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The effect of transient lateral internal gravity wave propagation on the resolved atmosphere in ICON/MS-GWaM

Supplementary material

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- a python toy code
  https://github.com/g-voelker/python-msgwam/tree/dev
- the ICON model
  https://gitlab.dkrz.de/icon/icon-model
- the ICON/MS-GWaM submodule
  https://github.com/DataWaveProject/msgwam

Mathematical model formulation

1. scale height separation $H_p/H_\theta = \mathcal{O}(\epsilon^\alpha)$

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2. small Rossby number $1 \gg \mathrm{Ro} = \epsilon$
   - quasi-geostrophic regime
   - hydrostatic scaling $H_w = \epsilon L_w$
   - stratification and rotation $f = \epsilon^{5-\alpha} N$

1. scale height separation $H_p/H_\theta = \mathcal{O}(\epsilon^\alpha)$
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   - quasi-geostrophic regime
   - hydrostatic scaling $H_w = \epsilon L_w$
   - stratification and rotation $f = \epsilon^{5-\alpha} N$
3. perturbation scale separation ϵ $(X, Y, Z, T) = \epsilon(x, y, z, t)$
4. perturbation expansion in small parameter (not shown)

- non-dimensional compressible Euler equations
- multiscale theory with $(\boldsymbol{X}, T) = \epsilon(\boldsymbol{x}, t)$, $\epsilon = \mathrm{Ro}$

$$\boldsymbol{v} = \sum_{j=0}^{\infty} \epsilon^j \boldsymbol{V}_0^{(j)} + \Re \sum_{\beta} \sum_{n=1}^{\infty} \epsilon^n V_\beta^{(n)} e^{\phi_\beta/\epsilon} \quad (1)$$

$$\theta = \sum_{j=0}^{\alpha} \epsilon^j \bar{\Theta}^{(j)}(Z) + \epsilon^{\alpha+1} \sum_{j=0}^{\infty} \epsilon^j \Theta_0^{(j)} + \epsilon^{\alpha+1} \Re \sum_{\beta} \sum_{n=1}^{\infty} \epsilon^n \Theta_\beta^{(n)} e^{\phi_\beta/\epsilon} \quad (2)$$

$$\pi = \sum_{j=0}^{\alpha} \epsilon^j \bar{\Pi}^{(j)}(Z) + \epsilon^{\alpha+1} \sum_{j=0}^{\infty} \epsilon^j \Pi_0^{(j)} + \epsilon^{\alpha+2} \Re \sum_{\beta} \sum_{n=1}^{\infty} \epsilon^n \Pi_\beta^{(n)} e^{\phi_\beta/\epsilon} \quad (3)$$

- field = reference + mean flow + superimposing waves β
- wave properties $\omega_\beta = -\partial_T \phi_\beta$, $\boldsymbol{k}_\beta = \nabla_{\boldsymbol{X}} \phi_\beta$

Achatz, Ribstein, et al. 2017

- dispersion relation $\dot{\xi} = (\partial_t + \boldsymbol{c}_g \cdot \nabla_r)\xi$

$$\omega = \Omega(\boldsymbol{X}, T, \boldsymbol{k}) = \boldsymbol{k} \cdot \boldsymbol{U}_0^{(0)}(\boldsymbol{X}, T) \pm \sqrt{\frac{f^2 m^2 + N^2(k^2 + l^2)}{k^2 + l^2 + m^2}} \tag{4}$$

- eikonal equations

$$(\partial_T + \boldsymbol{c}_g \cdot \nabla_{\boldsymbol{X}})\boldsymbol{k} = \dot{\boldsymbol{k}} = -\nabla_{\boldsymbol{X}}\Omega \qquad \boldsymbol{c}_g = \nabla_{\boldsymbol{k}}\Omega \tag{5}$$

- polarization relations (omitted)
- wave action conservation (no TRI) $\mathcal{N} = \mathcal{N}(\boldsymbol{X}, T, \boldsymbol{k})$

$$0 = \partial_T \mathcal{N} + \nabla_{\boldsymbol{X}} \cdot (\boldsymbol{c}_g \mathcal{N}) + \nabla_{\boldsymbol{k}} \cdot (\dot{\boldsymbol{k}}\mathcal{N}) + S \tag{6}$$

$$0 = \nabla_{\boldsymbol{X}} \cdot \boldsymbol{c}_g + \nabla_{\boldsymbol{k}} \cdot \dot{\boldsymbol{k}} \tag{7}$$

