Beating Nyquist

Unlocking insights in historic tidal records with analysis methods tailored to high-low tidal data

Introduction & objective

Conventional tidal analysis methods don't fully utilize the information embedded in high-low tidal records (e.g Fig 1). Thus, specialized analysis techniques tailored to high-low tide records are preferred.

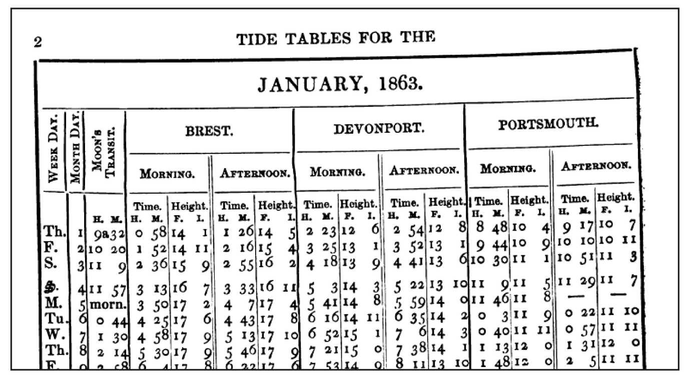

Fig 1. A page from 1863 tide tables for British and Irish Ports, giving daily high and low water predictions for Brest, Devonport, and Portsmouth.

Benefits of High-Low Tidal Analysis:

- **Understanding Historical Shifts**: Unveiling subtle changes in tidal hydrodynamics over time
- **Accurate Modeling**: Facilitating precise modeling of past tides and sediment transport patterns.
- **Estimating Hydroperiods**: Enabling better estimation of hydroperiods critical for ecological studies.
- **Extracting Subtidal Signals**: Extracting subtidal signals for studying long-term sea level trends, storm surges, and river-tide interactions.

This study aims to establish the degree to which the continuous tidal signal can be retrieved from high-low water level records

1

Why harmonic analysis doesn't work for high/low tidal records

2

Frequencies above M2 may alias into frequencies below M2 due to aliasing (Ji & Guohong, 1987):

$$\begin{cases}(\sigma_1+\sigma_2)\tau = m\cdot 2\pi, (m=1,2,\dots) \text{ or}\\ (\sigma_1-\sigma_2)\tau = m\cdot 2\pi, (m=\pm1,\pm2,\dots)\end{cases} \quad \text{(Eq. 1)}$$

Most major shallow water constituents form aliasing pairs (Fig 2), and should this be excluded according to the Nyquist-Shannon theorem.

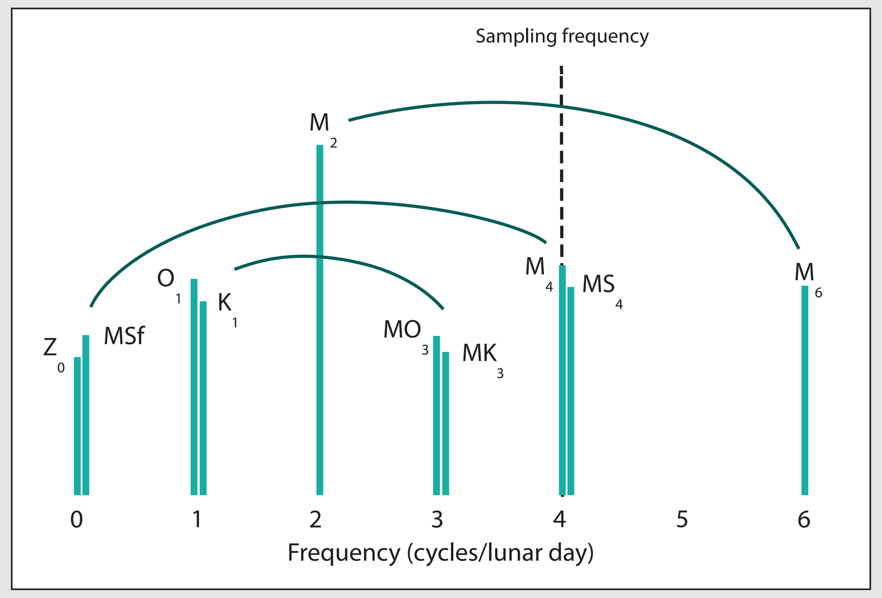

Fig 2. An example of several important aliasing pairs when sampling at an interval of 6 lunar hours.

