
Generation and pathways of internal waves on a subtropical continental shelf

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Hypothesis and Goals

It is expected that at the tropical and large South Brazil Bight (Fig 1) the interaction of slope generated internal tides with the coastal topography (Whalen et al., 2020) be the main responsible for observed internal waves near an ecological sanctuary. From an energy perspective, here we sought to:

- i) Quantify the kinetic energy at the (sub)tidal bands available to the internal processes;
- ii) Quantify the energy transfer from barotropic tidal dynamics to baroclinic ones and its loss in the water column;
- iii) Identify the mechanisms responsible for the energy transfers;
- iv) Locate hotspots of baroclinic tidal energy generation.

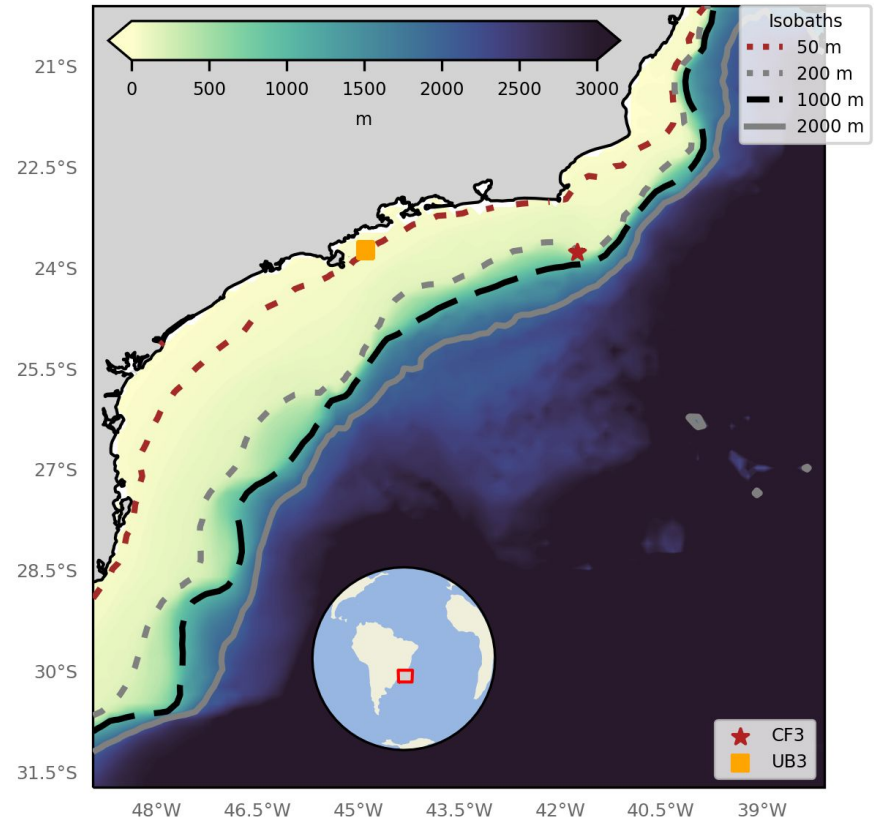


Fig 1: South Brazil Bight location and bathymetry, with 2 moorings (CF3 & UB3) located.

Diving into the data

Groups of internal waves with lengths from 1 to 5 km, spaced 30 km apart have been identified via True Color MODIS products (Jackson, 2012) during summers (Fig 2).

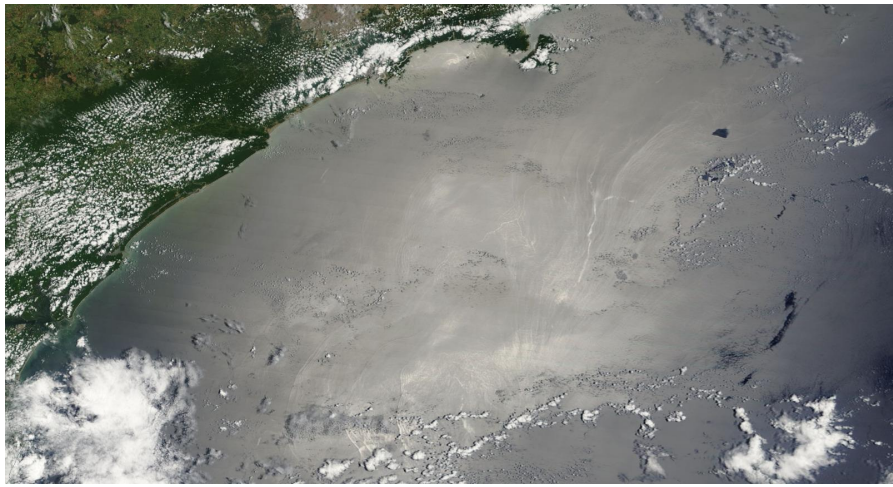


Fig 2: True-color image from MODIS at the SBB coast at 05.01.2022 with internal waves marked at the surface.