Grimshaw 1972; Grimshaw 1974; Muraschko et al. 2015; Achatz, Ribstein, et al. 2017

- changes in position

$$\dot{\xi} = (\partial_t + \boldsymbol{c}_g \cdot \nabla_r)\xi$$

$$\dot{\lambda} = \frac{c_{g\lambda}}{r\cos\phi} \qquad \dot{\phi} = \frac{c_{g\phi}}{r} \qquad \dot{r} = c_{gr}$$

- changes in wavenumber

$$\dot{k_\lambda} = -\frac{1}{r\cos\phi}\left(\boldsymbol{k}\cdot\partial_\lambda \boldsymbol{U} + \frac{|\boldsymbol{k}_h|^2\partial_\lambda N^2}{2\hat{\omega}|\boldsymbol{k}|^2}\right) - \frac{k_\lambda}{r}c_{gr} + \frac{k_\lambda\tan\phi}{r}c_{g\phi}$$

$$\dot{k_\phi} = -\frac{1}{r}\left(\boldsymbol{k}\cdot\partial_\phi \boldsymbol{U} + \frac{|\boldsymbol{k}_h|^2\partial_\phi N^2 + k_r^2\partial_\phi f^2}{2\hat{\omega}|\boldsymbol{k}|^2}\right) - \frac{k_\phi}{r}c_{gr} - \frac{k_\lambda\tan\phi}{r}c_{g\lambda}$$

$$\dot{k_r} = -\left(\boldsymbol{k}\cdot\partial_r \boldsymbol{U} + \frac{|\boldsymbol{k}_h|^2\partial_r N^2}{2\hat{\omega}|\boldsymbol{k}|^2}\right) + \frac{k_\lambda}{r}c_{g\lambda} + \frac{k_\phi}{r}c_{g\phi}$$

- wave action conservation

$$0 = \partial_t\mathcal{N} + \boldsymbol{c}_g\cdot\nabla_{\boldsymbol{r}}\mathcal{N} + \dot{\boldsymbol{k}}\cdot\nabla_{\boldsymbol{k}}\mathcal{N} + S$$

- relax assumption of geostrophic, horizontal, and hydrostatic leading order mean-flow
- direct flux approach

$$D_t\theta + N^2 w = -\nabla_h \cdot \overline{\boldsymbol{u}'\theta'} \tag{8}$$

$$D_t\boldsymbol{u} + f\boldsymbol{e}_z \times \boldsymbol{u} = -c_p\bar{\theta}\nabla_h\pi - \frac{1}{\bar{\rho}}\nabla \cdot (\bar{\rho}\overline{\boldsymbol{u}'\boldsymbol{v}'}) + \frac{f}{g}\boldsymbol{e}_z \times \overline{\boldsymbol{u}'b'} \tag{9}$$

entropy-flux convergence, momentum-flux convergence, elastic term

- for details see Achatz, Ribstein, et al. 2017; Wei et al. 2019

$$\bar{\rho}\overline{u'u'} = \mathcal{A}\left(kc_{gx} - f^2\frac{kc_{gx}+lc_{gy}}{f^2-\hat{\omega}^2}\right) \qquad \bar{\rho}\overline{v'u'} = lc_{gx}\mathcal{A} \tag{10}$$

$$\bar{\rho}\overline{u'v'} = kc_{gy}\mathcal{A} \qquad \bar{\rho}\overline{v'v'} = \mathcal{A}\left(lc_{gy} - f^2\frac{kc_{gx}+lc_{gy}}{f^2-\hat{\omega}^2}\right) \tag{11}$$

$$\bar{\rho}\overline{u'w'} = kc_{gz}\mathcal{A}\left(\frac{f^2}{f^2-\hat{\omega}^2}\right) \qquad \bar{\rho}\overline{v'w'} = lc_{gz}\mathcal{A}\left(\frac{f^2}{f^2-\hat{\omega}^2}\right) \tag{12}$$

