Supplementary Material

Reference

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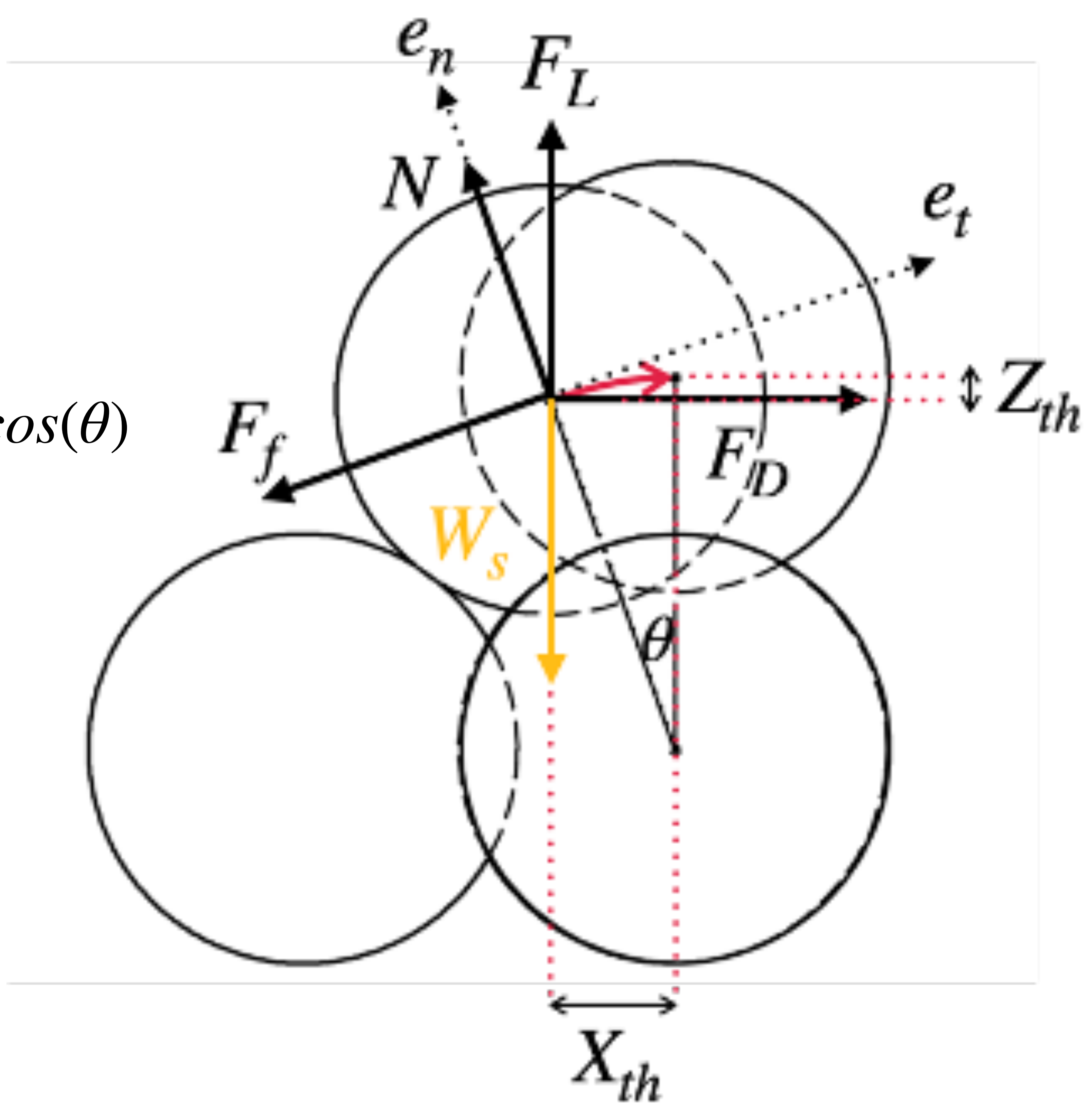
Supplementary Material

Method

A. Work-based Criterion (Lee et al., 2012)

1. Force balance equation (Tangential direction)
   1.1. Dimensional
      1.1.1. $(m_{*p} + \frac{1}{2}m_{*f})\frac{dv_{*t}}{dt_*} = \overbrace{F_{*D}(t_*)(cos\theta - \mu sin\theta) + F_{*L}(t_*)(sin\theta + \mu cos\theta)}^{F_{*e}(t_*)} - \overbrace{(m_{*p} - m_{*f})g_*(sin\theta + \mu cos\theta)}^{F_{*cr}}$
   1.2. Dimensionless [force scale: $(m_{*p} - m_{*f})g_*$; length scale: d_*; time scale: $\sqrt{d_*/g_*}$]
      1.2.1. $\frac{dv_t}{dt} = \frac{2(S_G - 1)}{2S_G + 1}\left(F_e(t) + F_{cr}\right)$;
   1.3. Integrating 1.1 to obtain $v_t(t)$
      1.3.1. $v_t(t) = \frac{2(S_G - 1)}{2S_G + 1}\left[\int_0^t F_e(\tau)d\tau - \int_0^t S(\tau)d\tau\right]$, where $S_G = \frac{\rho_p}{\rho_f}$; $S(t) = sin(\theta) + \mu cos(\theta)$



