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On coherent vortical structures in wave breaking

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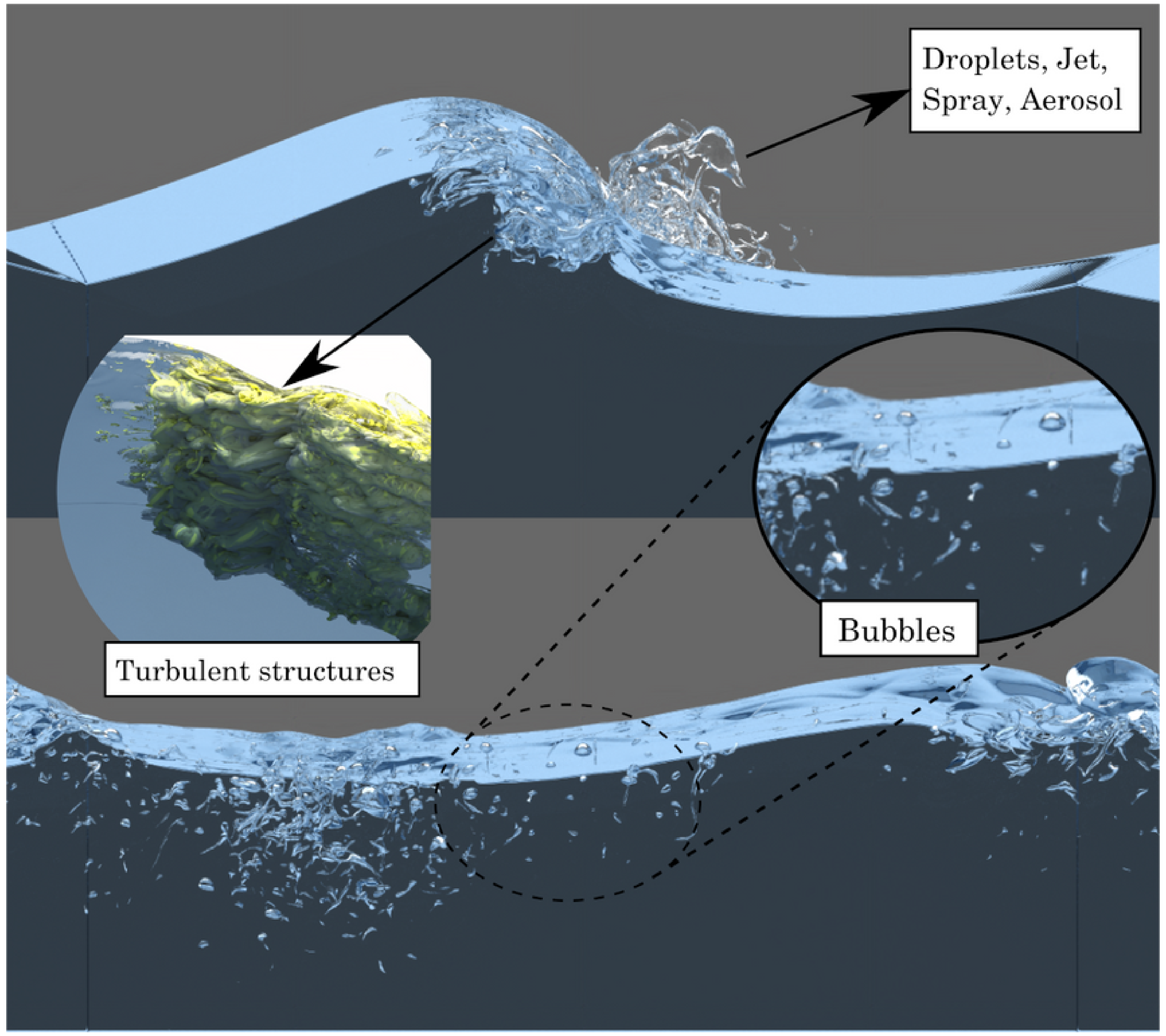
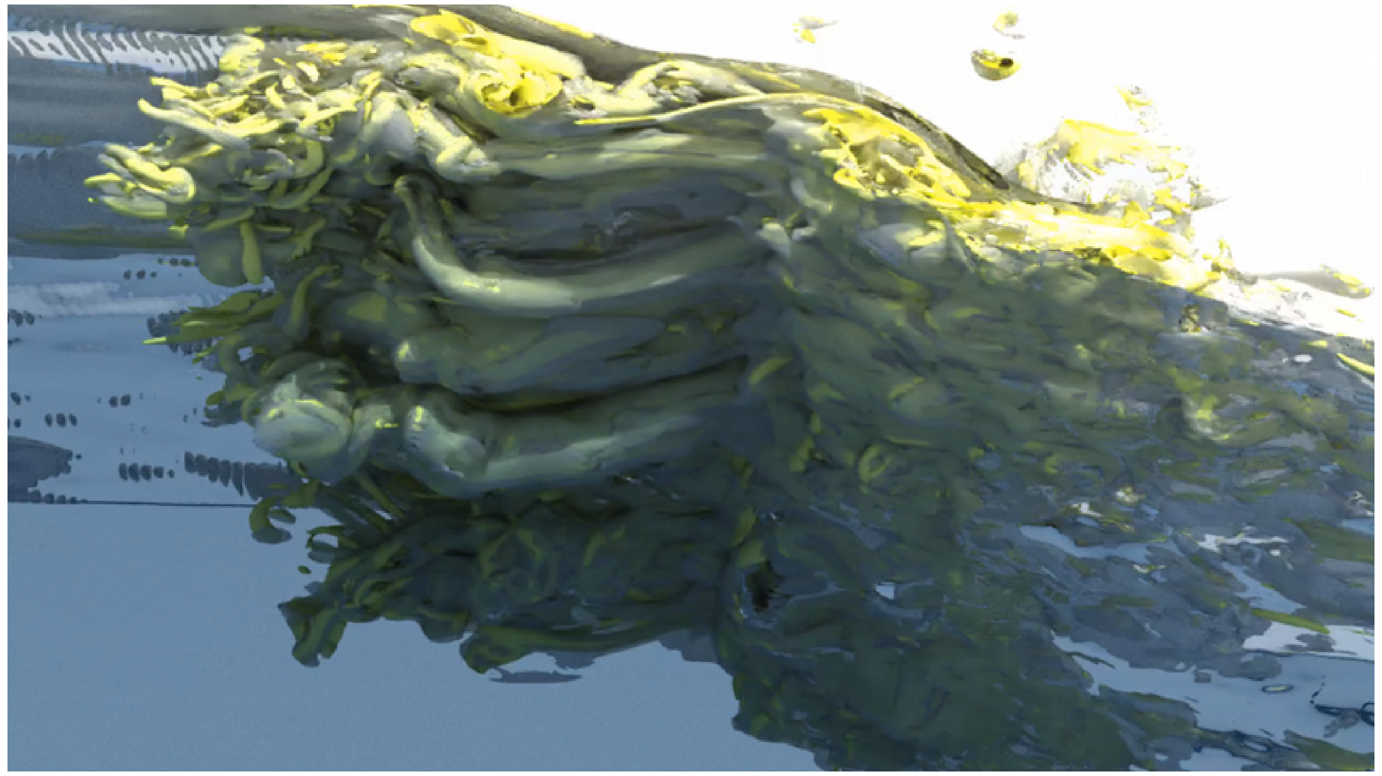
Breaking Waves

- Wave breaking is responsible for **energy dissipation**, and enhancement of momentum, heat, and gas transfer between air and water;
- Bubble fragmentation and **turbulence processes** making the breaking a challenging **multi-scale problem**.





Motivation

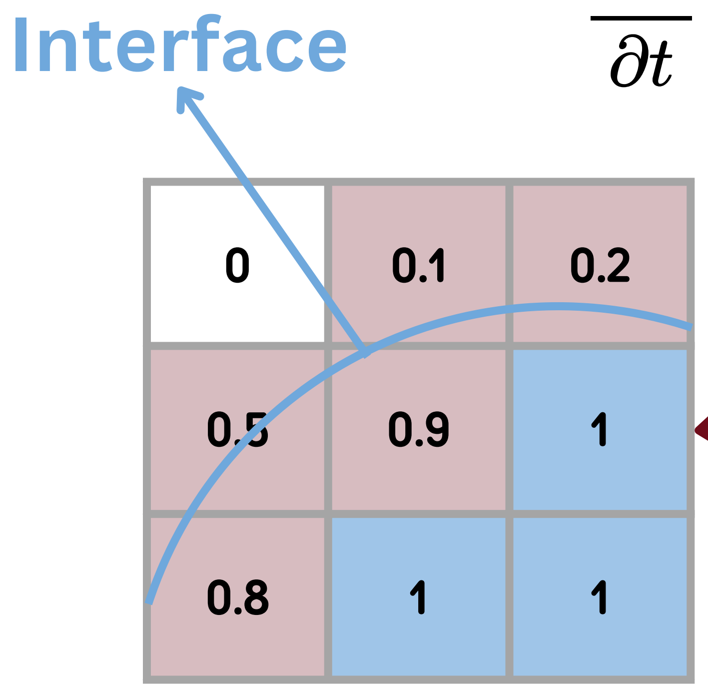
- Deeply understand the mechanism behind **energy dissipation** through **vortical structures** analysis.

Methodology - Volume Of Fluid, Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = \frac{1}{\rho}\left[-\nabla p + \frac{1}{Re}\nabla \cdot \left(\mu\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)\right) + \frac{1}{We}\mathbf{f}_\sigma\right] - \frac{1}{Fr}\mathbf{j}$$

Hydrodynamics



$$\frac{\partial \chi}{\partial t} + \mathbf{u} \cdot \nabla \chi = 0$$

$\chi \in [0, 1]$

Volume Fraction

Fluid properties

density $\rho = \rho_1 \chi + (1 - \chi)\rho_2$

viscosity $\mu = \mu_1 \chi + (1 - \chi)\mu_2$

Dimensionless numbers

$$Re = \frac{\tilde{\rho}\tilde{U}\tilde{L}}{\tilde{\mu}} \quad We = \frac{\tilde{\rho}\tilde{U}^2\tilde{L}}{\sigma}$$

$$Fr = \frac{\tilde{U}^2}{g\tilde{L}}$$

Surface tension

$$\mathbf{f}_\sigma = k\delta\left(\mathbf{x} - \mathbf{x}_s\right)\mathbf{n}$$

interface normal

curvature

Dirac delta function

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Simulation conditions

Steep wave in periodic domain (Third-order Stokes wave)

- **Initial wave profile as in Chen et al. (1999), Iafrati (2009), Deike et al. (2015, 2016), Wang et al. (2016)…**

Interface profile

$$\eta(x,z) = \frac{\varepsilon}{2\pi}\left(\cos\big(k(x - r(z)\Delta x)\big) + \frac{\varepsilon}{2}\cos(2k(x - r(z)\Delta x) + \frac{3}{8}\varepsilon^2\cos(3k(x - r(z)\Delta x)\right)$$

$r(z) \in (0,1)$

Random

Velocity field

$$u(x,y,z) = \Omega\frac{\varepsilon}{k}\exp(ky)\cos\big(k(x - r(z)\Delta x)\big) \qquad v(x,y,z) = \Omega\frac{\varepsilon}{k}\exp(ky)\sin\big(k(x - r(z)\Delta x)\big)$$

$$\Omega = \sqrt{gk(1+\varepsilon^2)}$$

- **Surface tension corresponding to 30 cm wavelength**

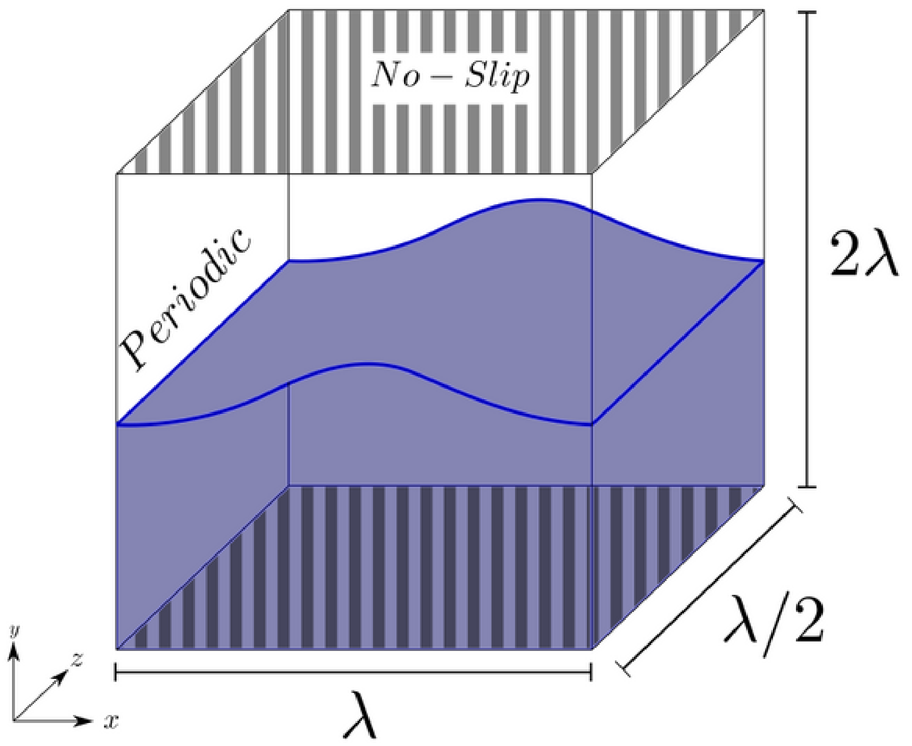
$\epsilon = 0.5$

$$We = \frac{\rho_w U_r^2 \lambda}{\sigma} = \frac{\rho_w g \lambda^2}{\sigma} = 12262.5$$

$$Re = \frac{\rho_w g^{1/2}\lambda^{3/2}}{\mu_w} = 5.1 \times 10^5$$ Real

$\rho_w/\rho_a = 800 \qquad \mu_w/\mu_a = 55$

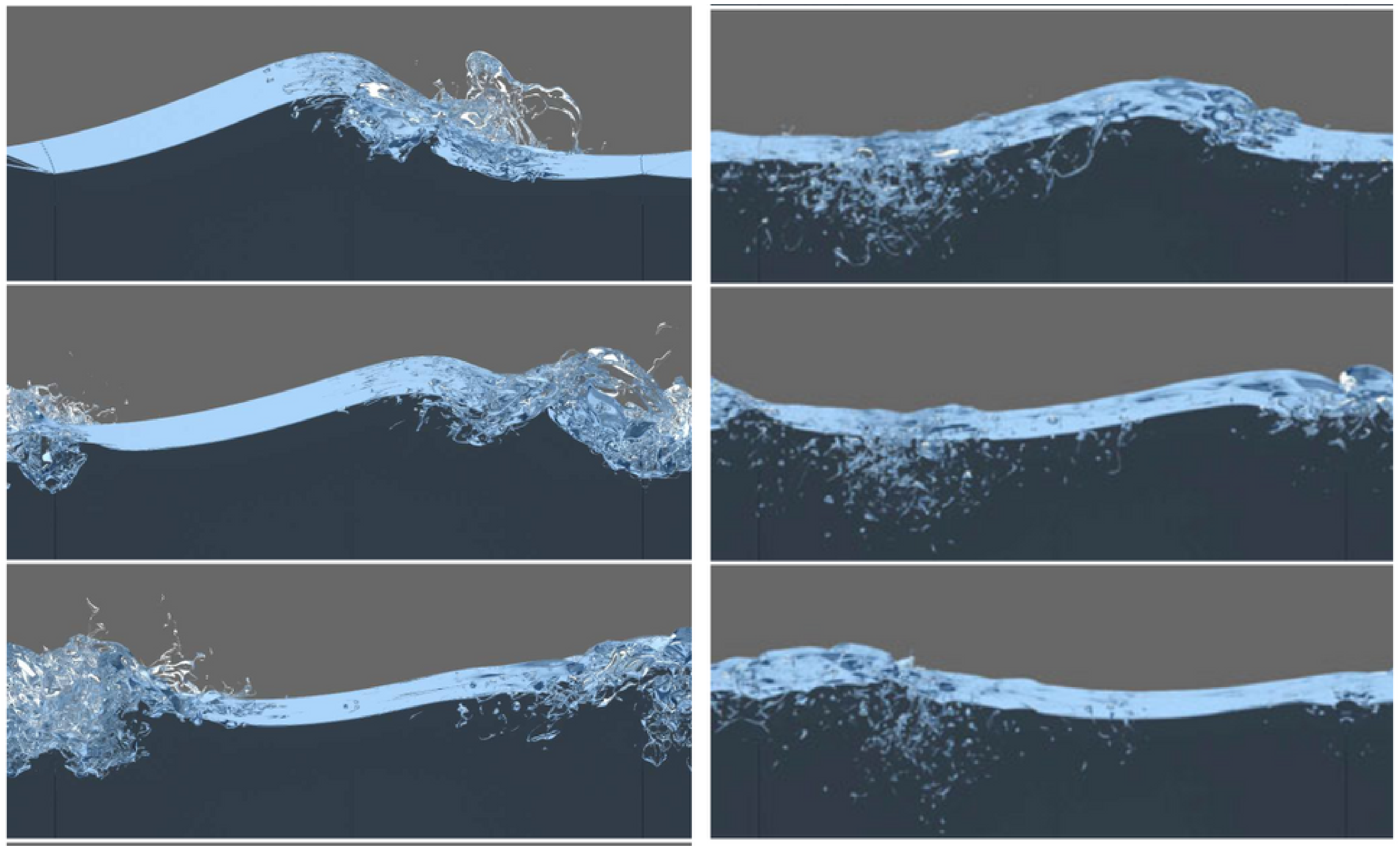
$$Re = 10000 - 40000$$ Simulation



- **Grid points ~ 400'000'000 - > Mesh Resolution ~ 0.3 mm**

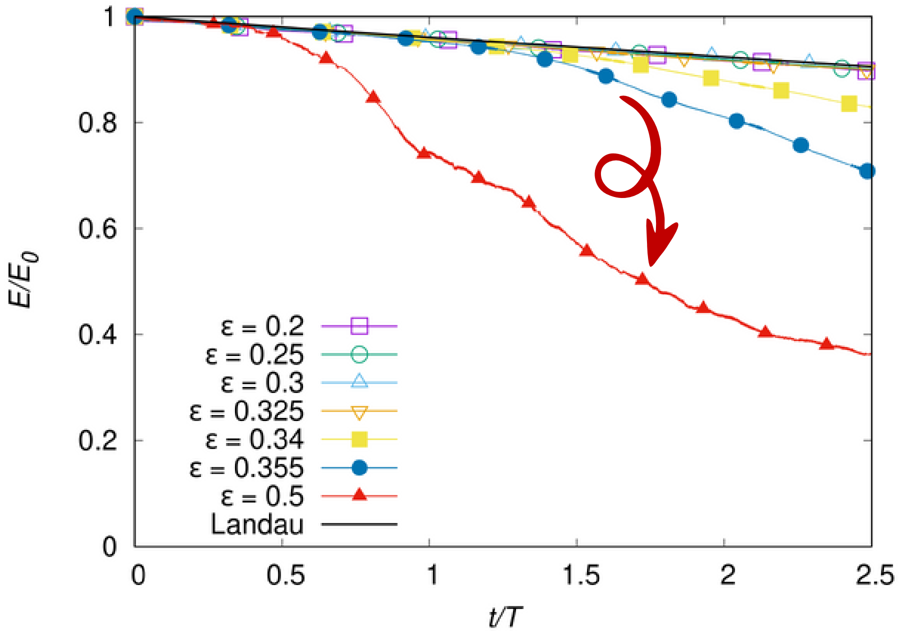
Air entrainment, bubbles fragmentation and turbulent structures

Wave breaking:
- Air entrainment
- Bubbles fragmentation



Generation of vorticity and turbulence

Energy dissipation

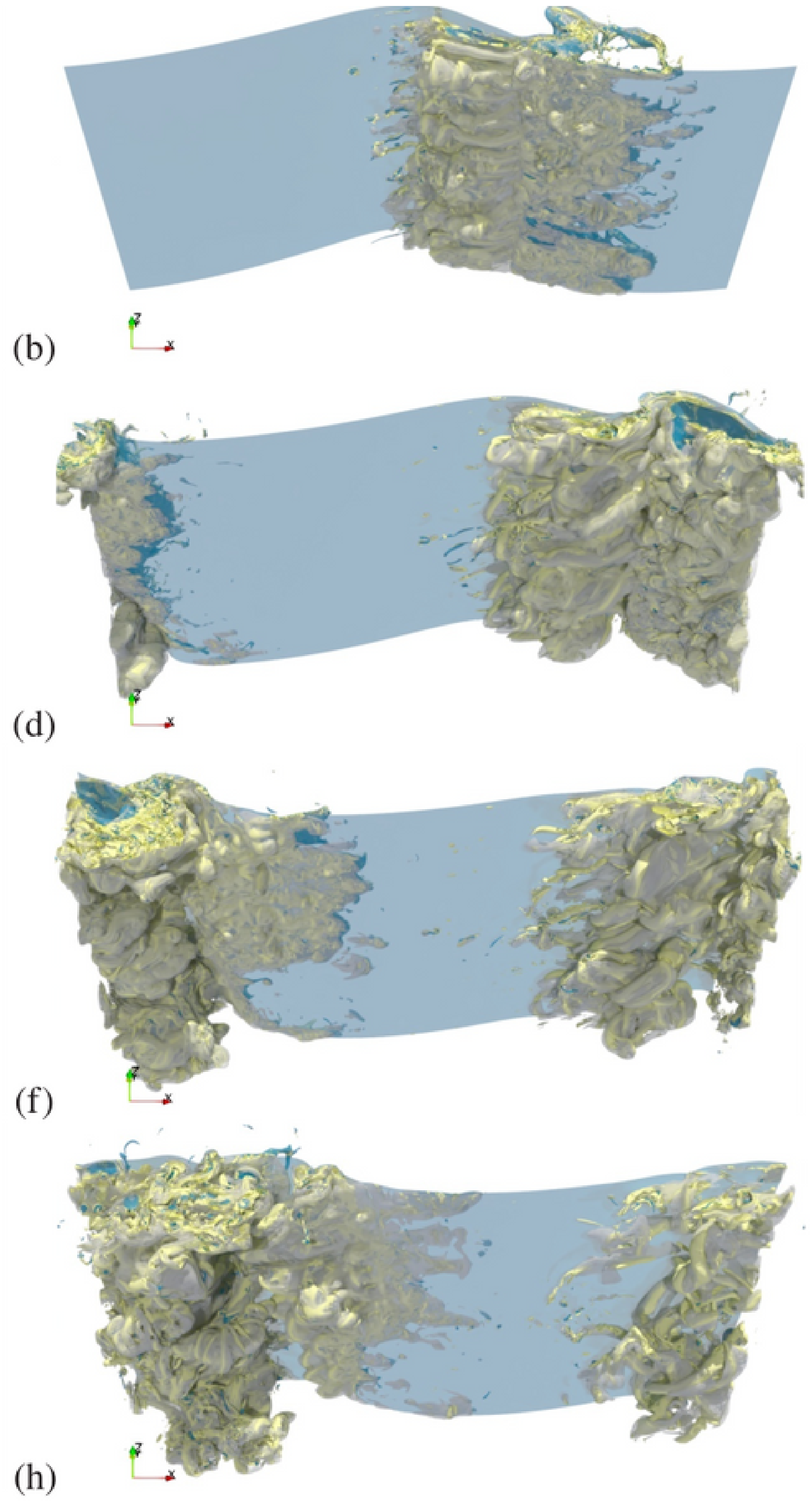


RE=40000

$$\epsilon = 0.5$$

Turbulent structures:
- Vortex Sheet
- Vortex Tubes

Horiuti & Takagi, 2005



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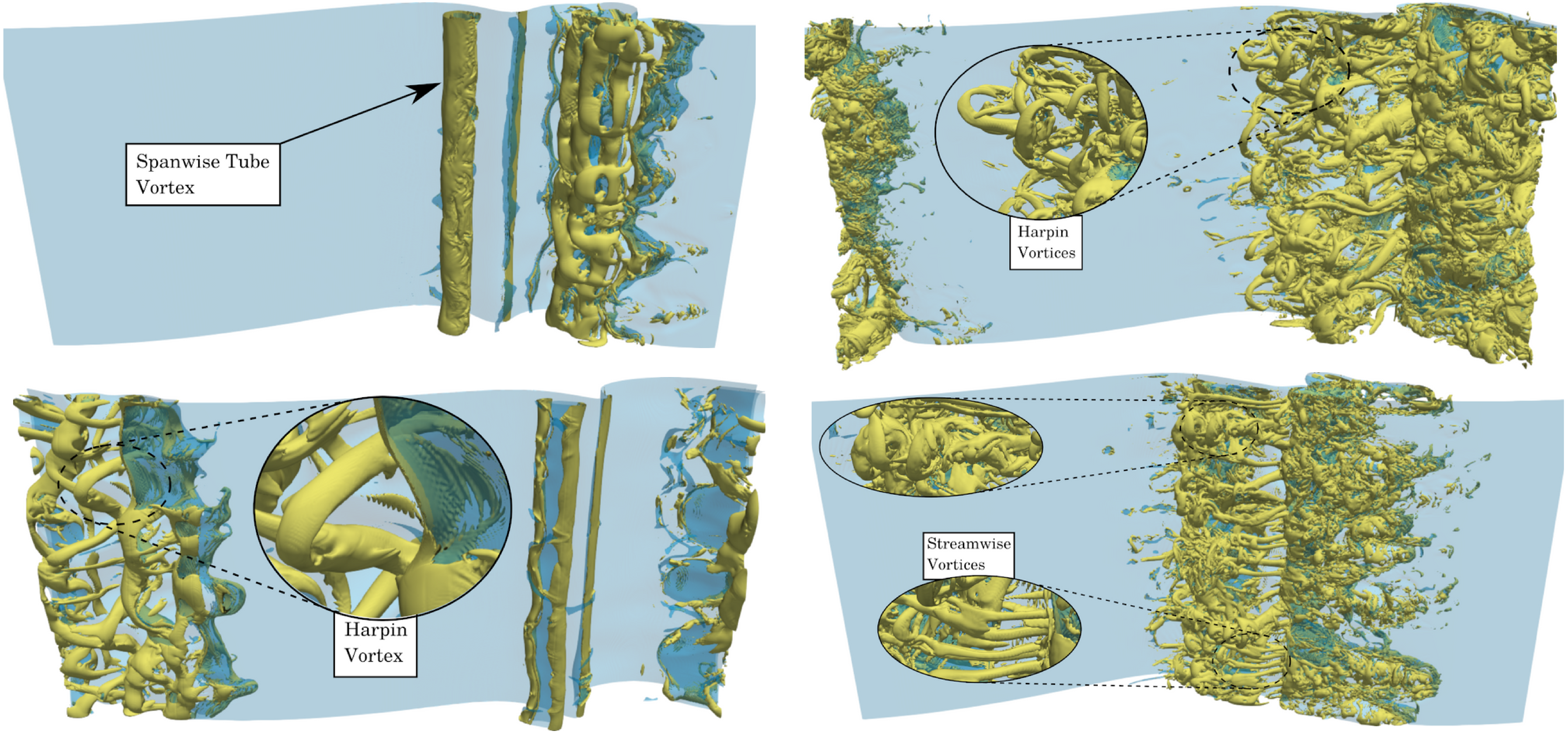
Vortex tubes