$$\bar{\rho}\overline{u'\theta} = \frac{\bar{\theta}fN^4k_h^2}{g\hat{\omega}K^2m(\hat{\omega}^2-f^2)}l\mathcal{A} \qquad \bar{\rho}\overline{v'\theta} = -\frac{\bar{\theta}fN^4k_h^2}{g\hat{\omega}K^2m(\hat{\omega}^2-f^2)}k\mathcal{A} \tag{13}$$

$$\frac{f}{g}\overline{u'b'} = \frac{f^2N^4k_h^2}{\bar{\rho}g\hat{\omega}k^2m(\hat{\omega}^2-f^2)}l\mathcal{A} \qquad \frac{f}{g}\overline{v'b'} = -\frac{f^2N^4k_h^2}{\bar{\rho}g\hat{\omega}k^2m(\hat{\omega}^2-f^2)}k\mathcal{A} \tag{14}$$

- wave action conservation (no TRI) $\dot{\xi} = (\partial_t + \boldsymbol{c}_g \cdot \nabla_r)\xi$

$$0 = \partial_T \mathcal{A}_\beta + \nabla_{\boldsymbol{X}}(\boldsymbol{c}_{g,\beta}\mathcal{A}_\beta) \tag{14}$$

Muraschko et al. 2015

- wave action conservation (no TRI) $\dot{\xi} = (\partial_t + \boldsymbol{c}_g \cdot \nabla_r)\xi$

$$0 = \partial_T \mathcal{A}_\beta + \nabla_{\boldsymbol{X}}(\boldsymbol{c}_{g,\beta}\mathcal{A}_\beta) \tag{14}$$

- wave action in *phase space*

$$\mathcal{A}_\beta = \int_{\mathbb{R}^3} \mathcal{N}(\boldsymbol{X}, T, \boldsymbol{k})\delta(\boldsymbol{k} - \boldsymbol{k}_\beta)d\boldsymbol{k} \qquad \boldsymbol{k}_\beta = \boldsymbol{k}_\beta(\boldsymbol{X}, T) \tag{15}$$

Muraschko et al. 2015

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- we then find for $\mathcal{N}(\boldsymbol{X}, T, \boldsymbol{k})$

$$0 = \partial_T \mathcal{N} + \nabla_{\boldsymbol{X}} \cdot (\boldsymbol{c}_{g,}\mathcal{N}) + \nabla_{\boldsymbol{k}} \cdot (\dot{\boldsymbol{k}}\mathcal{N}) \tag{16}$$

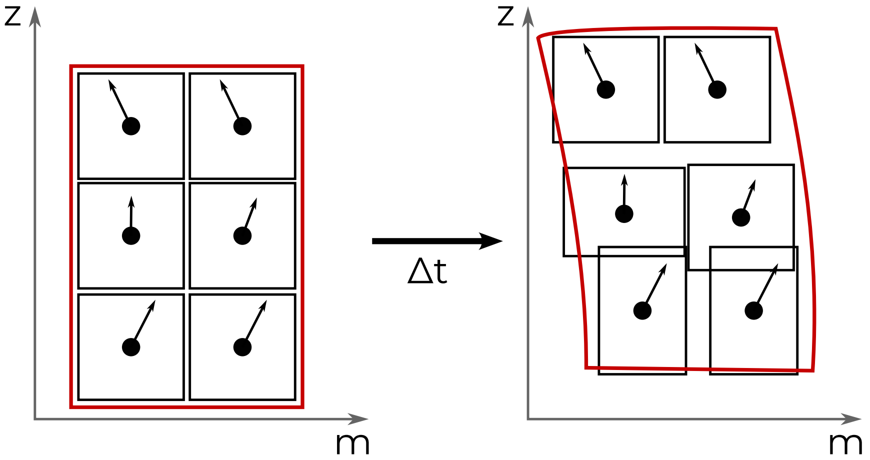
$$0 = \nabla_{\boldsymbol{X}} \cdot \boldsymbol{c}_{g,} + \nabla_{\boldsymbol{k}} \cdot \dot{\boldsymbol{k}} \tag{17}$$