Larger energy peaks associated with periods > 46 h and intense energy around the M2 tidal period near the 85 m depth (Fig 3) for mooring located at 200 m isobath.

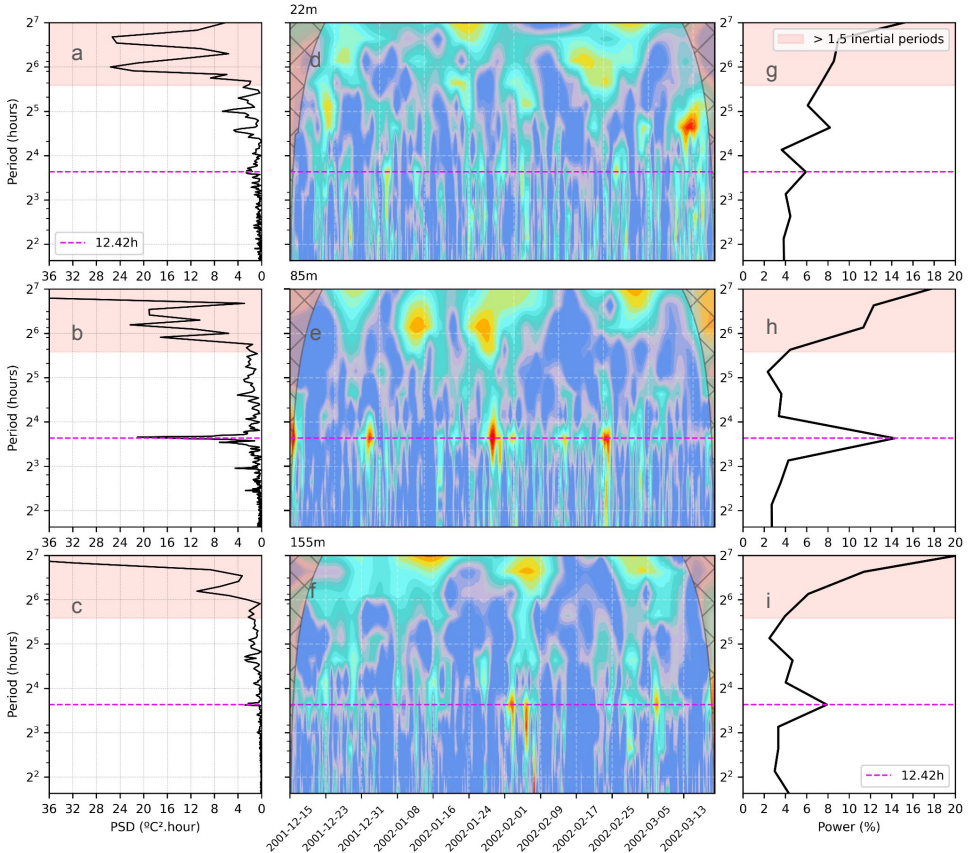


Fig 3: Power Spectrum Density of temperature data at CF3 mooring for 22 m, 85 m, 155 m depths (a,b,c). Wavelet to temperature data with 95% significance (d,e,f) and the respective summation of energy at the period relative to the total energy at the period interval (g,h,i).

The importance of Bathymetry

- α (Fig 4) as the ratio between the bottom slope and the ray-path angle (s) (Baines, 1982). $\longrightarrow s = \sqrt{\frac{\omega^2 - f^2}{N^2 - \omega^2}}$
 - $\alpha \sim 1$: critical regime near the coast until the 200 m isobath \rightarrow M2 ray to propagate along the isobaths;
 - $\alpha > 1$: supercritical regime along the 1000 m isobath \rightarrow generated internal tides will be reflected back to the open ocean.