t/T = 0.8

t/T = 1.0

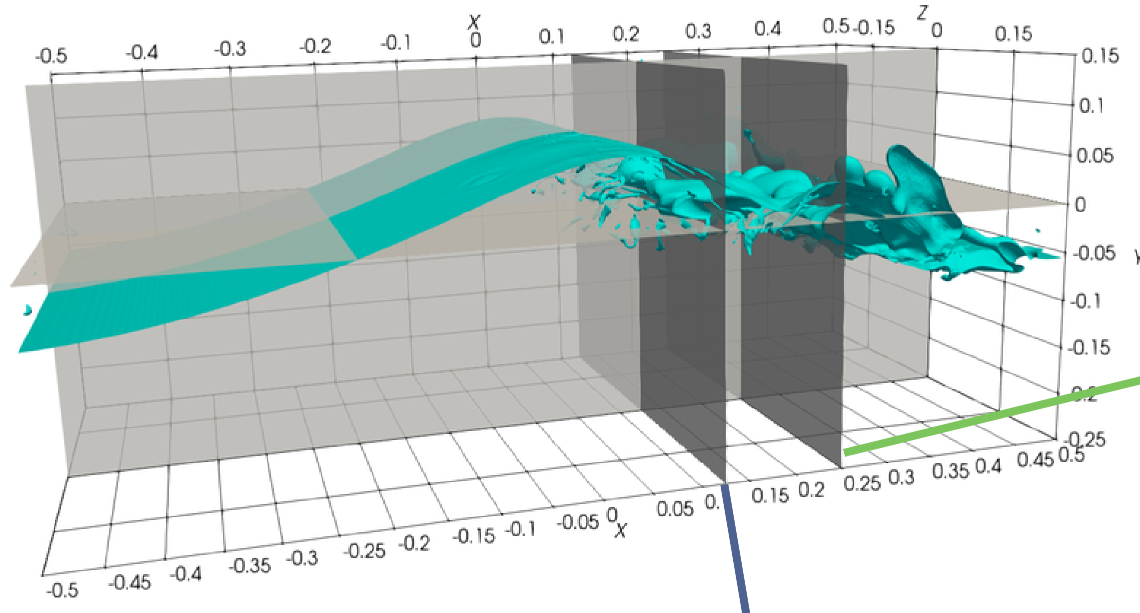


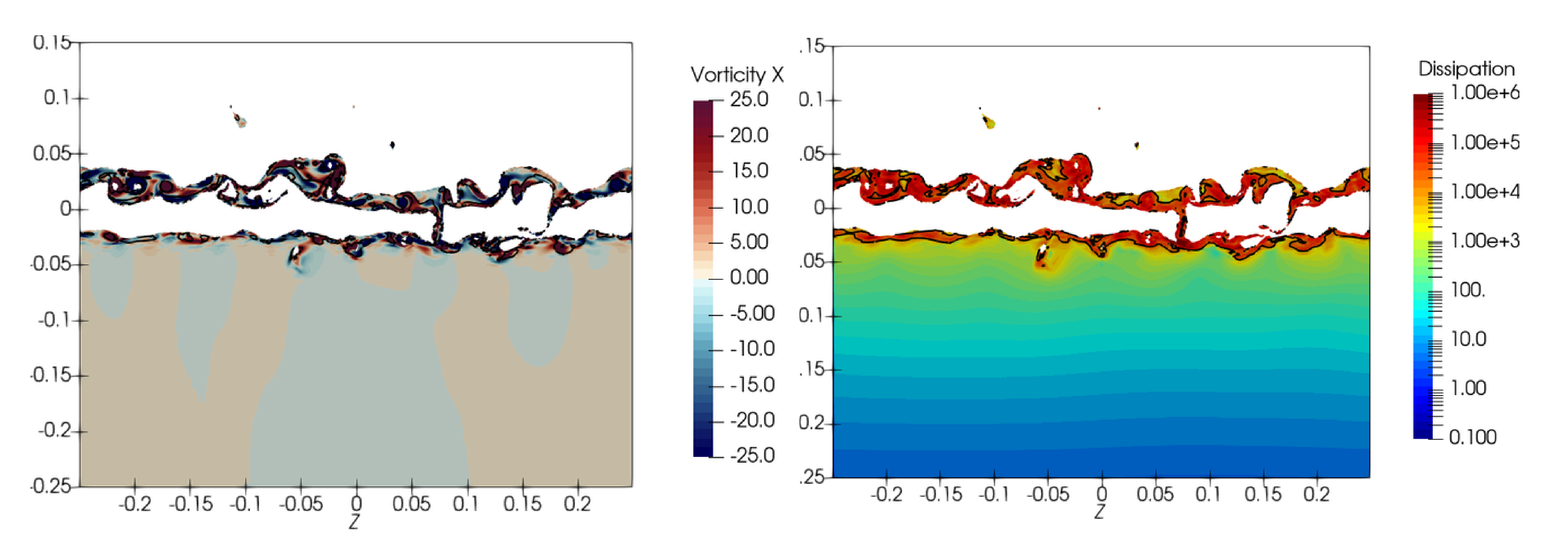
RE=10000

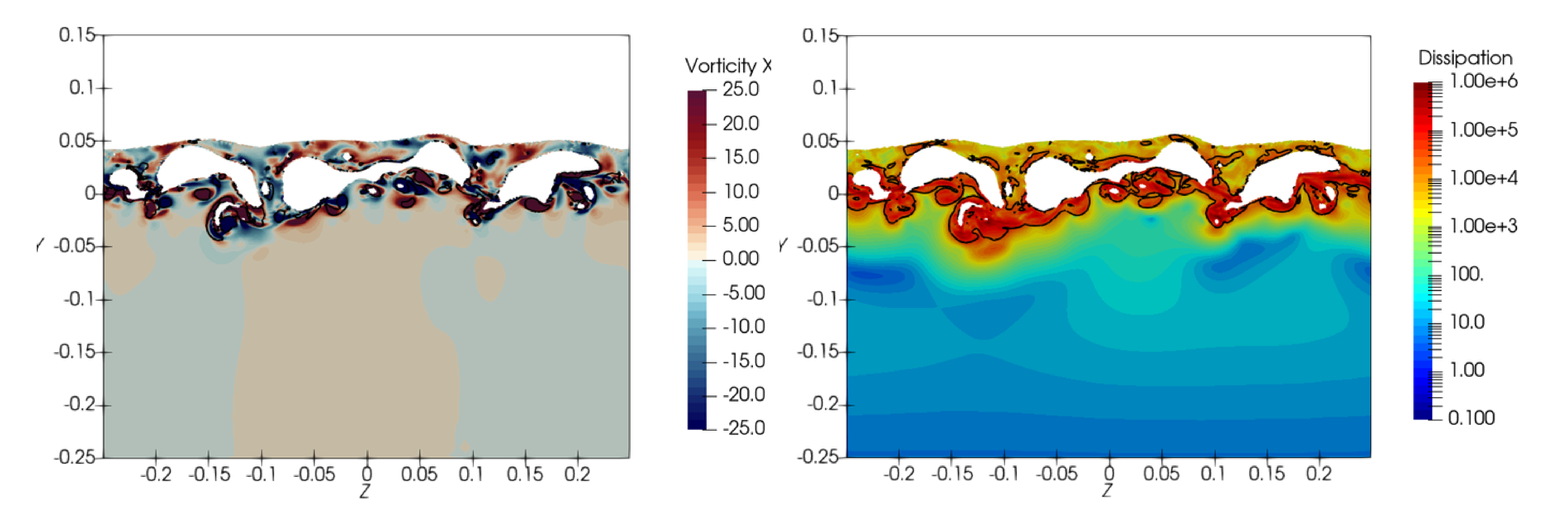
RE=40000

Vortex structures and air entrainment: Re = 40000

t/T = 0.878







Large correlation between vorticity and tubes

Large correlation between viscous dissipation and vortex sheets

Conclusions

- Breaking of a steep wave in a periodic domain
- Focus on energy dissipation and on vortex structures developing during breaking
- Vortex sheets closely correlated with dissipation zones
- Complex dynamics of sheets and tubes

- Deeper investigation of interplay between vortical structures and air entrainment needed
- Quantitative characterization of size and shapes of vortical structures

Di Giorgio S, Pirozzoli S, Iafrati A. On coherent vortical structures in wave breaking. Journal of Fluid Mechanics. 2022;947:A44. doi:10.1017/jfm.2022.674

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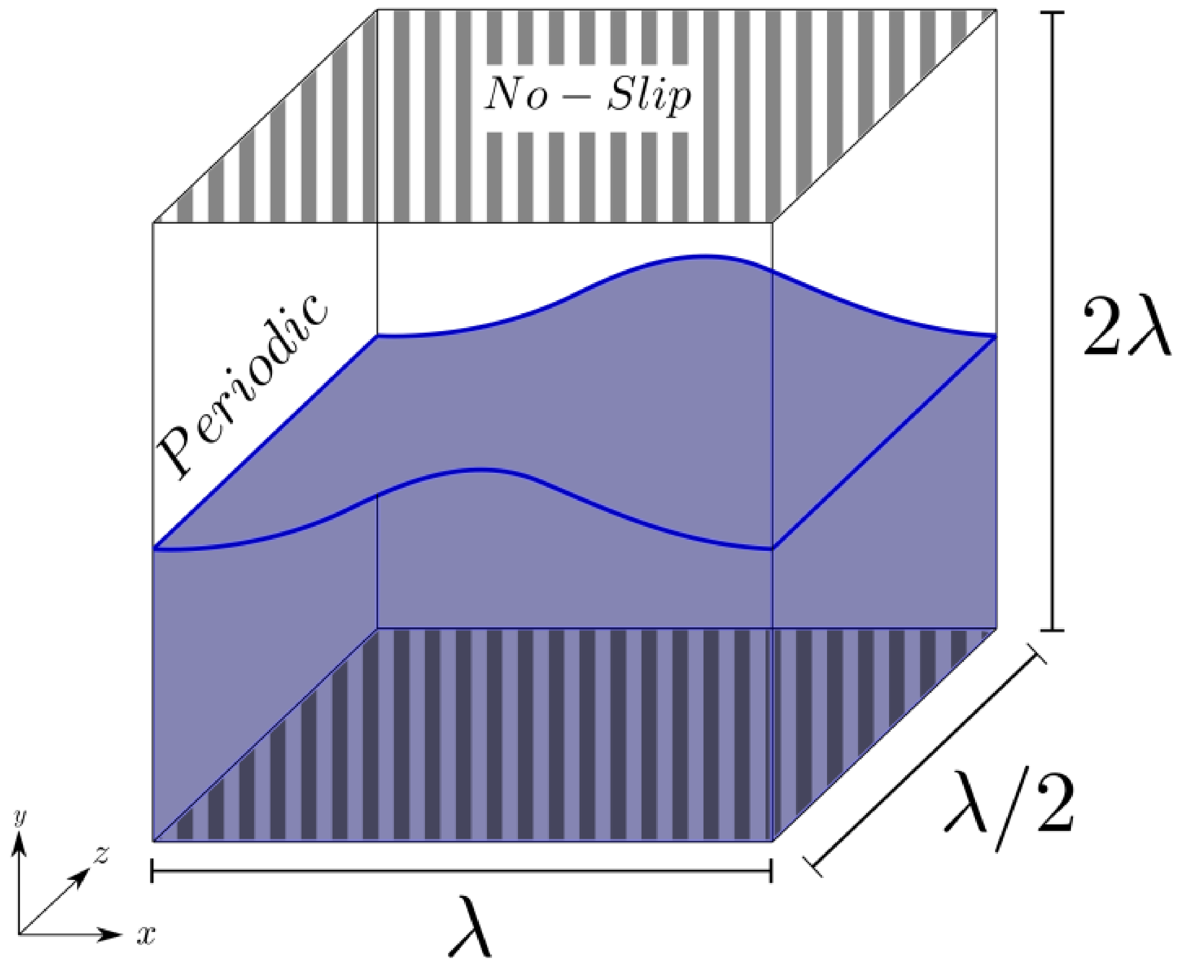
How to investigate the problem

Breaking of a steep periodic wave

- Simulations of the breaking of a steep periodic wave performed at different Reynolds numbers
- Analysis of air entrainment, energy dissipation. Particular focus on the analysis of vortical structures developing during the breaking with an attempt to identify regions where dissipation is mostly located



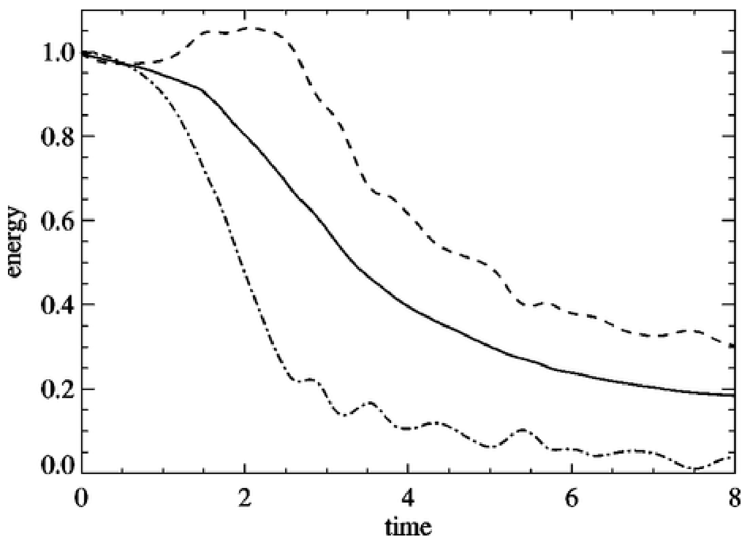
$$Re = \frac{\rho_w \tilde{U} \lambda}{\mu_w} \quad We = \frac{\rho_w \tilde{U}^2 \lambda}{\sigma} \quad Fr = \frac{\tilde{U}^2}{g\lambda} \quad \tilde{U} = \sqrt{g\lambda}$$

Flow case	$N_x \times N_y \times N_z$	$L_x \times L_y \times L_z$	ϵ	Re_w
3D1E4c	$512 \times 384 \times 256$	$1 \times 2 \times \frac{1}{2}$	0.50	10000
3D1E4f	$1024 \times 768 \times 512$	$1 \times 2 \times \frac{1}{2}$	0.50	10000
3D4E4c	$512 \times 384 \times 256$	$1 \times 2 \times \frac{1}{2}$	0.50	40000
3D4E4f	$1024 \times 768 \times 512$	$1 \times 2 \times \frac{1}{2}$	0.50	40000

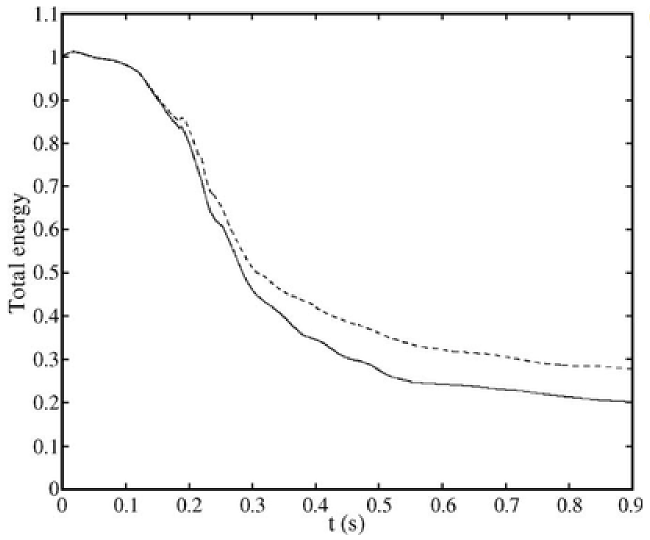
Motivation

Energy dissipation in breaking waves

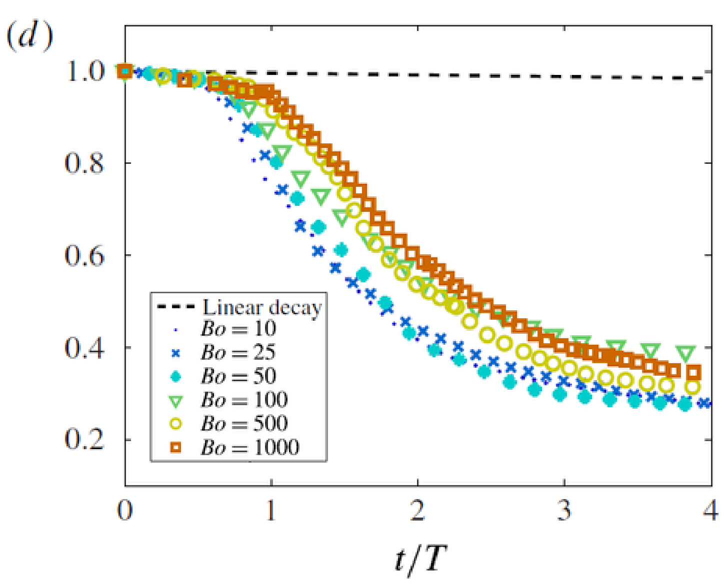
- Many studies investigated the breaking of an artificially steep periodic wave train;
- In the case of plunging breaking, energy fraction dissipated by the breaking seems rather independent of the air/water density ratio, Reynolds number, surface tension, steepness and breaking severity, 2D or 3D computations, and even grid resolution;
- Whereas a lot of studies have been done to investigate the bubble size, not much attention has been paid to understand the dissipation mechanisms of the different cases.



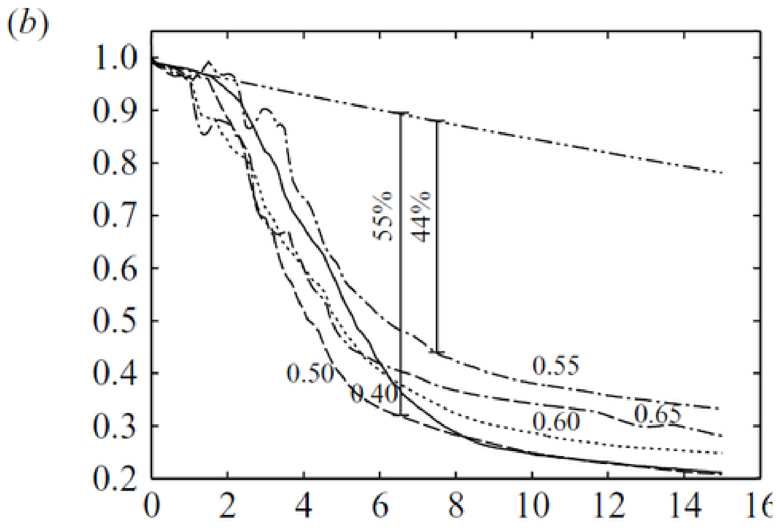
Chen et al., 1999
$\rho_a/\rho_w = 10^{-2}$