- phase space volume is conserved

Muraschko et al. 2015

Implementation of the coupled Lagrangian model

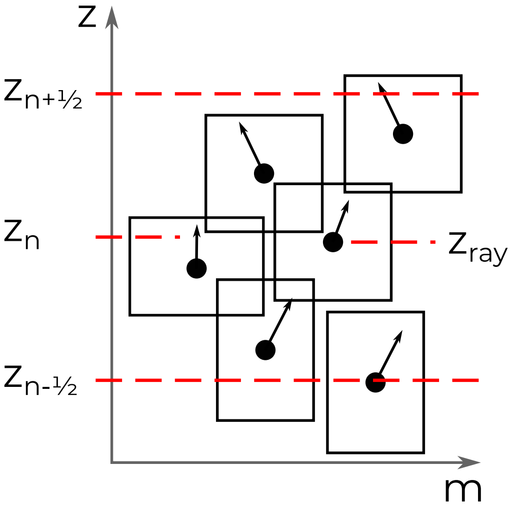
$$0 = \nabla_{\boldsymbol{x}} \cdot \boldsymbol{c}_{\boldsymbol{g},\beta} + \nabla_{\boldsymbol{k}_\beta} \cdot \dot{\boldsymbol{k}}_\beta \tag{18}$$



freely after Fig. 3 of Muraschko et al. 2015

- mean → wave: interpolation

$$N_{ray} = N_n + \frac{N_{n+1} - N_n}{z_{n+1} - z_n}(z_{ray} - z_n)$$



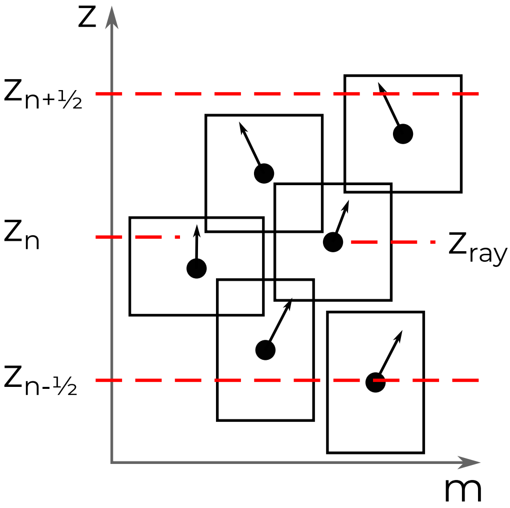
extended version of Fig. 4 of Muraschko et al. 2015

- mean → wave: interpolation

$$N_{ray} = N_n + \frac{N_{n+1} - N_n}{z_{n+1} - z_n}(z_{ray} - z_n)$$

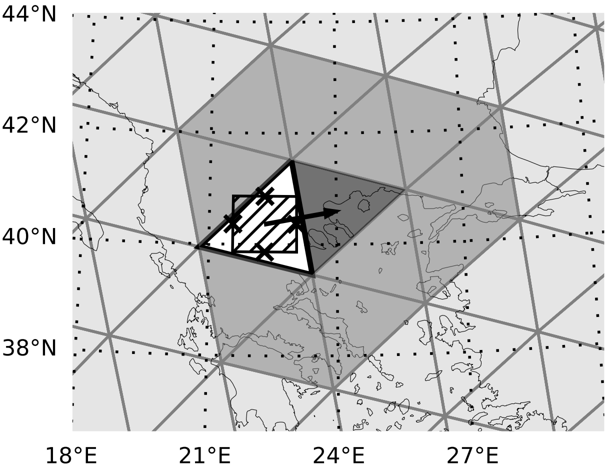
- wave → mean: integration and projection

$$D_t \boldsymbol{U} \propto -\frac{1}{\bar{\rho}} \nabla \cdot \int_{\mathbb{R}^3} (\hat{\boldsymbol{c}}_g \boldsymbol{k} \mathcal{N}) d\boldsymbol{k}$$