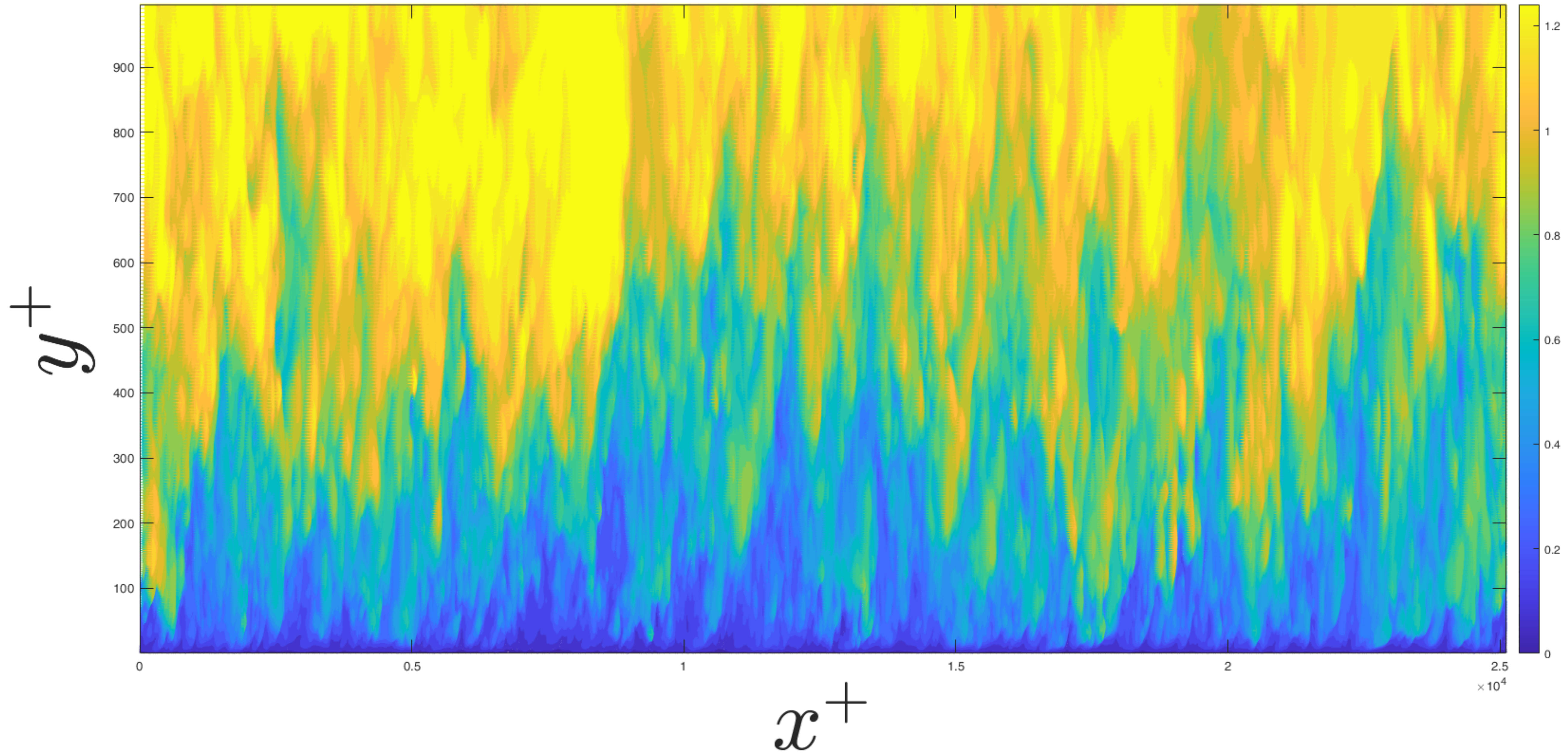
2. The work done by effective hydrodynamic force F_e (Dimensionless)
   2.1. $\int_0^t F_e(\tau)v_t(\tau)d\tau = \frac{2(S_G - 1)}{2S_G + 1}\left[\frac{I^2(t)}{2} - \alpha S_0\int_0^t F_e(\tau)\tau d\tau\right]$
3. The work needed to escape the pocket (Dimensionless)
   3.1. Resistant work from Gravity along vertical: $Z_{th} = sin\theta$
   3.2. Resistant work from Friction force along streamwise: $\mu X_{th} = \mu cos\theta$
   3.3. Total resistant work: $Z_{th} + \mu X_{th}$
4. Equating 2.1 and 3.3 And assuming $T_B \ll$ time scale of particle motion; therefore, $\alpha = 1$ and $\theta = \theta_0 = constant$
   4.1. $I^2(1 - F_{cr}/F_e) \approx \frac{2S_G + 1}{S_G - 1}(Z_{th} + \mu X_{th})$, where $I = F_e \cdot T_B$

B. Johns Hopkins Turbulence Databases

1. Direct Numerical Simulation (DNS) data set with $Re_\tau \approx 1000$
2. Domain Size: $[8\pi h,\ 2h,\ 3\pi h]$, where $h = \nu / u_* = 1.0006 \cdot 10^{-3}$
3. Grid points: $2048 \times 512 \times 1536$
4. Time stored: $[0,\ 25.9935]$
5. Selected Δt : 0.01

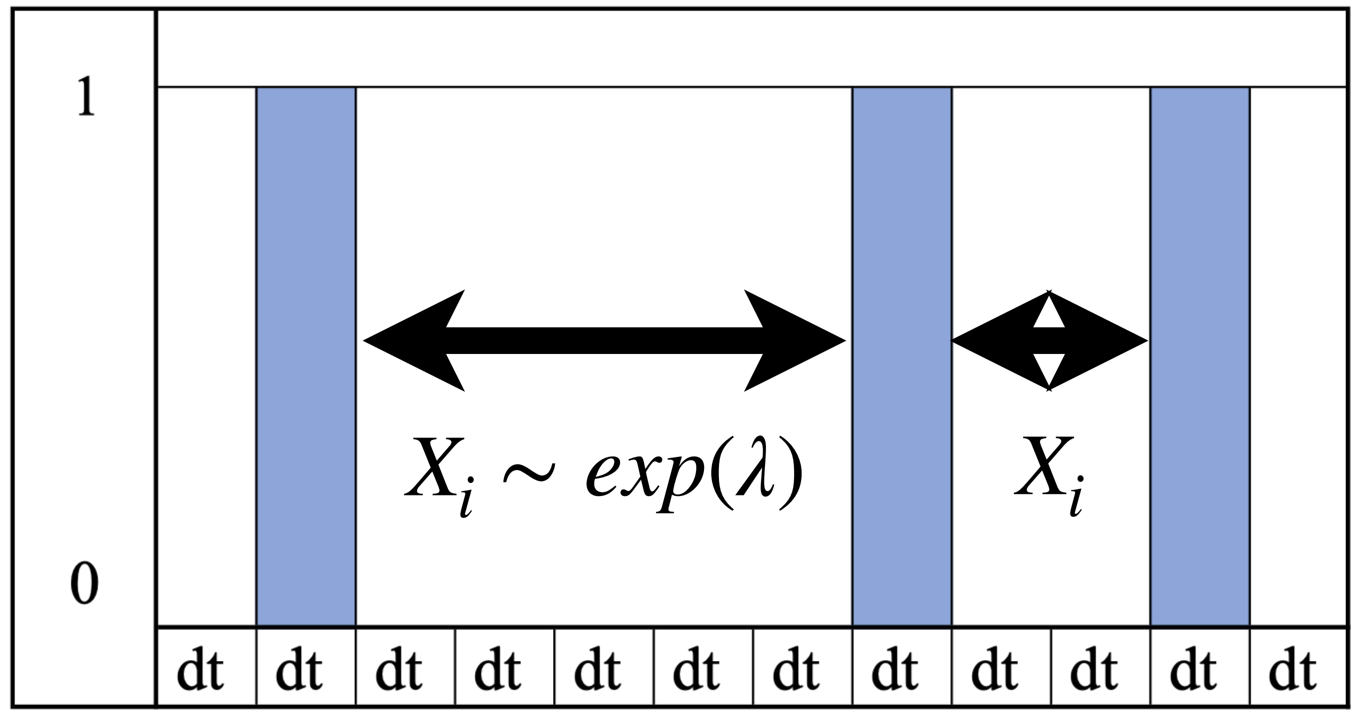
(Data obtained from the JHTDB at http://turbulence.pha.jhu.edu)