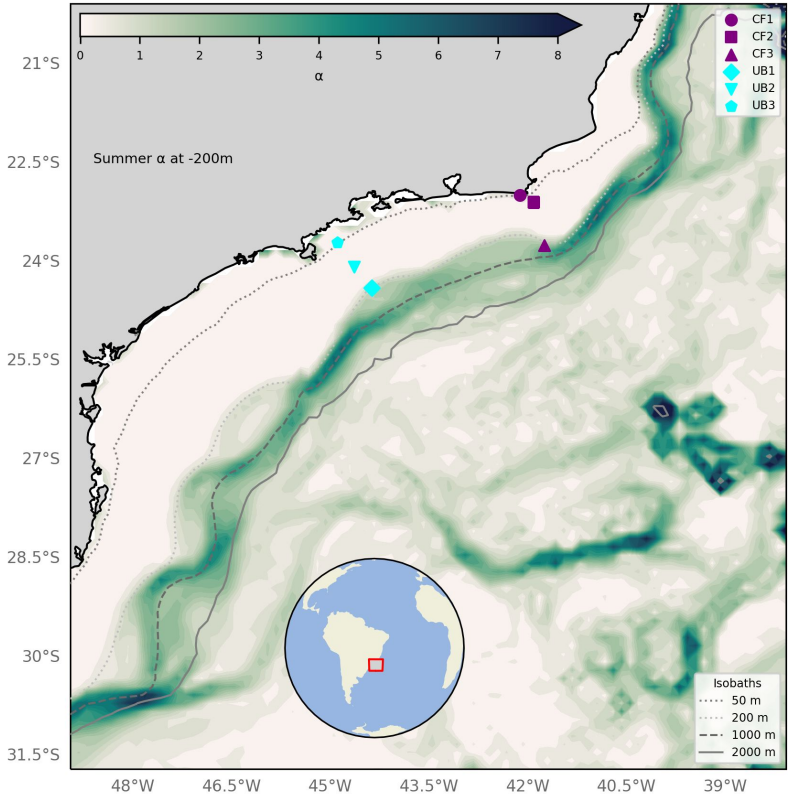


Fig 4: α ratio at 200 m depth for summer climatology at the SBB with purple (Cabo Frio) and Cyan (Ubatuba) markers indicating moorings locations.

The importance of Bathymetry

- Simulations with Regional Ocean Modeling System (ROMS) (Osborne, 2011) for Dec 2001 - Jan 2002, 3km horizontal resolution, smoothed bathymetry, 40 vertical sigma-layers (Fig 5);
 - Good enough to simulate the internal dynamics;
- ERA5, 11 tide components (TPXO 9.2). Boundary conditions from GLORYS' reanalysis; .

