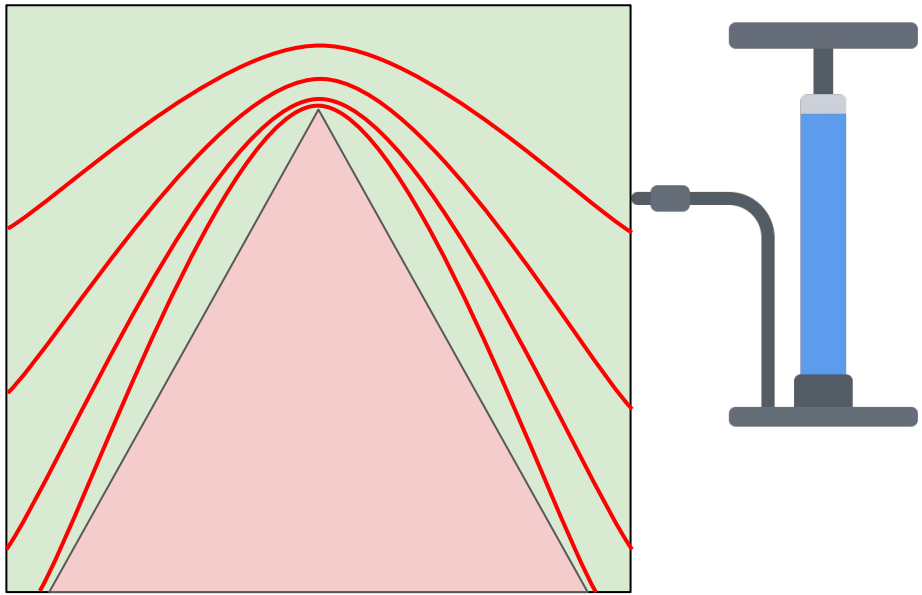
1. Uncertainty estimation
2. More accurate
3. Reduced overfitting
4. Prior information incorporation

Cons

1. MCMC is really, **really**, slow

Variational Bayesian HA

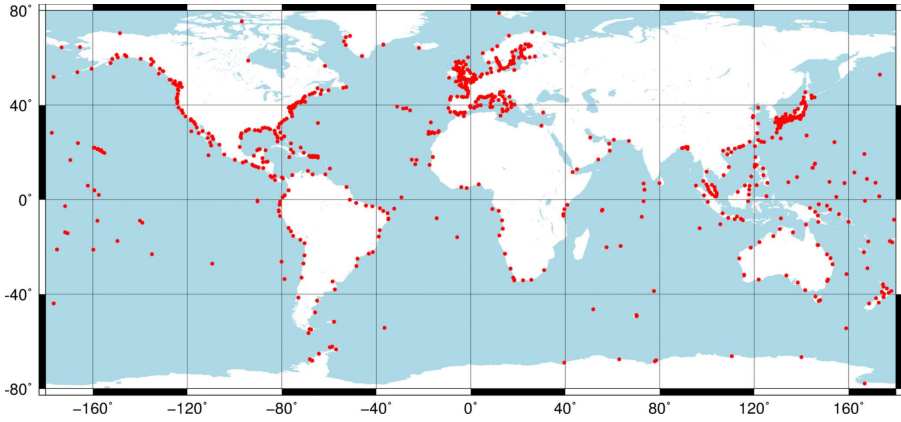
What if we could approximate the posterior in a different way?



- Honest
- Accurate
- Fast

Synthetic Data

- TICON is a tidal constituent dataset comprised of ~984 global tide gauges with harmonics computed for 40 tidal constituents.



TICON Tide Gauge Locations

Piccioni, Gaia, et al. "TICON: TIdal CONstants based on GESLA sea-level records from globally located tide gauges." *Geoscience Data Journal* 6.2 (2019): 97-104.

Constituent Root Mean Squared Error:

$$\mathrm{RMS}_k = \sqrt{[(A_{\mathrm{model}}\sin(\omega_k t) + B_{\mathrm{model}}\cos(\omega_k t)) - (A_{\mathrm{true}}\sin(\omega_k t) + B_{\mathrm{true}}\cos(\omega_k t))]^2}$$

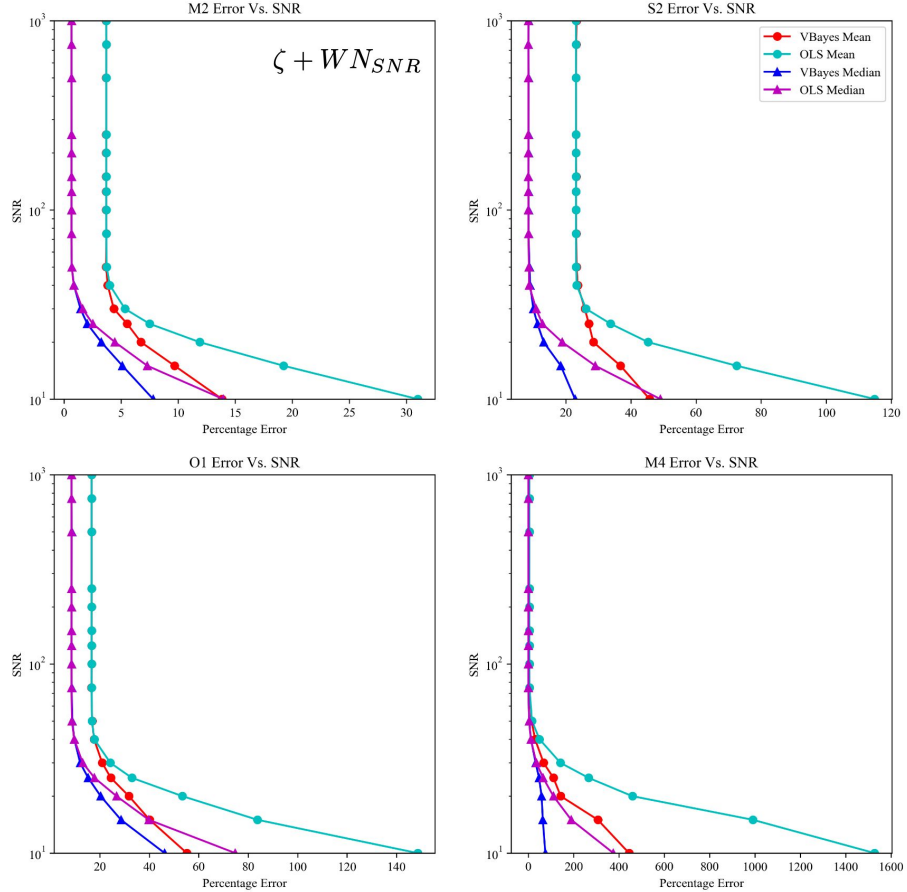
Root Sum of Squares:

$$\mathrm{RSSE} = \sqrt{\sum_{k=1}^{n}(\mathrm{RMS}_k)^2}$$

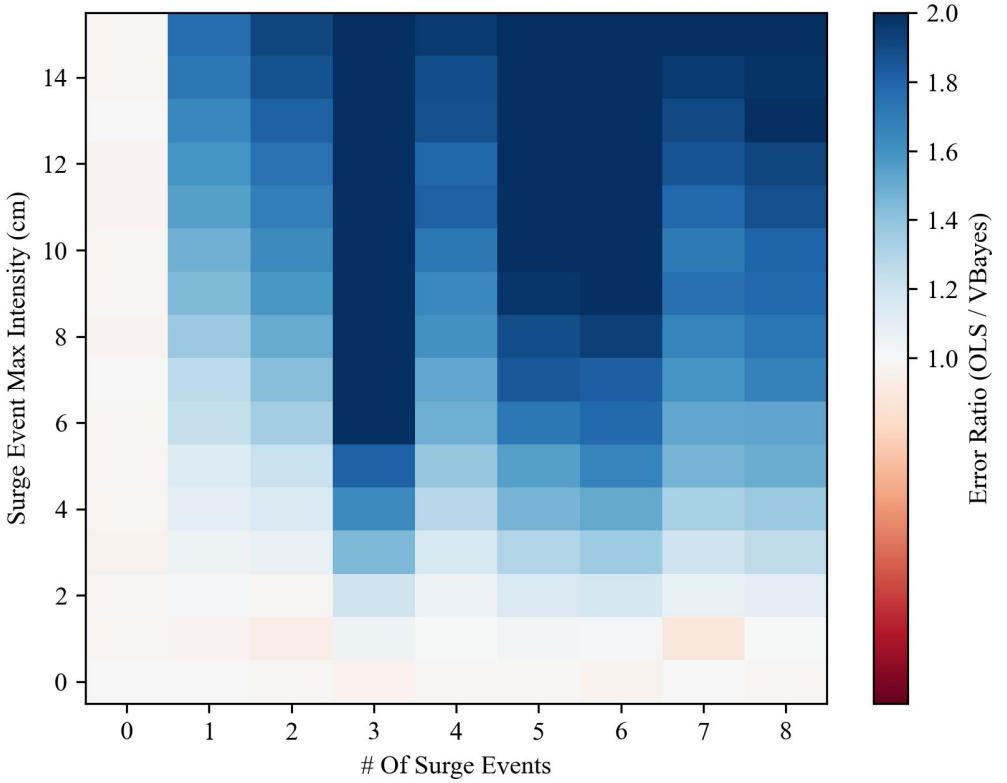
DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD

How does this compare to OLS?