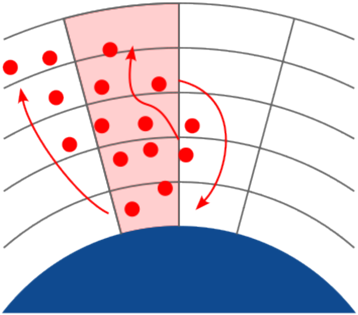


extended version of Fig. 4 of Muraschko et al. 2015

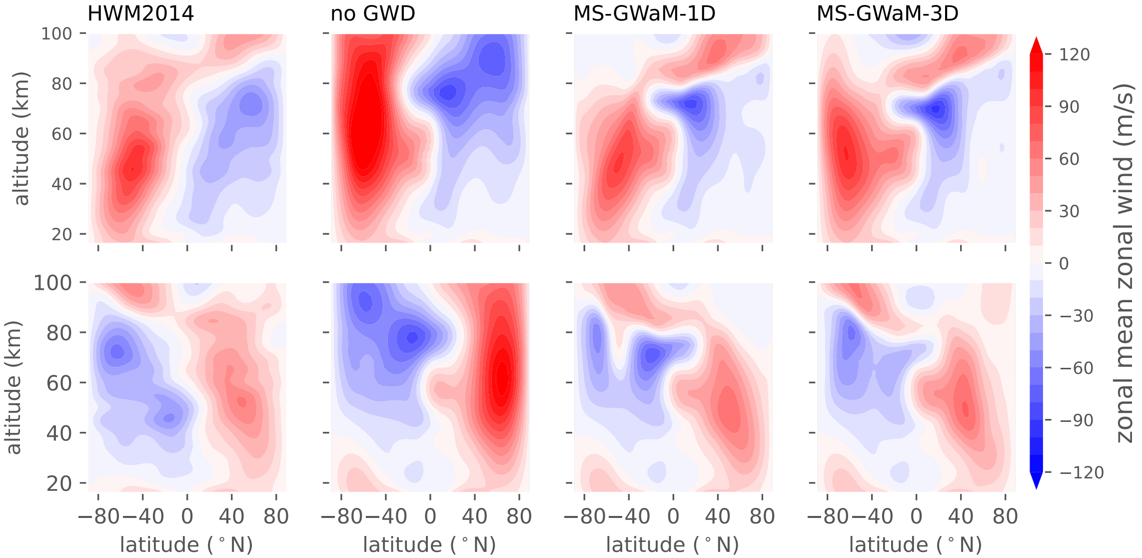
Coupling the Eulerian and the Lagrangian Model



- Eulerian dynamical core
- Lagrangian wave model
- two-way coupling
- massively parallelized