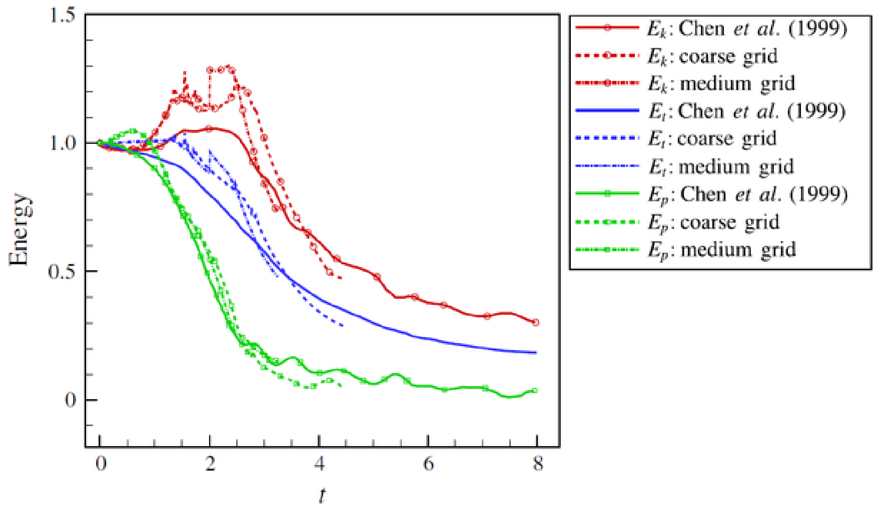
Lubin et al., 2006
2D vs 3D



Deike et al., 2015
Bond number



Iafrati, 2009
Initial steepness

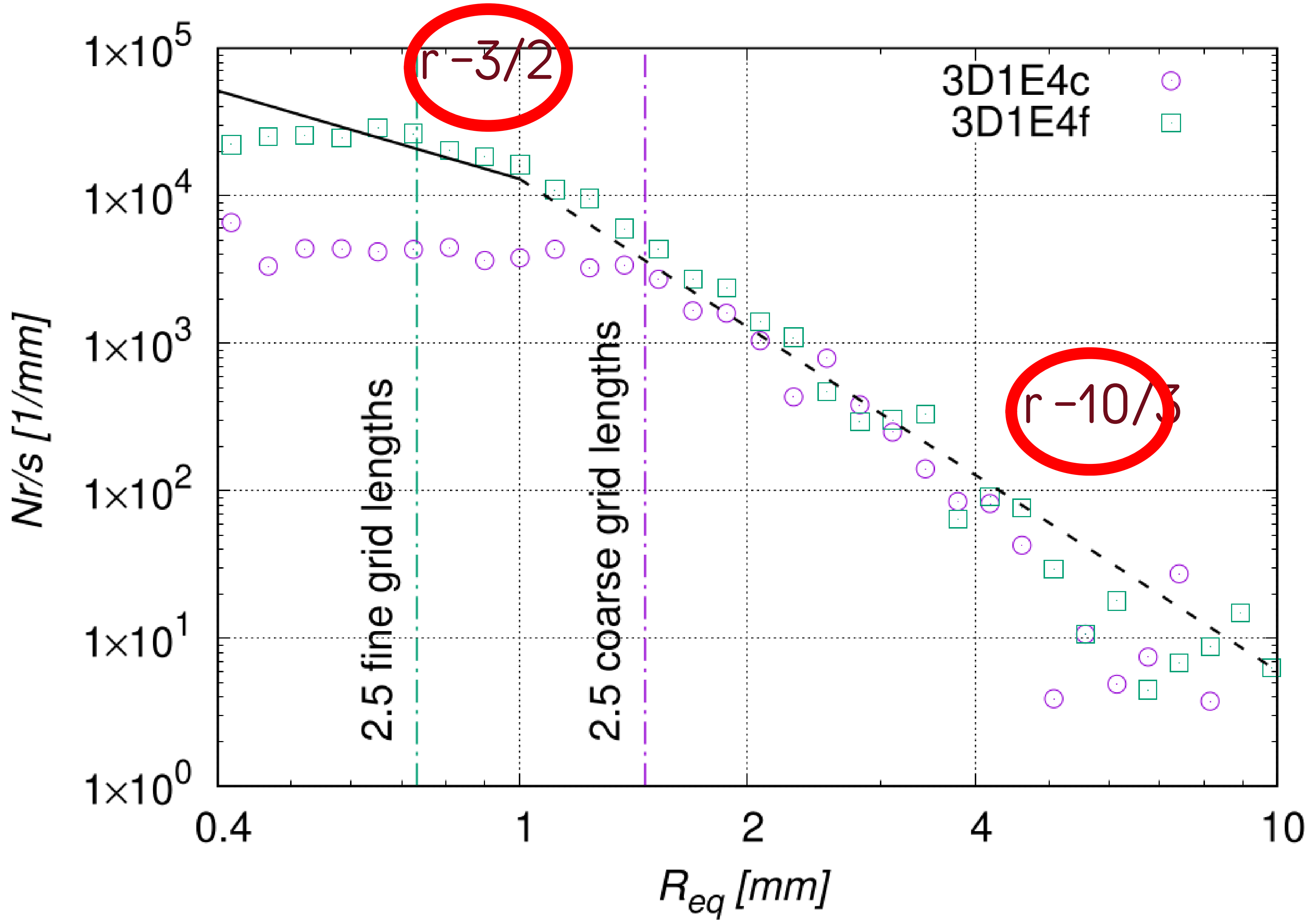


Wang et al., 2016
Grid resolution

- Total energy versus time

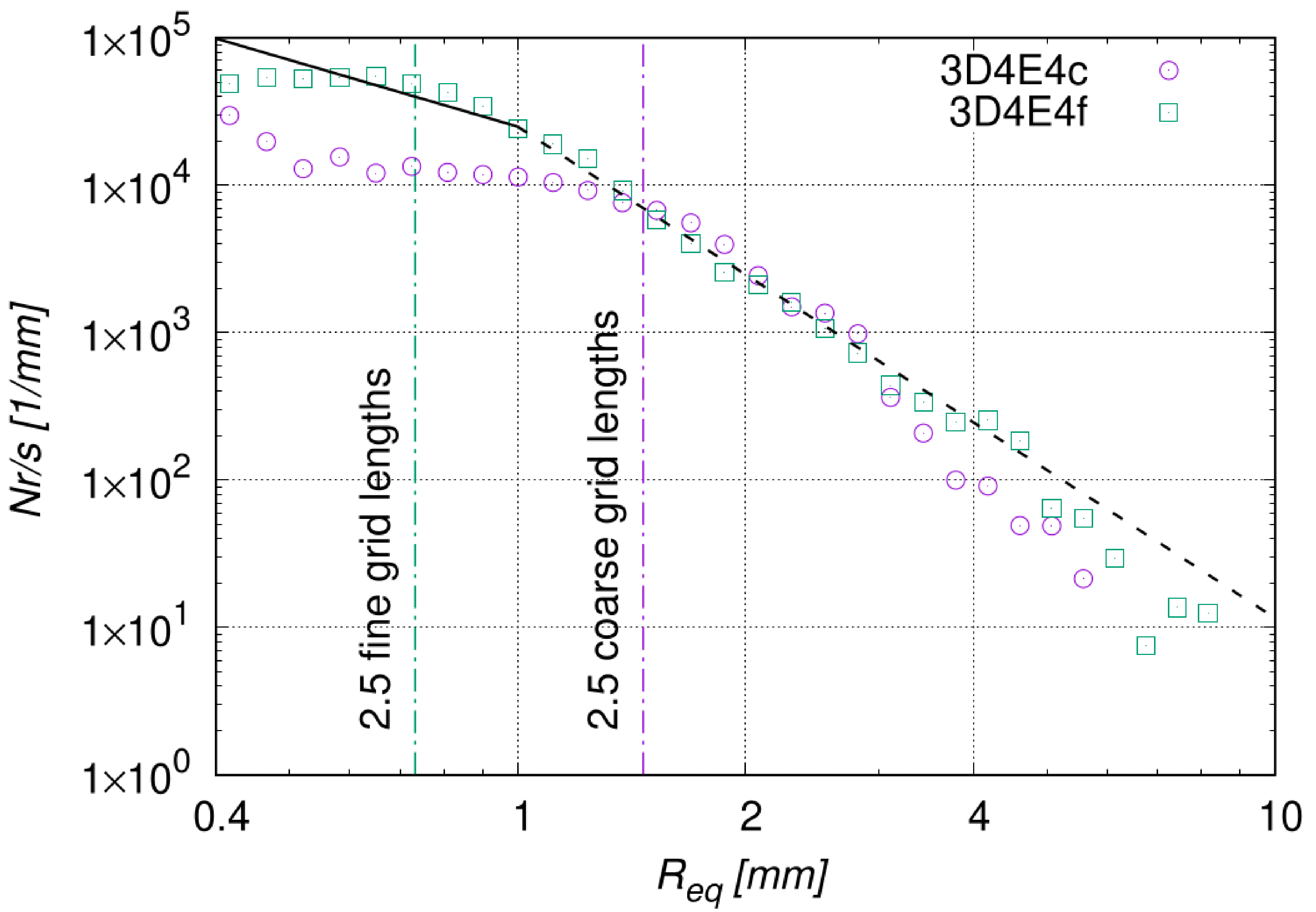
Bubble size distribution

Number of bubbles scaled by bin size and averaged over frames



RE=10000

Hinze scale



RE=40000

- Deane & Stokes 2002
- Hinze 1955

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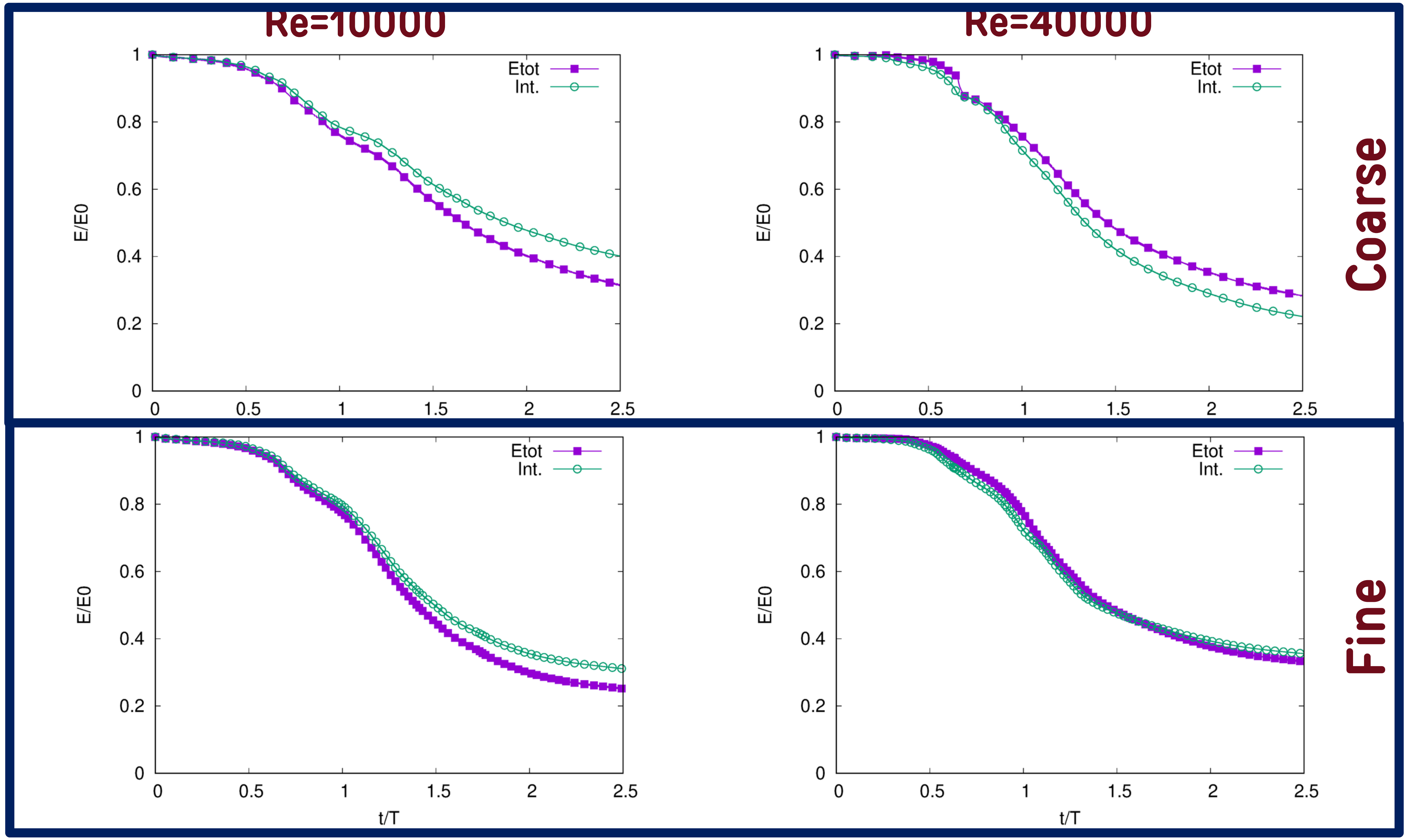
Energy budget

$$\frac{d}{dt}\int_V \rho\frac{u^2}{2}+\rho g y\,\mathrm{dV} = -\oint_{\partial V}\left(p+\rho\frac{u^2}{2}+\rho g y\right)\mathbf{u}\cdot\mathbf{n}\;\mathrm{dS} + \oint_{\partial V}(\Sigma\cdot\mathbf{u})\cdot\mathbf{n}\;\mathrm{dS} - \int_V \boldsymbol{\Sigma}:\nabla\mathbf{u}\;\mathrm{dV} + \int_V \mathbf{u}\cdot\mathbf{f}_\sigma\,\mathrm{dV}$$

LHS

zero for periodic BC

RHS



- Balance of the integrals of LHS and RHS
- Quantify numerical error
- MAC scheme: lower numerical error

Numerical model

Two-Fluids Navier-Stokes solver

- **Incompressible N-S equations** solver via a classical Fractional Step approach.
- Adams-Bashforth used for explicit terms, C-N for the diffusive part.
- Staggered layout, second-order discretization with energy preserving properties (MAC)

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -\left(\frac{3}{2}\mathbf{N}_h^n - \frac{1}{2}\mathbf{N}_h^{n-1}\right) + \frac{1}{2\rho^{n+\frac{1}{2}} Re}\left(\mathbf{D}_h^n + \mathbf{D}_h^*\right) + \frac{1}{\rho^{n+\frac{1}{2}}}\mathbf{f}^n \qquad \frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho^{n+\frac{1}{2}}}\nabla_h p$$

Poisson Equation

- Solved via HYPRE library. Either geometric multigrid (PFMG or SMG) or Krylov methods (BiCGStab or GMRES) are employed

$$\nabla_h \cdot \left(\frac{1}{\rho^{n+\frac{1}{2}}}\nabla_h p\right) = \frac{1}{\Delta t}\nabla_h \cdot \mathbf{u}^*$$

Numerical model

Algebraic VOF

- Interface reconstructed via an algebraic TVD-VOF method based on a extra-bee limiter (Pirozzoli et al., 2019)

$$C_i^{n+1} = C_i^n - \frac{1}{\Delta x_i}\left(\hat{f}_{i+1/2} - \hat{f}_{i-1/2}\right) \qquad \hat{f}_{i+1/2} = u_{i+1/2}\left(C_I^n + \frac{\nu}{2}(1-\sigma_{i+1/2})\varphi(\theta_{i+1/2})\delta C_{i+1/2}\right)$$

$$\varphi_{EB} = \max\left(0, \min\left(\frac{2}{1-\sigma_{i+1/2}}, \frac{2\theta_{i+1/2}}{\sigma_{i+1/2}}, 2 + s(\theta_{i+1/2} - 1)\right)\right)$$

Pirozzoli et al., 2019

Surface Tension

- **Classical Height function method (e.g. Francois et al, 2006; Hernández et al. 2008; López & Hernández 2010), wherever it converges**
- **Least-square, finite-difference, method when necessary**

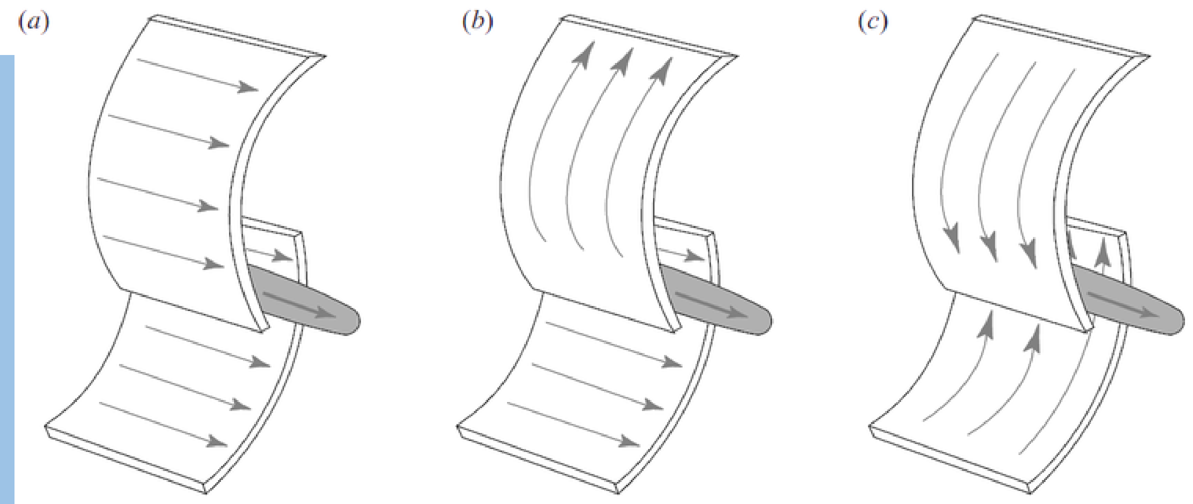
$$k = \frac{\nabla\tilde{C} * H(\tilde{C}) * \nabla\tilde{C}^T - |\nabla\tilde{C}|^2 Trace\,(H(\tilde{C}))}{2|\nabla\tilde{C}|^3}$$

Goldman, 2005

Vortex structures: sheets and tubes

- Various vortical structures can be identified in turbulent flows. Basically, we may have tubular or filamentary structures, which are the *tubes*, and structures with nonfilamentary, flat, vorticity distribution, named as *sheets* (Horiuti & Takagi, 2005).
- In tubes the vorticity predominates the strain rate, whereas in the sheets strain rate and vorticity are comparably large and correlated (Horiuti & Fujisawa, 2008).
- Since vortex tubes are mostly responsible for intermittency, they are rather popular and several methods have been developed to identify them, generally based on the invariants of the velocity gradient tensor
- DNS of isotropic turbulence have shown that tubular structures of strong axial vorticity, are responsible for only a negligible part of the energy dissipation (Jiménez & Wray, 1998) most of it being located about the spiraling vortex sheets (Kuwahara, 2005)

- In contrast, vortex sheets, consisting of zones lo locally nearly two-dimensional shearing motion, provide a dominant contribution to the enstrophy production through vortex stretching and to energy dissipation (Pirozzoli et al., 2010)
- Tubes and sheet are related: tubes often formed as Kelvin-Helmholtz instability of the vortex sheet. Tubes may also form by multiple vortex sheets forming a recirculating flow with a pressure minimum



Horiuti & Fujisawa, 2008

Vortex structures

Vortex tubes identification

Connected regions where $\nabla \mathbf{u} = \frac{\partial u_i}{\partial x_j}$ has one real eigenvalue, λ_r (straining motion), and two complex conjugate eigenvalues, $\lambda_c^{\pm} = \lambda_{cr} \pm i\, \lambda_{ci}$ (spiralling motion).

Tubes identified as regions with $\omega_t = 2\lambda_{ci}$ equal to a given value

Vortex sheets identification

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Strain rate

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

Vorticity

$$A_{ij} = \left(S_{ik}\Omega_{kj} + S_{jk}\Omega_{ki}\right)$$

Symmetric 2nd order velocity gradient

Connected regions where the largest eigenvalue (λ_A) of A_{ij} (discarding the one associated with the eigenvector that is most aligned with the vorticity vector) is positive. For a two-dimensional parallel flow (i.e. pure shear), λ_A is proportional to the square of the vorticity modulus.

Sheets identified as regions with $\omega_s = \sqrt{2\lambda_A}$ equal to given value

Vertical profile of vortex structures and dissipation

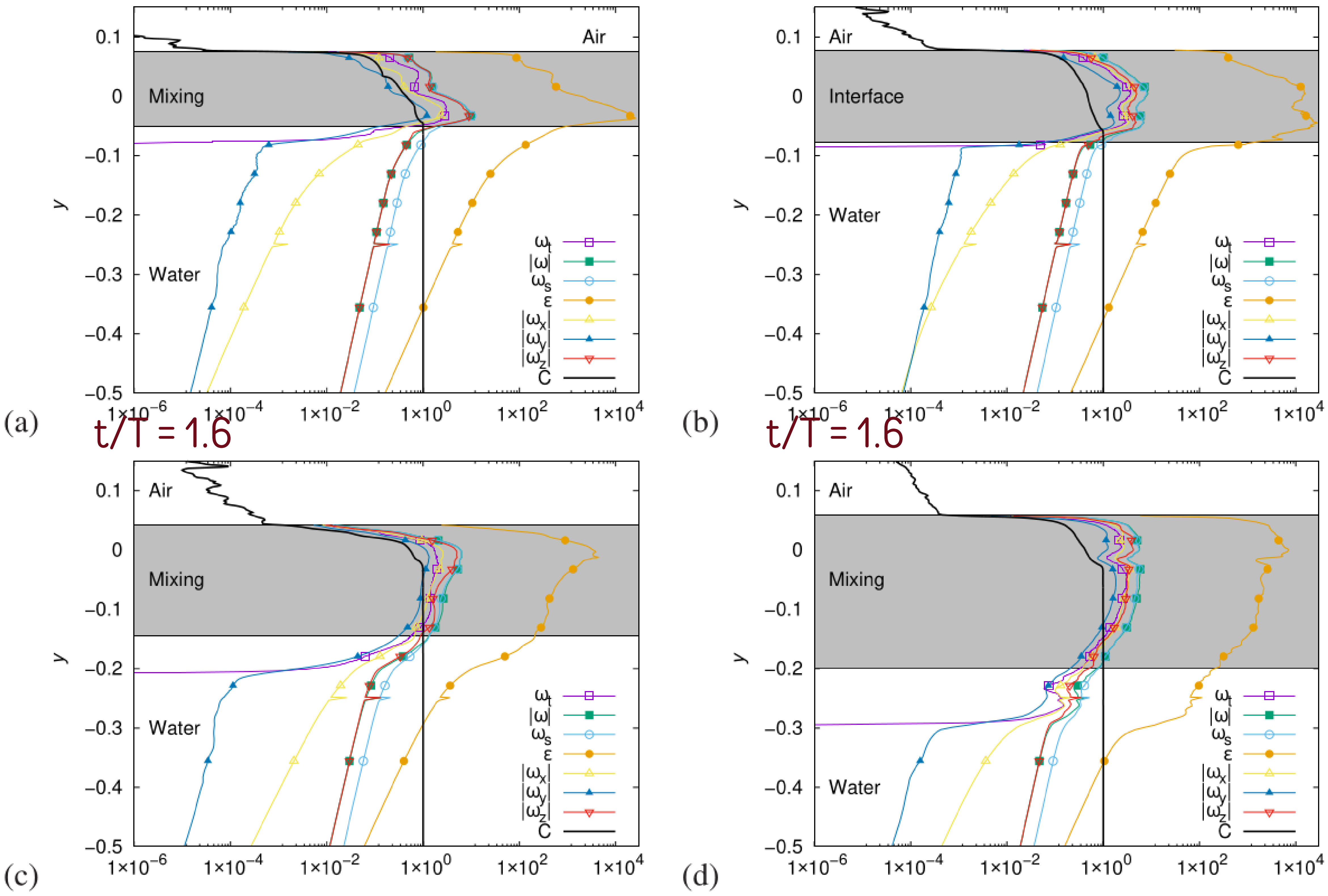
Horizontal averages of the vorticity components and energy dissipation

t/T = 0.938

t/T = 0.878

t/T = 1.6

t/T = 1.6



Tubes

Sheets

Dissipation

0.001 < C < 0.999

T

t decay quite sharply beneath the mixing region whereas s follows the viscous dissipation

RE=10000

RE=40000

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Di Giorgio et al., JFM 2022

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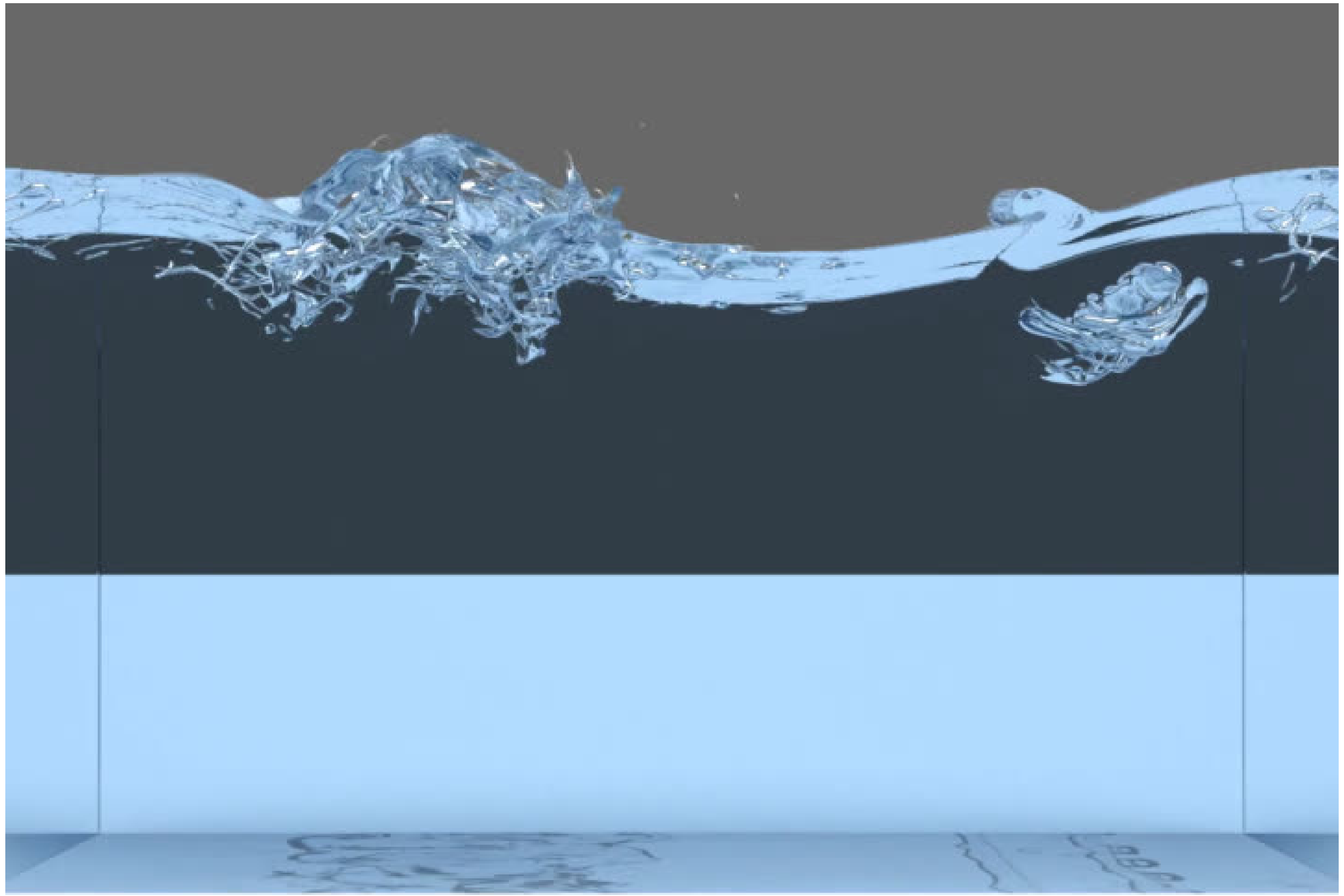
Air entrainment and bubble fragmentation



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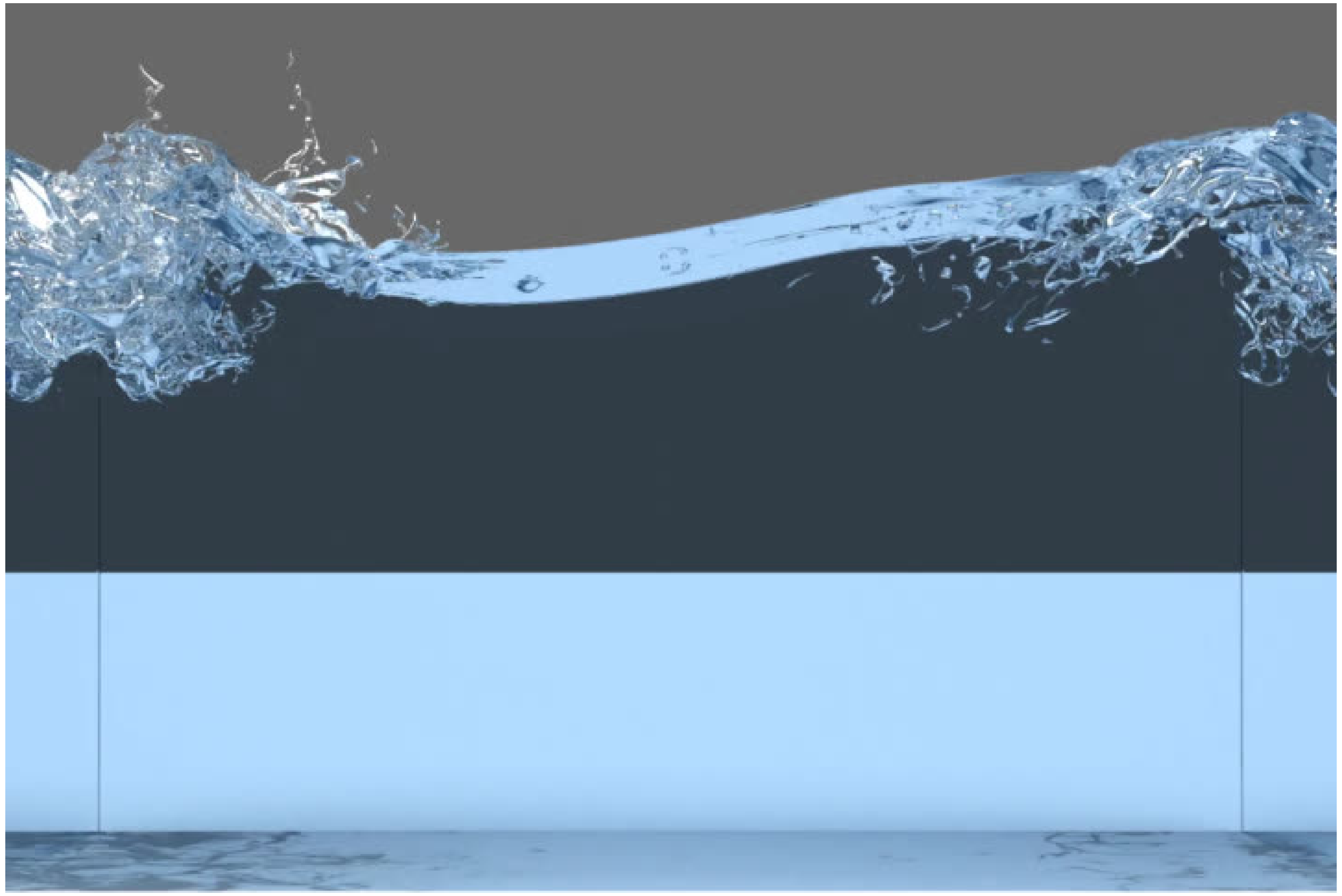
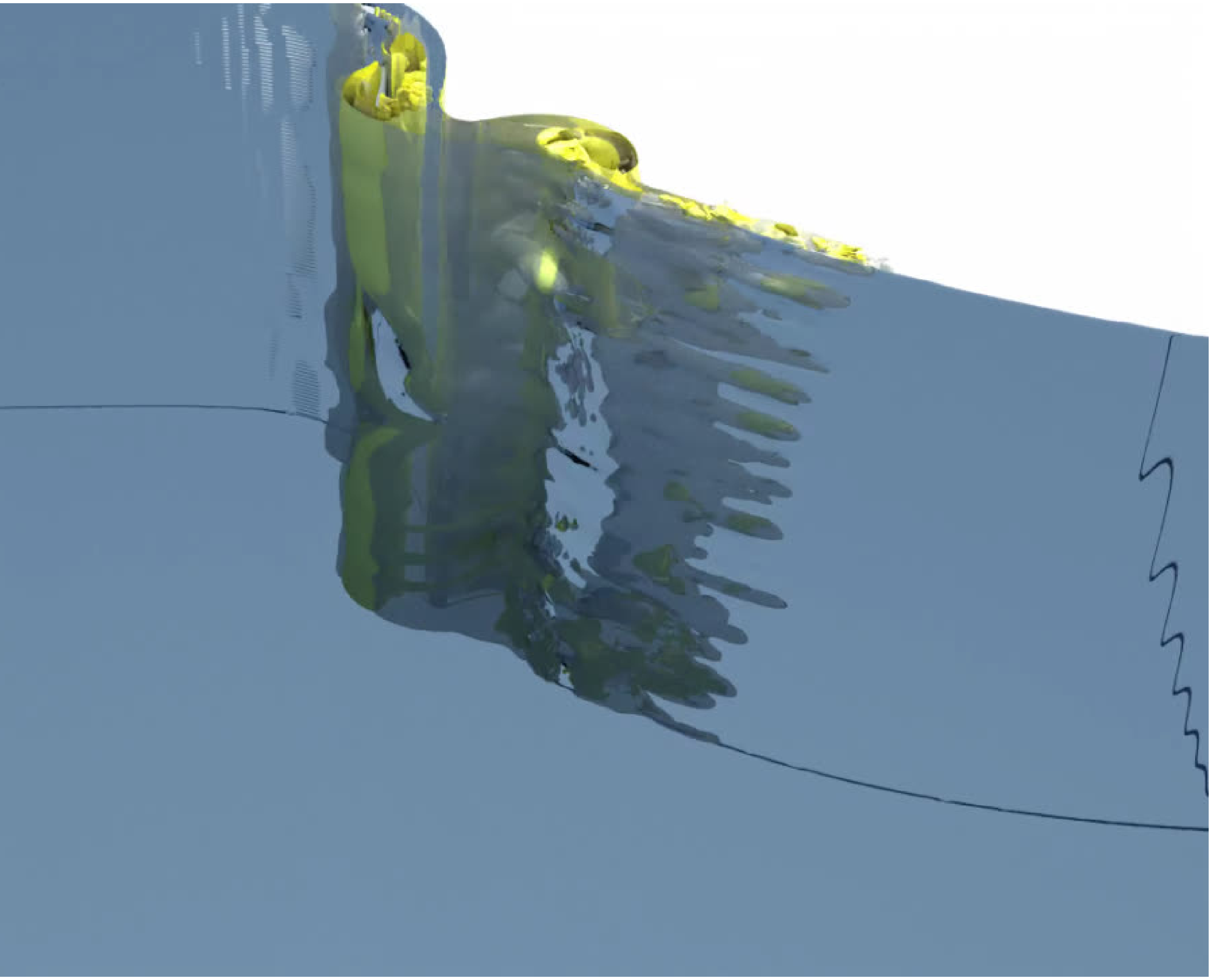
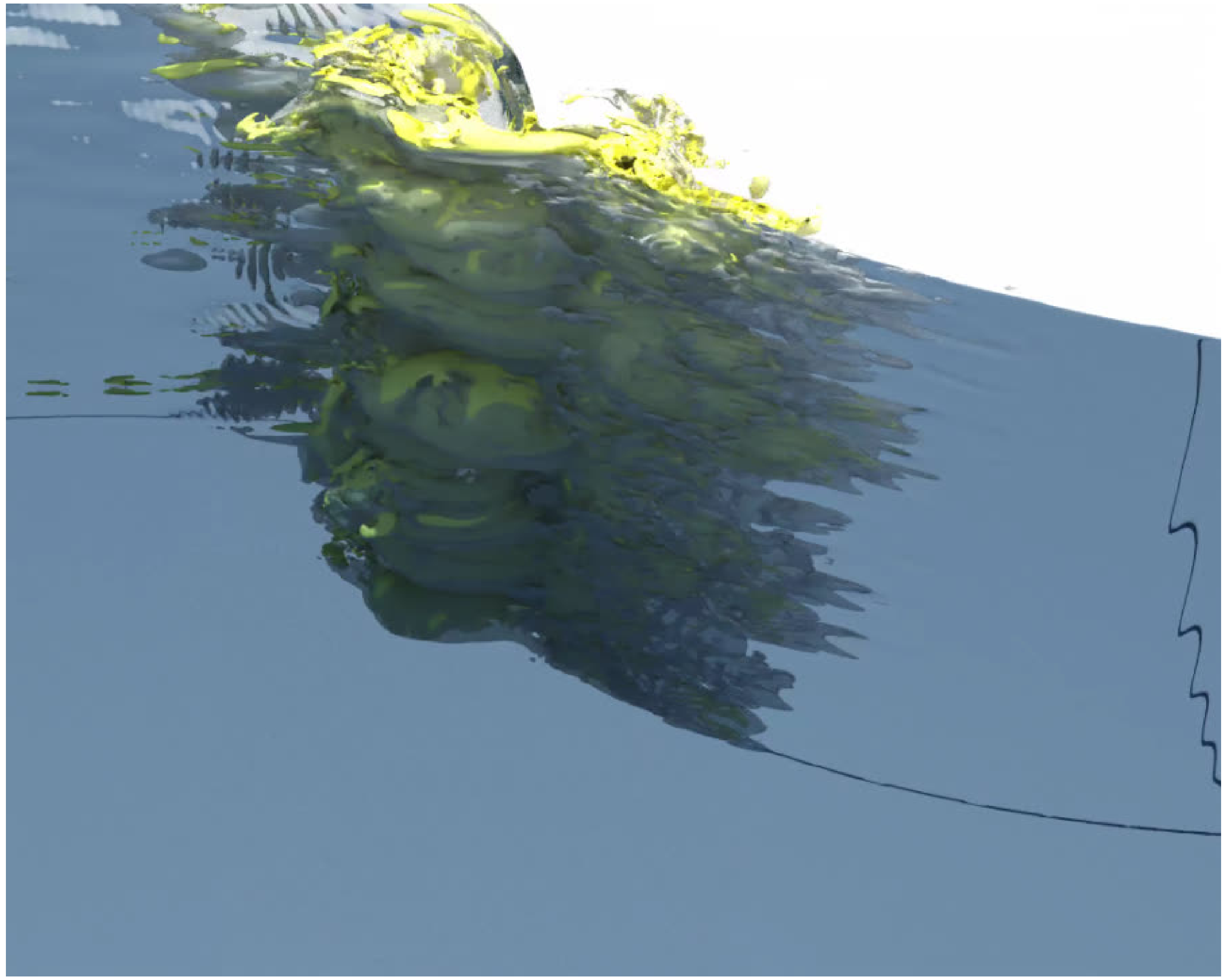