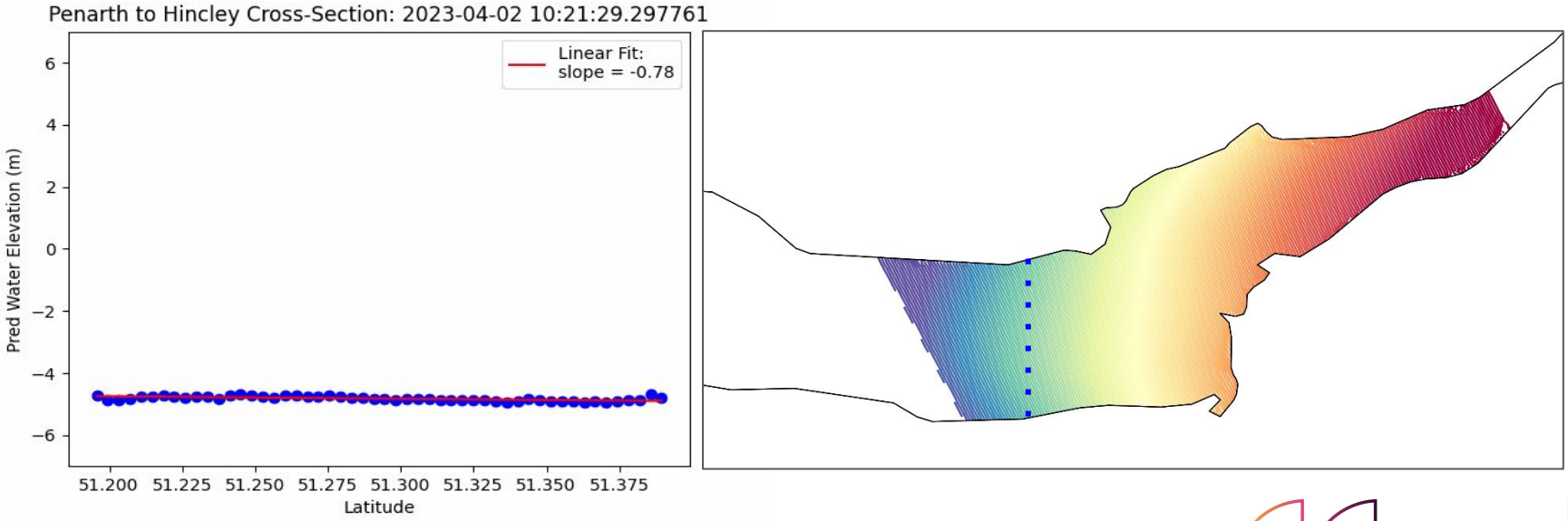


$$\zeta = \zeta_T + \zeta_S + B * \zeta_S(\zeta''_T + \zeta'_T)$$

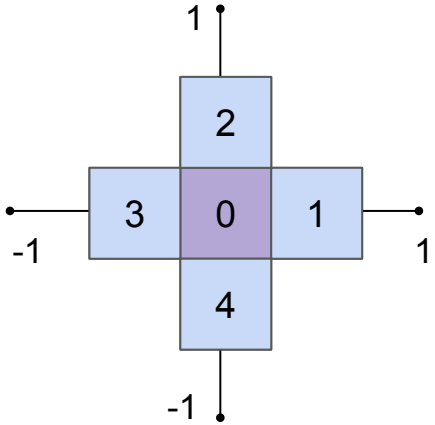


Spatial Coherence

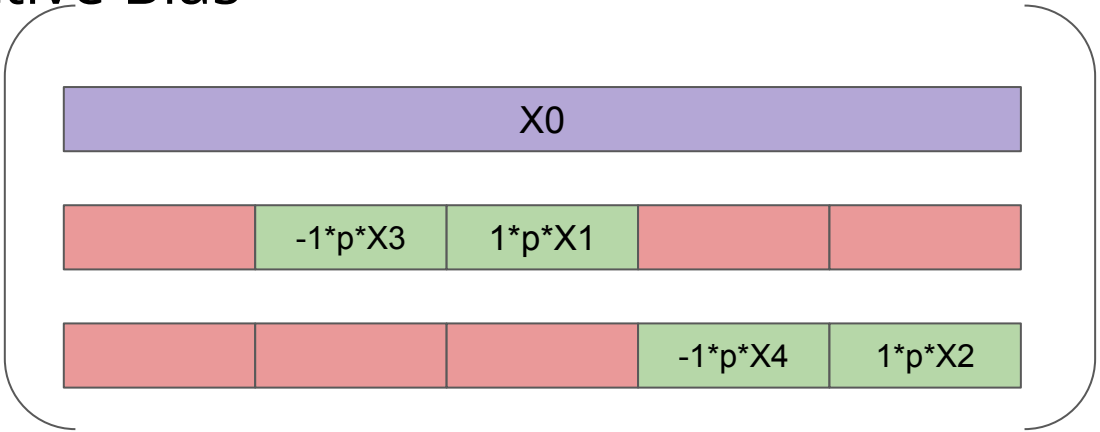
What if we could leverage the information from adjacent points?



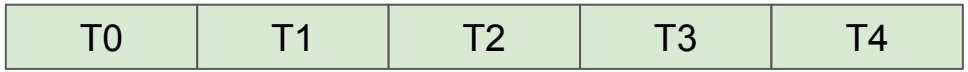
Spatially Coherent Inductive Bias



x =



t =



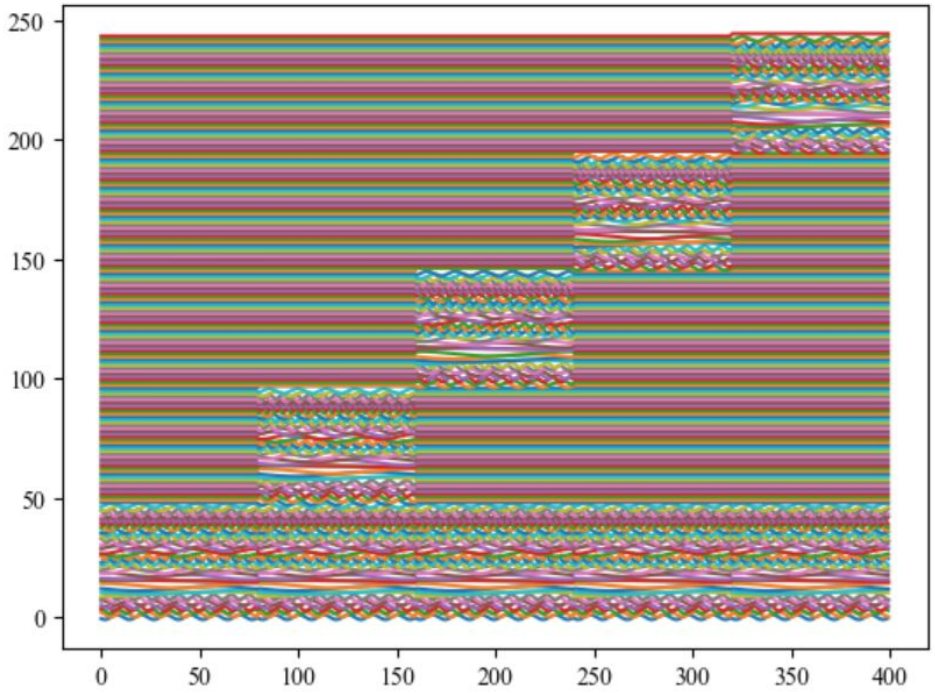
data =

0 = X0.T*A

vert = (Mv*d*p + A)*X0.T

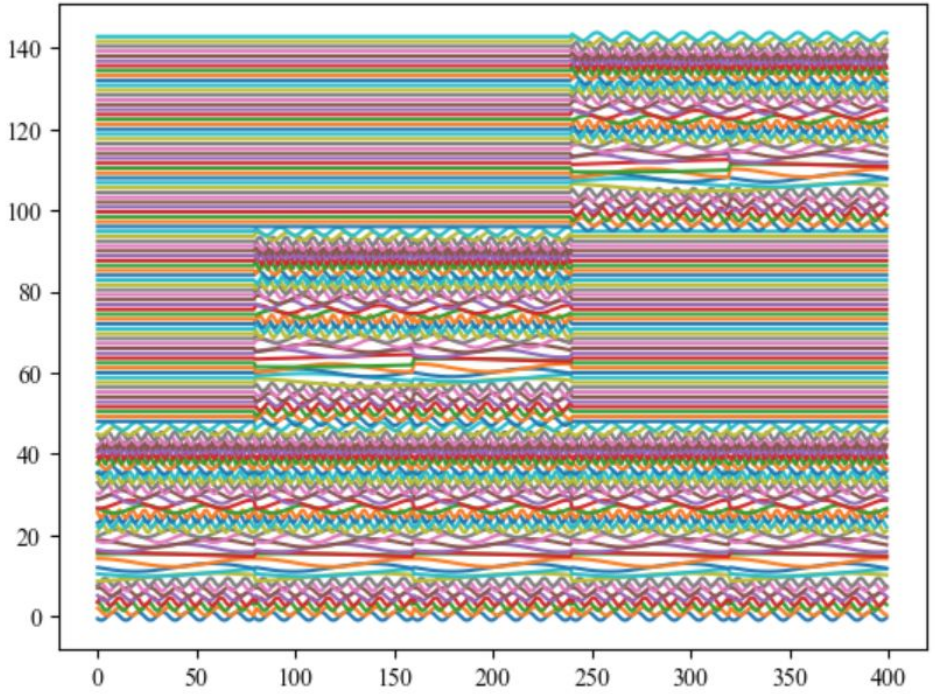
hori = (Mh*d*p + A)*X0.T

Original (X)