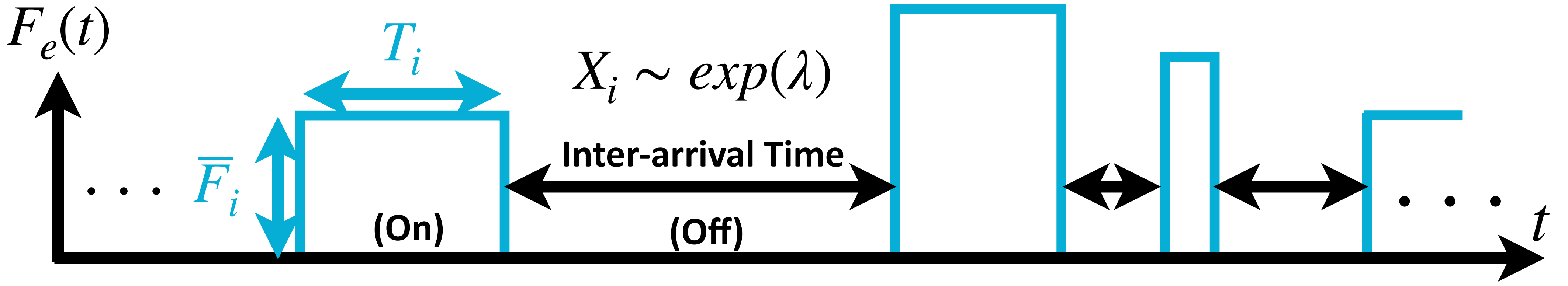
$$u^+(x, y, z_0, t_0)$$



C. Poisson Process

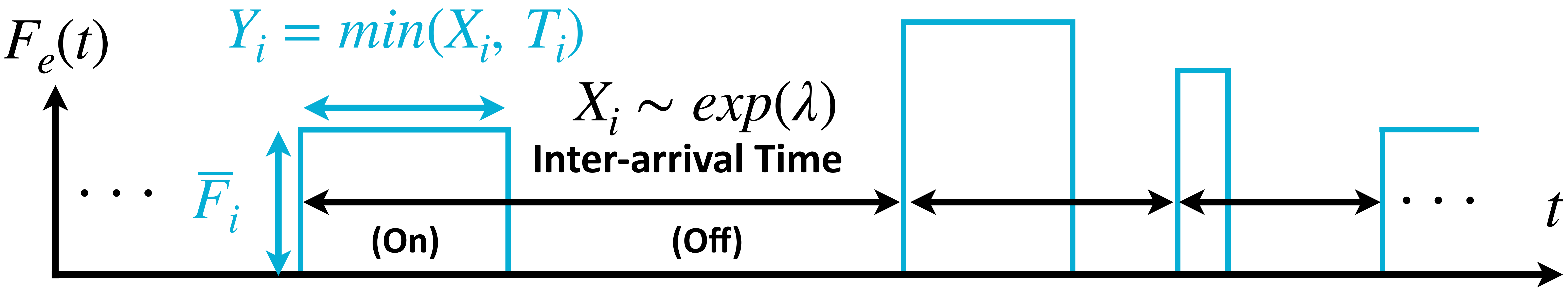
1. General description
    1.1. One of the most widely-used counting process
    1.2. Can be used to count the No. occurrence of certain events given a **constant rate** λ
2. Poisson distribution
    2.1. A discrete random variable N follows a Poisson distribution $N \sim P(\lambda\tau)$ if its PMF is given by $P_N(N = k) = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$ for $k \in \{0,1,2,3,\ldots\}$
    2.2. $E(N) = Var(N) = \lambda\tau$
    2.3. $N_1 + N_2 + \ldots + N_n \sim P(\lambda_1 + \lambda_2 + \ldots + \lambda_n)$ as all X_i are independent
3. Definition of the Poisson process $\{N(t),\ t \in [0,\ \infty)\}$ with given rates λ:
    3.1. $N(0) = 0$
    3.2. $N(t)$ has independent increments
    3.3. The number of arrivals in any interval of length $\tau > 0$ follows $P_N(\lambda\tau)$
    3.4. The inter-arrival time distribution is $f_X(X = t) = \lambda exp(-\lambda t)$
4. Second definition of the Poisson process with a very short interval of length dt
    4.1. $P(N(dt) = 0) = 1 - \lambda dt + o(dt)$
    4.2. $P(N(dt) = 1) = \lambda dt + o(dt)$
    4.3. $P(N(dt) \geq 2) = o(dt)$
5. Simulating random arrival of impulse event by using 4.2
    5.1. The probability of occurrence during dt is λdt. Here, λ is defined as $\Sigma T_i / Simulation\ Time$
    5.2. If the event occur at a specfic dt, we assign it as 1; otherwise we assign it as 0.
    5.3. A train of spikes forms!