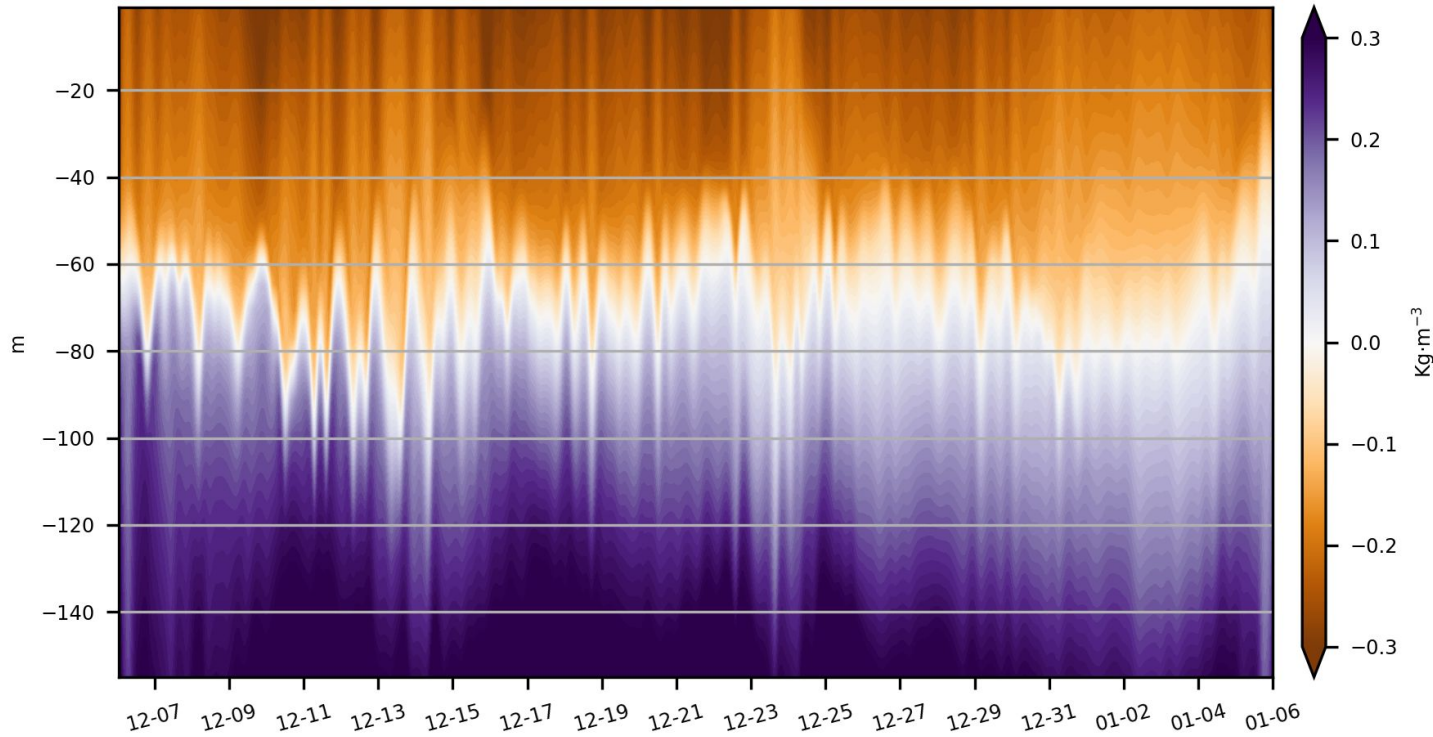


Fig 5: High-passed density perturbation at CF3 (Fig 1) mooring location through simulated period.

Baroclinic M2 Energetics: Energy between frequencies

- 30.6% of the total kinetic energy comes from tidal/supertidal processes (Fig 6);
 - The kinetic energy (KE) ratio is calculated following Equation 2 where the δ ratio as the ratio between the subinertial KE by the total.
 - The tilde denotes the low-pass filter of the baroclinic velocities u and v .

$$\delta = \frac{1}{2} \left(\frac{\tilde{u}_{bc}^2 + \tilde{v}_{bc}^2}{u_{bc}^2 + v_{bc}^2} \right) \times 100\%$$

