Support slides - Machine learning synthesis and inversion method for Stokes parameters in the solar context

Juan Esteban Agudelo[1], Germain Nicolás Morales[1], Santiago Vargas[1] and Sergiy Shelyag[2]

National University of Colombia[1], Flinders University[2]

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MHD - Continuity equation

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \,. \tag{1}$$

where ρ is the mass density y $\vec{v}$ velocity field of the fluid.

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MHD - Motion equation

- Motion Equation

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} P + \vec{\nabla} \cdot \overleftrightarrow{\sigma} + \vec{J} \times \vec{B} + \vec{f} \tag{2}$$

where
- P is the pressure of the fluid.
- $\overleftrightarrow{\sigma}$ is the viscosity stress tensor.
- $\vec{J}$ is the electric currents density and $\vec{B}$ is the magnetic field, so that the term $\vec{J} \times \vec{B}$ represents the Lorentz force (There is not an electric force term, $\rho_q \vec{E}$, because, in the case of non-relativistic fluids, this one has negligible magnitude).
- $\vec{f}$ represents other present volumetric forces. For example, one could include the gravitational force (per unit volume), $\rho \vec{g} = -\rho \vec{\nabla} \phi$.

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MHD - Energy equation

- Energy equation.

$$\rho \frac{de}{dt} = -P\vec{\nabla} \cdot \vec{v} - \vec{\nabla} \cdot \vec{q} + \vec{f} \cdot \vec{v} - \vec{\nabla} \cdot \vec{F}_{rad} + Q^{(vis)} + \eta_e J^2 \tag{3}$$

where
- e is the inner energy of the fluid.
- $-P\vec{\nabla} \cdot \vec{v}$ represents the work exerted by pressure.
- $-\vec{\nabla} \cdot \vec{q}$ is the ratio of thermal energy transported inside the gas (by the random movement of particles)
- $\vec{f} \cdot \vec{v}$ is the work exerted by the volumetric force $\vec{f}$
- $-\vec{\nabla} \cdot \vec{F}_{rad}$ is the ratio of loss of radiation energy by emission (or augmented by absorption)
- $Q^{(vis)}$ is the viscosity heating (volumetric) ratio.
- $\eta_e J^2$ is the ratio of ohmic heating (volumetric). The function η_e es the electrical resistivity of the fluid.

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MHD - Induction equation

- Induction equation.

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \left(\vec{v} \times \vec{B} - \frac{\eta_e}{\mu_0} \vec{\nabla} \times \vec{B} \right)$$
$$\vec{\nabla} \cdot \vec{B} = 0$$

with η_e the electrical resistivity of the fluid and μ_0 the void magnetic permitivity.

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MHD - Closure equations

- Radiative flux equation

$$\vec{F}_{rad} = \vec{F}_{rad}(\rho, P). \tag{4}$$

- Heat conductive flux equation

$$\vec{q} = \vec{q}(\rho, P, \vec{B}). \tag{5}$$

- Stress tensor equation.

$$\sigma_{ij} = 2\eta\tau_{ij} \tag{6}$$

with

$$\tau_{ij} = \frac{1}{2}\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v_k \delta_{ij}\right) \tag{7}$$

where $\eta = \eta(\rho, P)$ is a property of the medium named *dynamic viscosity coefficient*.

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MHD - Closure equations

- State equation between density and pressure.

$$P = P(\rho). \tag{8}$$

The most common state equation is the relation of ideal gases, which allow us to model pretty well the fluids in the astrophysical context. This equation is

$$P = \frac{k_B T}{\mu m_H}\rho, \tag{9}$$

where
- k_B is the Boltzmann constant.
- T is the gas temperature.
- $m_H \sim m_P$ is the Hydrogen atom mass (which can be aproximated to that of the proton).
- μ is the average molecular weight.

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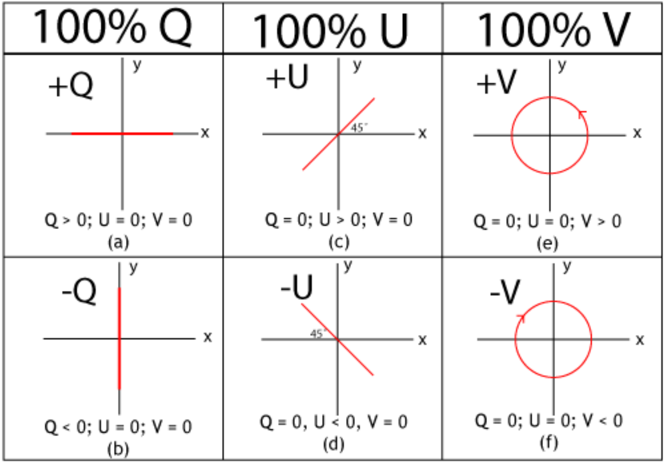
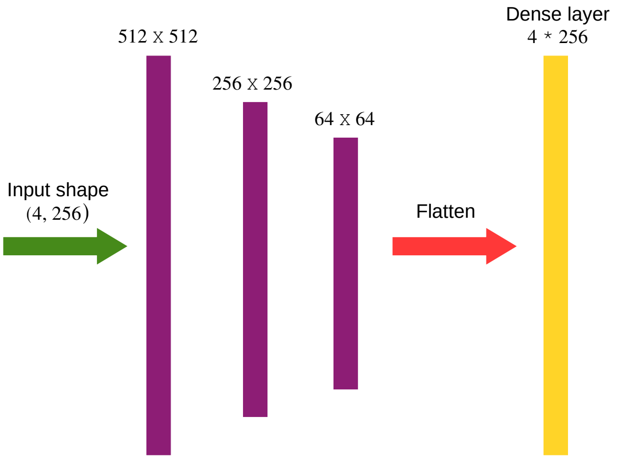
Stokes parameters representation



Neural network structure - Stokes



Neural network structure - Atmosphere magnitudes