Measurements / Parameters ≤ 1.6

Linear (X)

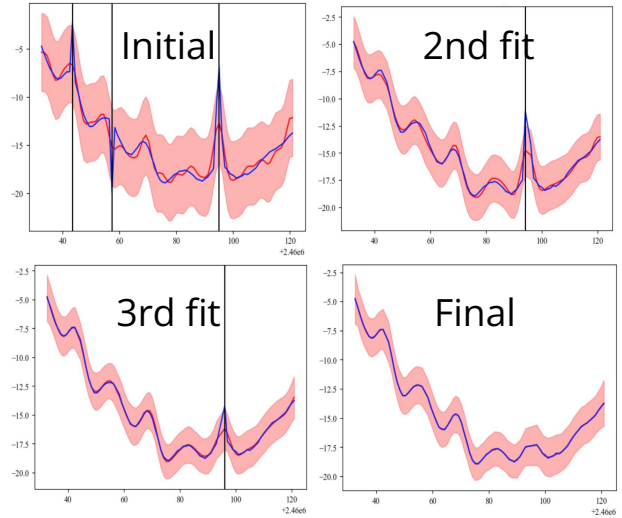


Measurements / # Params ≤ 2.9

** Note these are expanded for visualization purposes

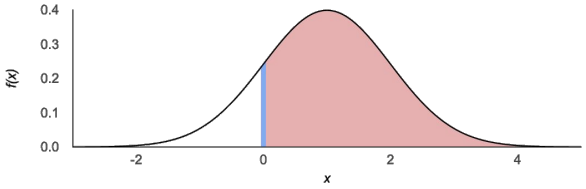
Other Bayesian Tricks

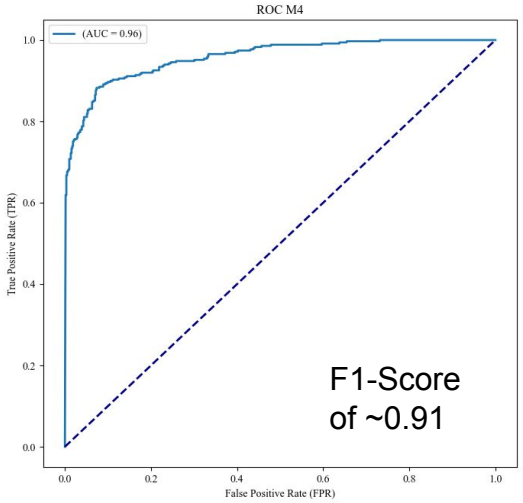
Dataset Pruning



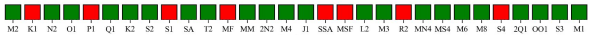
VBayes Observed Identified Bad Points

Significance Testing



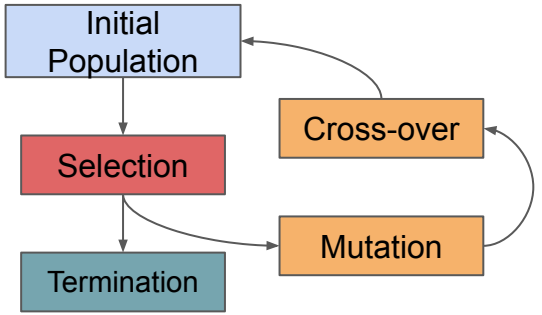


Evolutionary Constituent Optimization



= Constituent Included

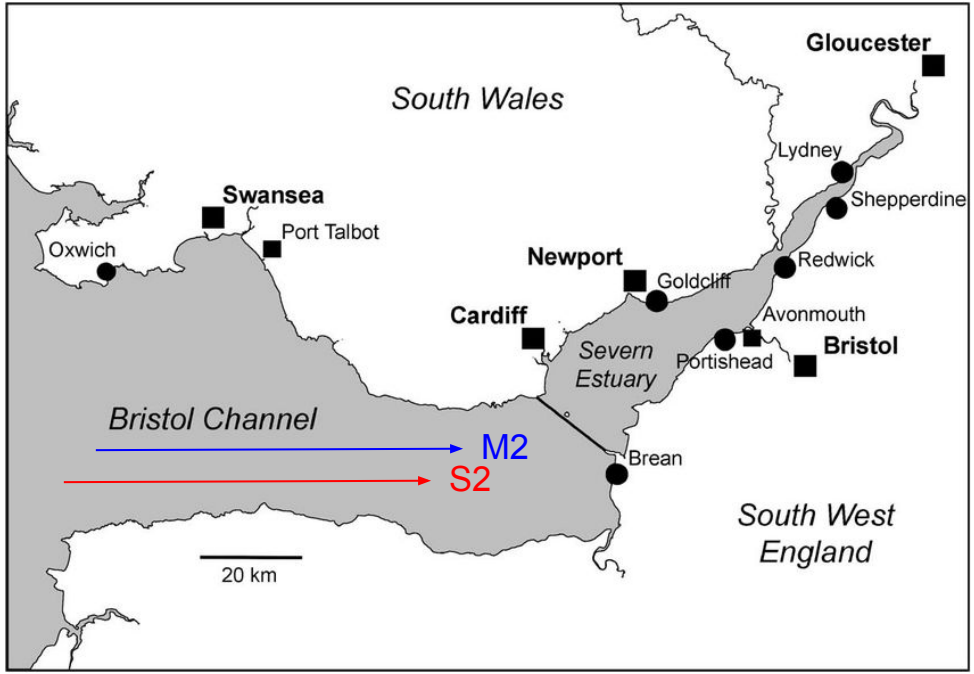
= Constituent Excluded



DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD

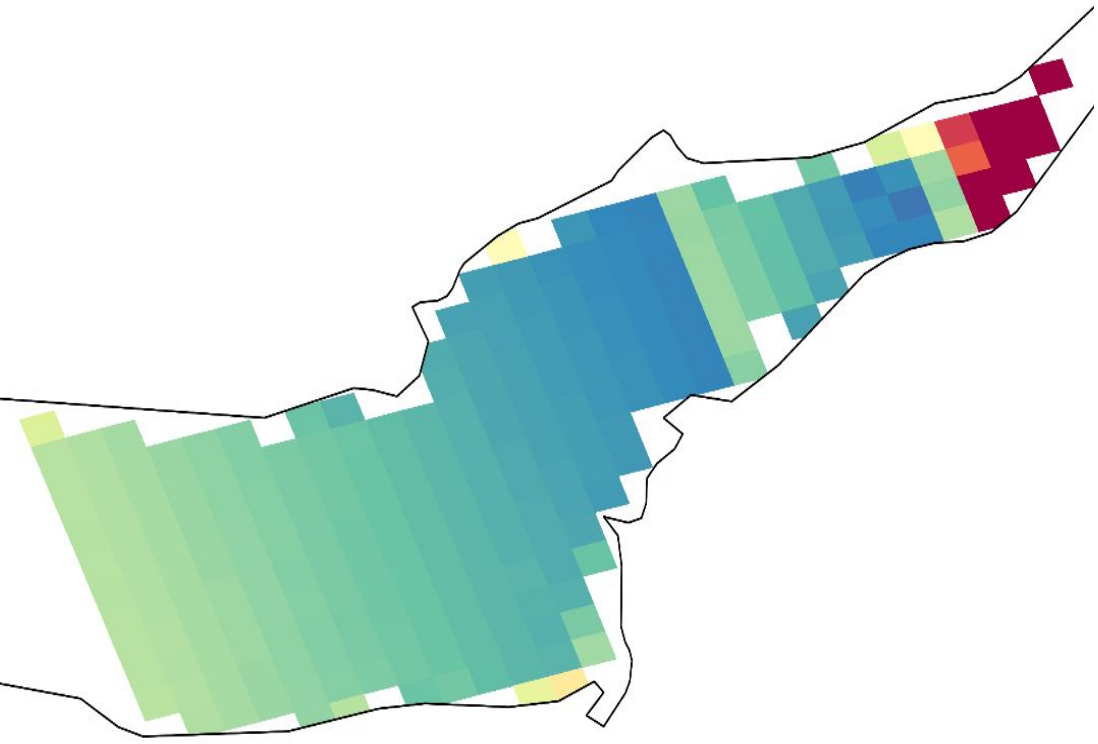
Bristol Channel (Overview)

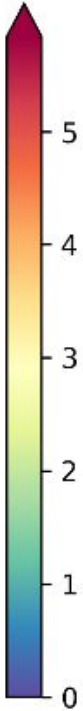


Source: Duquesne et al. Evidence for declining levels of heavy-metals in the Severn Estuary and Bristol Channel, UK and their spatial distribution in sediments

- Characterized by the second largest tidal range in the world.
- Spring tidal range exceeding 12-14 meters!
- Significant nonlinearity…
- Unique challenges for satellite altimetry.
- Cal/Val phase