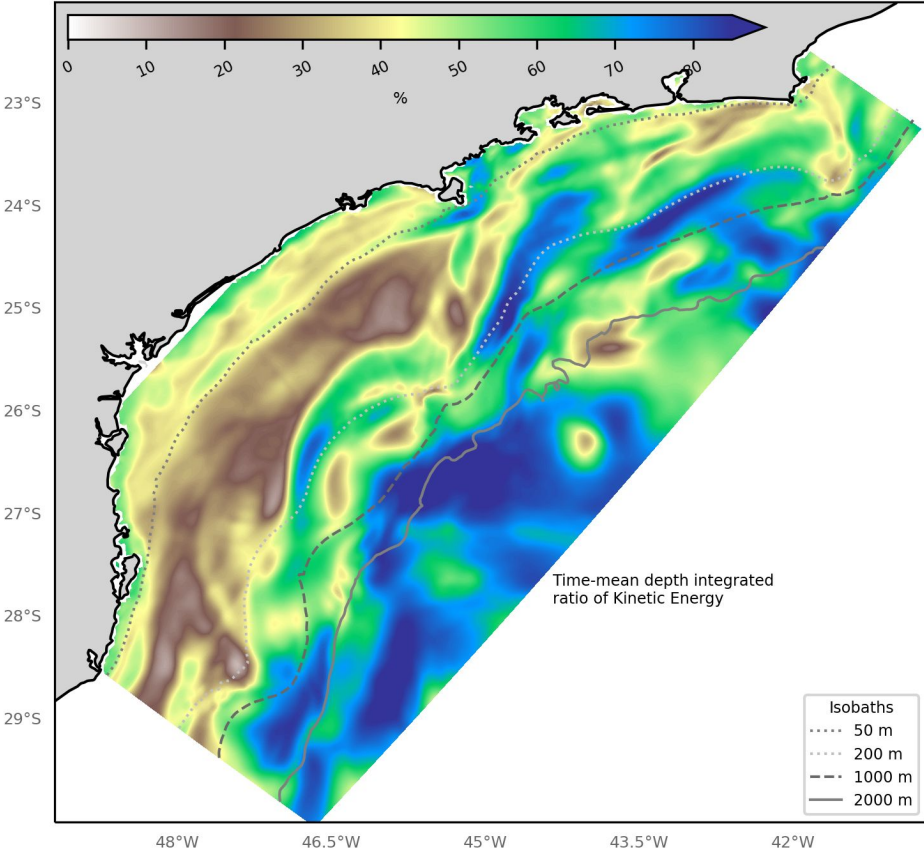


Fig 6: Time-mean depth integrated δ ratio of Kinetic Energy at the SBB.

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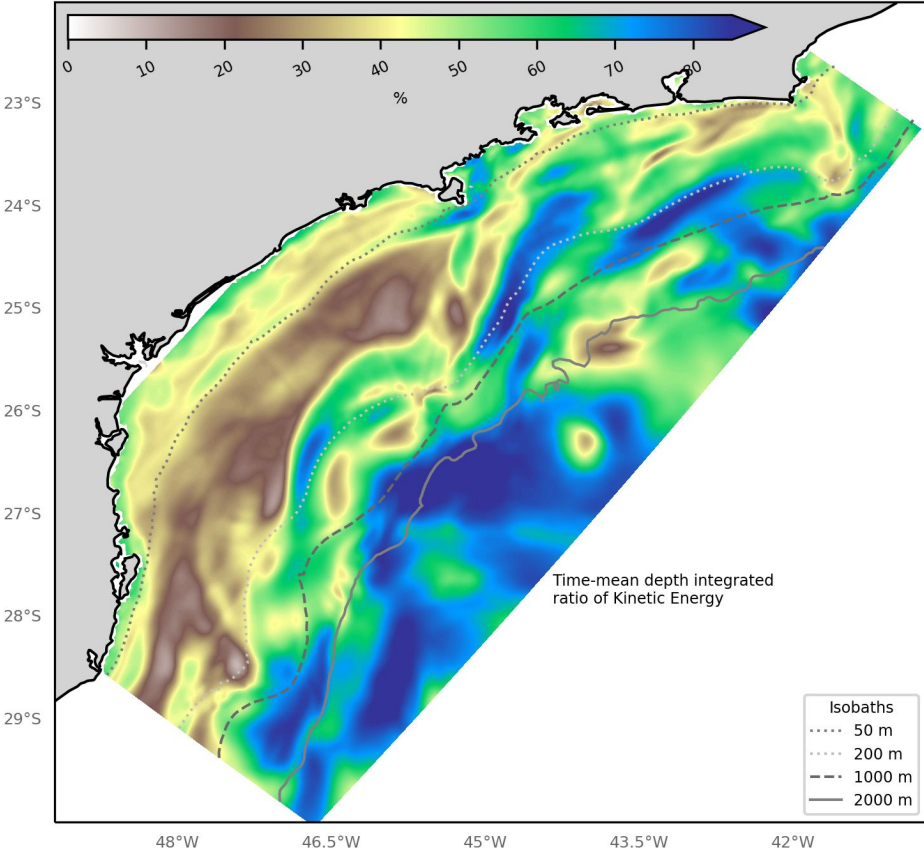


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Baroclinic M2 Energetics

For the analysis of baroclinic tidal energy, two approaches are common in the literature: (i) the separation into vertical modes by the Sturm-Liouville decomposition; (ii) assume baroclinicity as the deviation from the vertical mean (Kurapov et al., 2003).

Here we deal with the second approach, once the SBB has its tidal movements mostly barotropic (Pereira and Castro, 2007), and the first two dynamical modes (barotropic and 1st baroclinic) are responsible for answering more than 60 % of all the observed structures in the SBB. Furthermore, the Depth Mean Total Kinetic Energy at the middle SBB is 70 % baroclinic (mainly 1st mode) (Amorim et al., 2023).

Based on the linear theory (Kurapov et al., 2003), the depth-integrated, tidally averaged energy balance for the harmonically varying ocean is approximately, where TEC is the topographic conversion of barotropic to baroclinic tidal energy (i.e the conversion from barotropic to baroclinic) and EF is the depth-integrated baroclinic tidal energy flux (EF) vector.

