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Decomposing Realistic Ocean Flow in Balanced and Unbalanced Parts Using a Novel Balancing Approach

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1. Flow Decomposition

Animation:

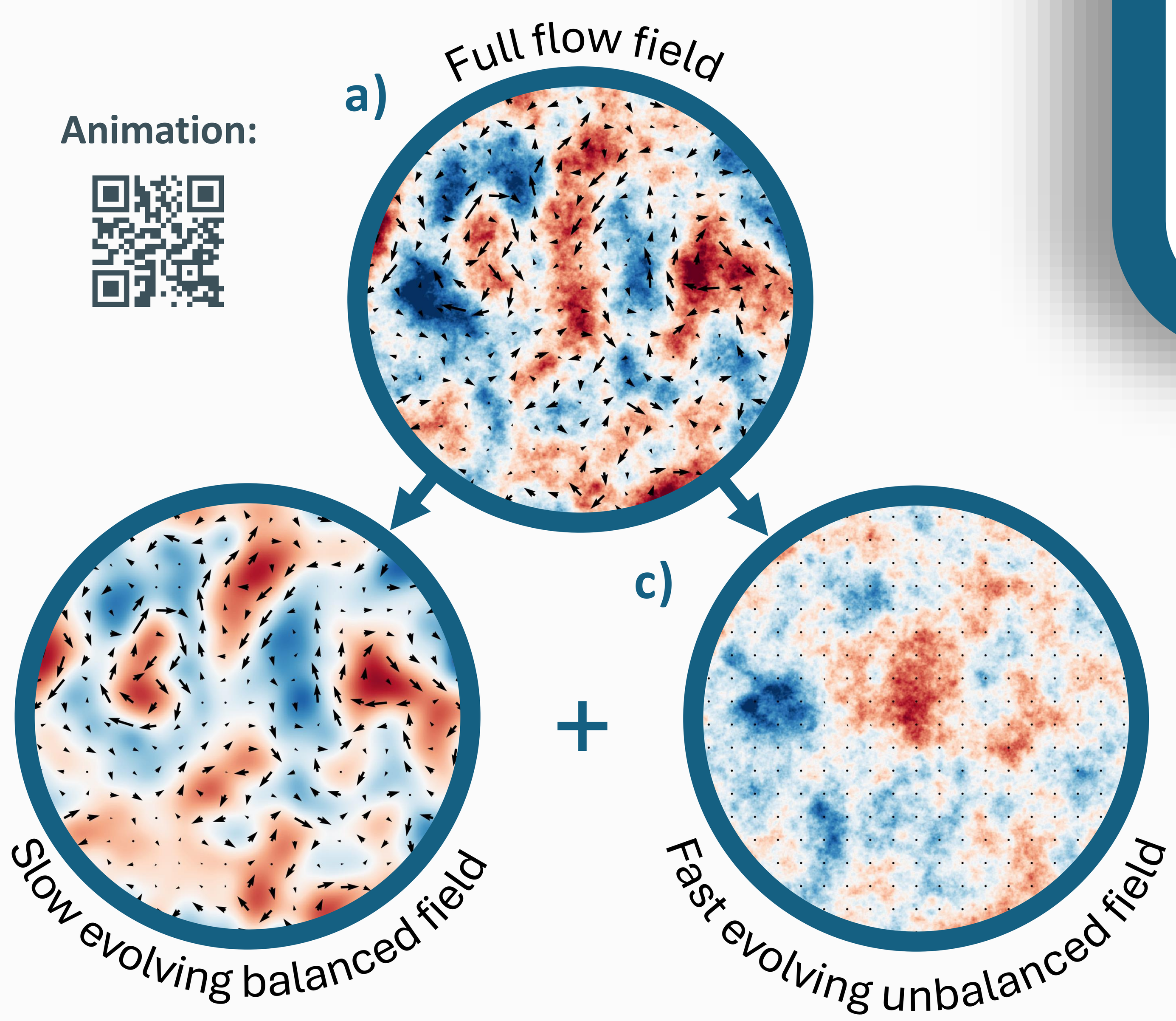

Fig. 1: Decomposition of a 2D shallow water flow field in balanced and unbalanced fields using optimal balance. The colors show the layer thickness.