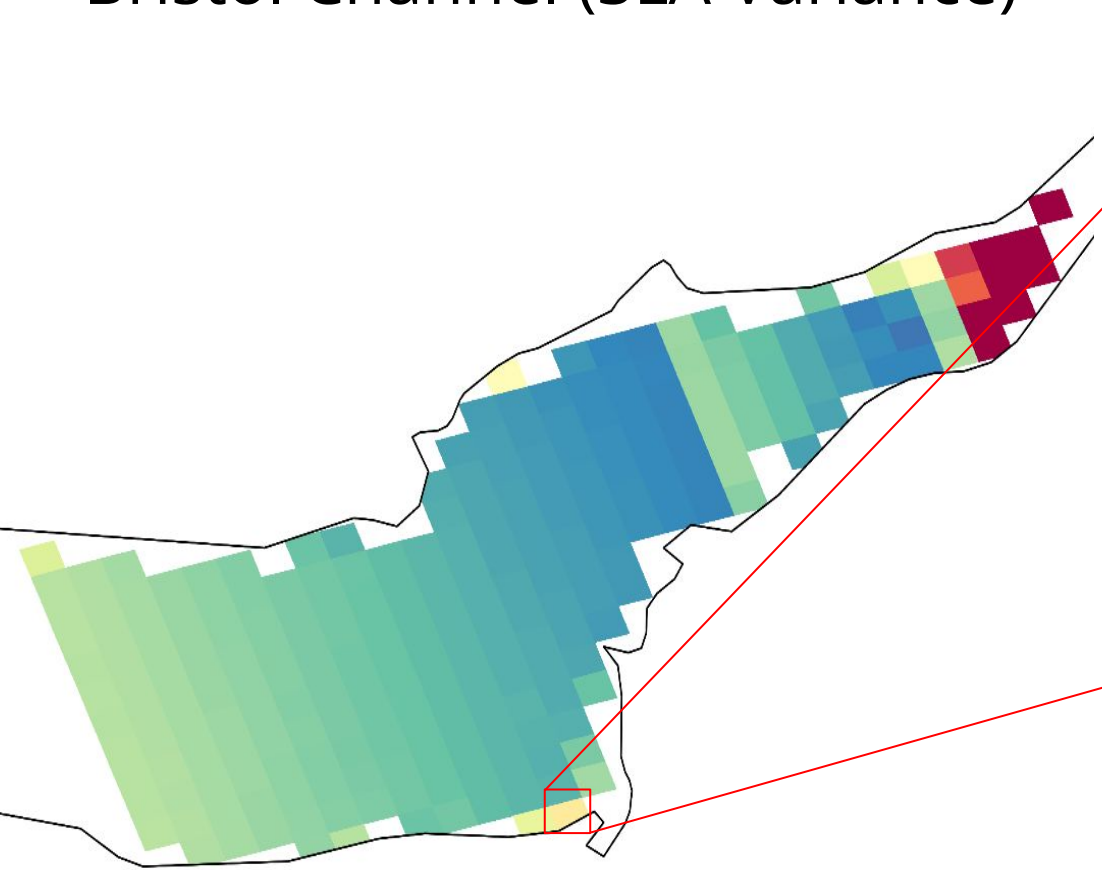
Bristol Channel (SLA Variance)

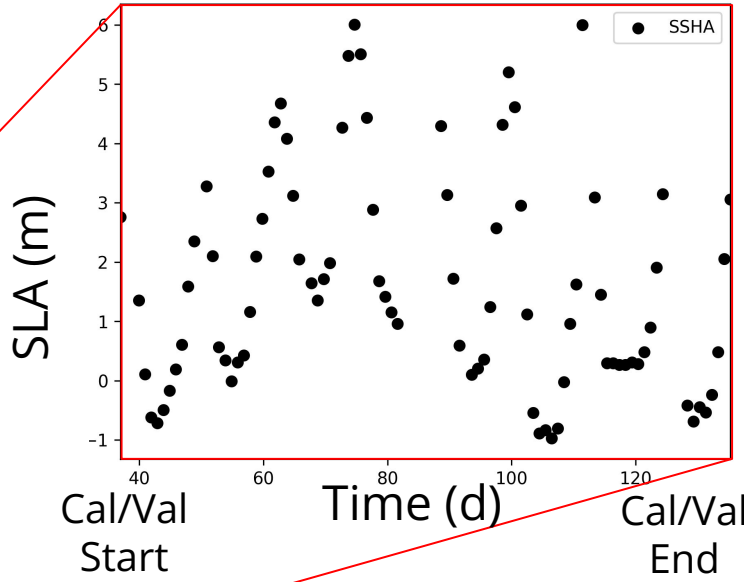




SLA
Variance (m)

Bristol Channel (SLA Variance)

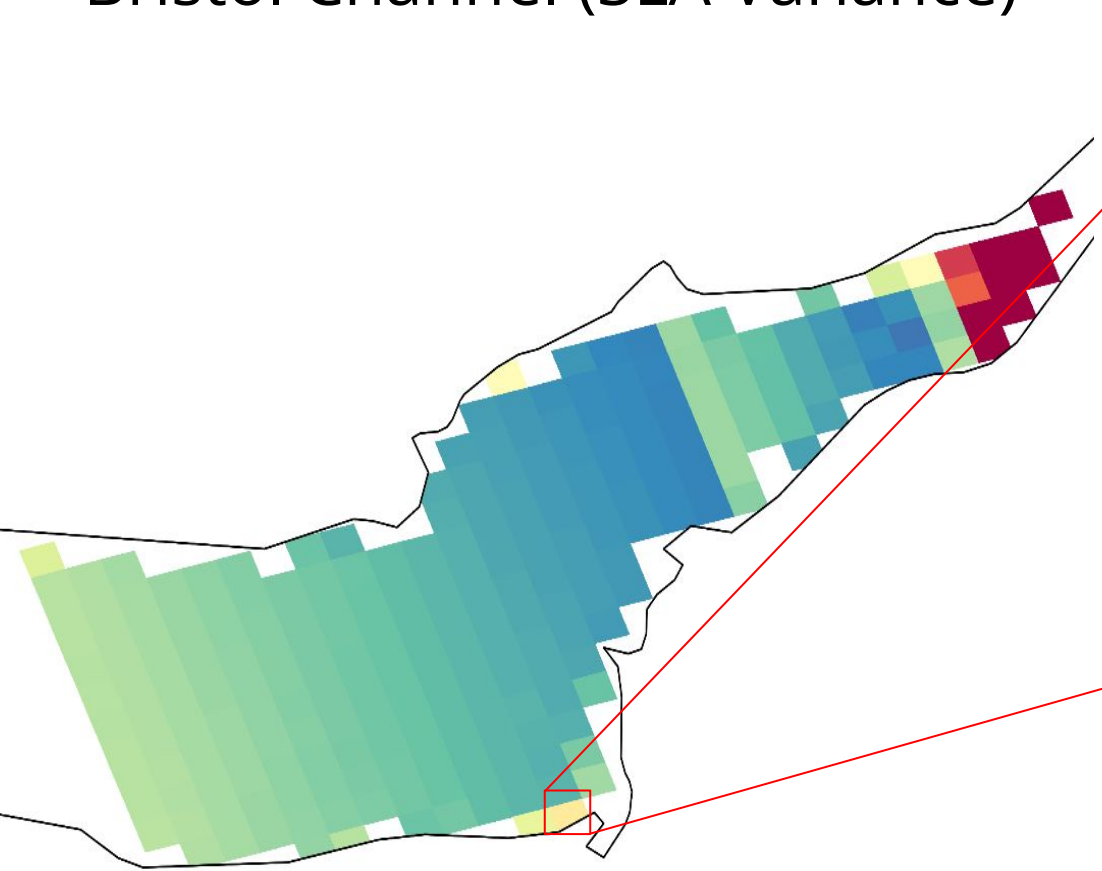


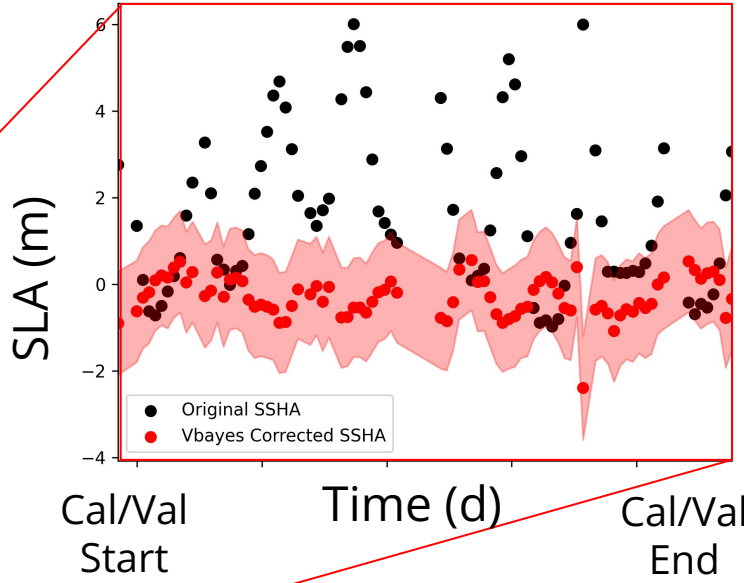


DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD

Bristol Channel (SLA Variance)