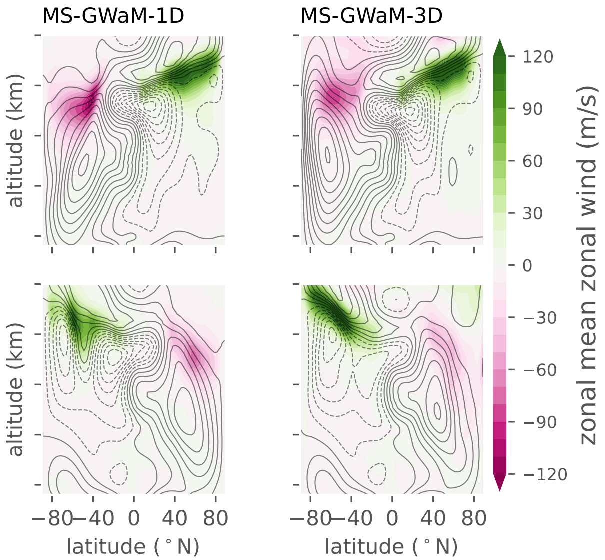


Comparing momentum flux distribution: 1D vs. 3D

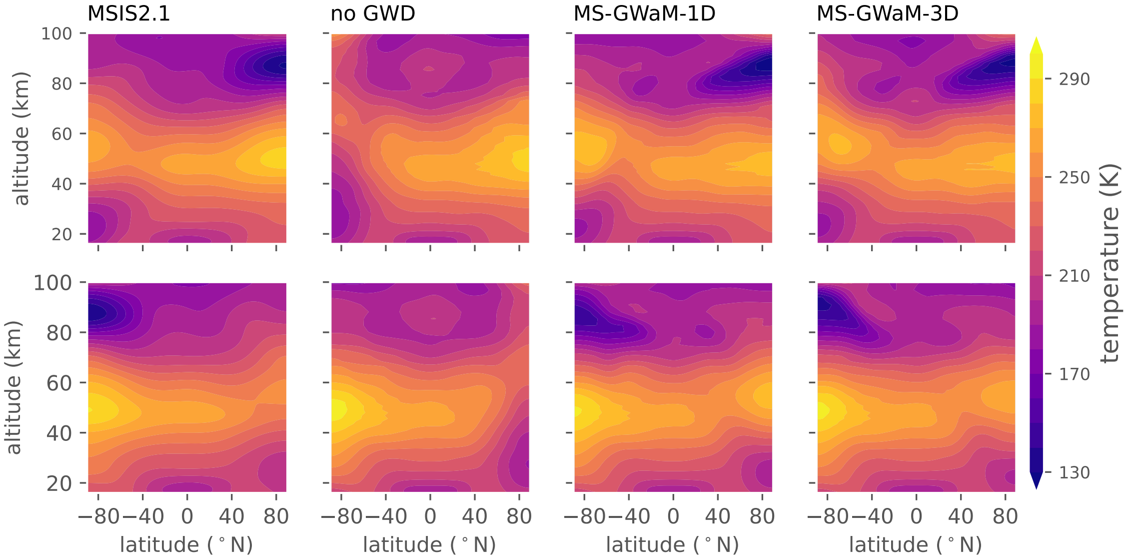


Drob et al. 2015

Zonal mean zonal winds and temperatures

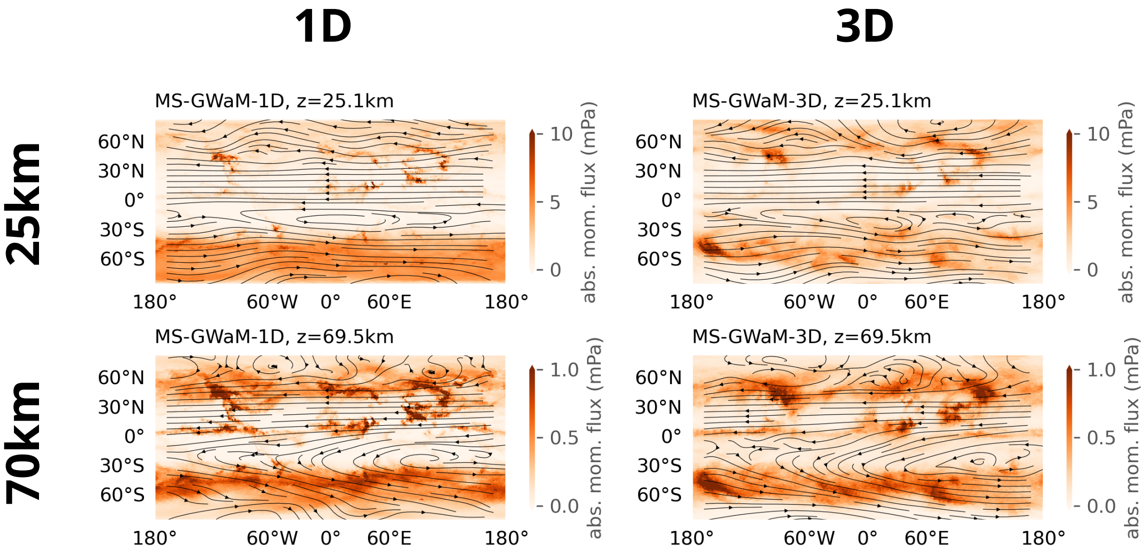


Zonal mean zonal winds and temperatures



Emmert et al. 2022

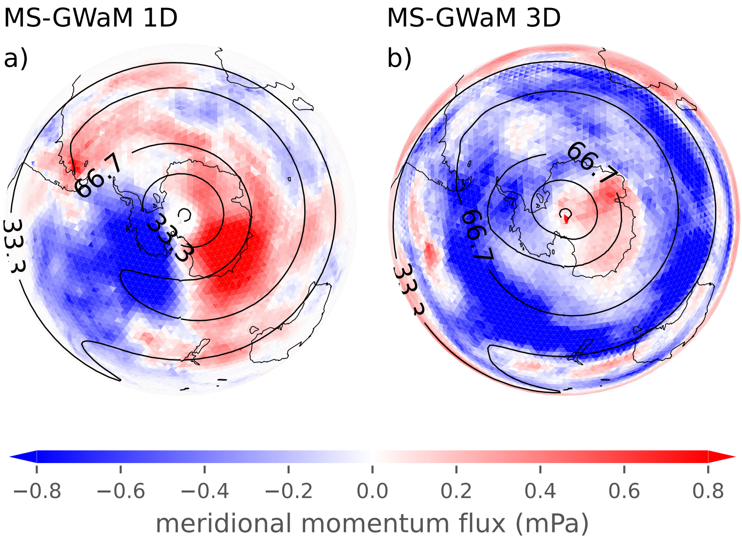
Horizontal distribution of absolute momentum fluxes (June 10-20 1991)