RE=40000

Vortex sheets and tubes (intensity level 20)



RE=10000



RE=40000

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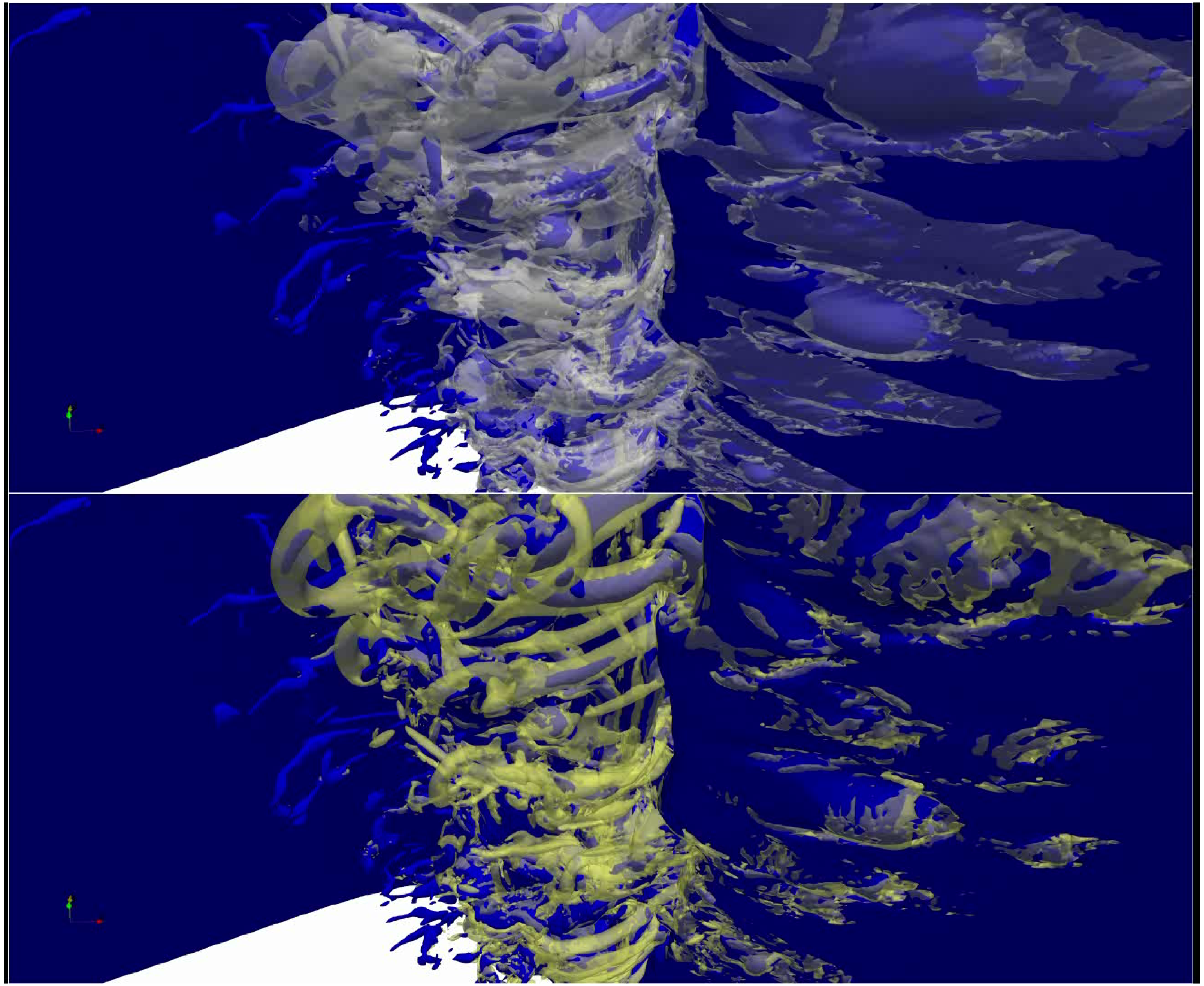
Vortex sheets and tubes (intensity level 80)

Vortex Sheets and Tubes vs air ligaments

- Most of the tubes are around long air ligaments
- Vortex sheets are outside the tubes



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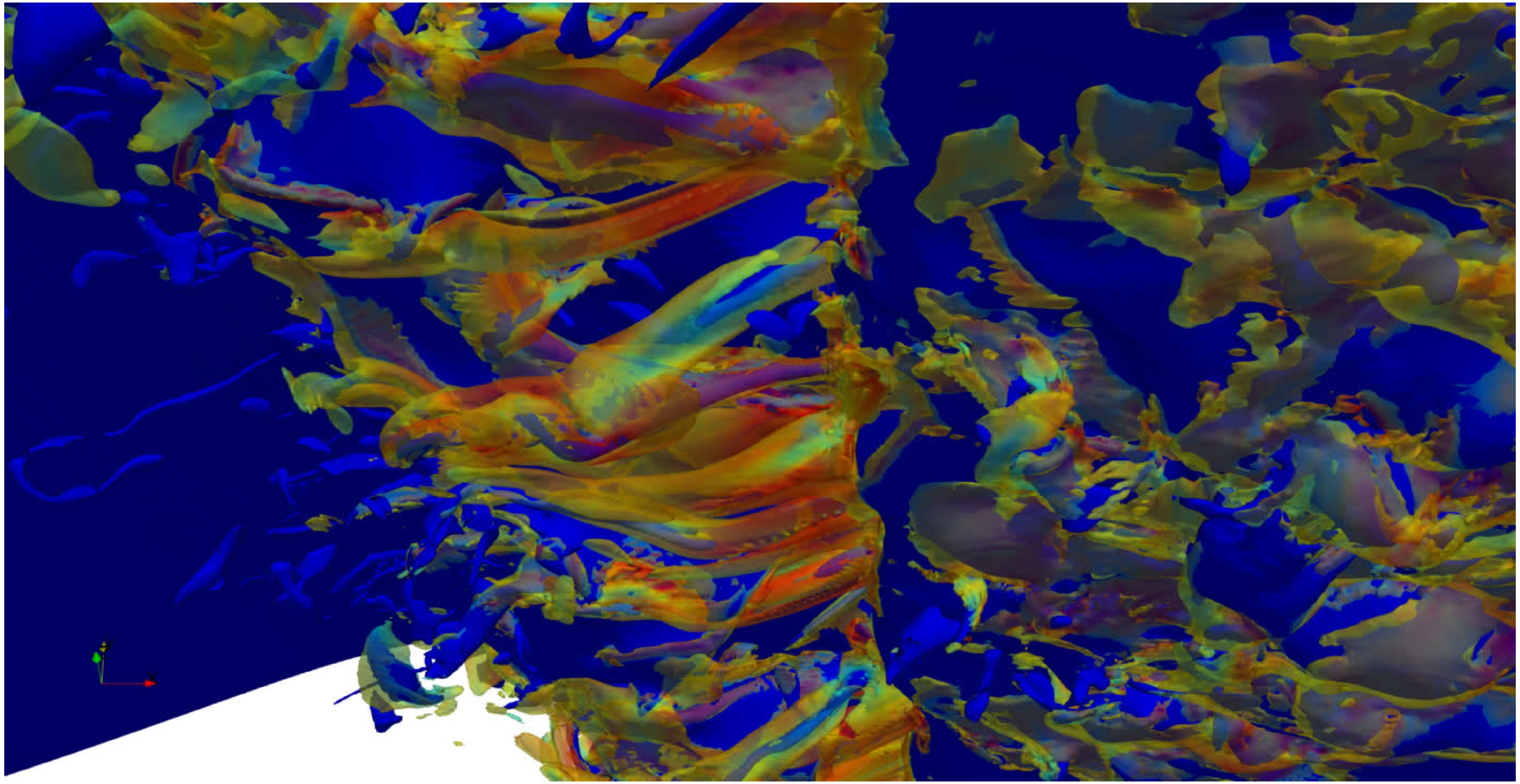
Vortex sheets and dissipation (intensity level 80)

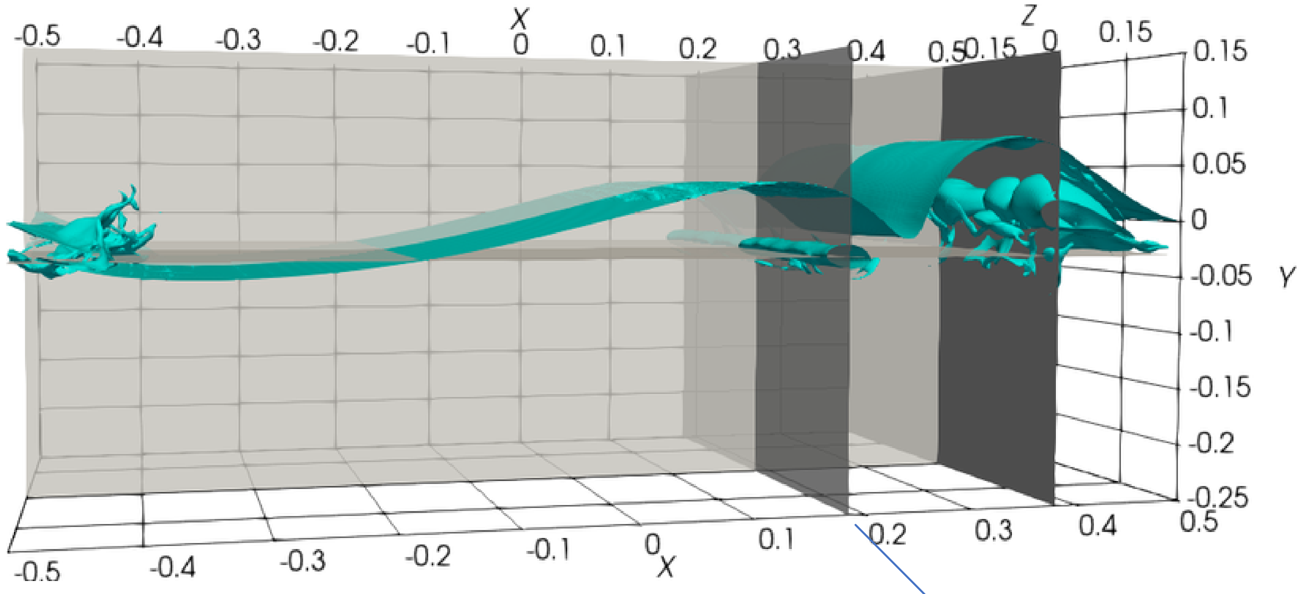
Vortex Sheets color scale by dissipation

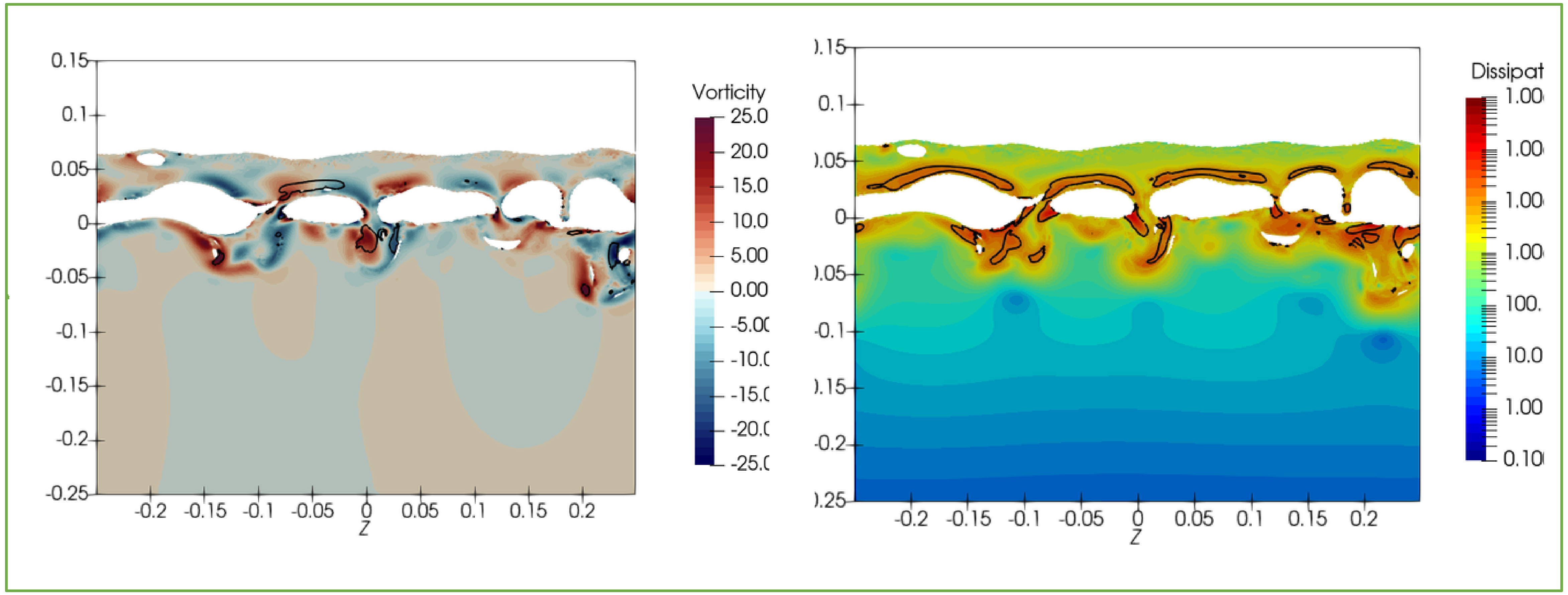
- At the beginning, the dissipation is mostly associated with the entrainment of air ligaments
- In the later stage, there is evidence of high dissipation levels just below the free surface ahead of the breaker

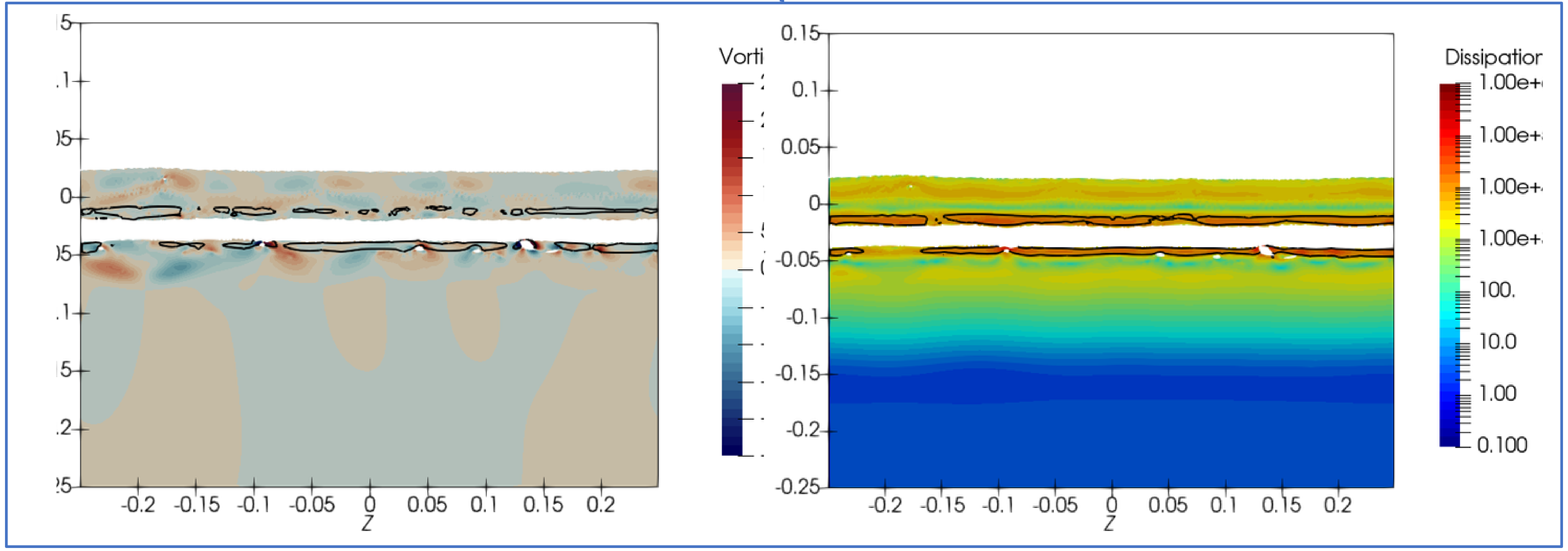


Vortex structures and air entrainment: Re = 10000

t/T = 0.938







Large correlation between vorticity and tubes

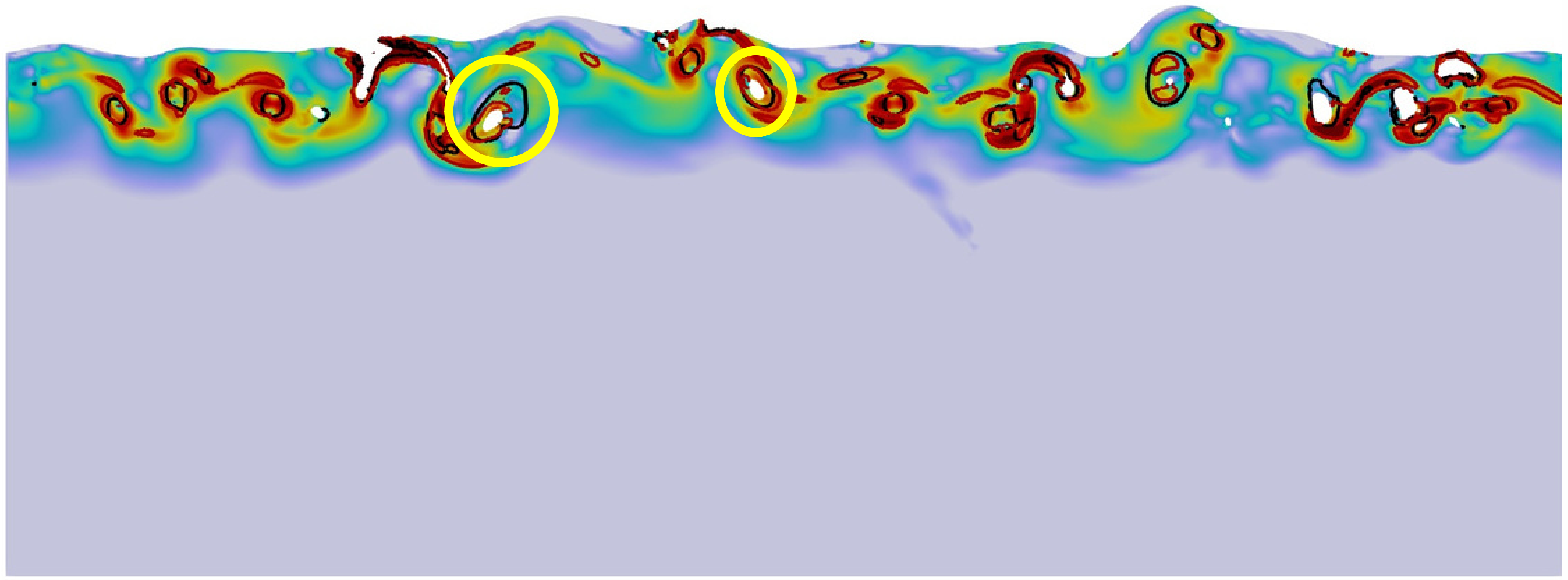
Large correlation between viscous dissipation and vortex sheets

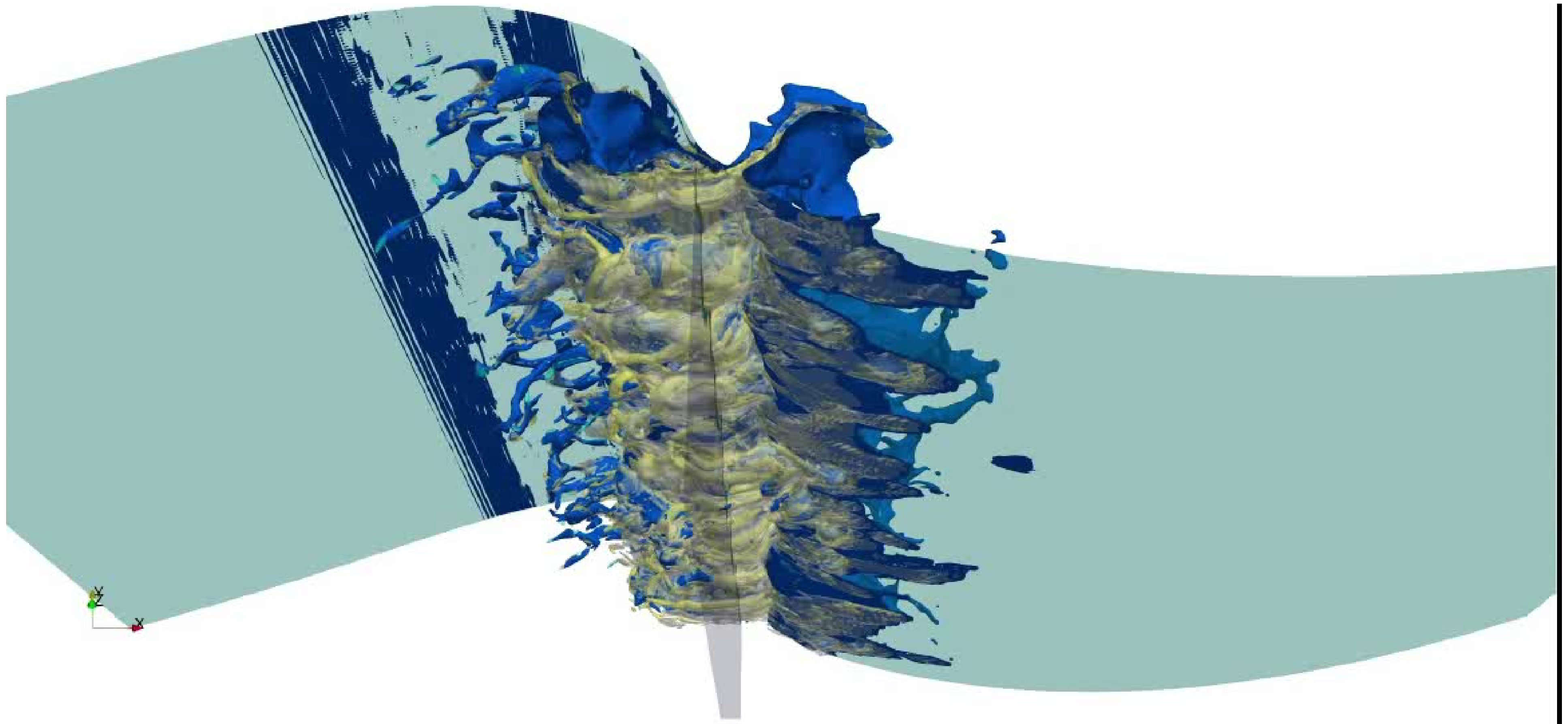
Vortex sheets and tubes and dissipation (intensity level 80)