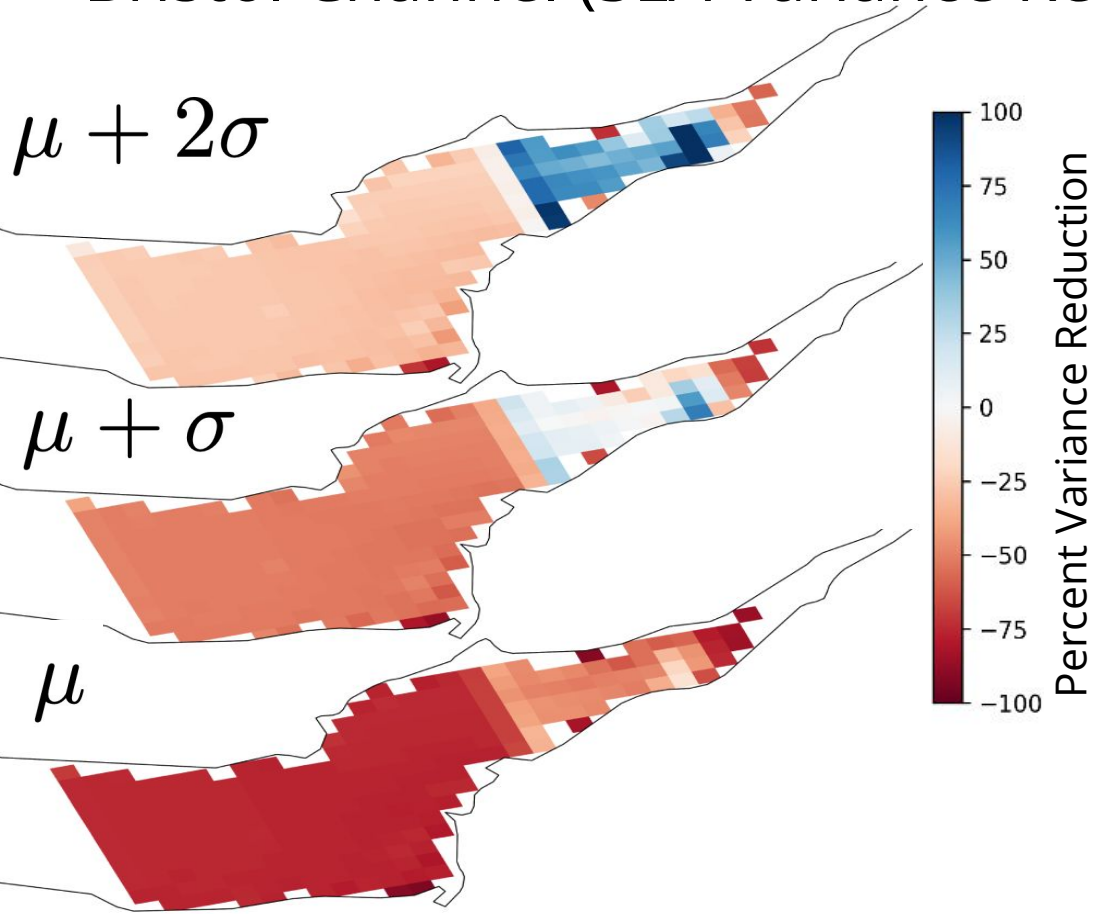




DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD

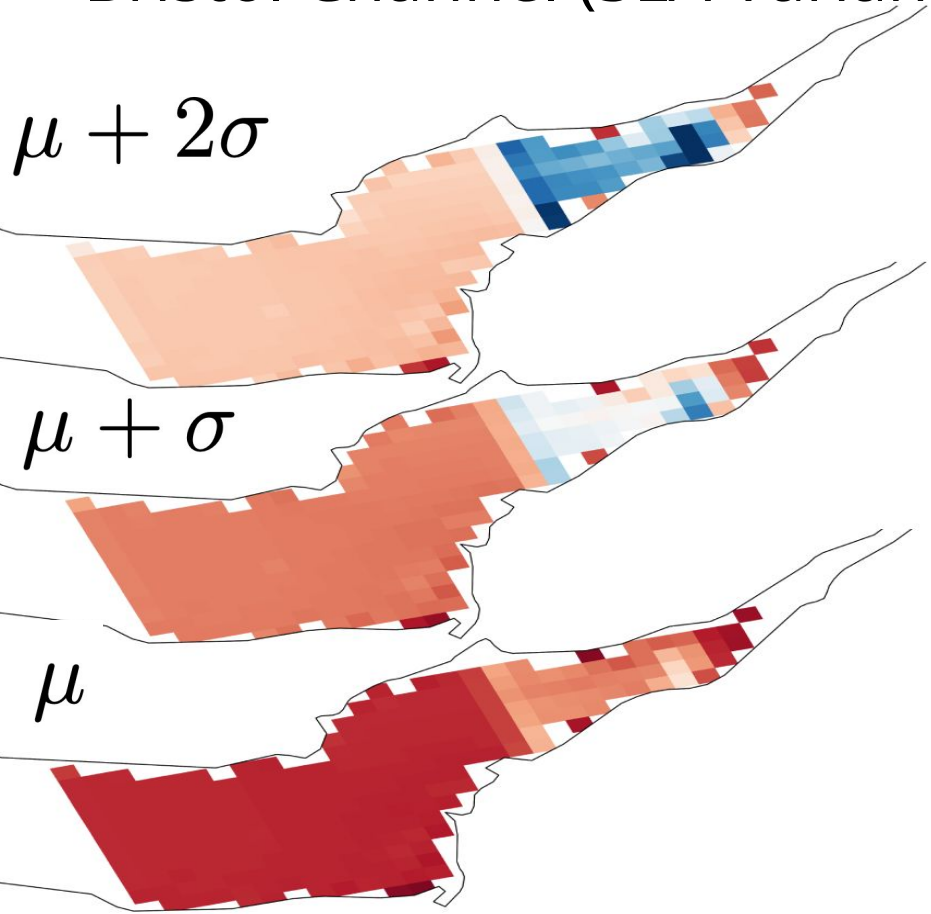
Bristol Channel (SLA Variance Reduction)

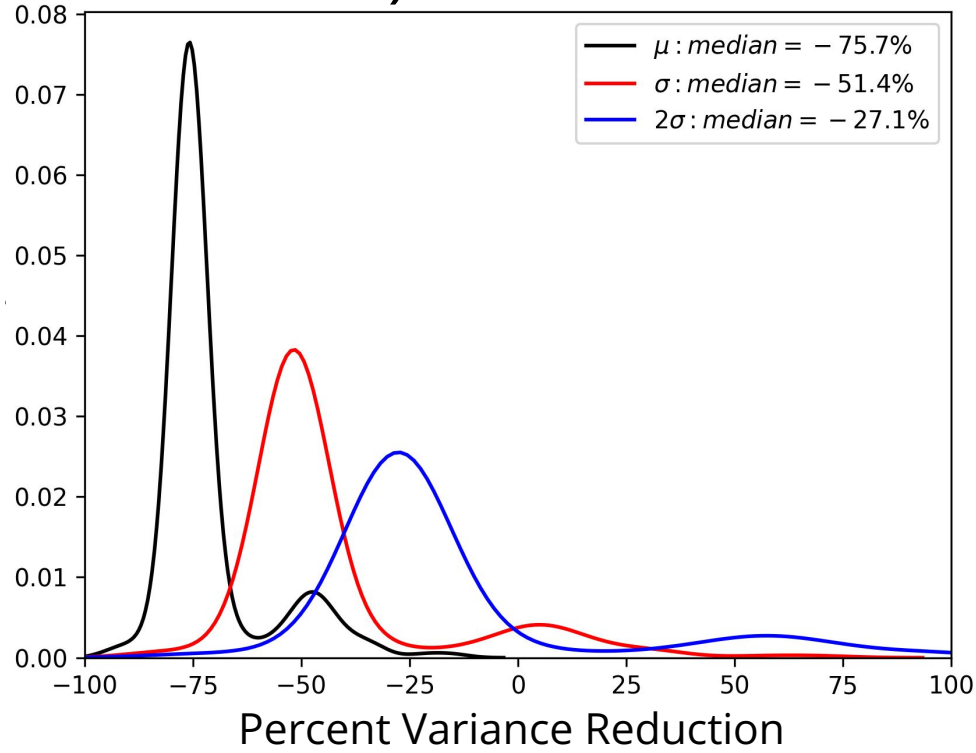


DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD

Bristol Channel (SLA Variance Reduction)