D. Alternating Renewal Process (Ross, 2014)

1. A generalization of the Poisson process
2. Assuming $\{Y_i\}$ to be *on* state, whereas $\{Z_i\}$ to be *off* state
    2.1. $X_i = Y_i + Z_i \sim P_X(t) = \lambda exp(-\lambda t)$
    2.2. $Y_i = min(X_i, T_i)$

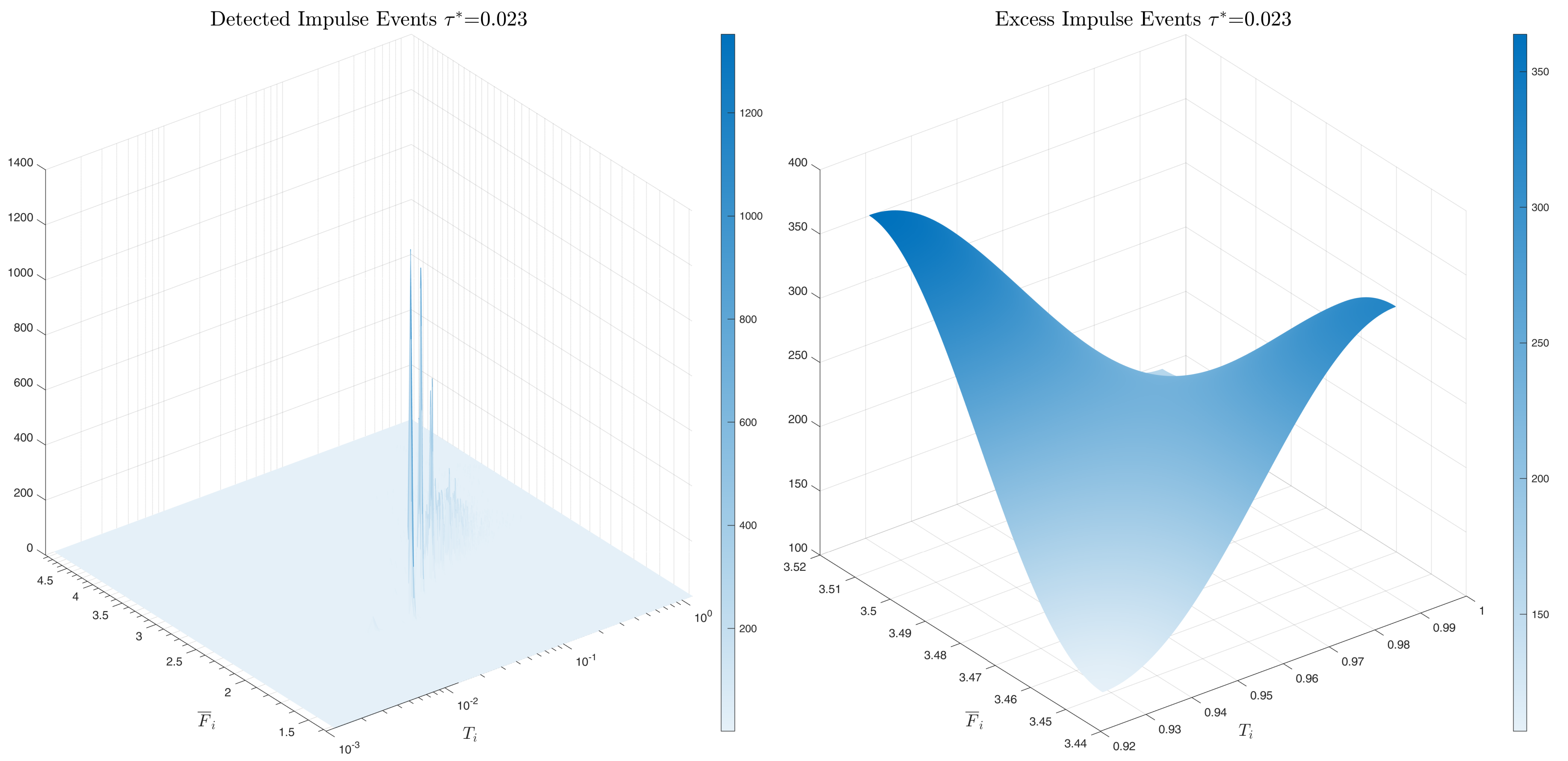


Supplementary Material

Preliminary Result

Joint PDF of $\overline{F}_i$ and T_i at different τ^*

(1)



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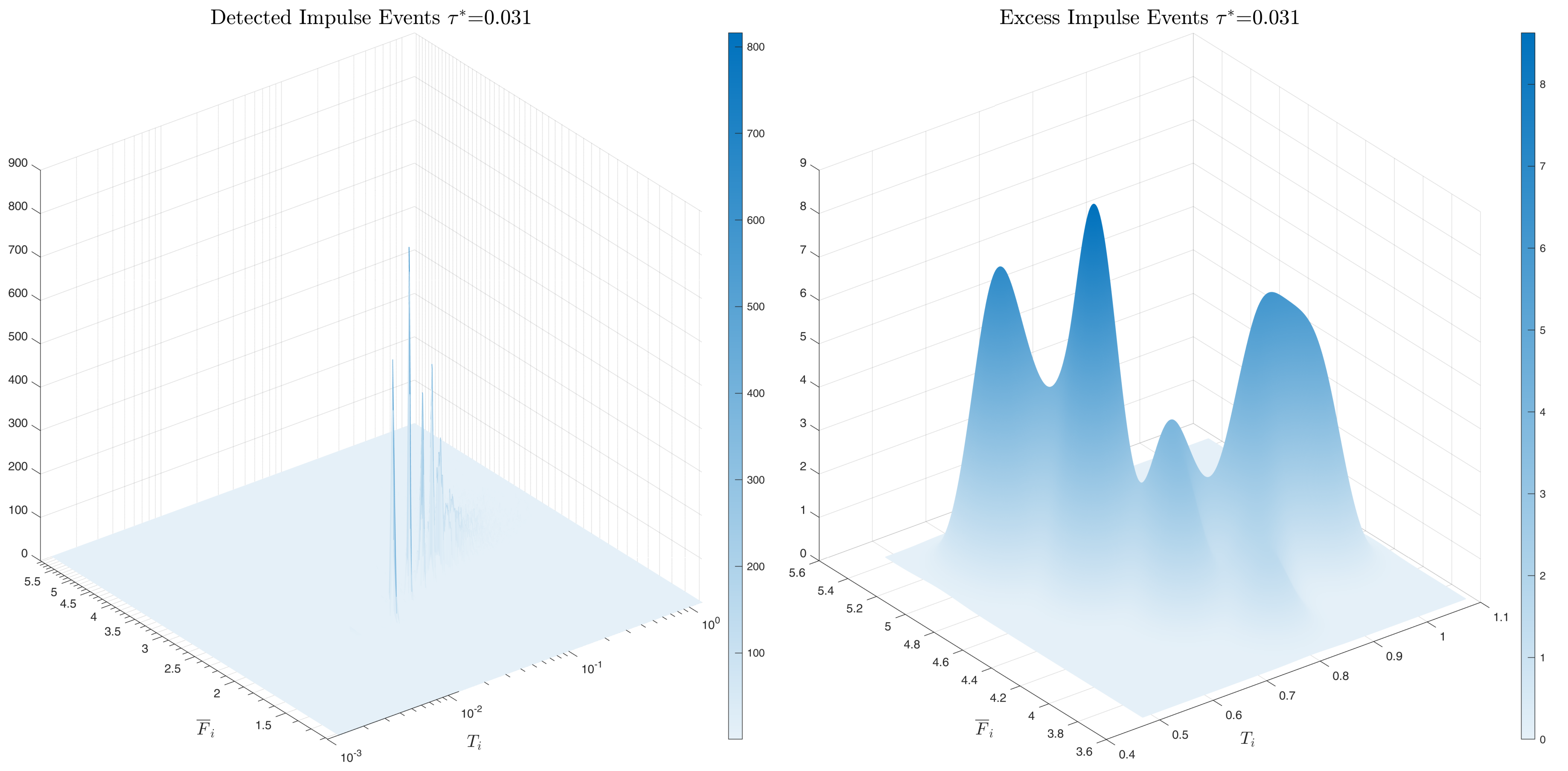