In the equations the tilde denotes complex harmonic constants, the asterisk complex conjugates. The residual term accounts for the effects of bottom friction, eddy dissipation throughout the water column, and nonlinear advection effects. Here it is estimated as $TEC - \nabla \cdot \mathbf{EF}$.

$$TEC \approx \nabla \cdot \mathbf{EF} + Res$$

$$TEC = -\frac{1}{2} \text{Re} \left\{ \tilde{P}_{M2bc}^* \Big|_{z=-h} \mathbf{u}_{M2bt} \right\} \cdot \nabla h$$

$$\mathbf{EF} = \frac{1}{2} \int_{-h}^0 \text{Re} \left\{ \tilde{\mathbf{u}}_{M2bc} \cdot \tilde{P}_{M2bc}^* \right\} dz$$

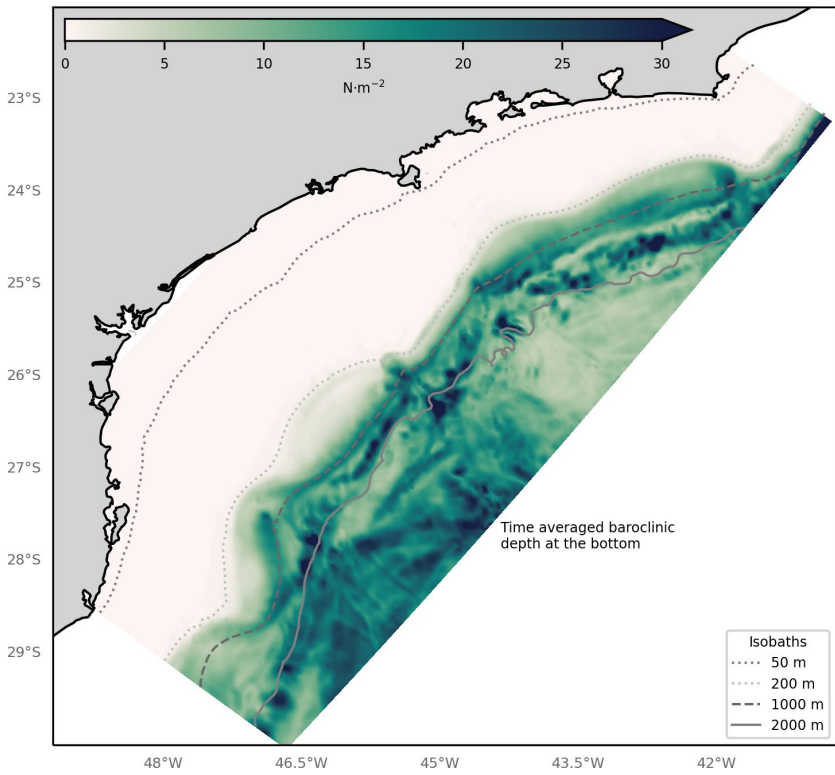
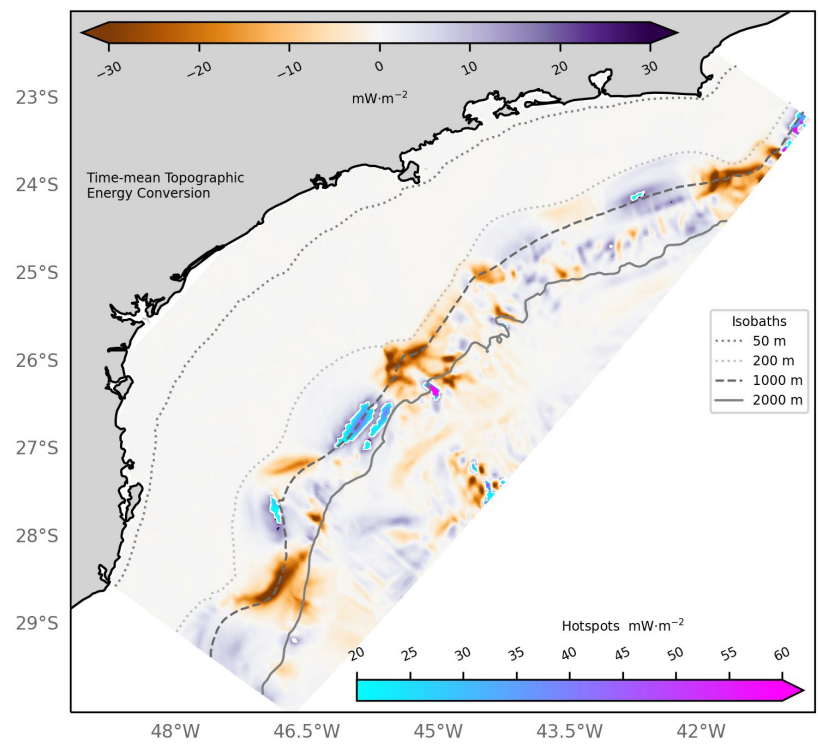