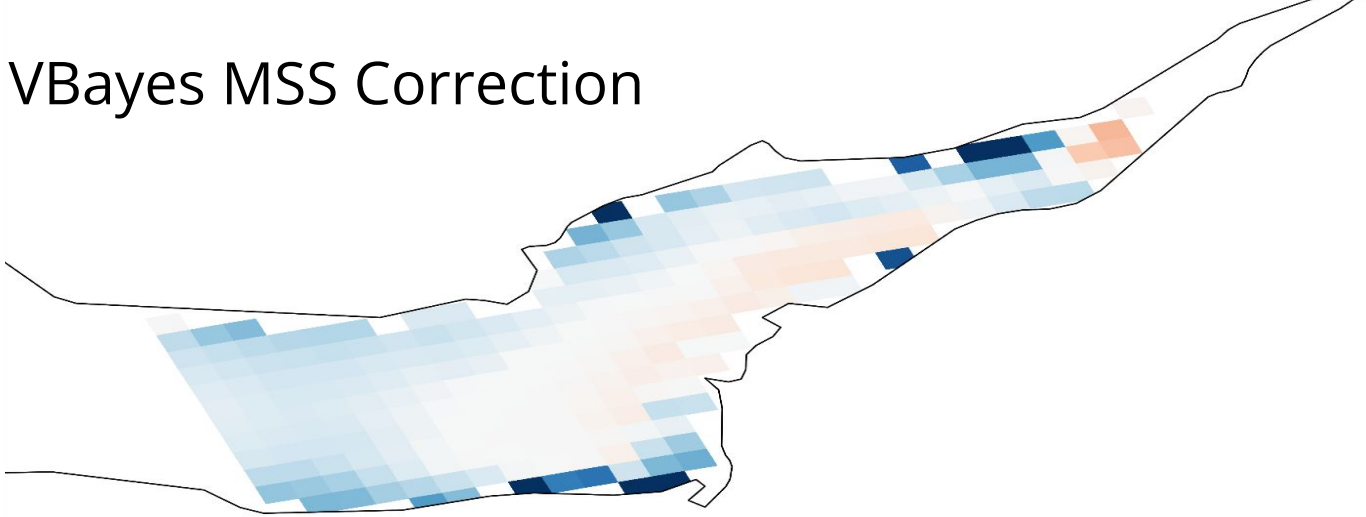




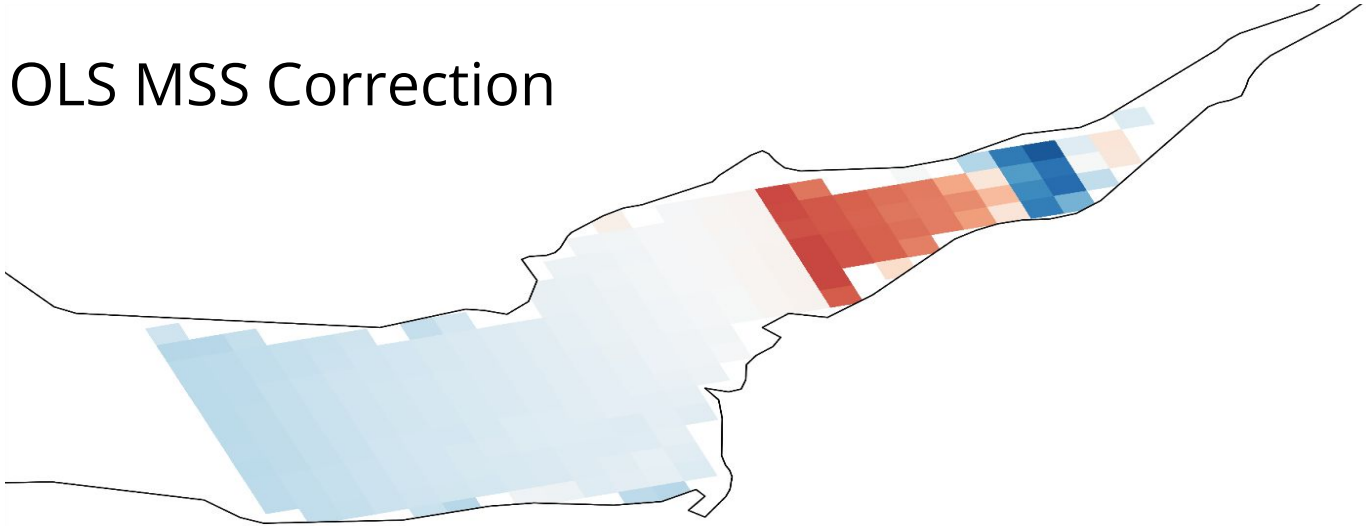
DEPARTMENT OF ENGINEERING SCIENCE

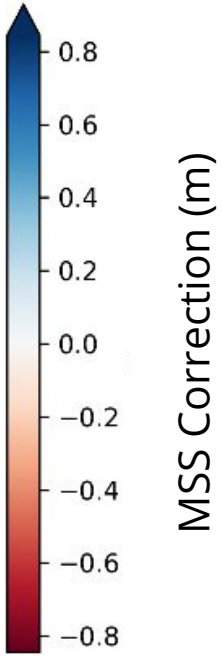
UNIVERSITY OF OXFORD

VBayes MSS Correction

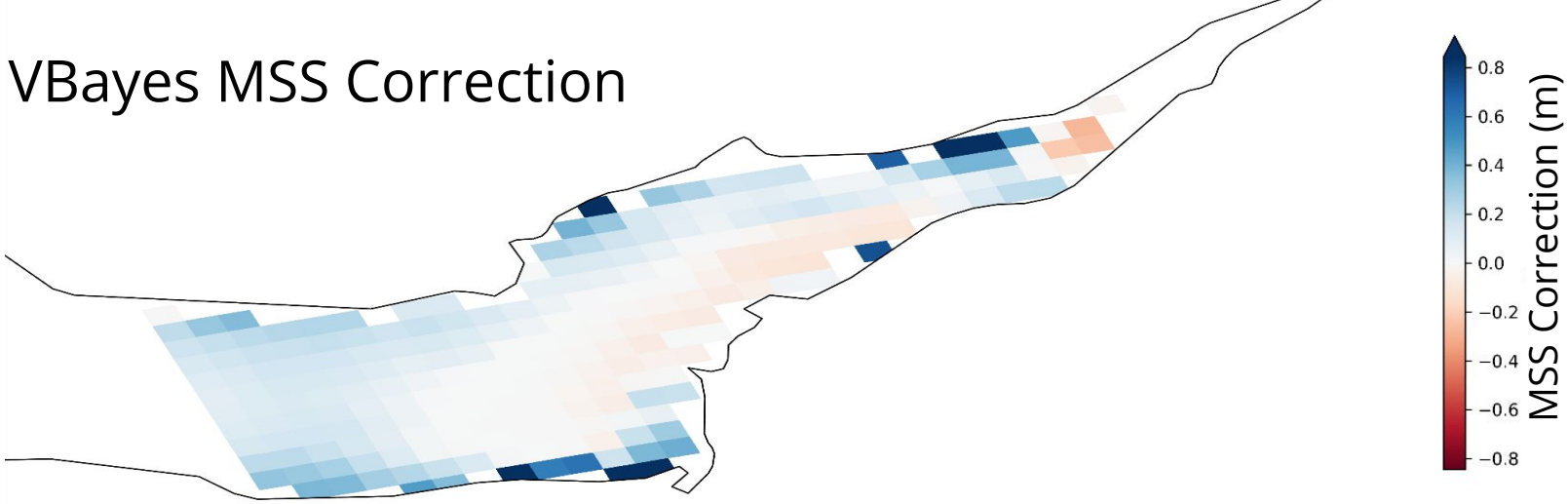


OLS MSS Correction

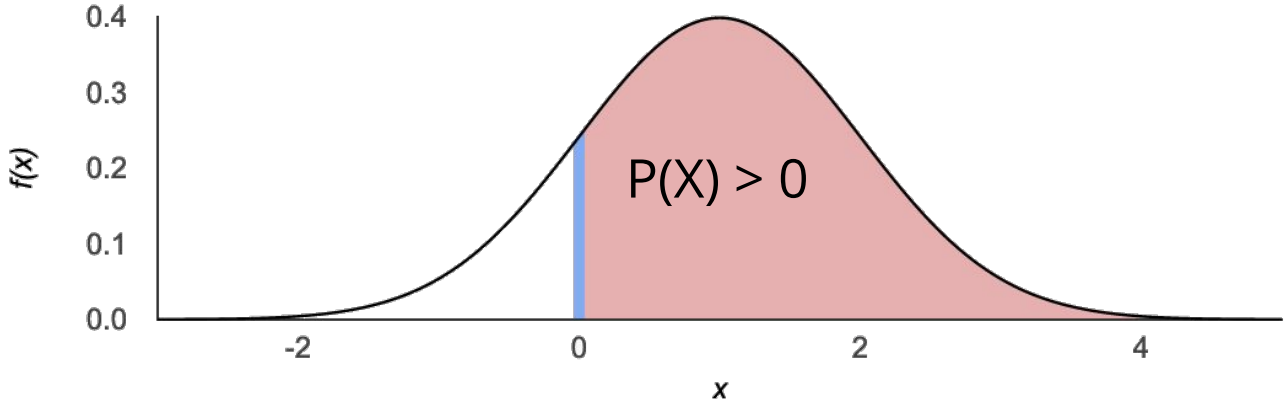




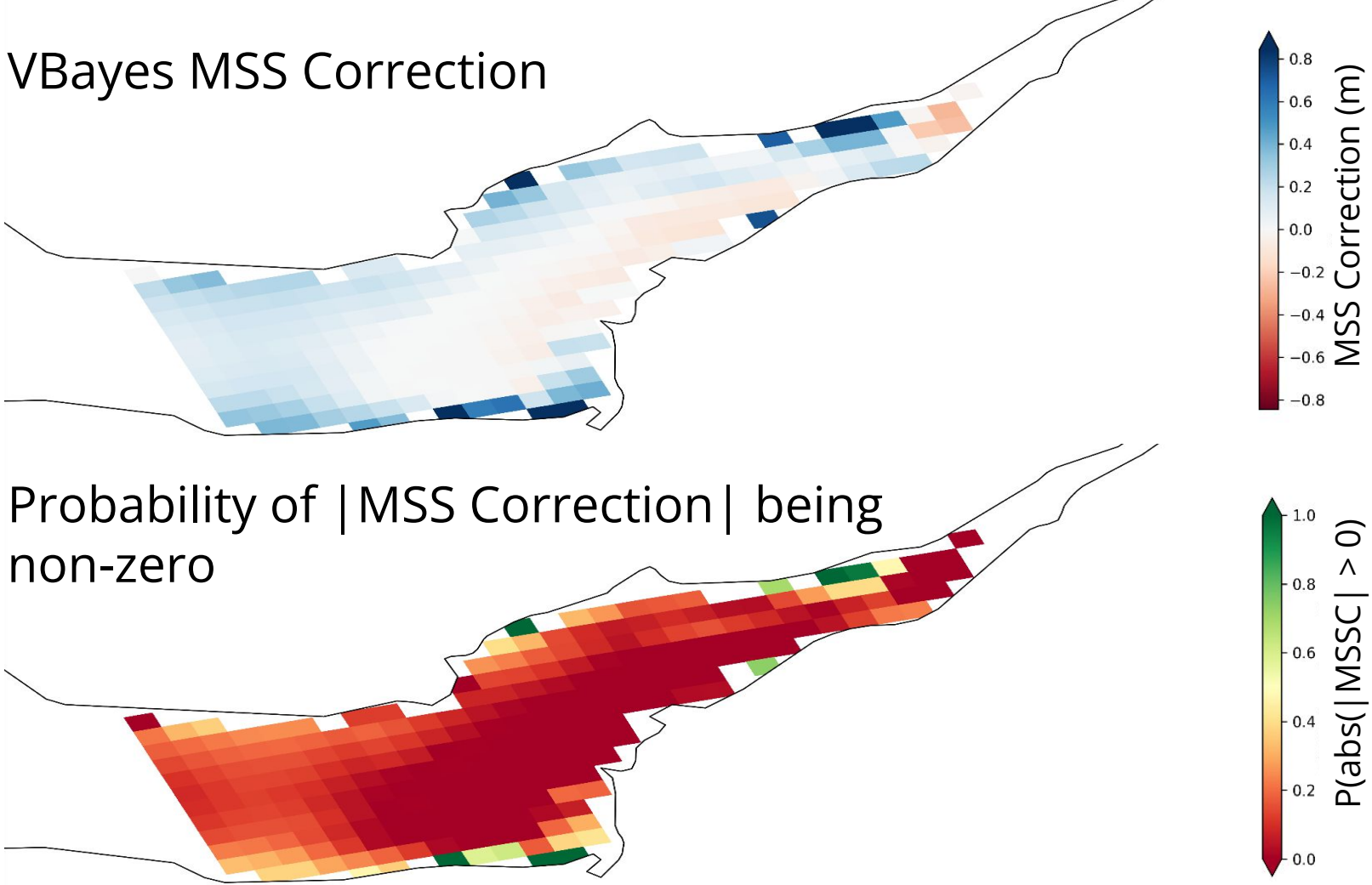
VBayes MSS Correction



How much can we trust this result?