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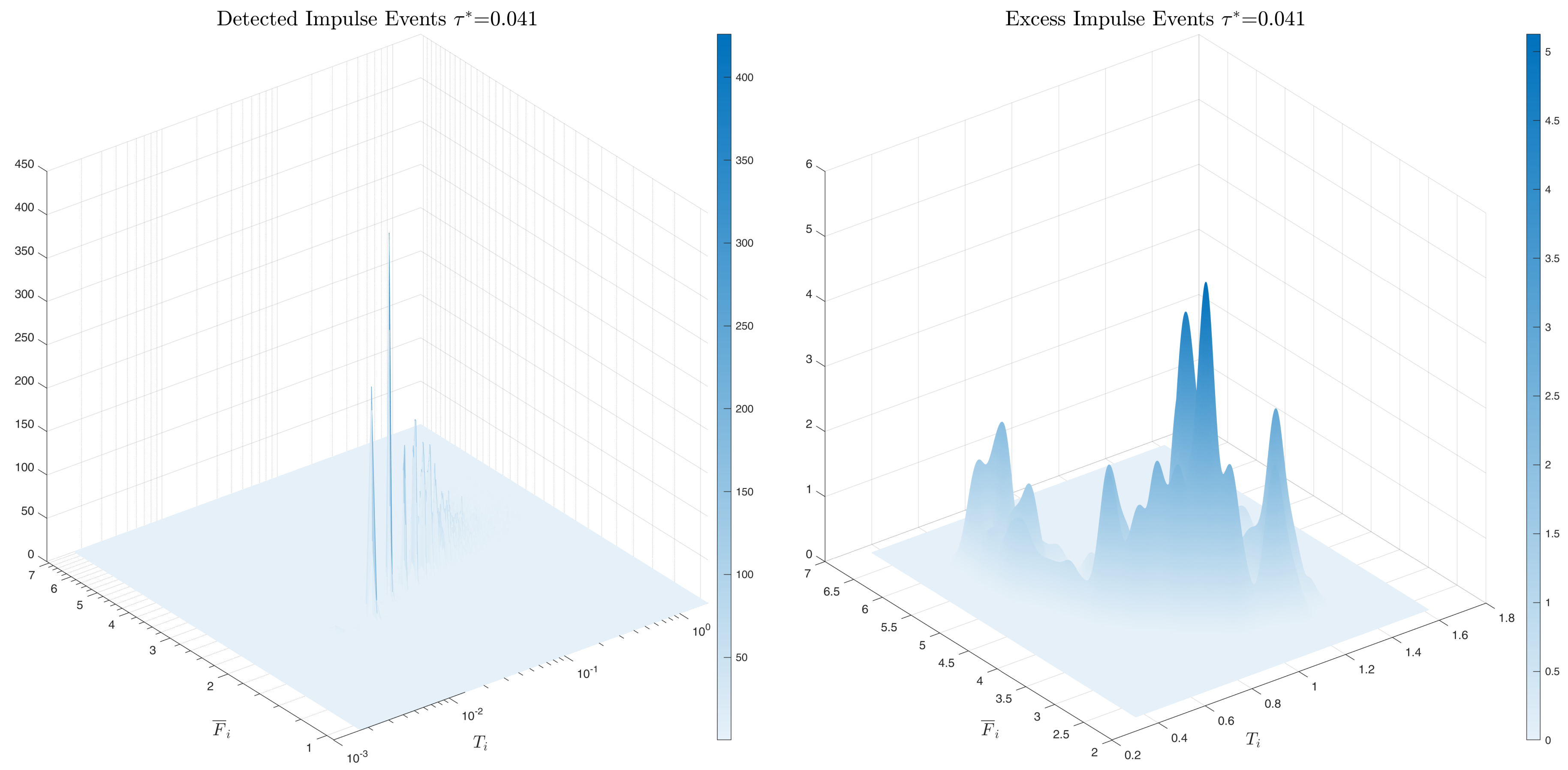