Why do we need flow decomposition?

- To understand and quantify energy transfer between different scales (e.g. dissipation of mesoscale eddies)

What high accuracy decomposition method exists?

- Nonlinear normal mode decomposition (Machenhauer, 1977)
- Optimal Balance (Viúdez et al., 2004; Masur et al., 2020)

2. Optimal Balance (OB)

- Transforms nonlinear decomposition problem into linear problem
- **Linear problem**: Projection onto linear geostrophic eigenspace
- Solve linear problem with spectral decomposition
- Spectral decomposition require idealized model setups:
  - **⇒ no topography, no lateral boundaries**
  - **⇒ constant coriolis parameter.**
- **We replace spectral decomposition with time averaging**

Animation:

Existing high accuracy decomposition methods are limited to idealized models (e.g. no topography).

- **We modify the "Optimal Balance" method, to make it applicable to realistic model setups**

3. Modification of OB:

Geostrophic projection with time averaging

- Define cumulative linear time average:

$$\langle z\rangle_T = \frac{1}{T}\int_{t_0}^{t_0+T} z(t)dt \qquad (1)$$

z stands for u, v, w, etc. — linear evolution (no advection)

- Linearized shallow water / non-hydrostatic boussinesq equations on f-plane can be decomposed into two modes*:

$$z(\boldsymbol{k},t) = \underbrace{z_{geo}(\boldsymbol{k})}_{\text{geostrophic mode}} + \underbrace{z_{wave}(\boldsymbol{k})e^{-i\omega(\boldsymbol{k})t}}_{\text{inertia-gravity wave mode}} \qquad (2)$$

⇒ Time average converge to geostrophic mode:

$$\langle z\rangle_T = z_{geo} + \delta_T \cdot z_{wave} \quad \text{with} \quad \delta_T = \frac{i}{\omega T}\left(e^{-i\omega T} - 1\right) \qquad (3)$$

- Faster convergence with multiple time averages:

$$\langle \dots \langle z\rangle_{T_1} \dots \rangle_{T_n} = z_{geo} + \left(\delta_{T_1} \cdot \dots \cdot \delta_{T_n}\right) z_{wave} \qquad (4)$$

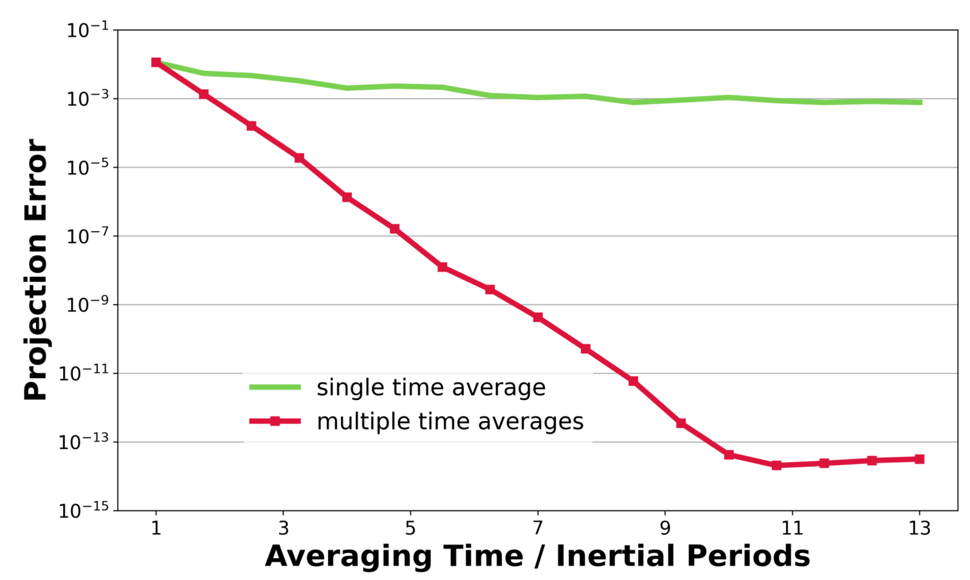

Fig. 2: The projection error is the norm of difference between the time averaged state and the geostrophic mode obtained with spectral decomposition. Experiments are performed with shallow water model with the initial condition shown in Fig. 1.