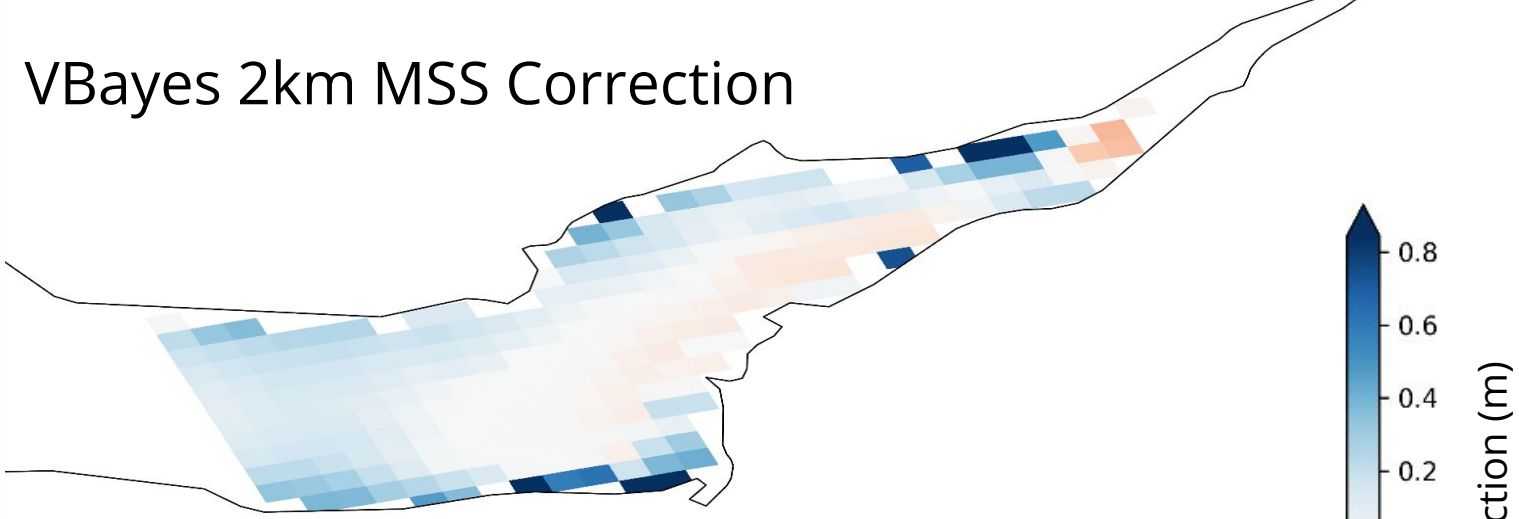


VBayes MSS Correction

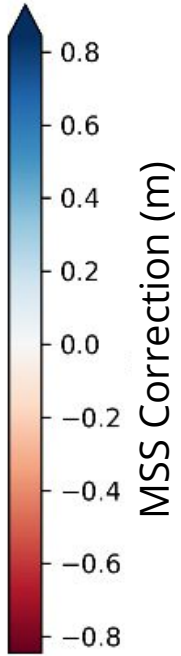


Probability of |MSS Correction| being non-zero

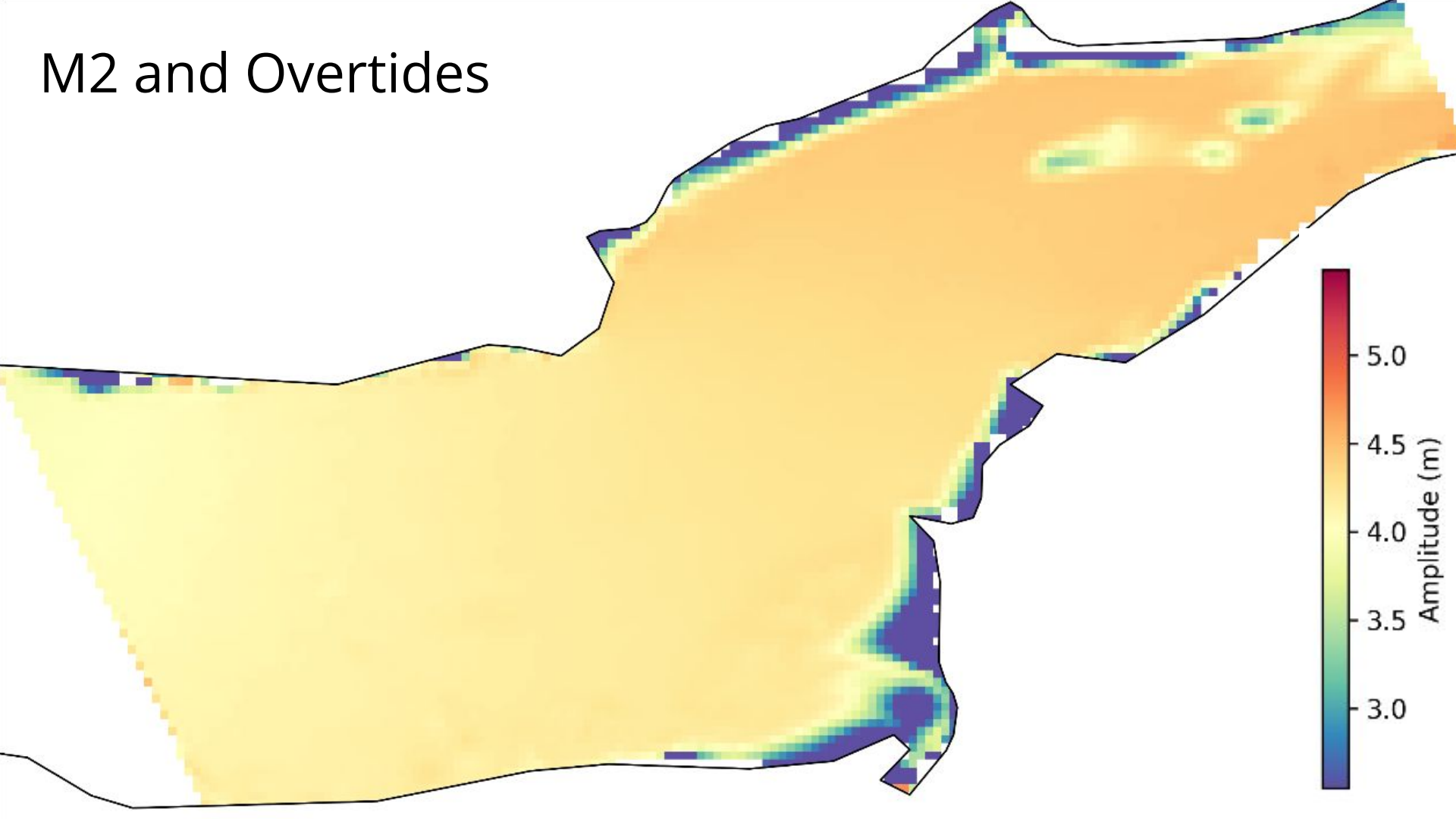
VBayes 2km MSS Correction



VBayes 500m MSS Correction

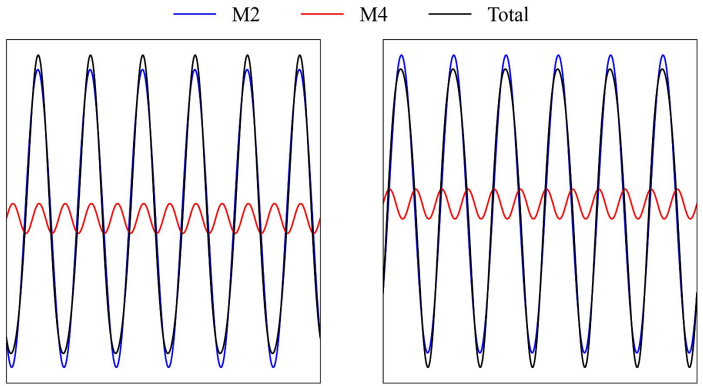


M2 and Overtides



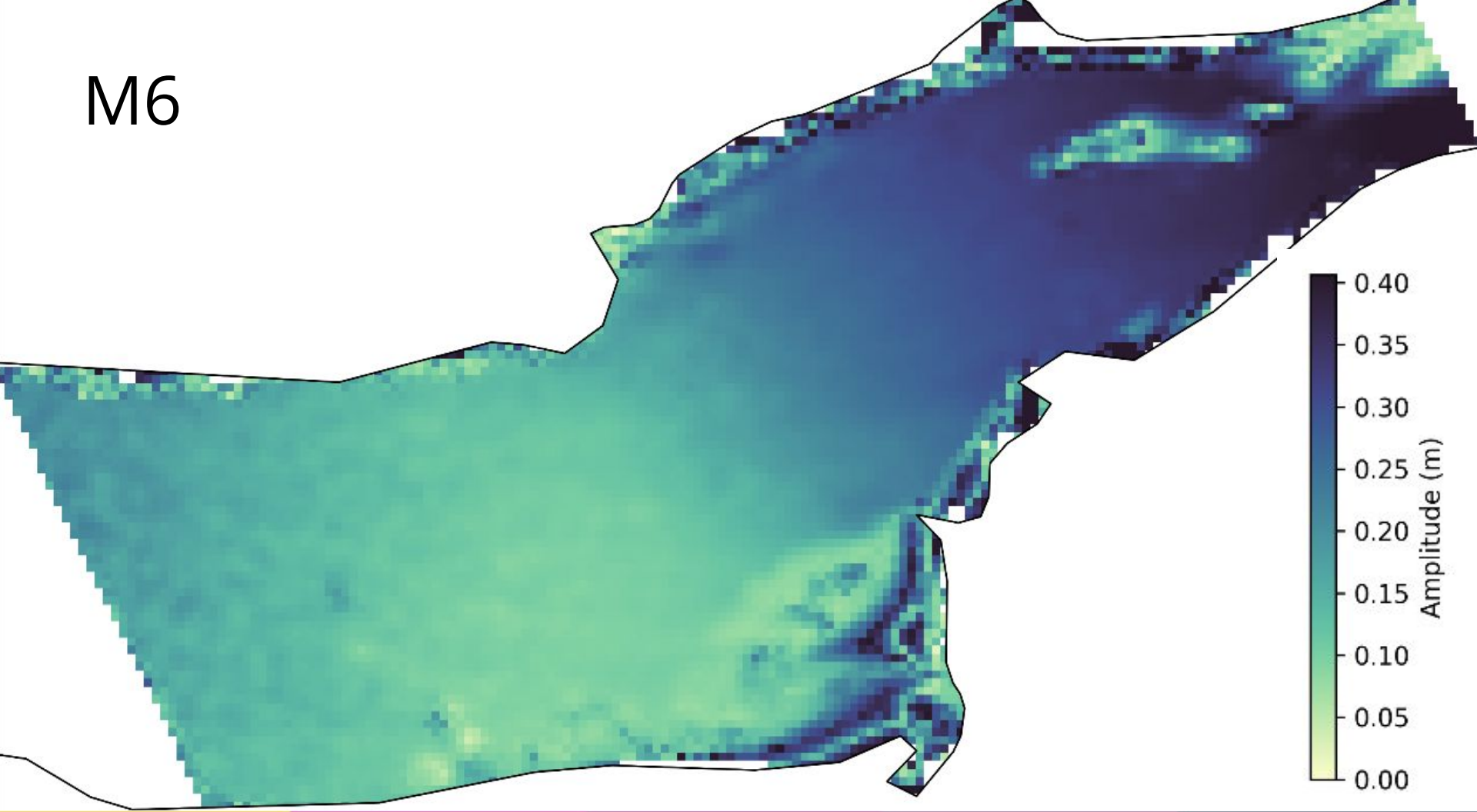
Tidal Asymmetry

$$Asymmetry \approx A_{M4}/A_{M2}$$



Also why MTL != MSS

M6