How it could work...

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Sampling fluctuations:
Tidal & non-tidal variations in sampling rates can alleviate aliasing problems (e.g. Fig 3a).

Additional Constraints:
- Derivative equals zero
- Intermediate points between hw & lw
- Intermediate derivatives should correspond to rise/fall
- Other?

Leveraging Equilibrium Tide:
Beyond dictating dominant frequencies, the equilibrium tide offers more (Fig 3bc; Munk & Cartwright, 1966). Equilibrium amplitudes and phases can be exploited e.g. through regularization techniques.

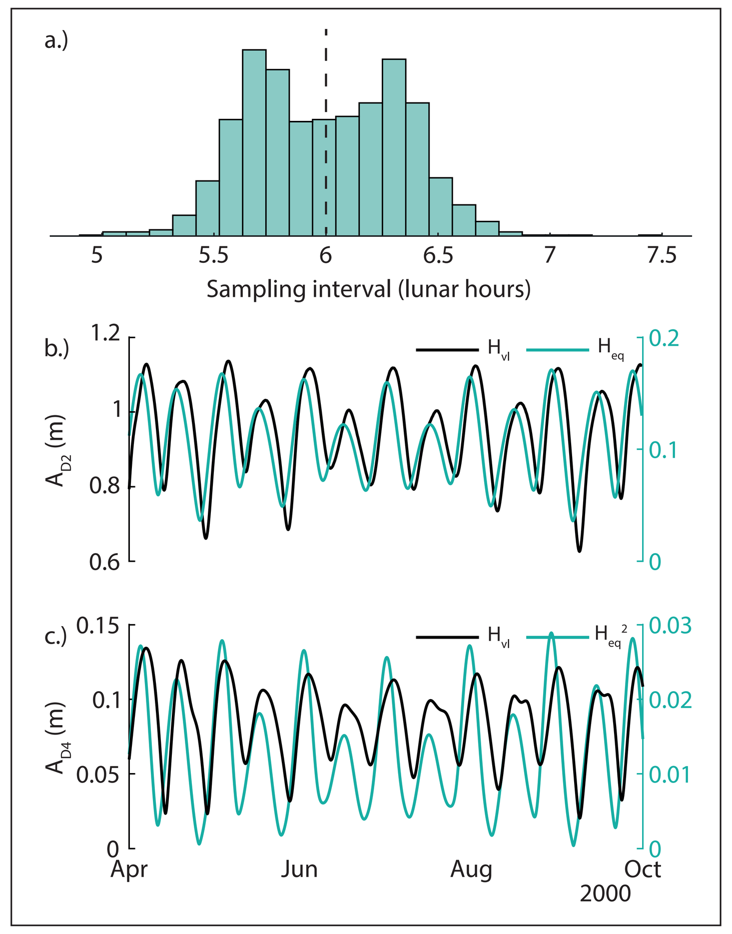

Fig 3. a.) Distribution of sampling intervals for high-low water observations at Vlissingen, The Netherlands. The dashed line represents the average sampling rate; b.) semidiurnal amplitude variations and Vlissingen and of the equilibrium tide; c.) fourth-diurnal amplitude variations at Vlissingen and of the squared equilibrium tide.