* There are actually two inertia-gravity wave modes which are here simplified to a single one

4. Optimal Balance with time averaging (OBTA)

Theory:

$$\underbrace{\mathcal{B}_{OBTA}(z)}_{\text{balanced state obtained with OBTA}} = \underbrace{\mathcal{B}_{OB}(z)}_{\text{balanced state obtained with OB}} + \underbrace{\delta\,(\overbrace{z - \mathcal{B}_{OB}(z)}^{\text{unbalanced state}})}_{\text{balancing error}} \qquad (5)$$

(δ: time averaging damping factor)

⇒ $\mathcal{B}_{OBTA}$ converges to $\mathcal{B}_{OB}$ when δ converges to zero

⇒ Apply $\mathcal{B}_{OBTA}$ iteratively to reduce the balancing error

Numerical Experiments:

- Two different models
  - 2D shallow water model
  - 3D non-hydrostatic boussinesq model
- Different initial conditions (unstable jet, random phase)
- Rossby numbers ranging from 0.03 to 0.5

GitHub: FRIDOM: Framework for Idealized Ocean models

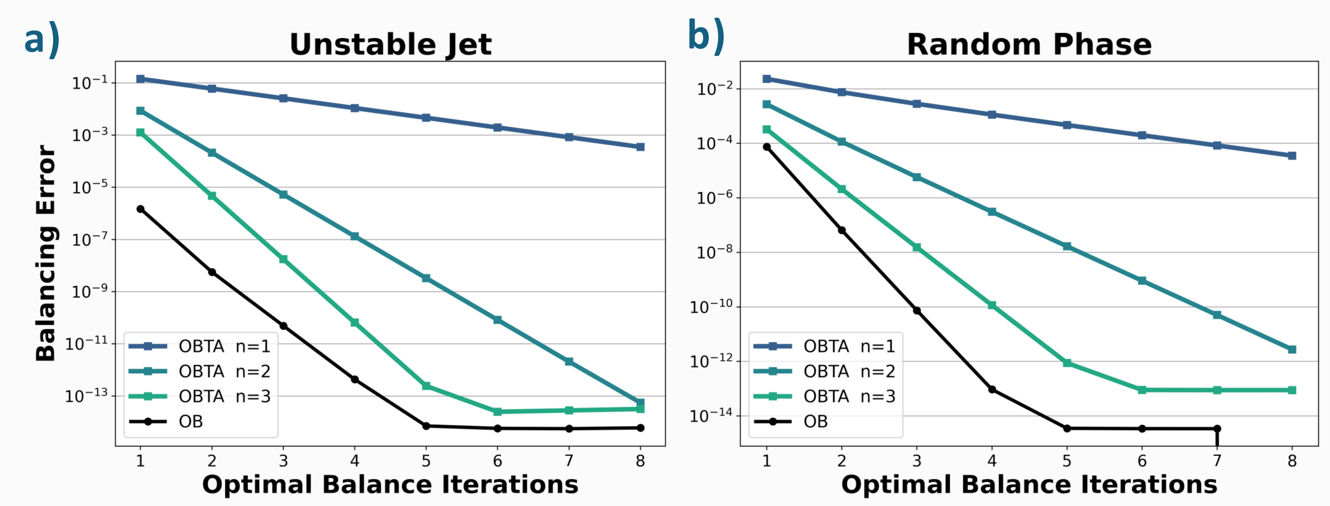

Fig. 3: The balancing error is the norm of difference between the balanced state of the corresponding method and $\mathcal{B}_{OB}(z)$ for ten iterations. Experiments are performed in the shallow water model with a Rossby number of 0.1. The number n refers to the number of time averages.

- **All numerical results show general agreement with Eq. (5)**
- Differences between $\mathcal{B}_{OBTA}(z)$ and $\mathcal{B}_{OB}(z)$ get extremely small

New method OBTA can be applied to models with topography and lateral boundaries!

- **Solution of OBTA approaches solution of OB**

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