SWOT empirical tide + MSS corrections with uncertainty.

May ‘24: L2/L3 2km Cal/Val Tide + MSS Uncertainty Estimates

Jun ‘24: L2/L3 2km Cal/Val Tide + MSS Full Corrections

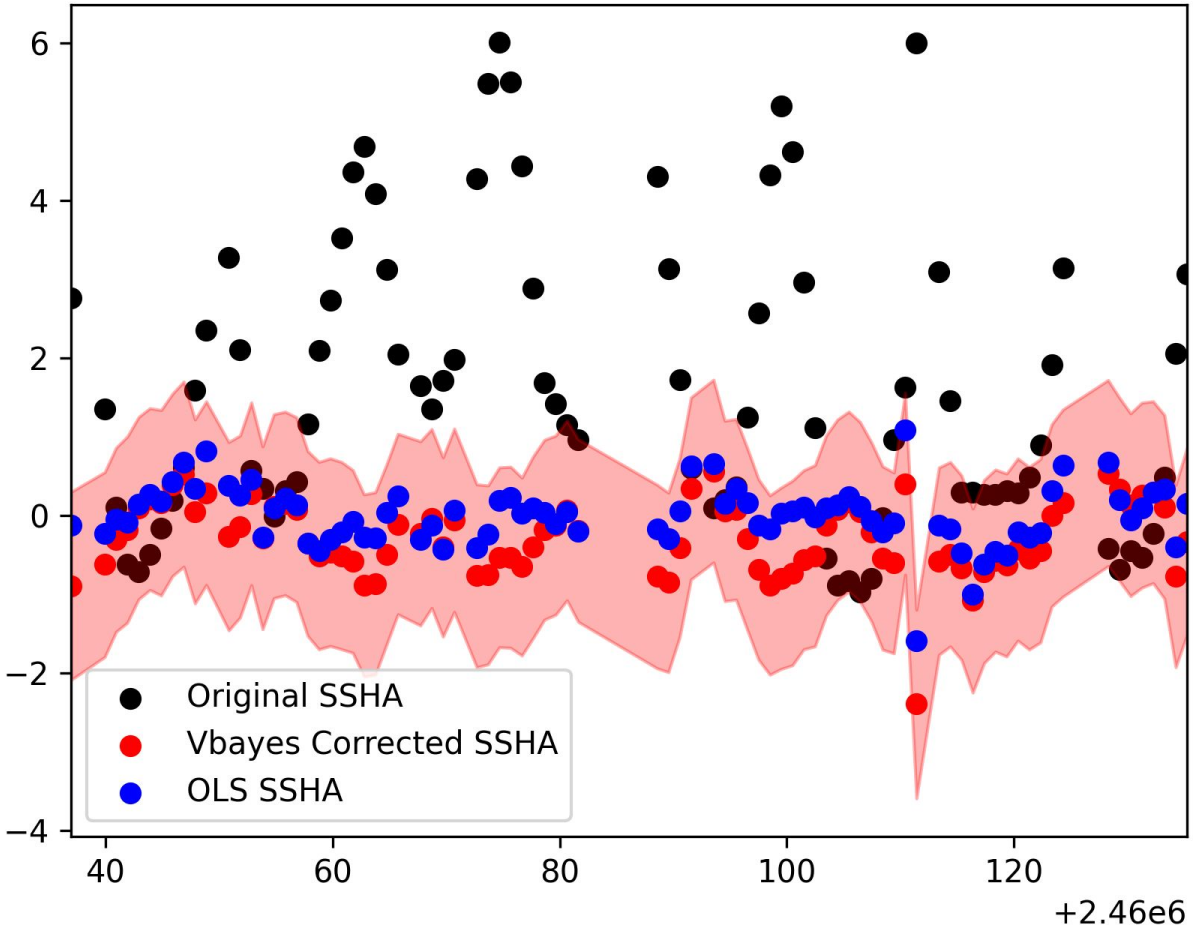
Jul ‘24: L2 250m Cal/Val Tide + MSS Corrections + Uncertainty

Thank you!

thomas.monahan@eng.ox.ac.uk

DEPARTMENT OF ENGINEERING SCIENCE

UNIVERSITY OF OXFORD


6
4
2
0
−2
−4
40
60
80
100
120
+2.46e6
Original SSHA
Vbayes Corrected SSHA
OLS SSHA