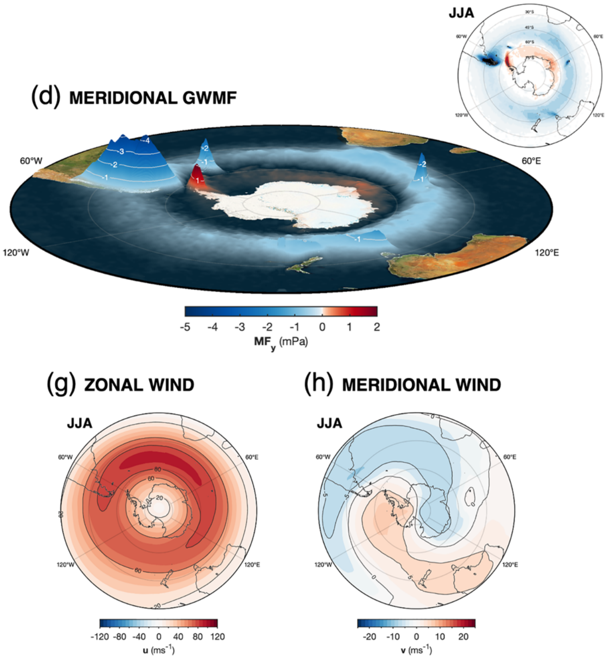
$$\mathcal{F}_{abs} = \bar{\rho}\left(\overline{u'w'}^2 + \overline{v'w'}^2\right)^{\frac{1}{2}}$$

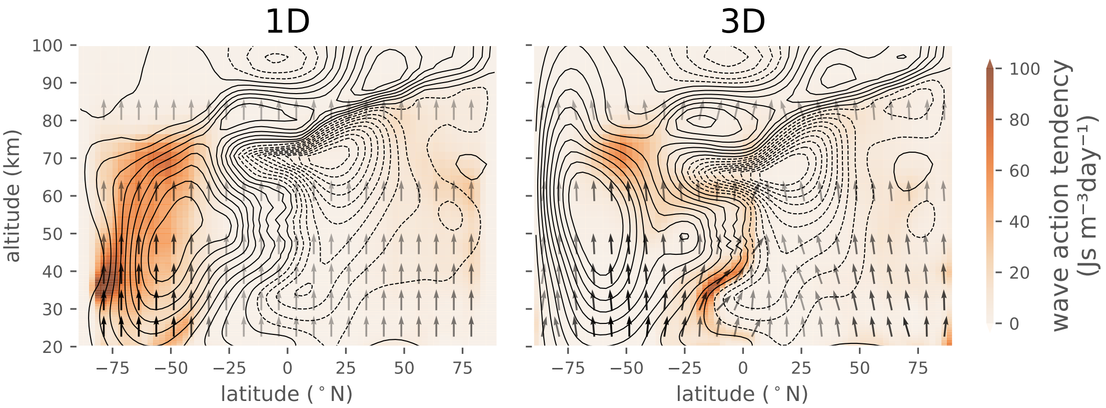
Meridional momentum flux distribution - Southern hemisphere

June (mean of 91-98)