Fig 7: Time-mean M2 baroclinic pressure at the bottom

Baroclinic M2 Energetics

High TEC along the 1000 and 2000 m isobaths. Horizontal resolution of the model insufficient to estimate conversion and EF in regions shallower than 200m.

Interestingly, the TEC patterns are almost organized in regularly spaced alongshore banding patterns with separations of $0.5^\circ - 1^\circ$ along the slope and outer shelf, compatible at 5 - 10 times the M2 internal tidal wavelengths for such depths. Crossshore these patterns concern 2 - 4 times the M2 internal tide wavelength for the same depths.



The TEC hotspots were computed from the values higher than 3 standard deviations.

Fig 8: Time-averaged Topographic Energy Conversion (TEC) at the SBB (orange to purple upper colorbar), and Hotspots of TEC highlighted (cyan to fuchsia lower colorbar).

Baroclinic M2 Energetics

The residual may contain both true baroclinic energy dissipation and scattering to the incoherent tide (when the tide is not phase locked constant in amplitude and phase). Accounts, then, for the effects of bottom friction, eddy dissipation throughout the water column, nonlinear advection effects and heat dissipation due to viscous forces (Kurapov, 2003).

To further map the energy cascade from tidal periods to the supratidal ones, the energy transfers as a function of time and space in ROMS with a coarse-graining approach (Eyink and Aluie, 2009) that was used for the first time at the scope of internal waves by Solano et al. (2023), and it is used to quantify energy transfer across an arbitrary time scale (τ) here defined as the 12h cutoff period of Lanczos filtering.

The coarse-grained kinetic energy transfer (W/m^2) (Fig 8) can be written as the Equation 3. Time-mean area summation yields the transfers from tidal to higher-harmonic frequencies <-- what comes from the residue due to the nonlinear advection effects. I.e, forward at the energy cascade + inverse in the energy cascade.

$$\Pi_{\tau}(x, t) = -\rho_0 (\overline{u_i u_j} - \overline{u_i} \overline{u_j}) \frac{\partial \overline{u_i}}{\partial x_j}$$