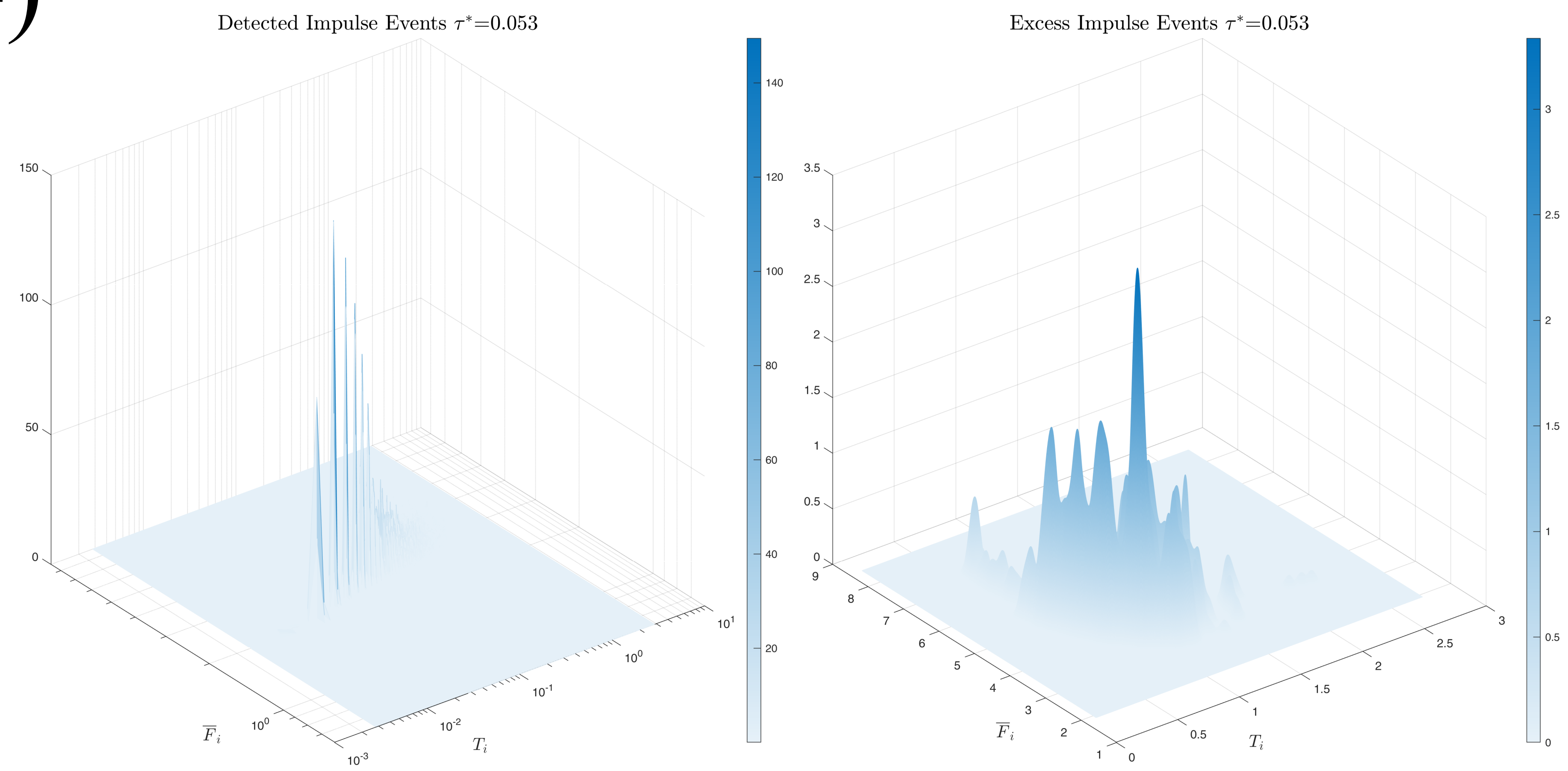
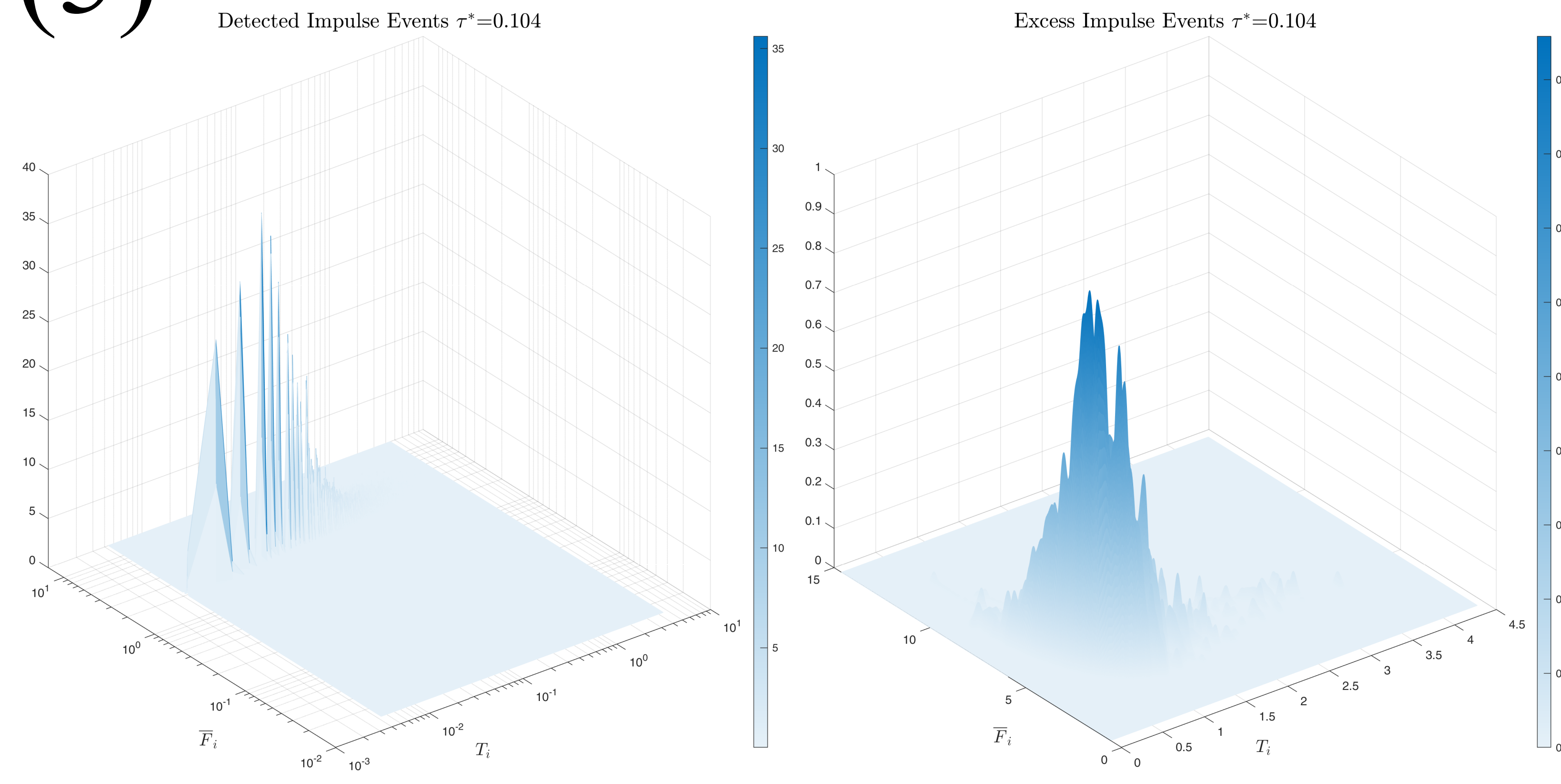