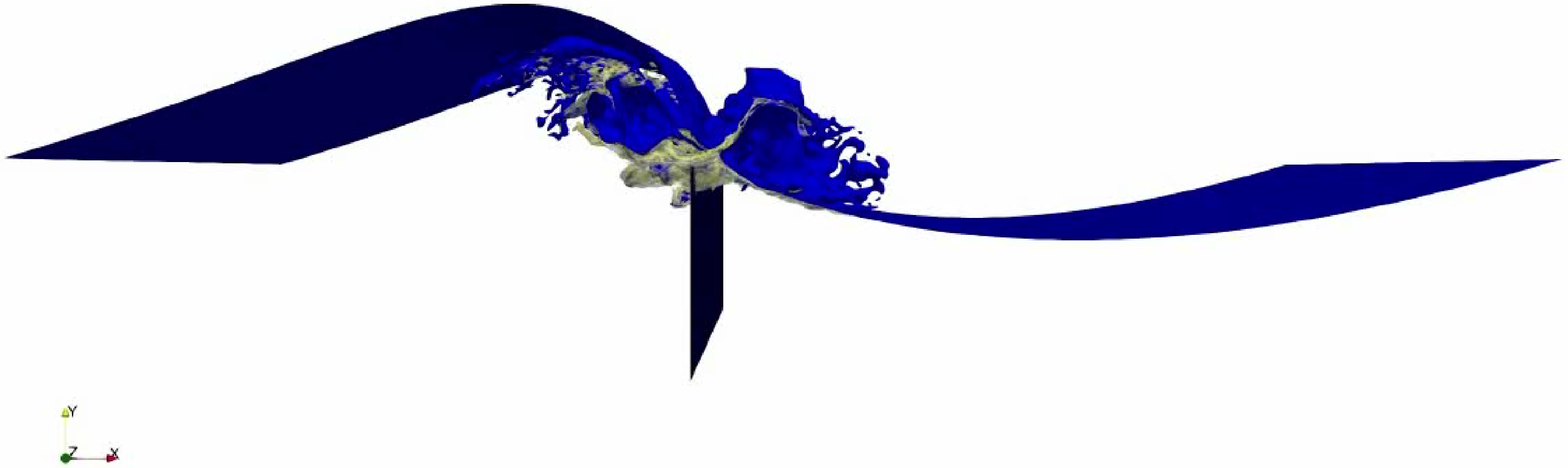
- Vortex sheets and tubes and air entrainment
- Dissipation in color scale on the slices

Vortex tube 6.6 mm

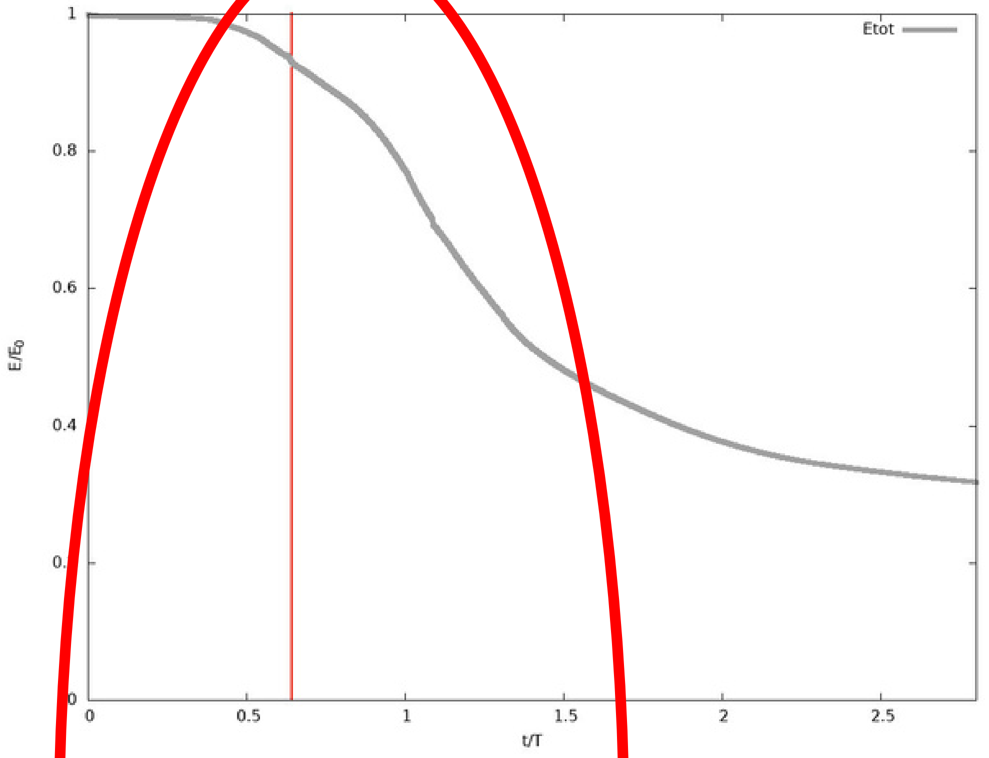
Air tube 2.29 mm

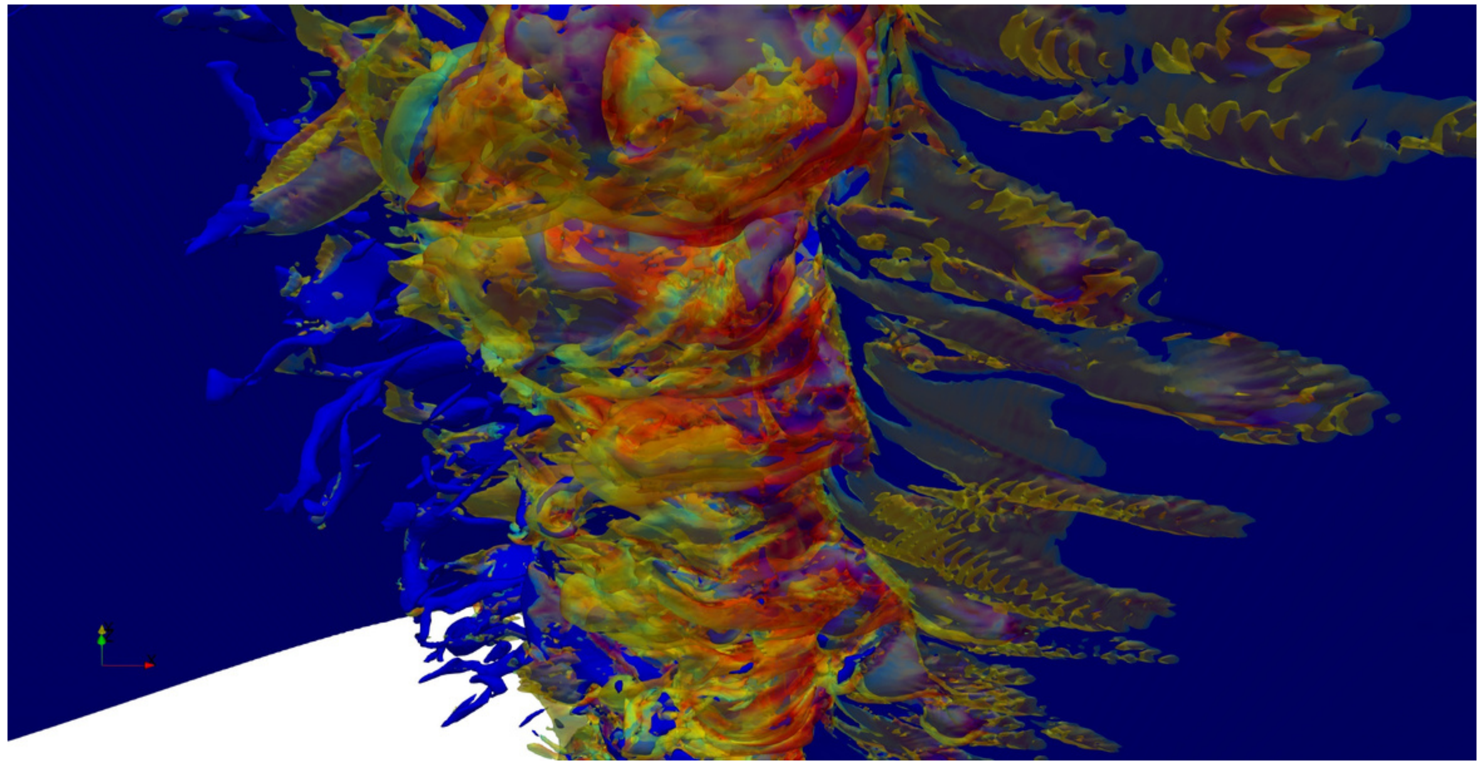


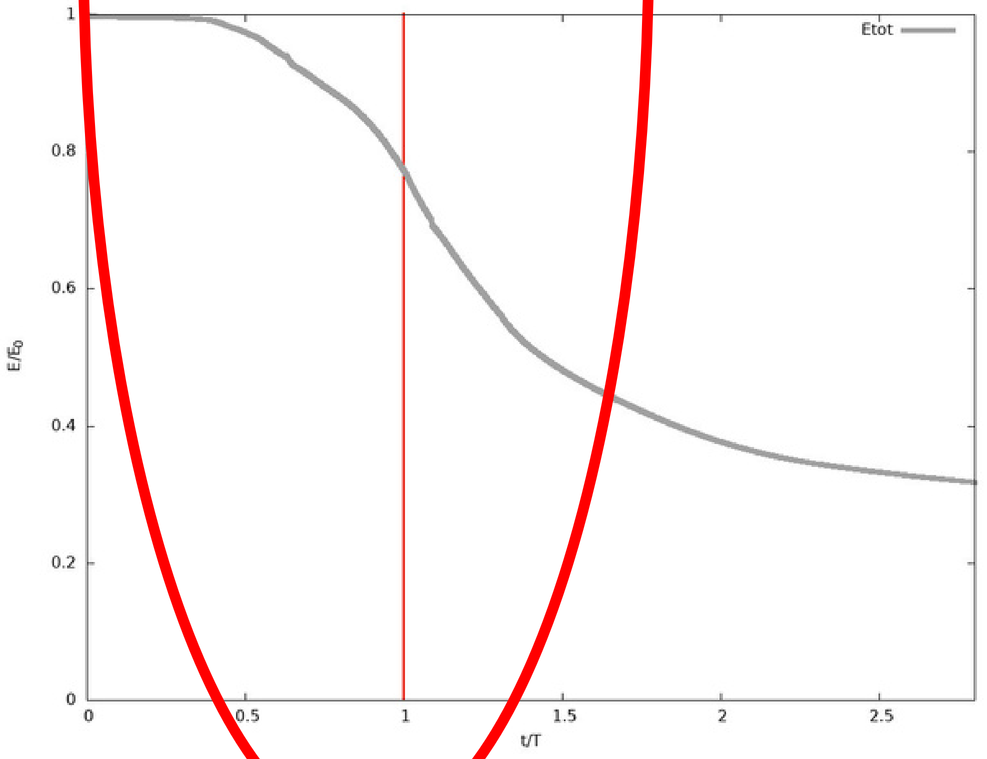


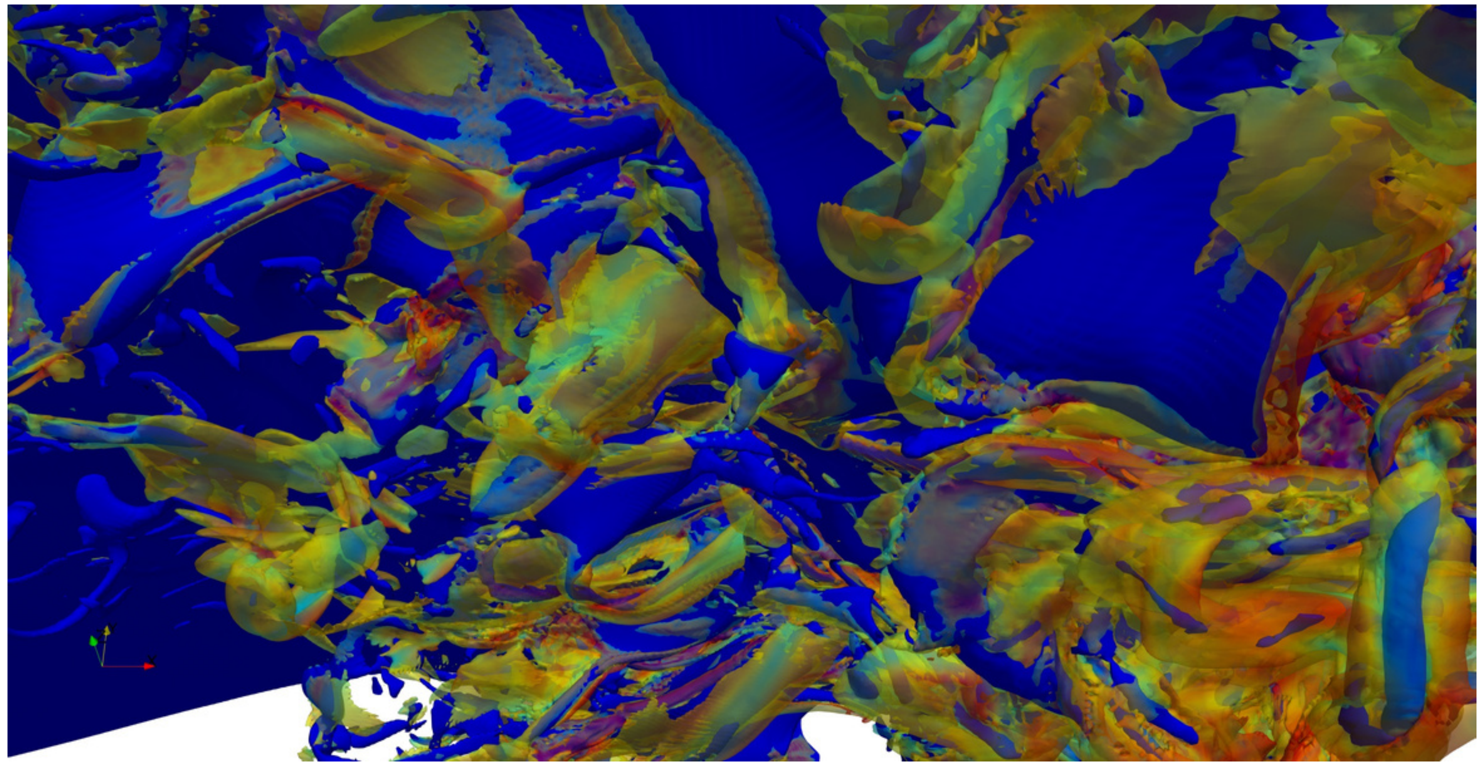


Vortex sheets and tubes and dissipation (intensity level 80)









The analysis covers only the early stage of the breaking process.

Additional investigation are to be done for the later stage

The analysis is made complicated by the highly fragmented interface as well as by the unsteadiness of the process that makes it difficult to perform statistics (without using ensemble averaging)

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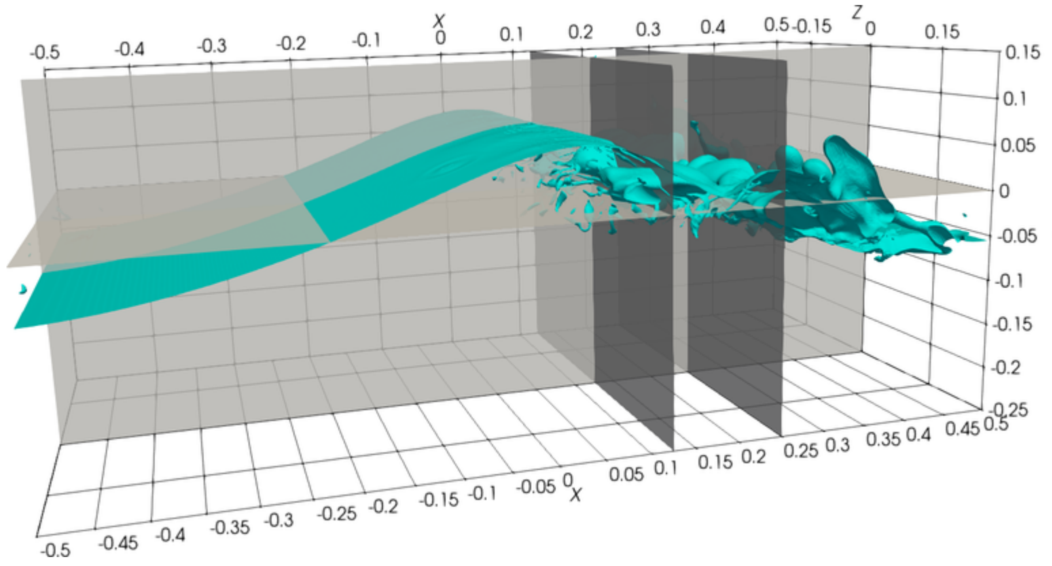
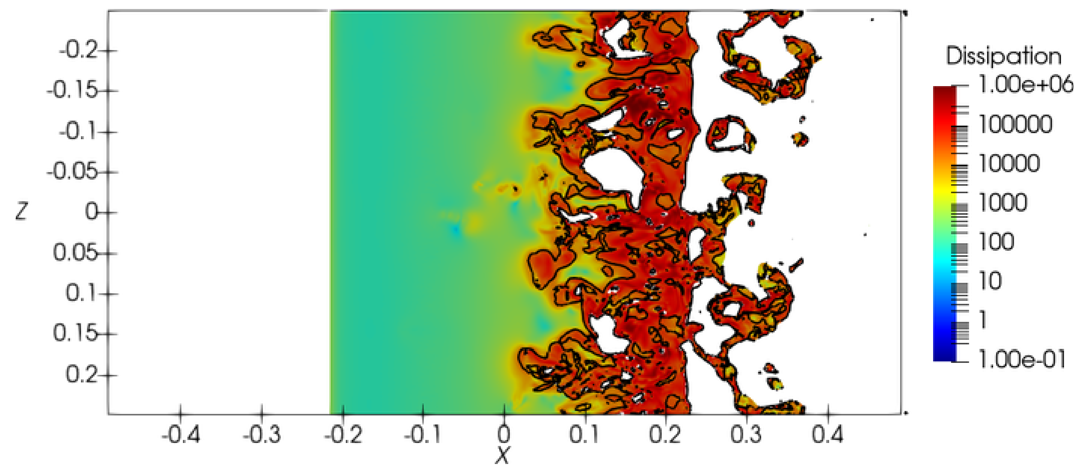
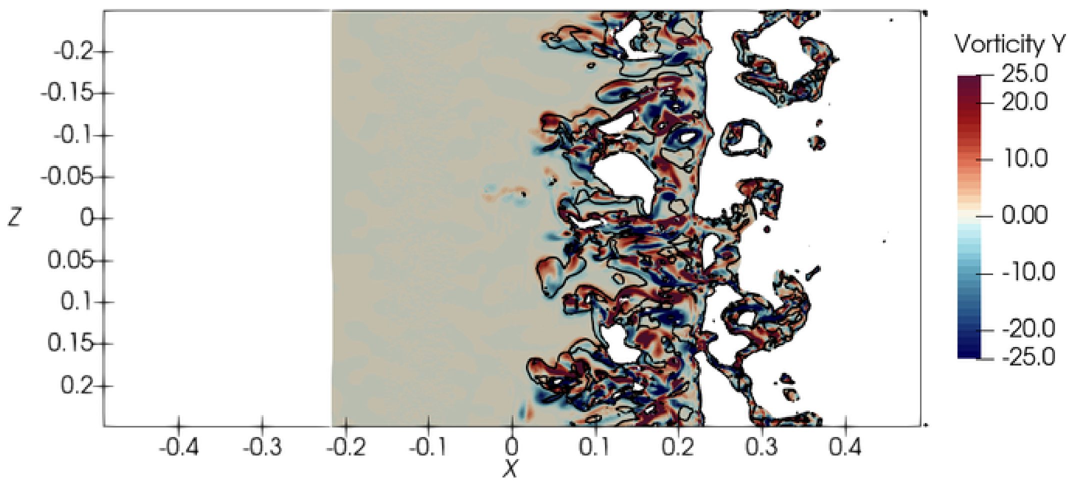
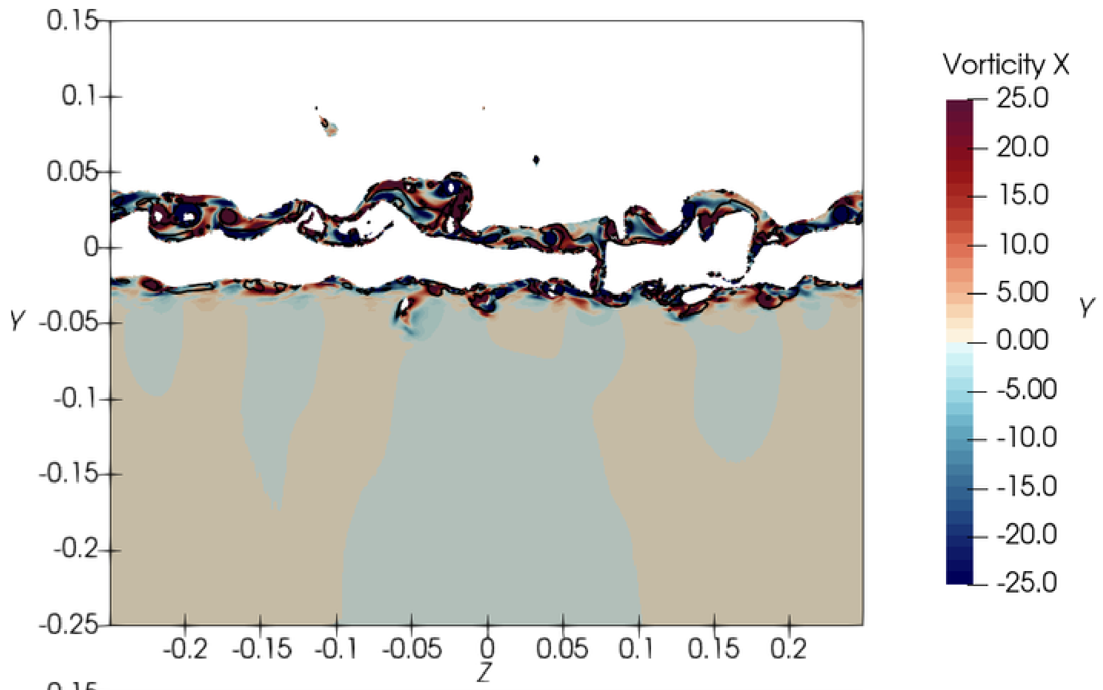
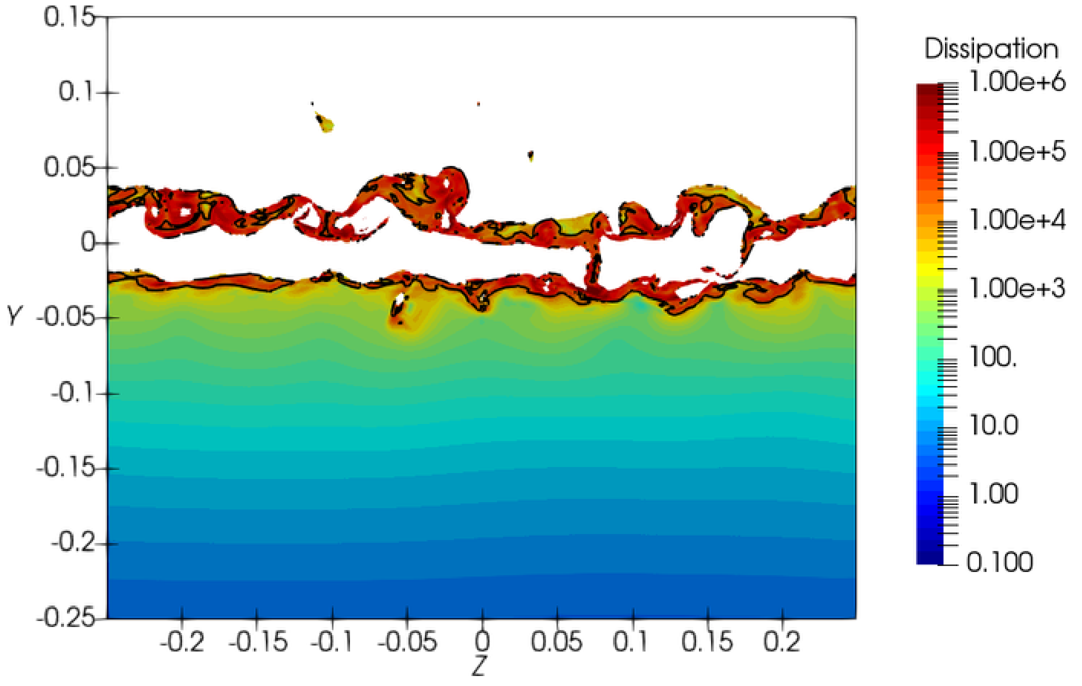
Vortex structures and air entrainment: Re = 40000