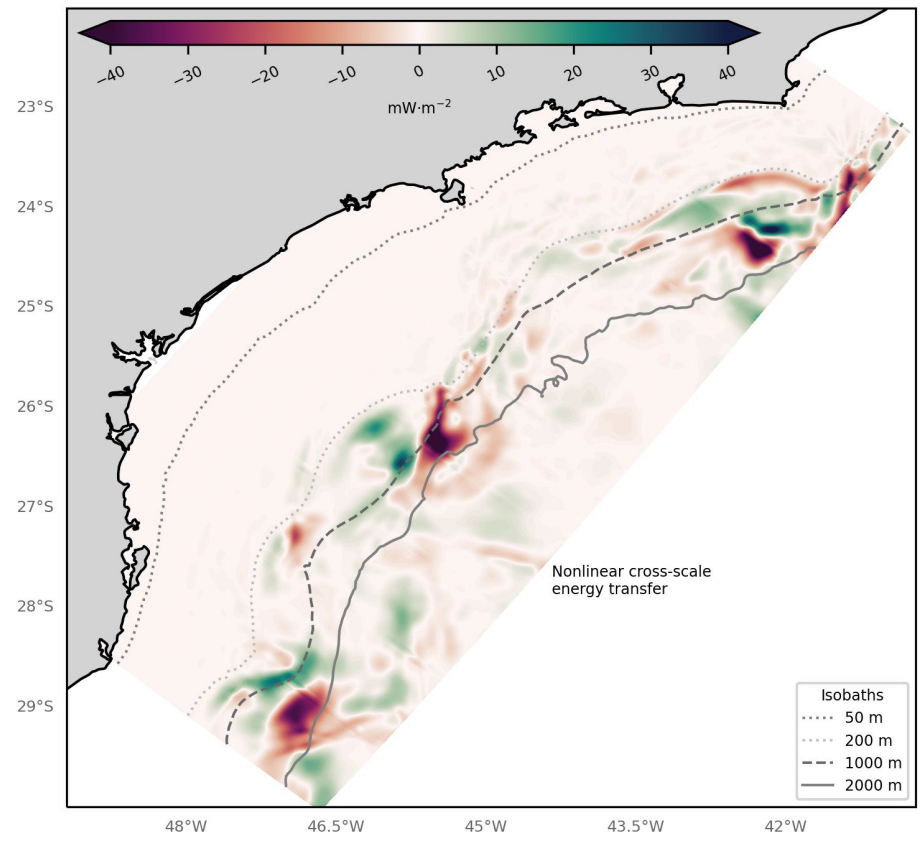


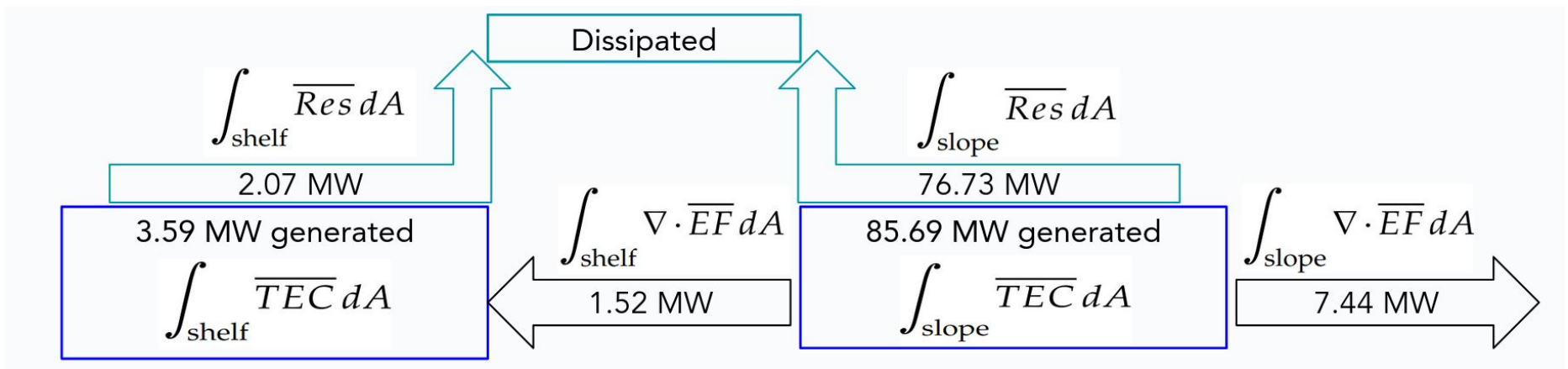
Fig 9: Time-mean depth integrated coarse-graining energy transfer,

Energy Budget

Due to the rapidly topographic variation, the slope region is expected to allow mostly TEC, and is also the region used to define where end the domain of the shelf while the offshore regime starts over the energy equations. Here the depth of 200 m is selected as the shelf SBB shelf break (Castro, 2014).

Integrating the terms from the energy equation over the slope ($h > 200$ m) and over the shelf ($h < 200$ m), results the off and onshore fluxes, as well the dissipated energy. More specifically, integrating the divergence of $\nabla \cdot \overline{EF}$ over the slope returns the offshore flux, hence integrating the divergence of $\nabla \cdot \overline{EF}$ over the shelf yields the onshore flux.

The integral of TEC yields the total energy conversion available to the previously said fluxes and to the dissipation calculated as the integral of residual over the slope and shelf, respectively.



Summary

- ❑ The few Hotspots account for more than 38% of all transferred energy
- ❑ 87.53% of the dissipation is due to nonlinear advection effects.;
- ❑ ~2% of the energy flow to the shelf, yet at the SBB the internal tide shows great potential to steep into nonlinear solitary waves.

Future work

- Resolution finer than 1 km at the shelf:
 - Account for less nonlinear dissipation (Solano et al., 2023).
- Subtidal forcing investigation:
 - Quantify the importance of wind and tide to the Internal Waves generation.

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