Preliminary results

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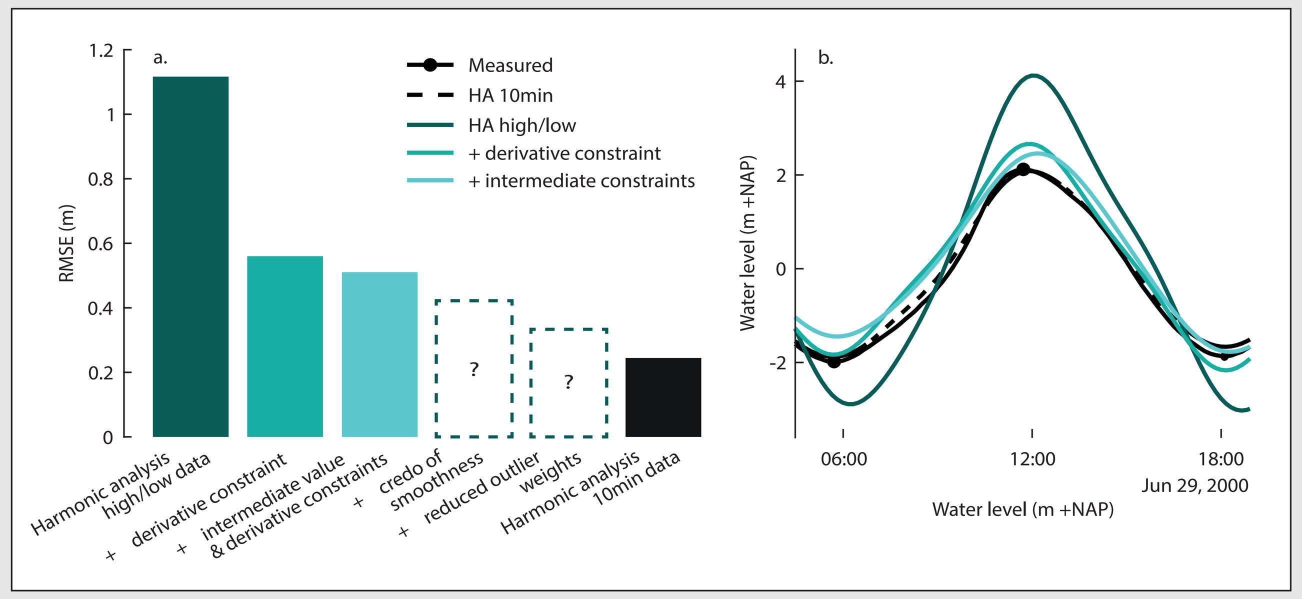

Fig 4. Comparison of analysis techniques in capturing measured water levels at Vlissingen, The Netherlands; (a) Root mean square error of the different methods for the year 2000, (b) Tidal hydrographs for the different methods.

Regular HA: Tidal amplitude is vastly overestimated (see Fig. 4b) for ordinary harmonic analysis

Derivative constraint: Results in a remarkable 50% reduction in RMSE (see Fig. 4a).

Further Enhancements: Integration of intermediate value and derivative constraints leads to an additional 4.9 cm decrease in RMSE.

Future Prospects: We anticipate further advancements by incorporating additional constraints (e.g., smoothness) and optimizing weights for improved accuracy.

Discussion & future outlook

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Enhanced Model Performance: Tailoring harmonic analysis techniques to high/low tidal records significantly improves model performance; incorporating derivative and intermediate constraints yields a RMSE reduction of >50%

Assessing Predictability: Assessing model performance across various tidal stations and noise levels promises further insights into the tidal predictability limits based on high/low water observations.

Other Promising Methodologies:
- Employing statistical models to replicate tidal responses to the equilibrium tide, requiring less data, could prove valuable.
- Utilizing "regular" harmonic analysis after interpolation of high/low water observations may also offer benefits.

Deltas out of Shape

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Munk, W. H., & Cartwright, D. E. (1966). Tidal spectroscopy and prediction. Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences, 259(1105), 533–581. https://doi.org/10.1098/rsta.1966.0024

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