(4)



Joint PDF of $\overline{F}_i$ and T_i at different τ^*

(5)



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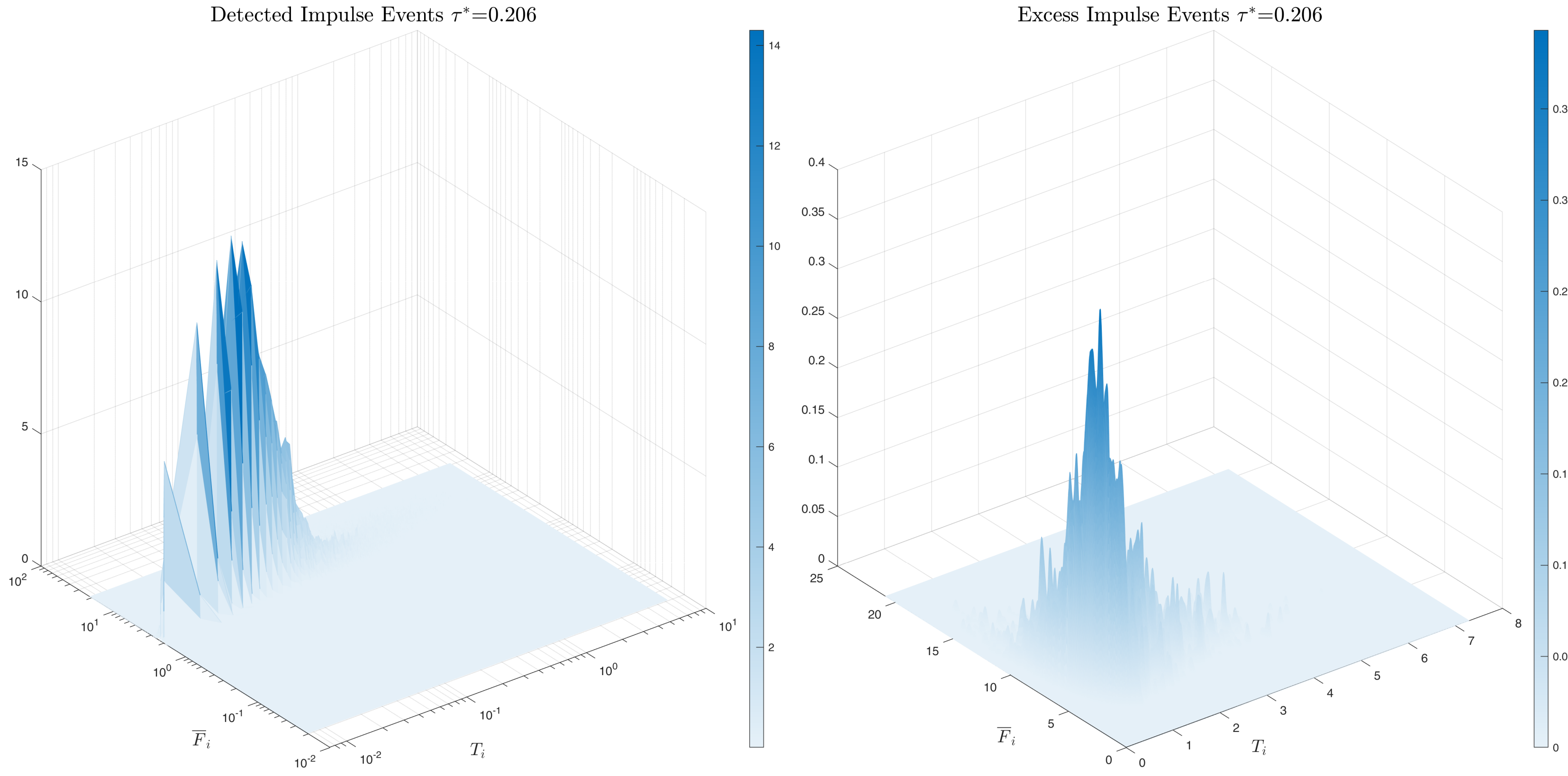