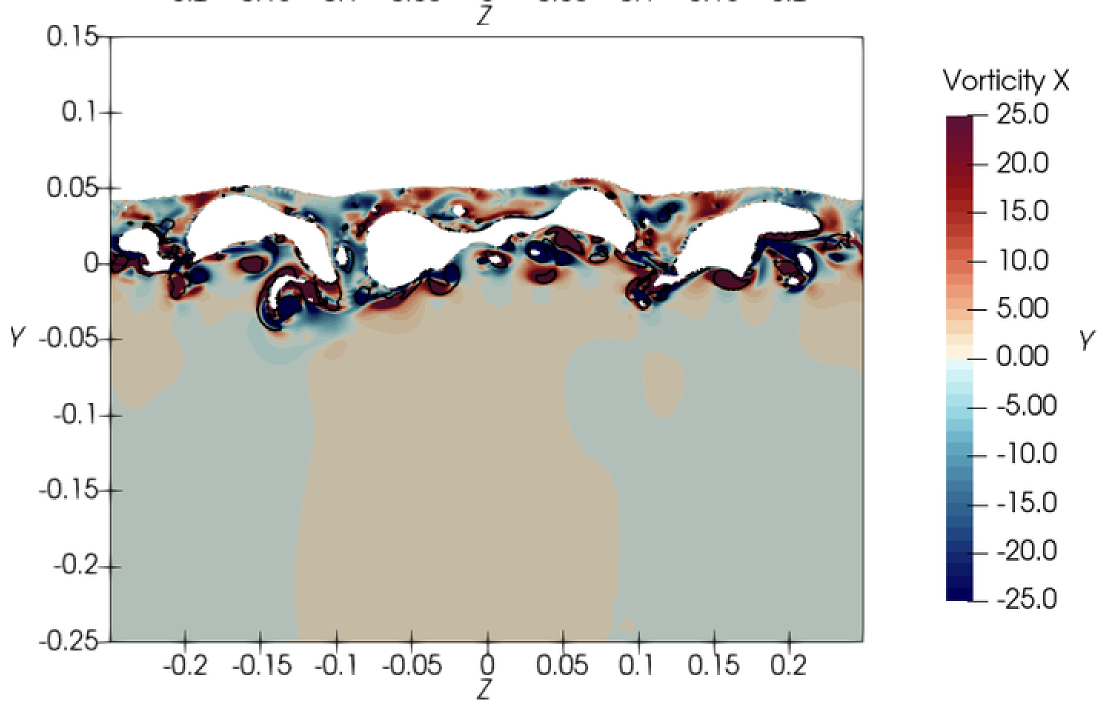


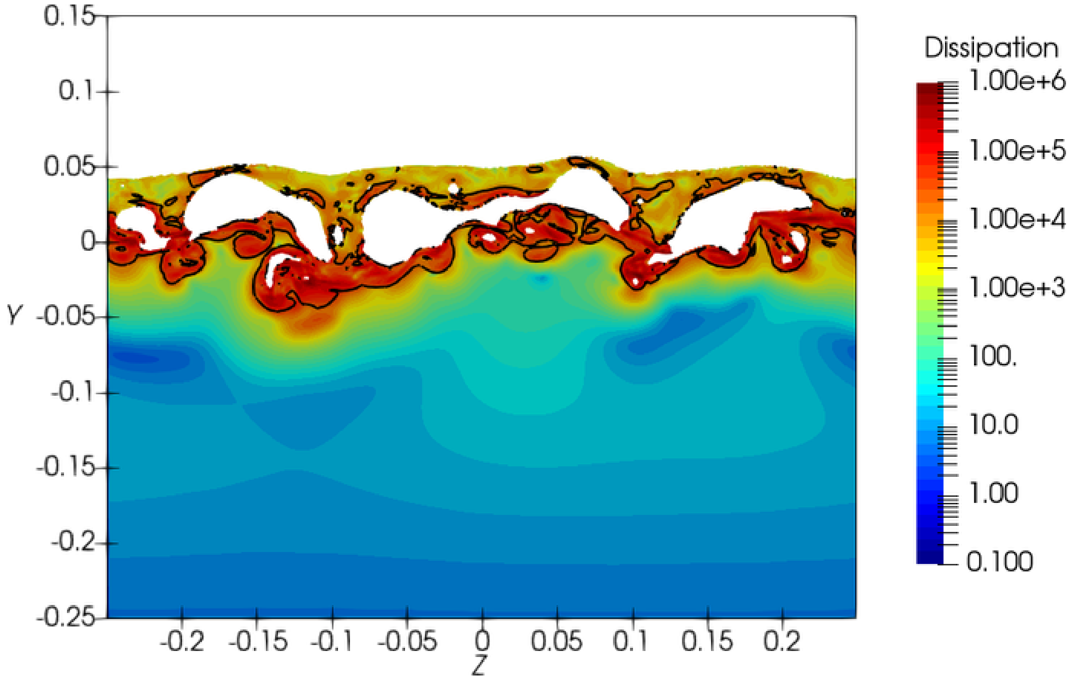


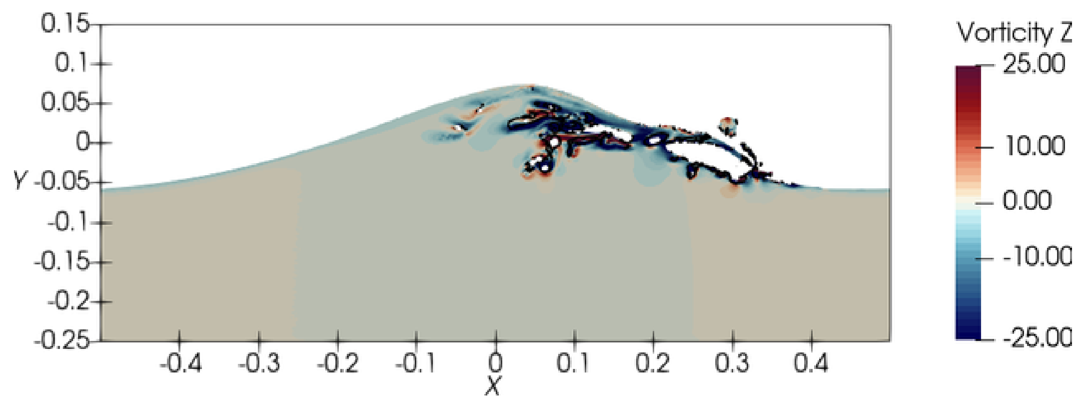


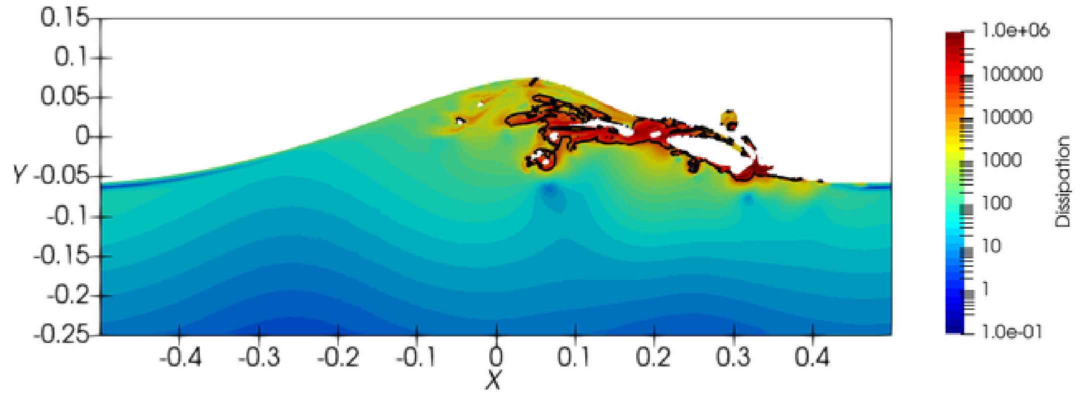








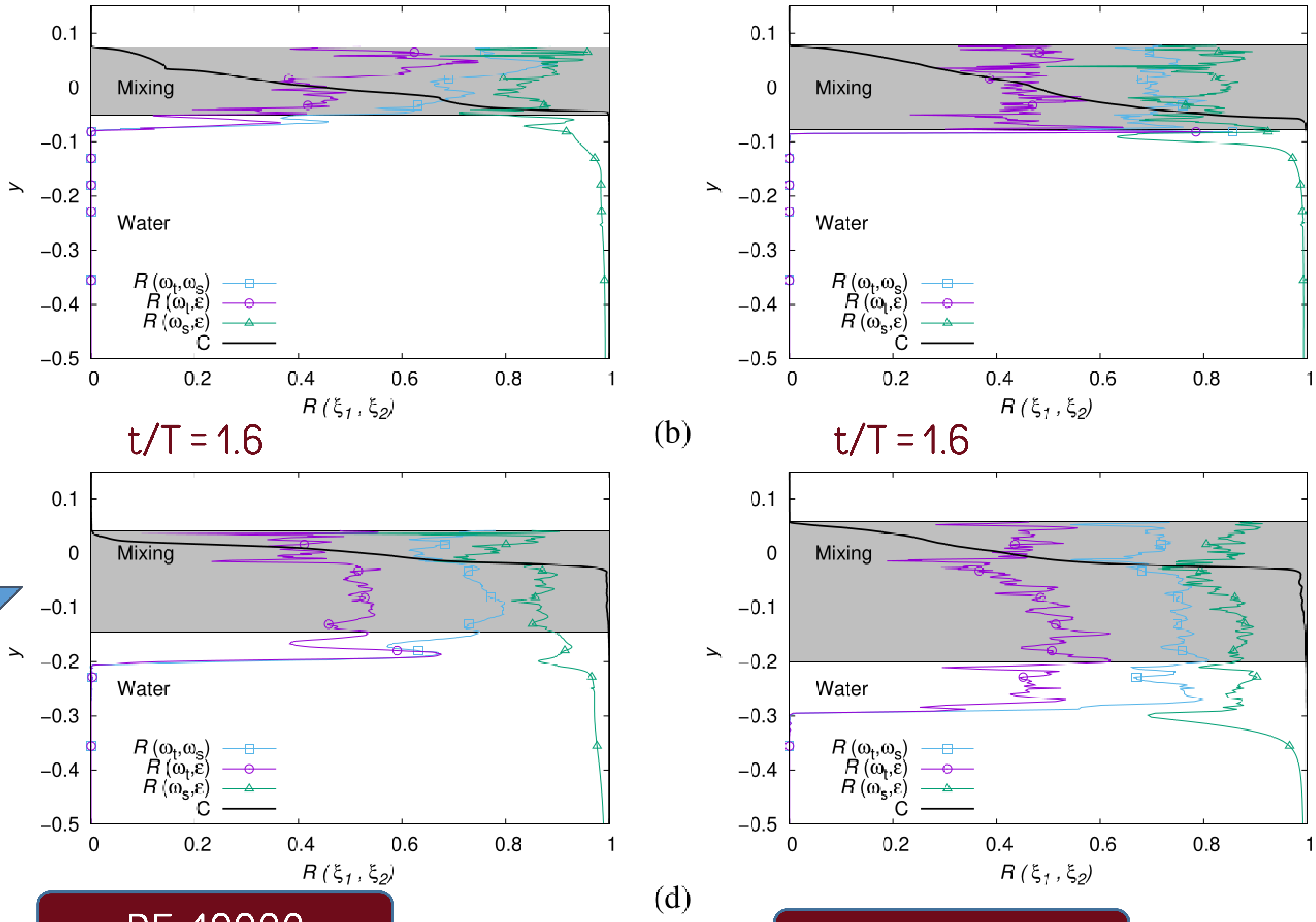




Correlation of vortex structures and dissipation

t/T = 0.938

t/T = 0.878



T

Tubes - Dissipation

Low Correlation

Tubes - Sheets

Sheets - Dissipation

High correlation

$0.001 < C < 0.999$

RE=10000

RE=40000

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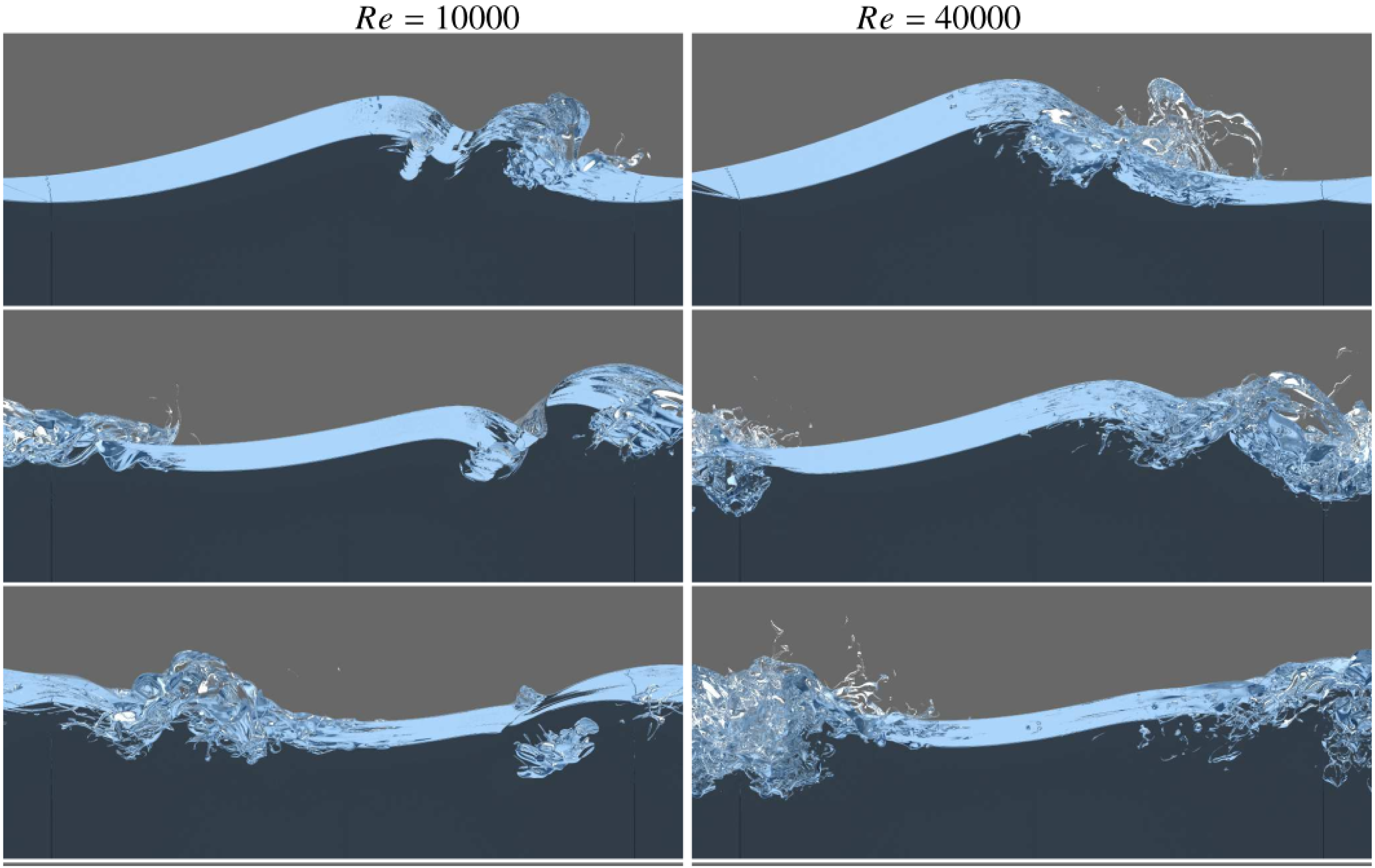
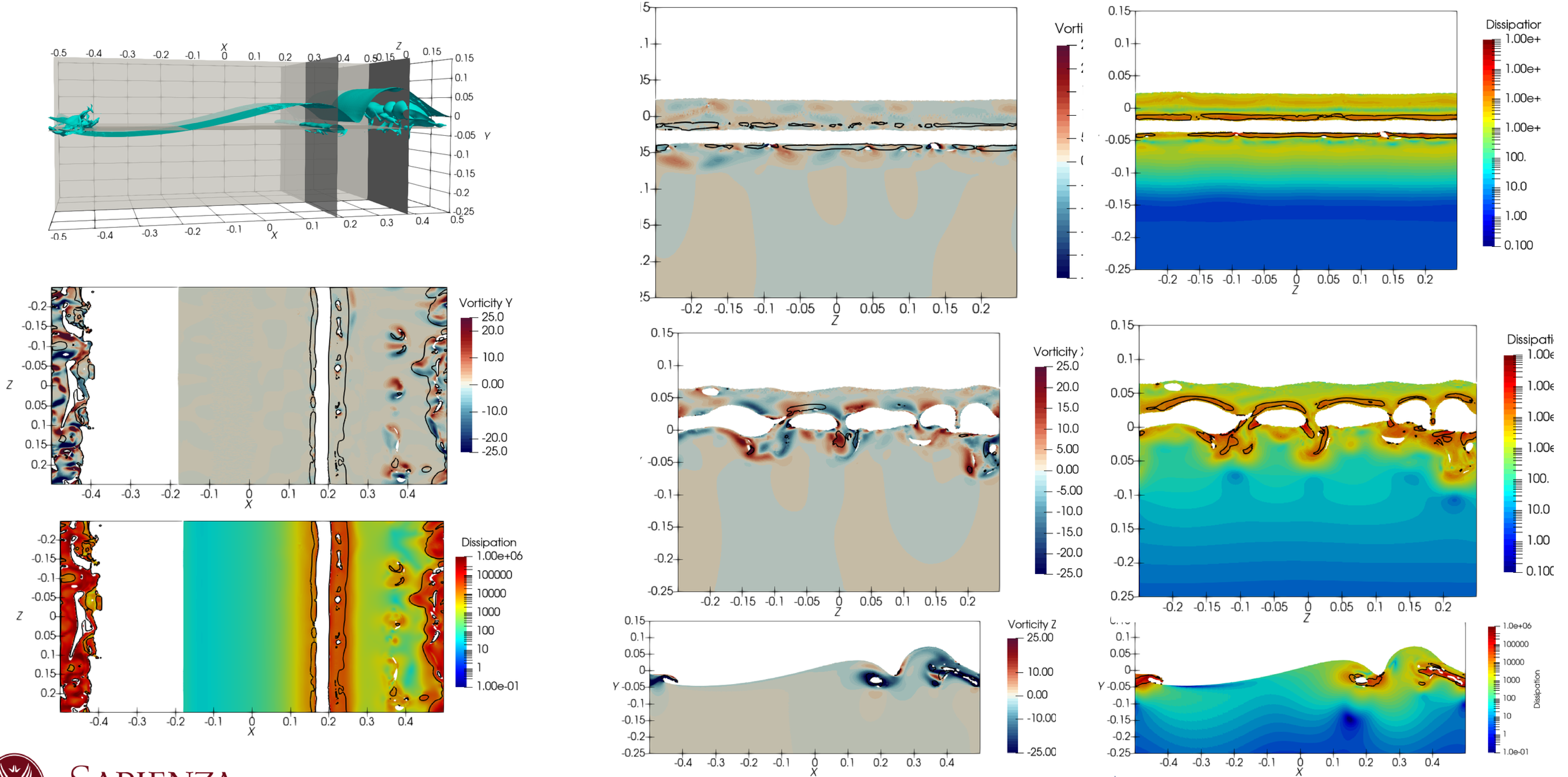


Re = 10000
Re = 40000

Vortex structures and air entrainment: Re = 10000



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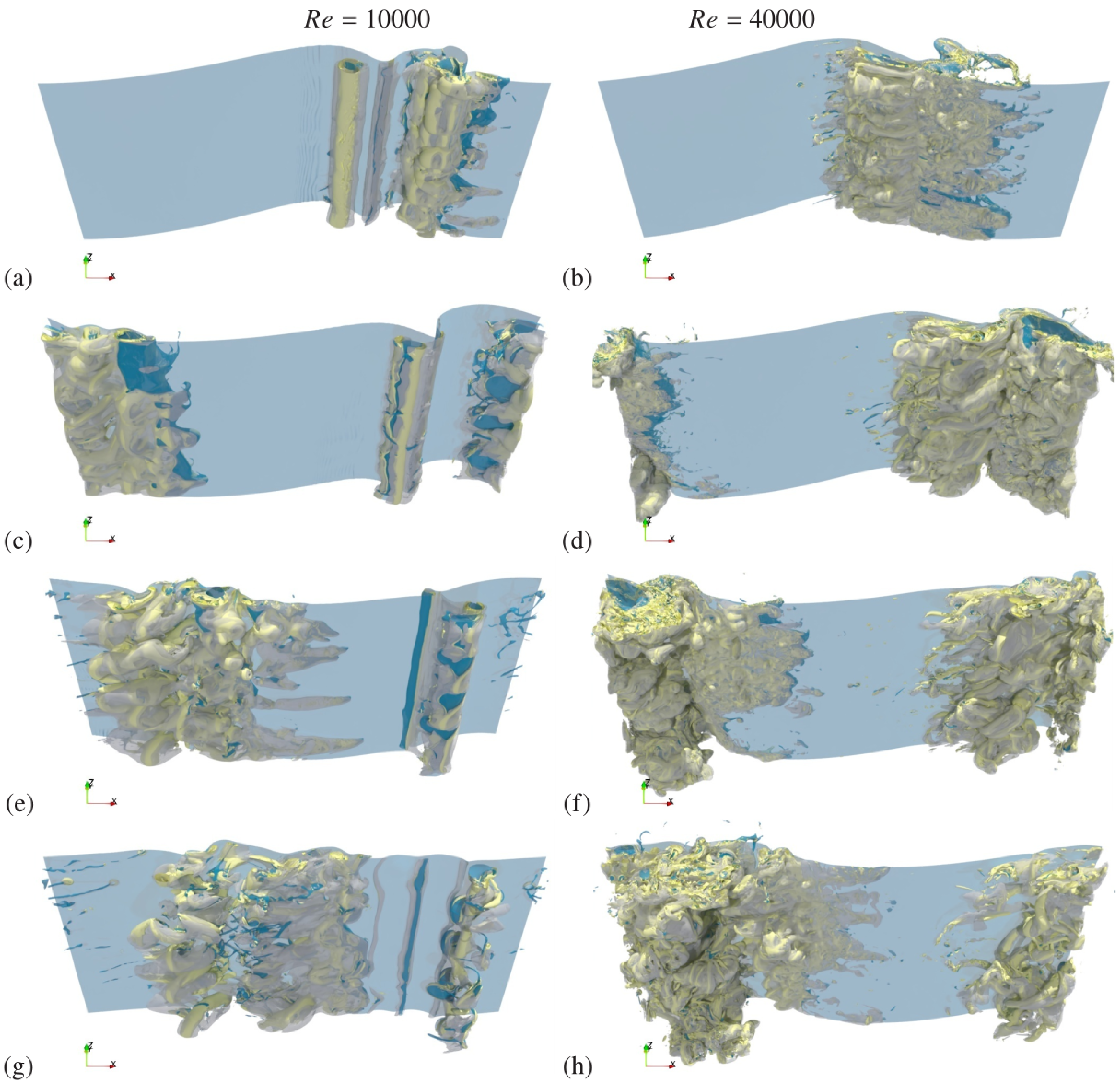

Re = 10000
Re = 40000
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)

Motivation of the study

Breaking modeling and parameterization

- Operational ocean models, albeit rather developed, still display some important drawbacks (e.g. WISE group, 2007)
- Parameterization of the breaking is often based on empirical models but there are not so many data available with enough detail

Breaking dissipation is accounted for both in large scale forecasting models

$$\frac{\partial N}{\partial t} + (c_g + U) \cdot \nabla N = S_{nl} + S_{in} + S_{diss}$$

… as well as in phase resolving HOS approaches (Seiffert et al. 2017, 2018, **Ducrozet's presentation B'WAVES 2023**)

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} + 2\nu_{edd}\frac{\partial^2 \eta}{\partial x^2}$$

$$\frac{\partial \phi}{\partial t} = -g\eta + 2\nu_{edd}\frac{\partial^2 \phi}{\partial x^2}$$

(Tian et al., 2010)

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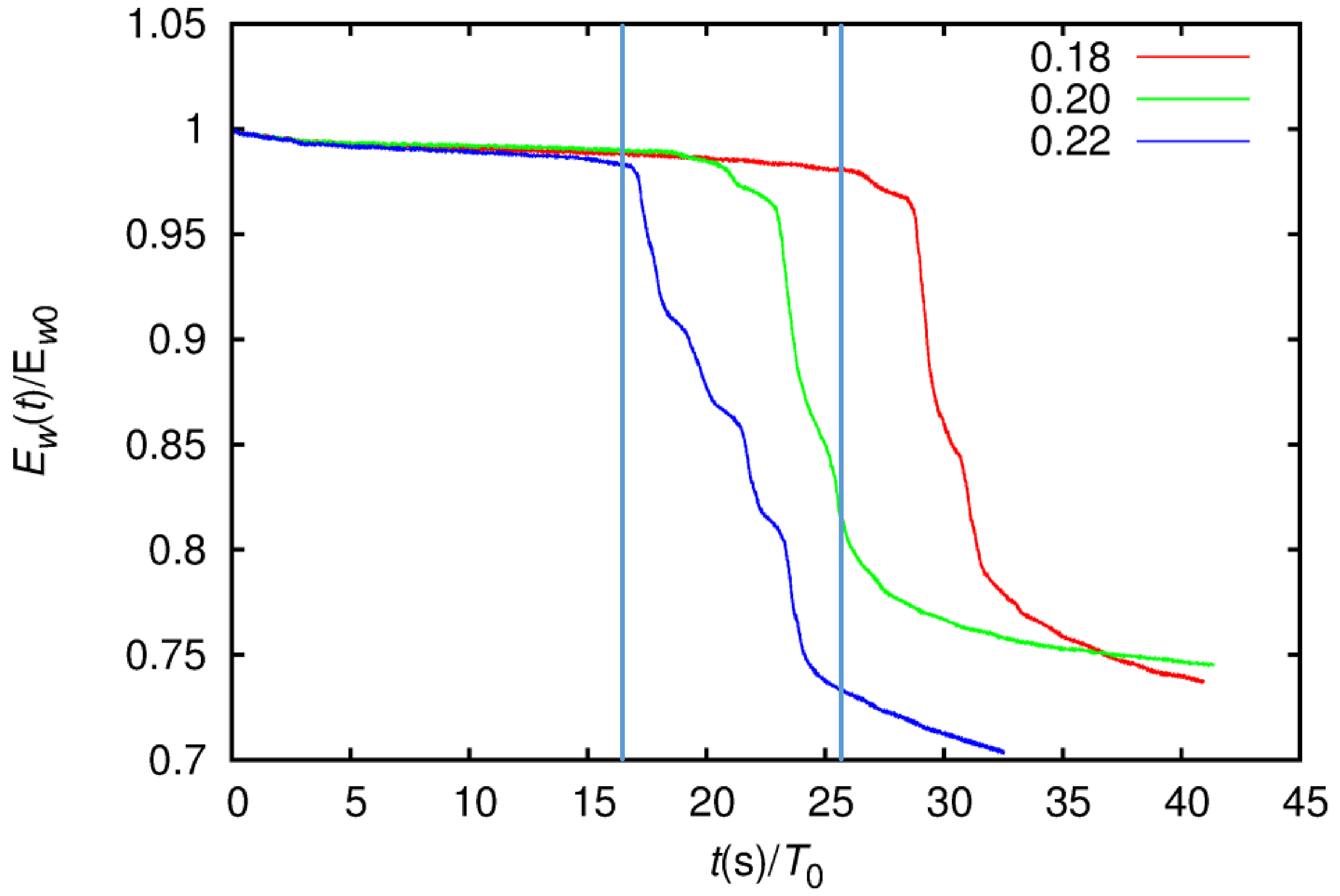
Motivation

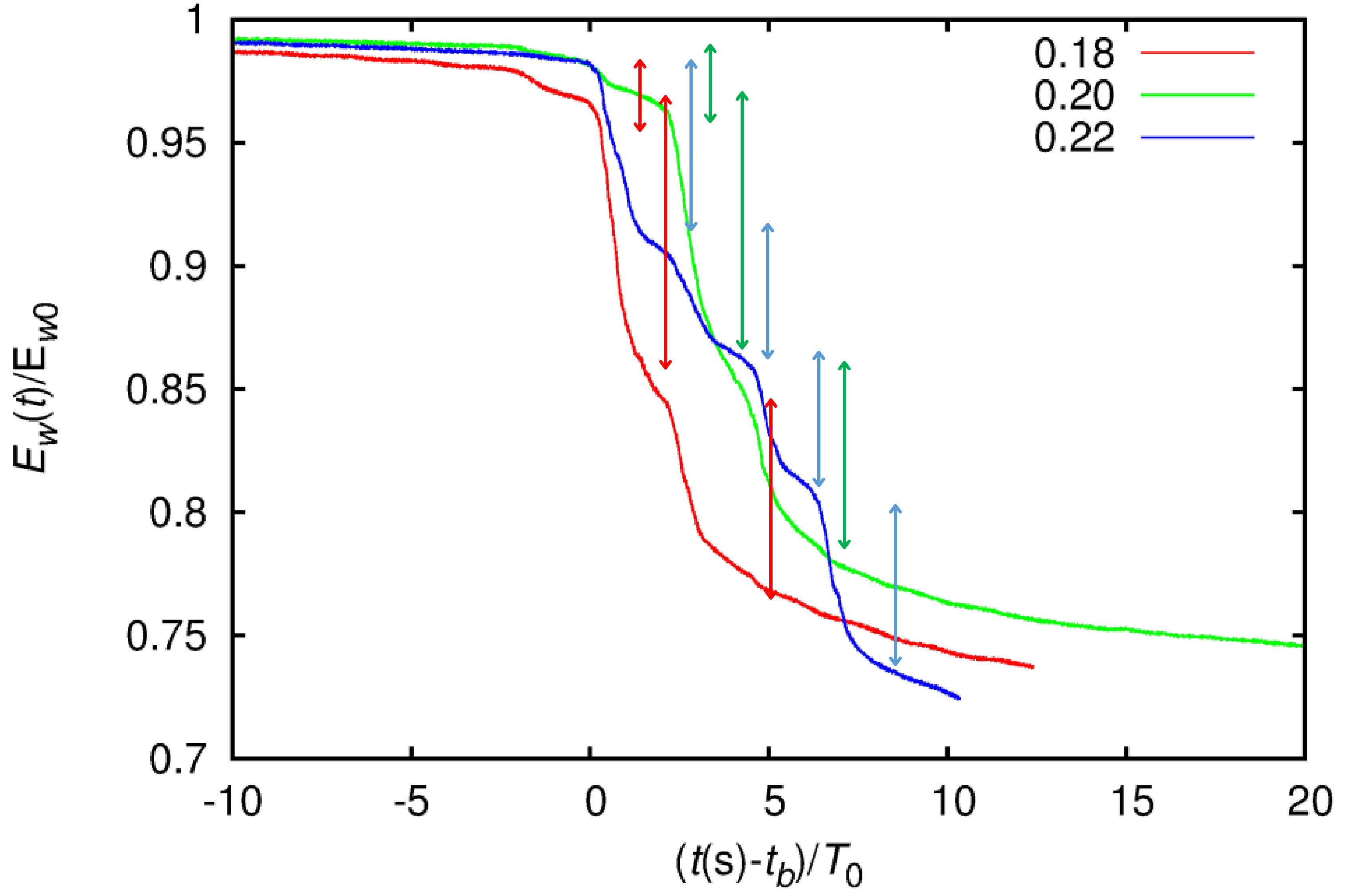
Energy dissipation in breaking waves

- **Similar results, in terms of dissipated energy fraction, is found in the case of modulational instability although the dissipation rate is averagely lower. In this case, attention should be paid at the single breaking event**