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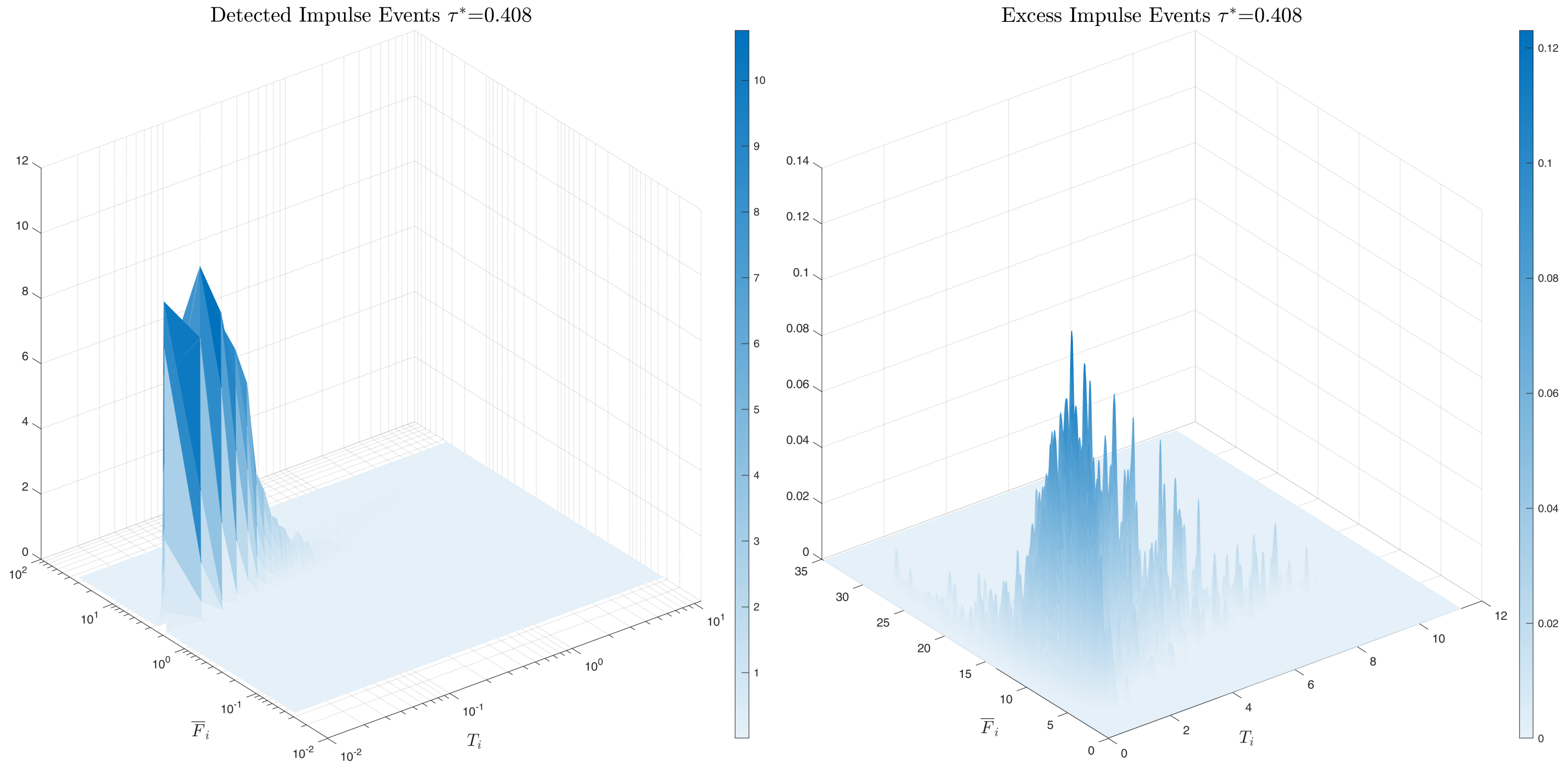

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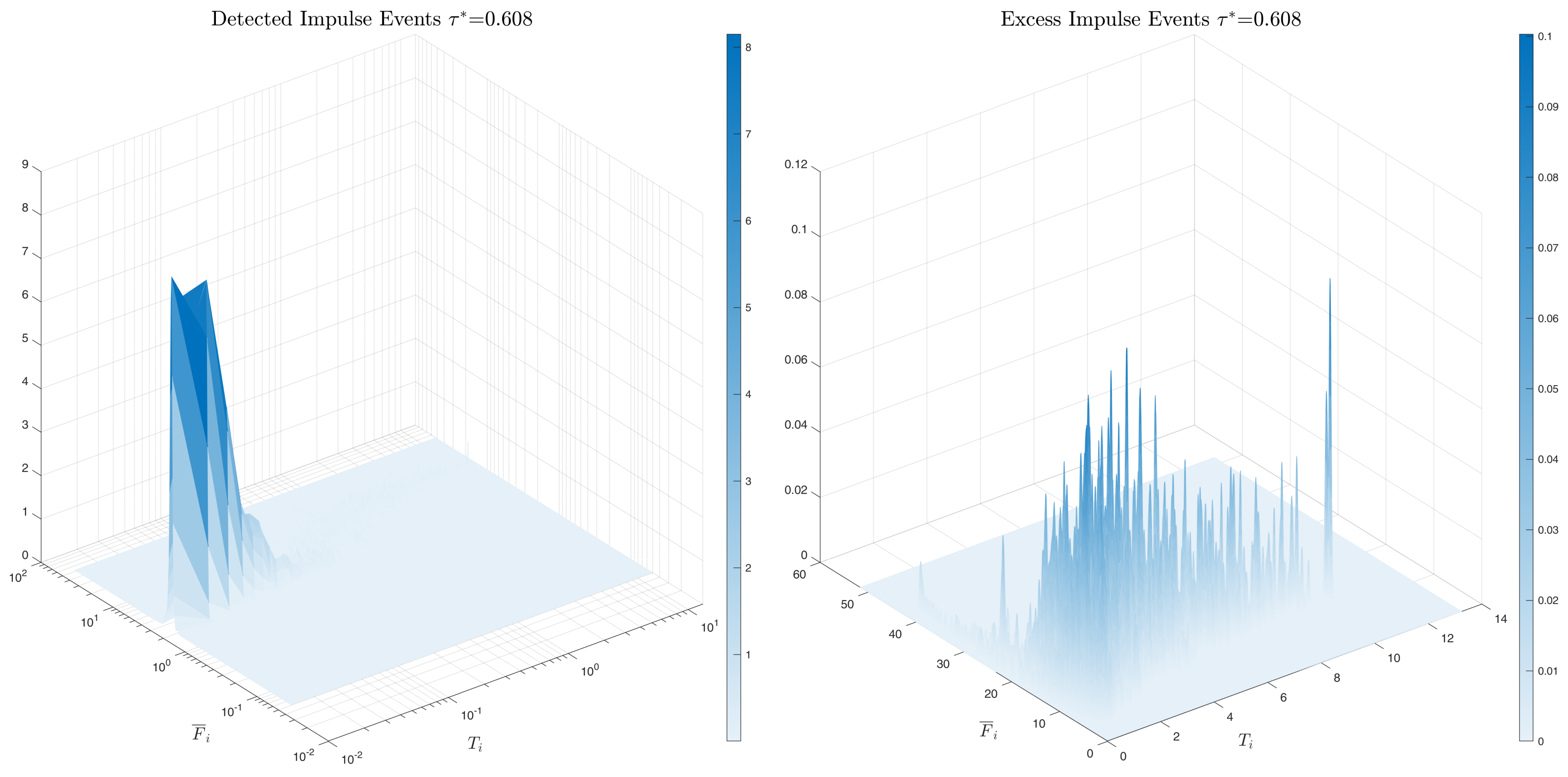

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