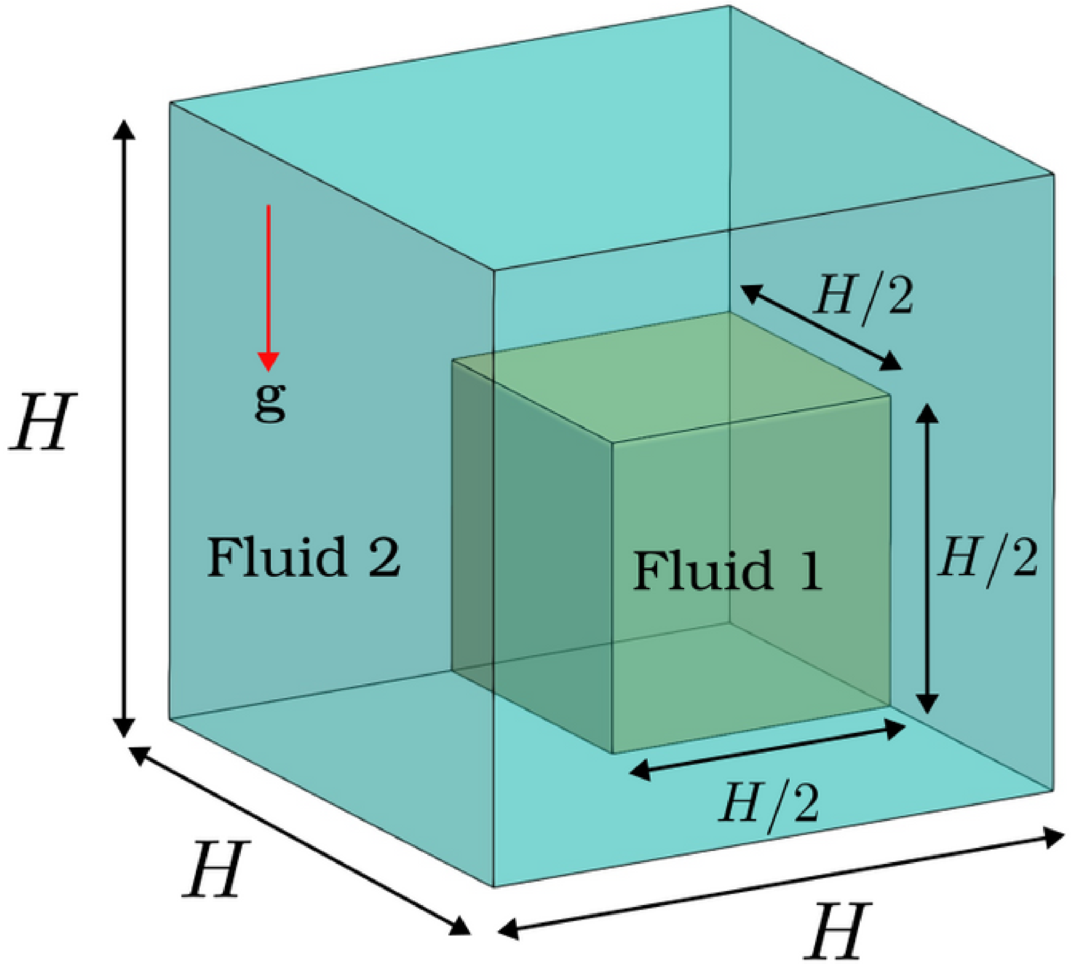


- Total energy versus time

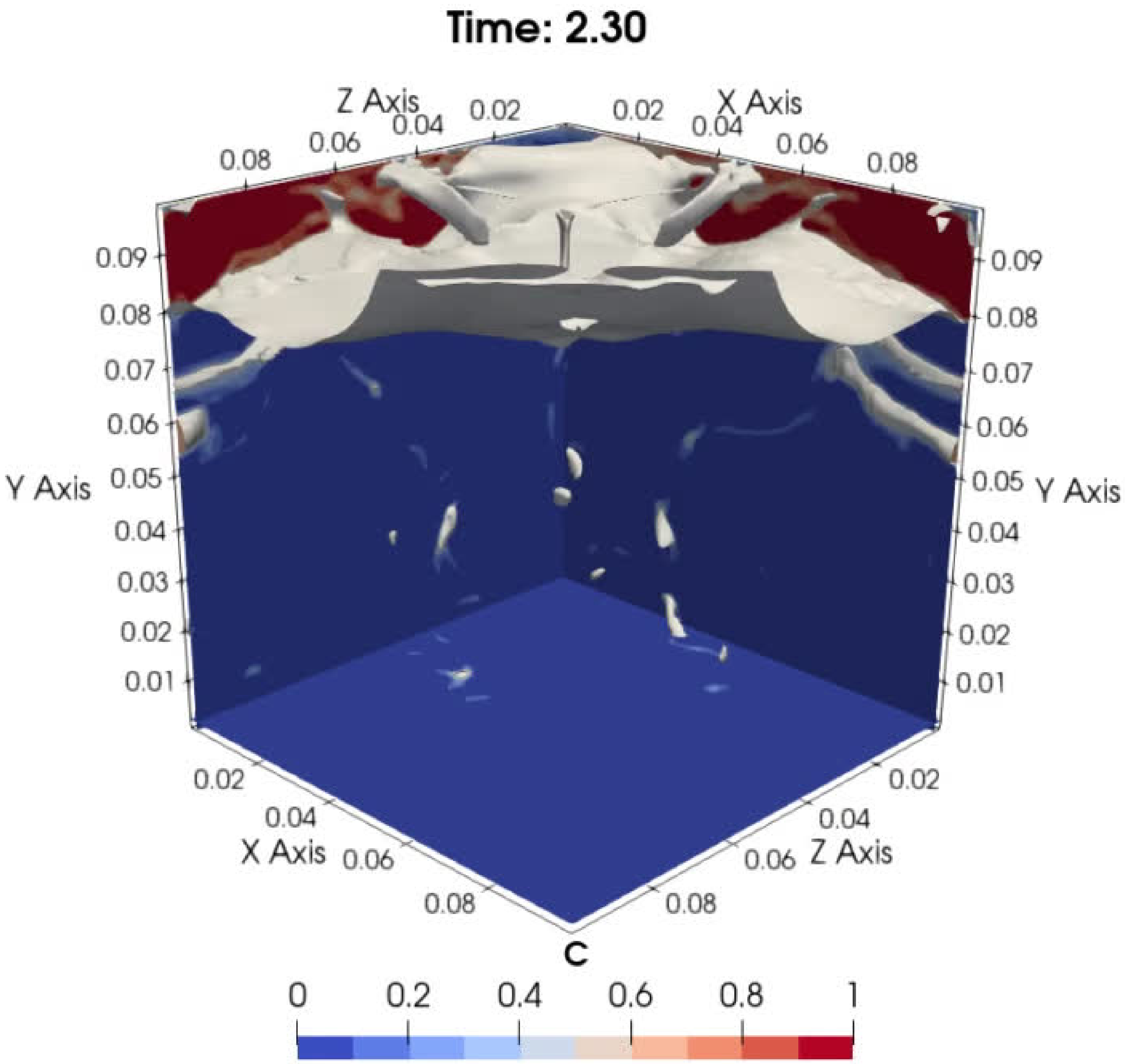
tb is first time with multivalued free surface



Validation of the VOF model and estimate of artificial dissipation

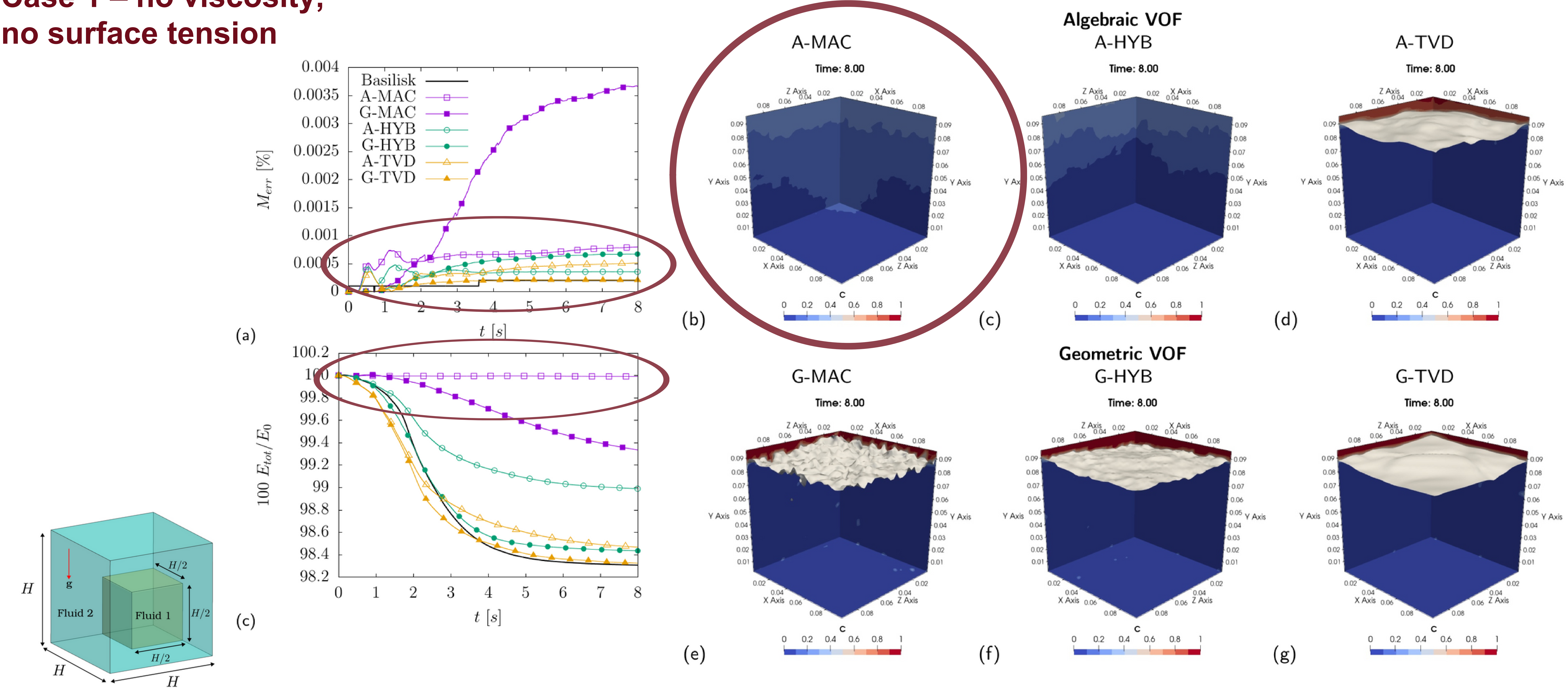


ESTIVALEZES, J.-L., et al. A phase inversion benchmark for multiscale multiphase flows. Journal of Computational Physics, 2022, 450: 110810



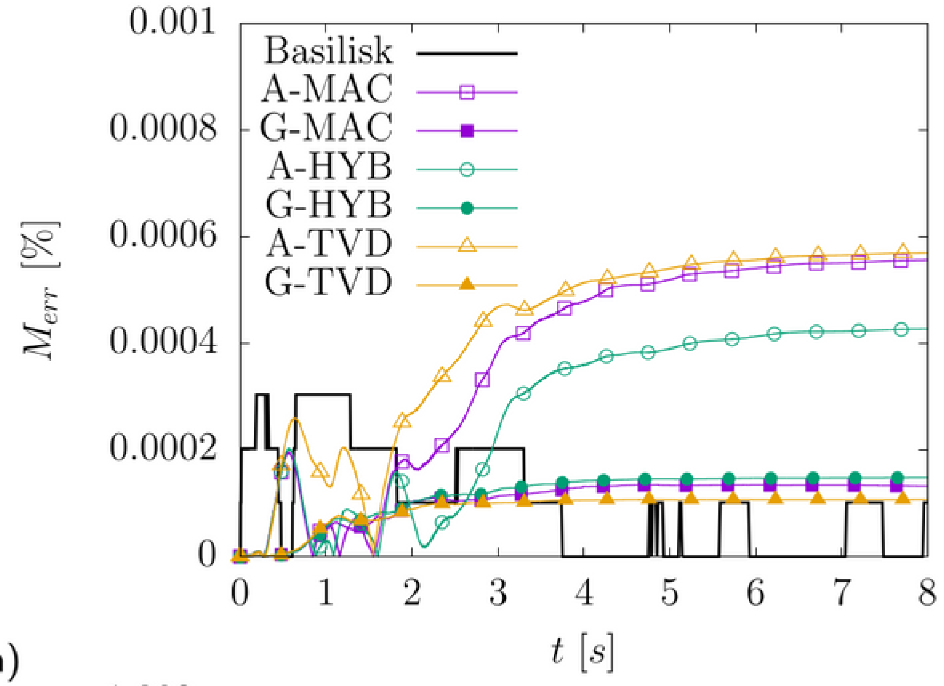
Validation of the VOF model and estimate of artificial dissipation

Case 1 – no viscosity, no surface tension

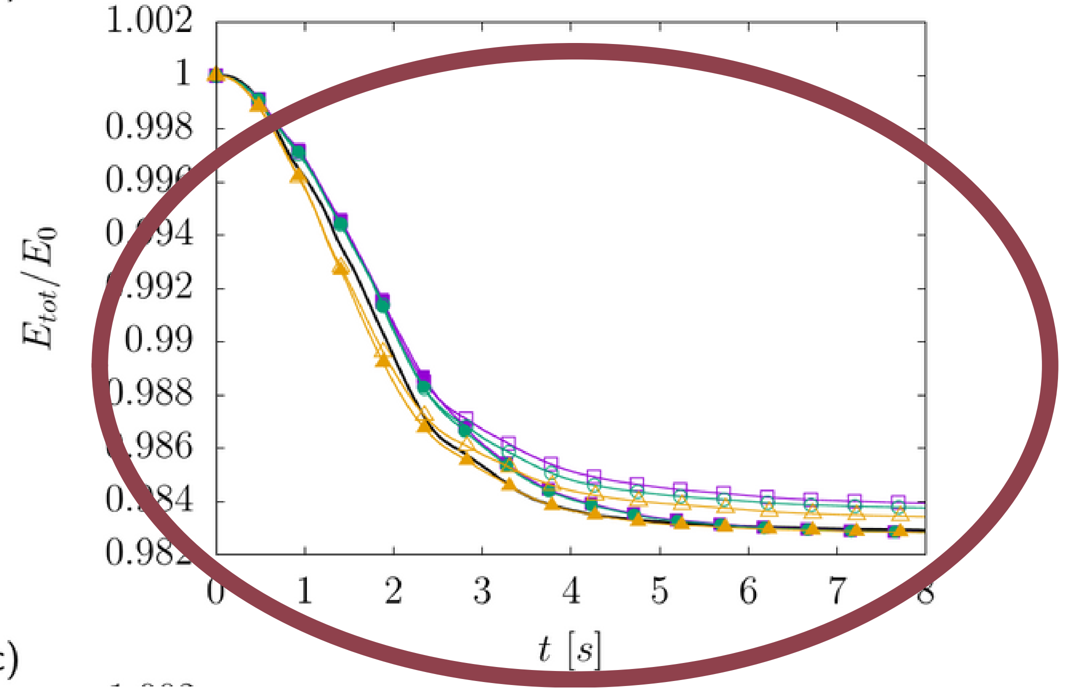


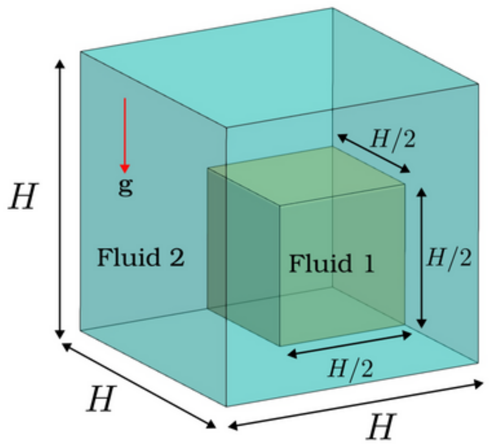
Validation of the VOF model and estimate of artificial dissipation

Case 2 – viscosity, surface tension

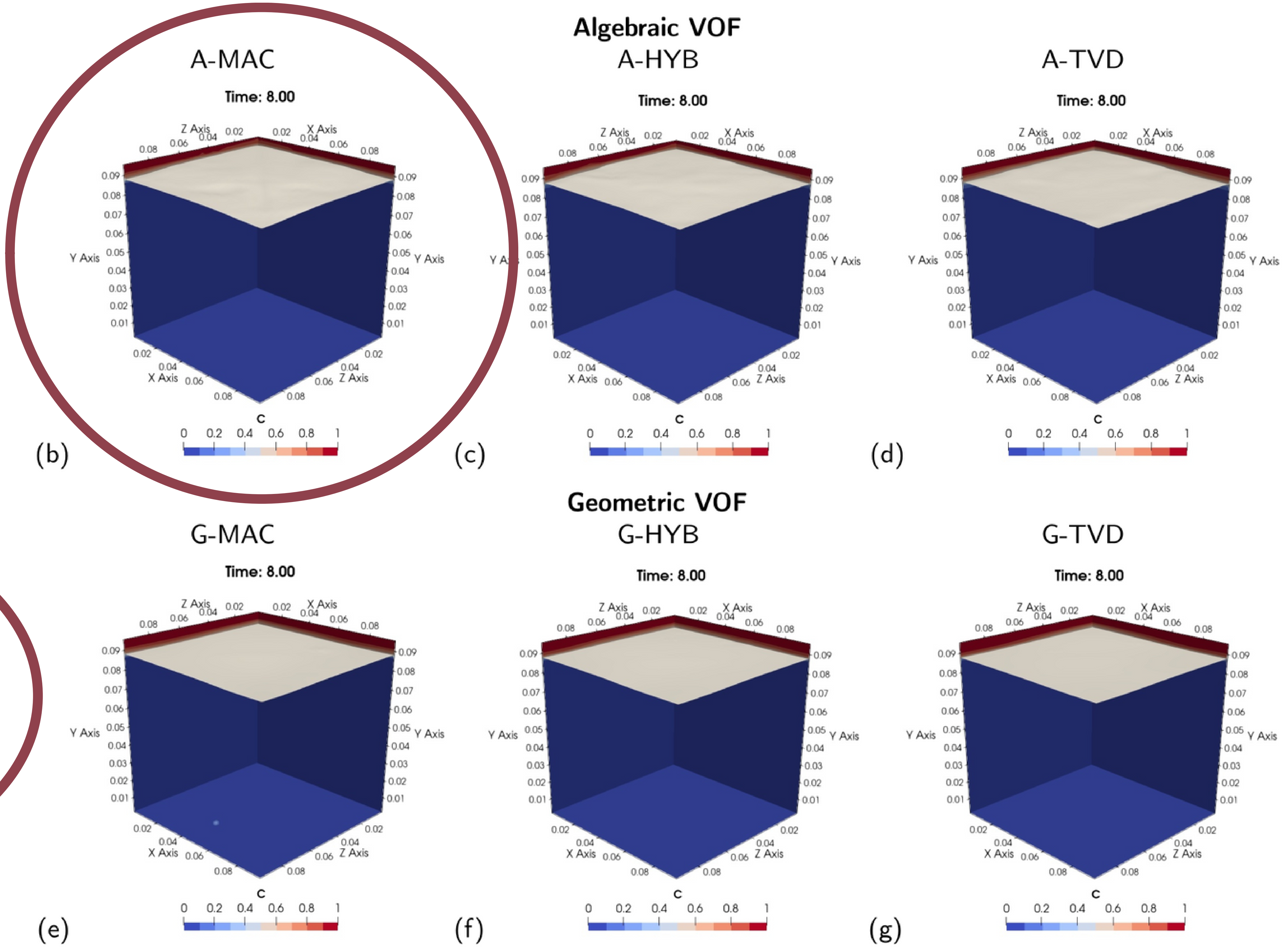


(a)

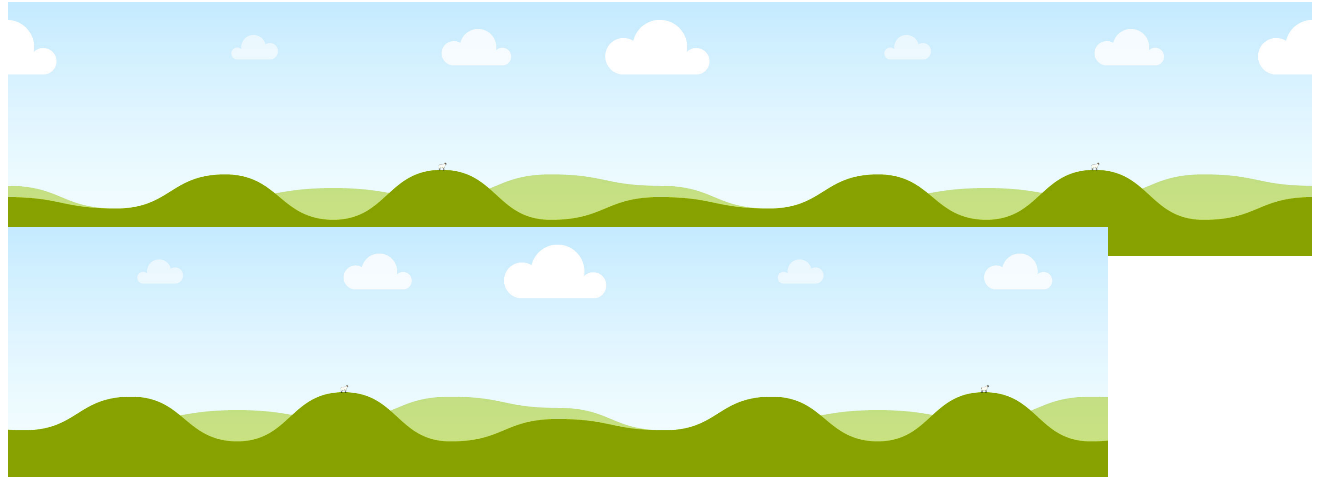




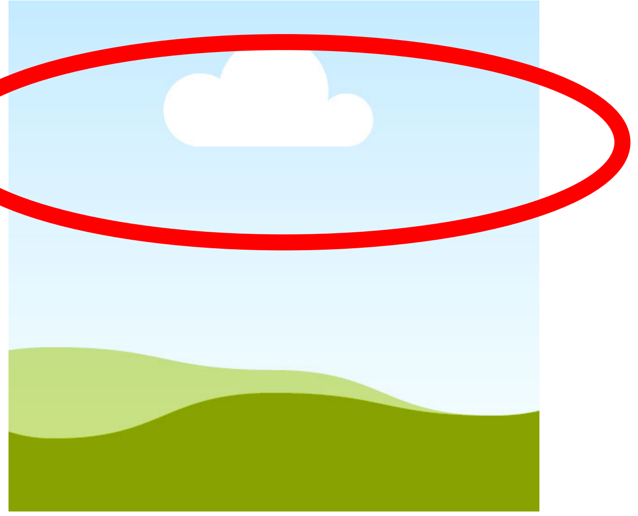
(c)

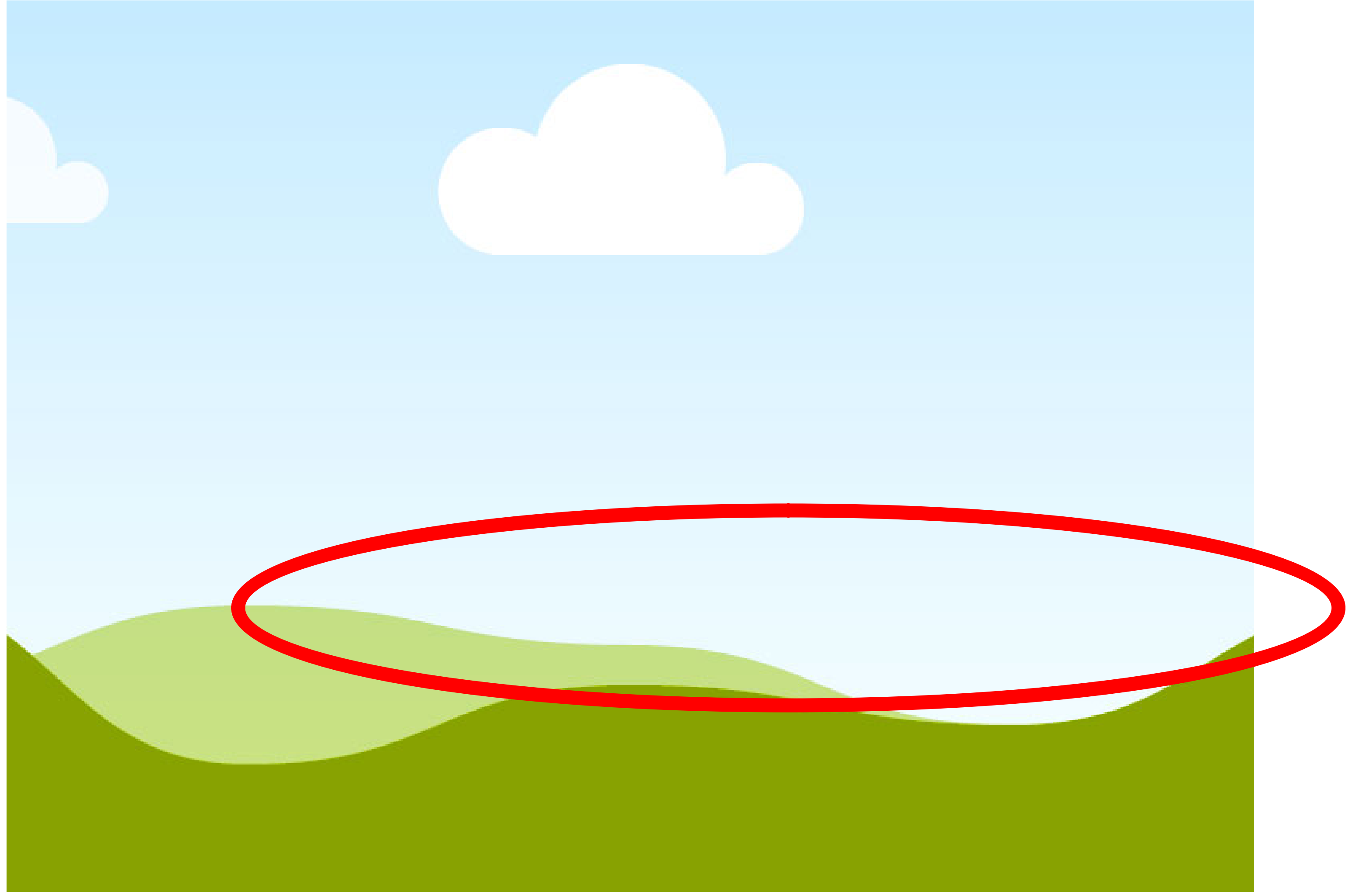


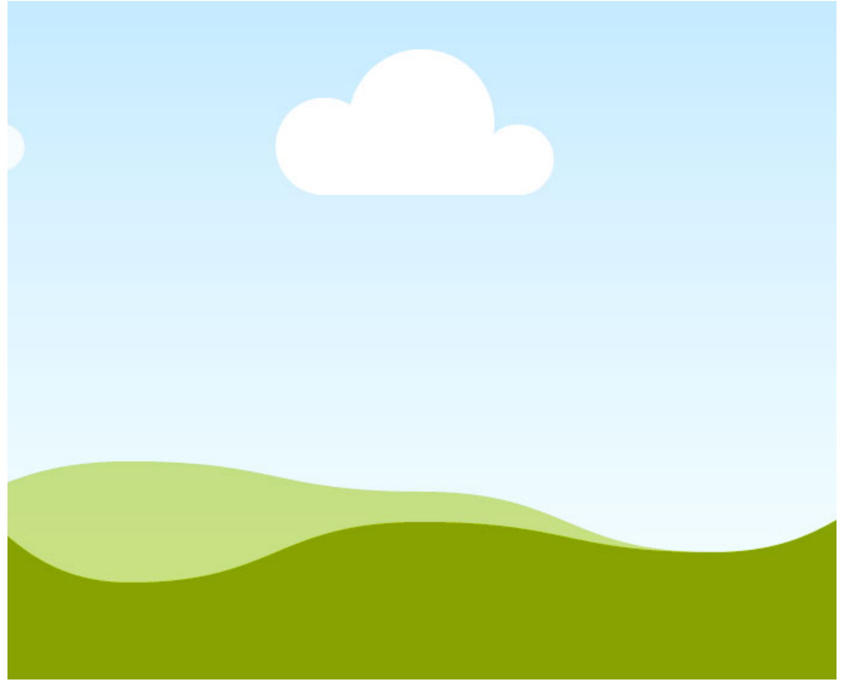
Validation of the VOF model – Energy budget and artificial dissipation



Surface tension contribution neglected, as in other studies

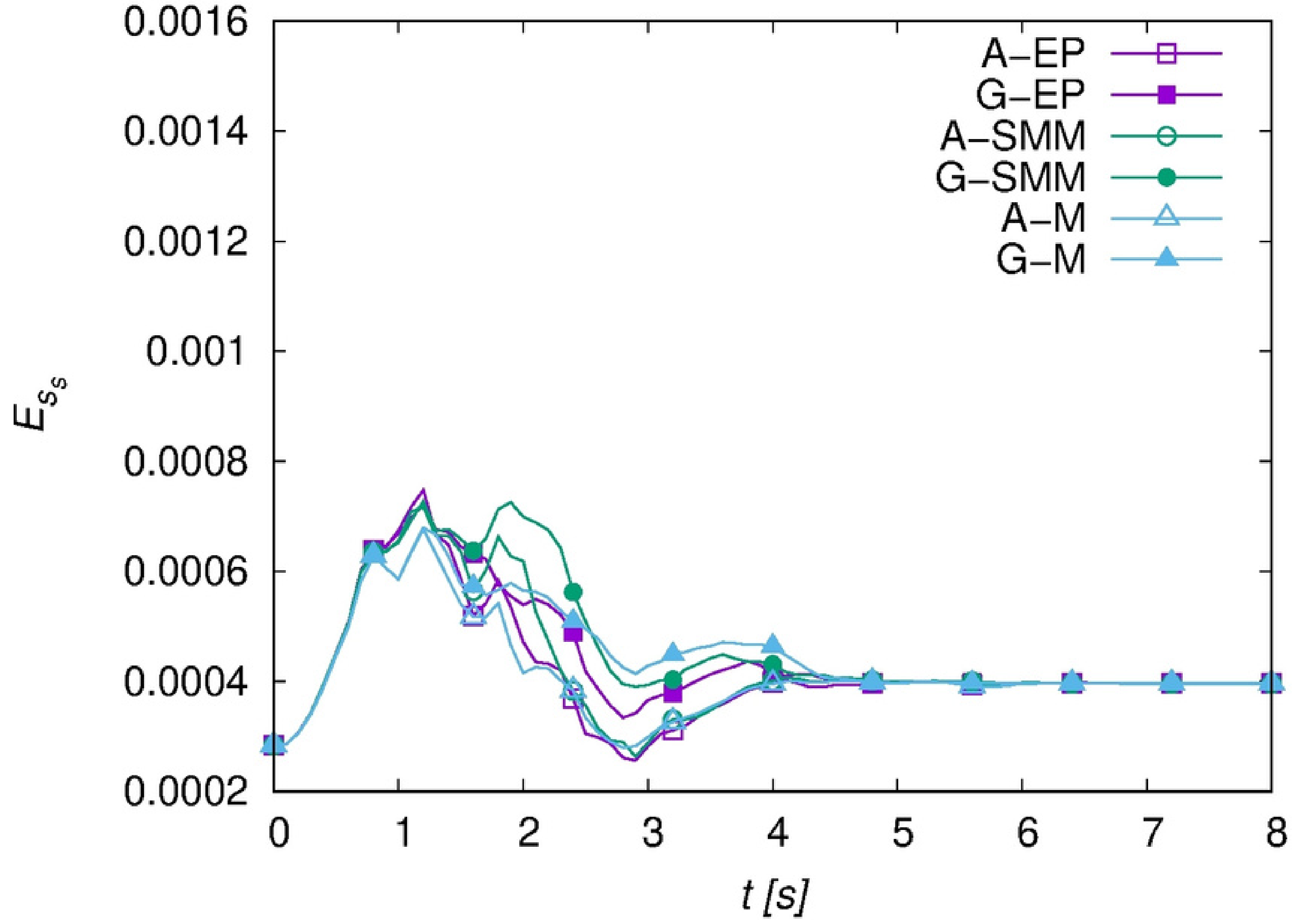


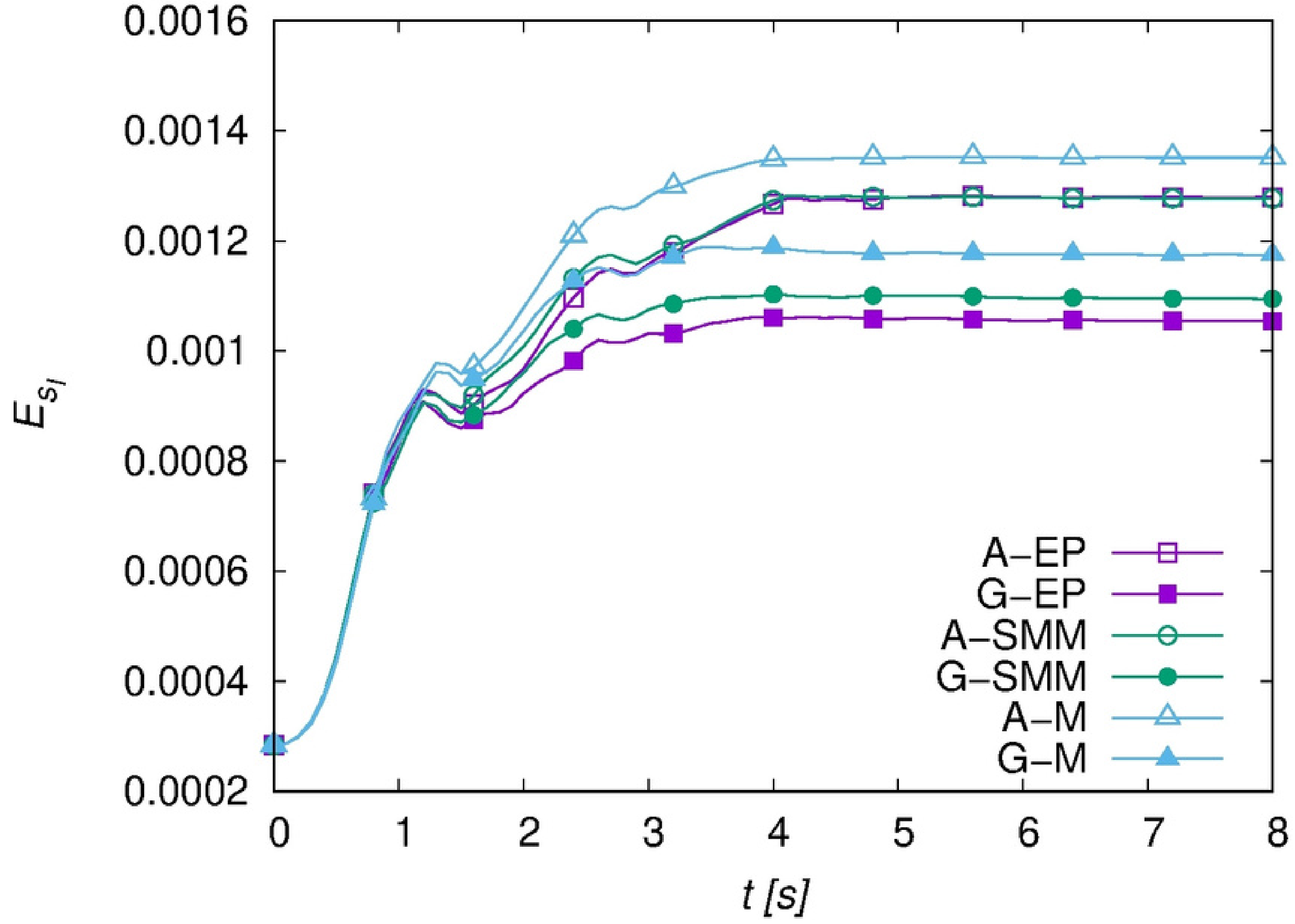


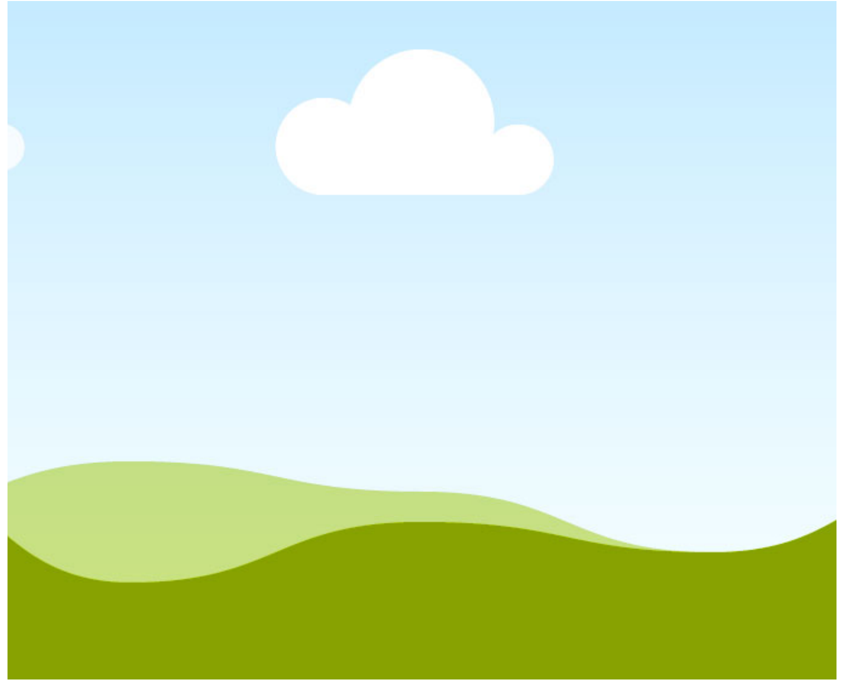


Validation of the VOF model – Energy budget and artificial dissipation

Surface tension contribution to the energy



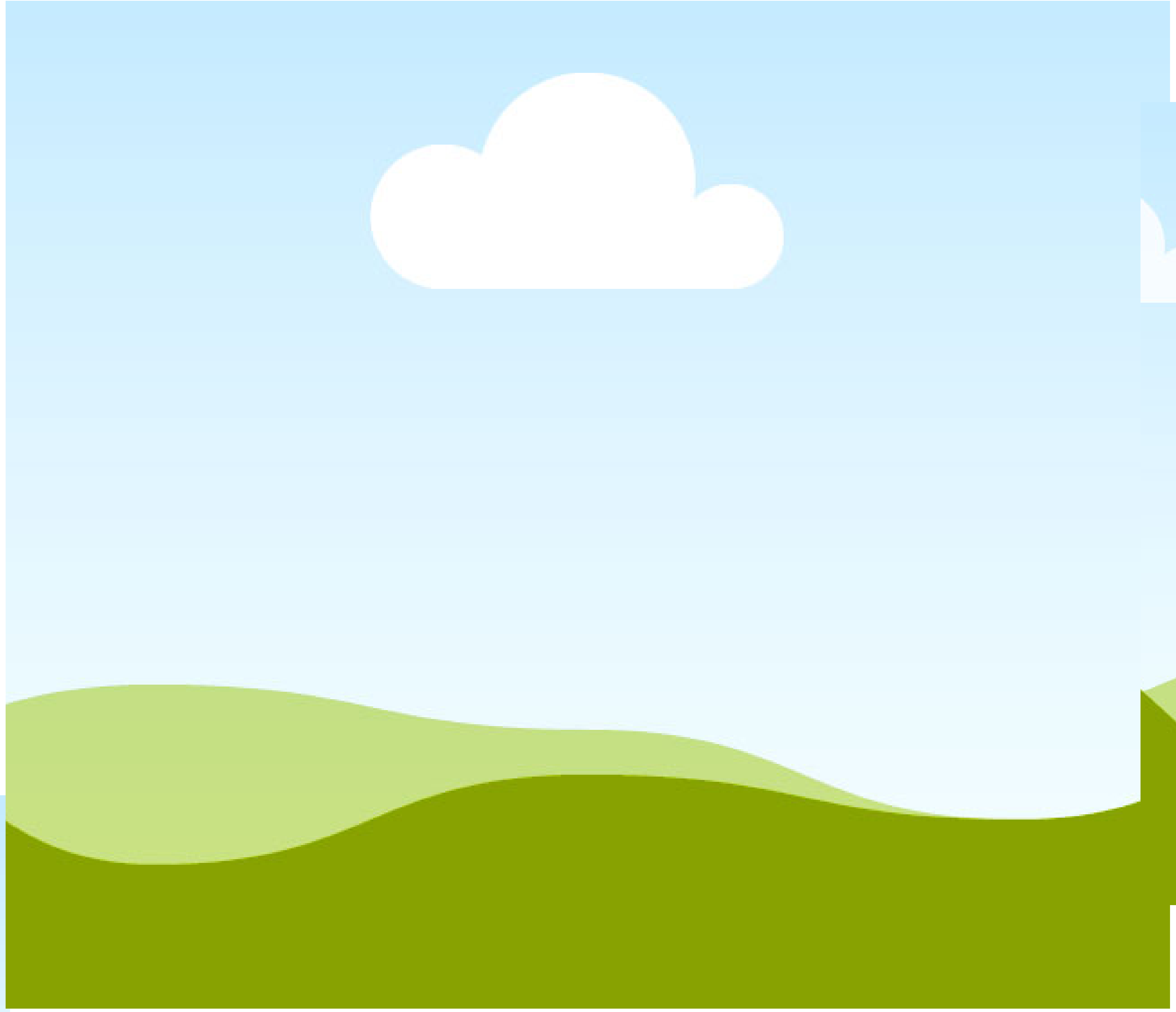


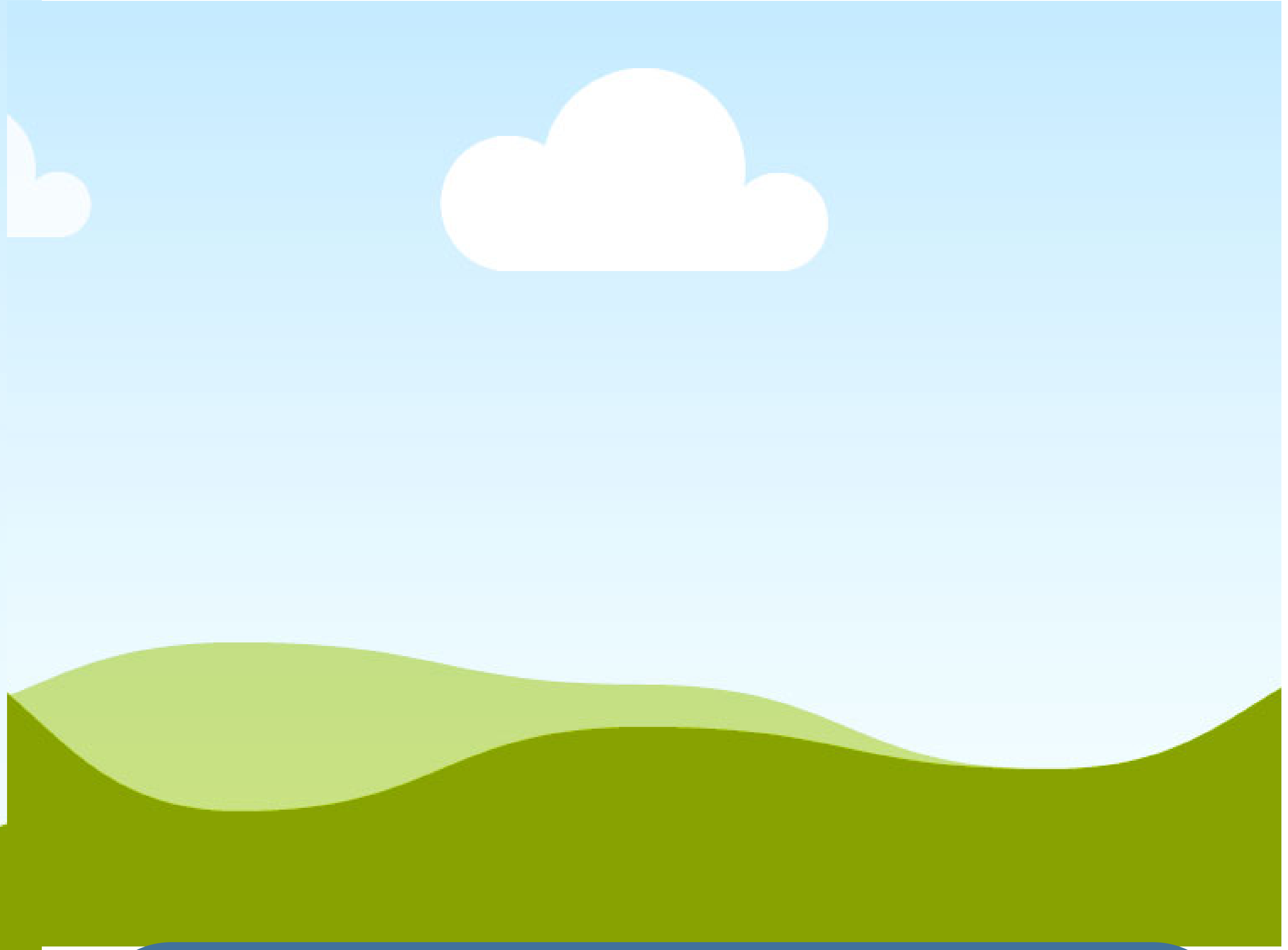


Large discrepancies after t = 1

Validation of the VOF model – Energy budget and artificial dissipation

Surface tension contribution to the energy



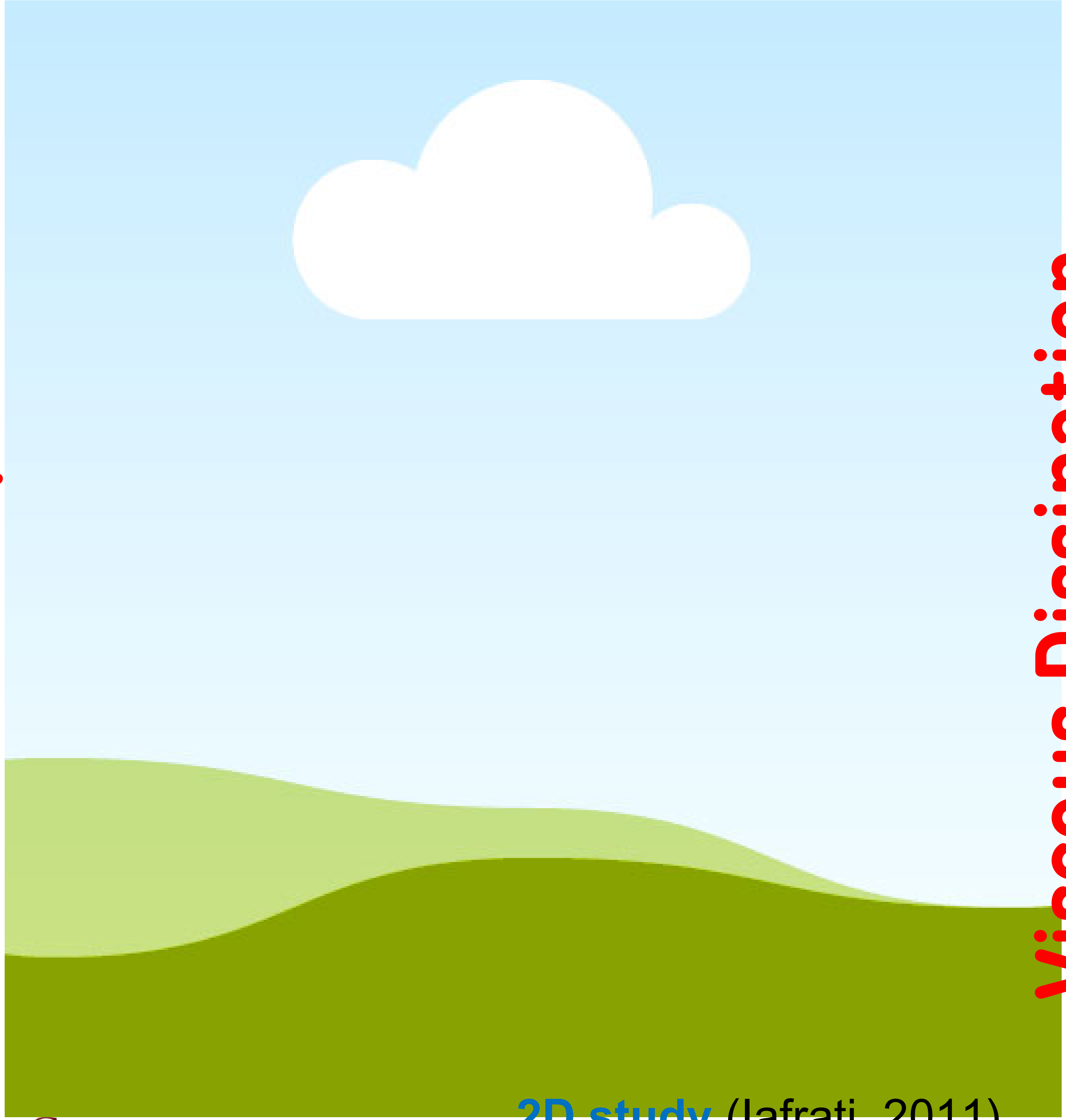


Differences due to the limits in the model in resolving surface tension forces

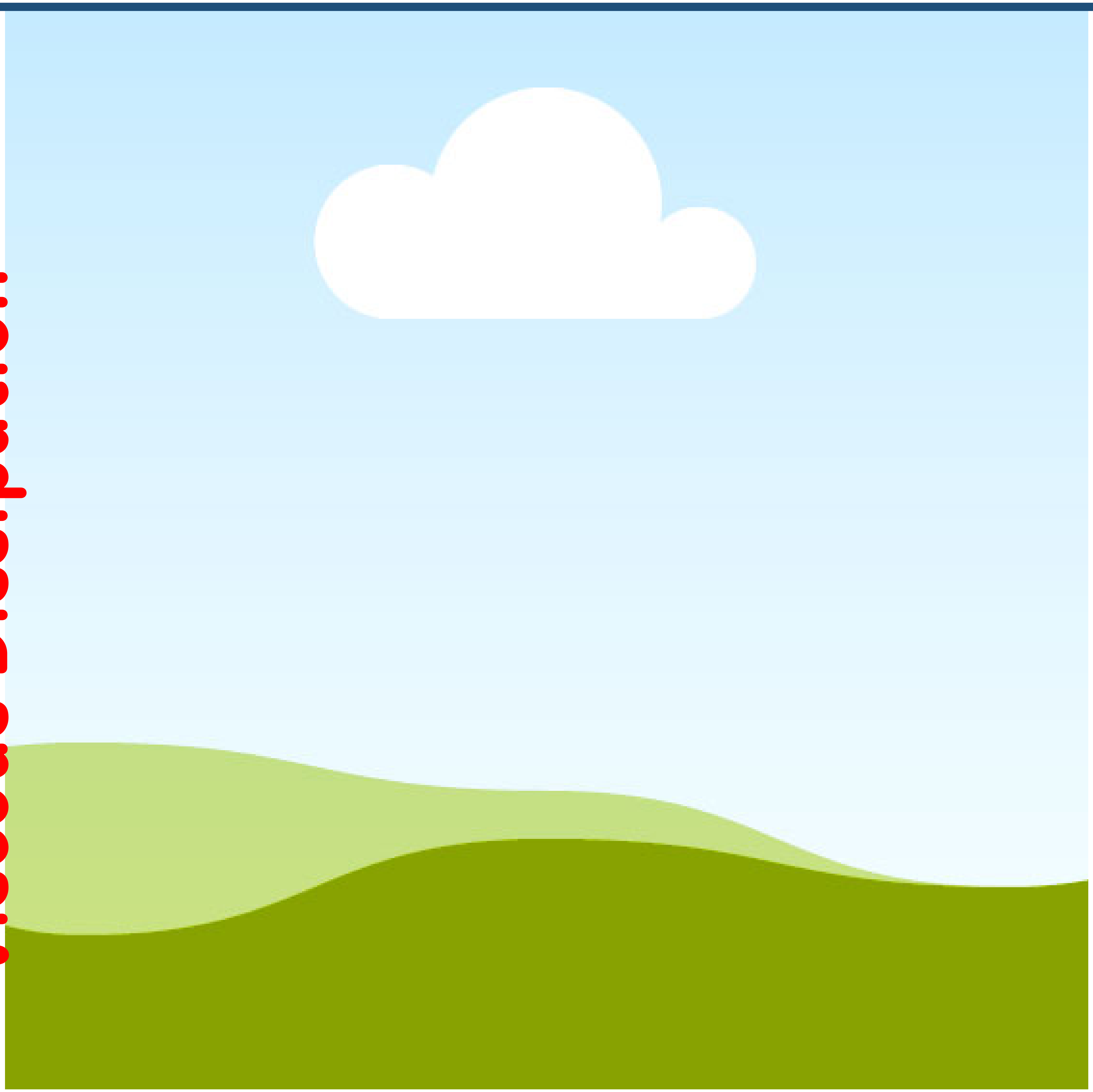
Integral of surface tension power during the forced expansion and contraction of a circular cylinder: limits in describing the contraction at low resolution

Energy Dissipation & bubble fragmentation

Vorticity



Viscous Dissipation



2D study (Iafrati, 2011)