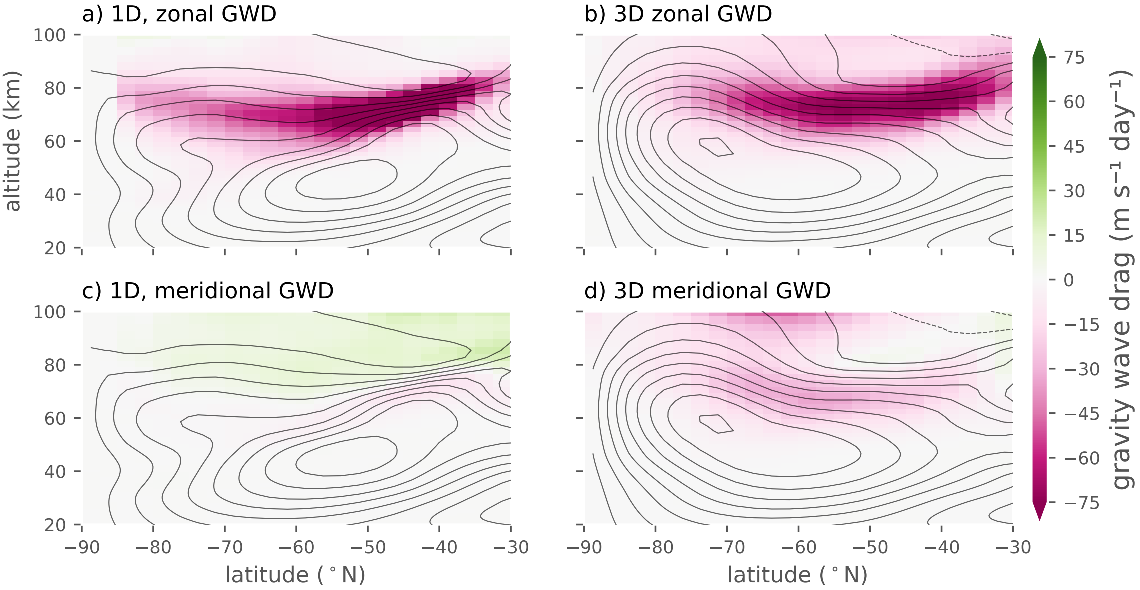
Meridional momentum flux distribution in Hindley et al. 2020



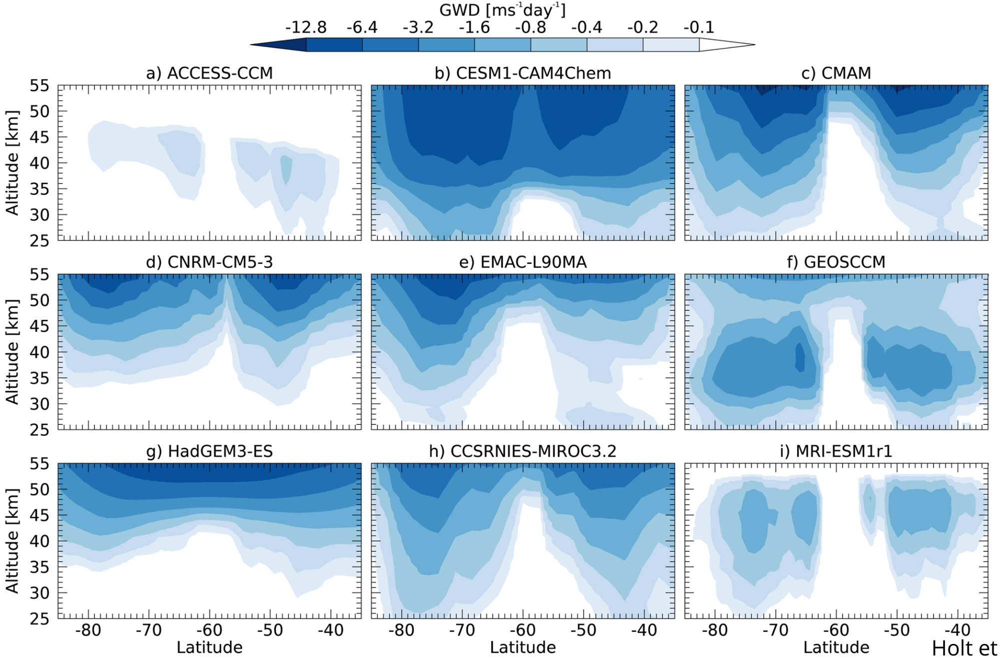


- refraction redistributes wave action dissipation

Wave drags around the SH winter jet



The (still) missing wave drag at 60°S

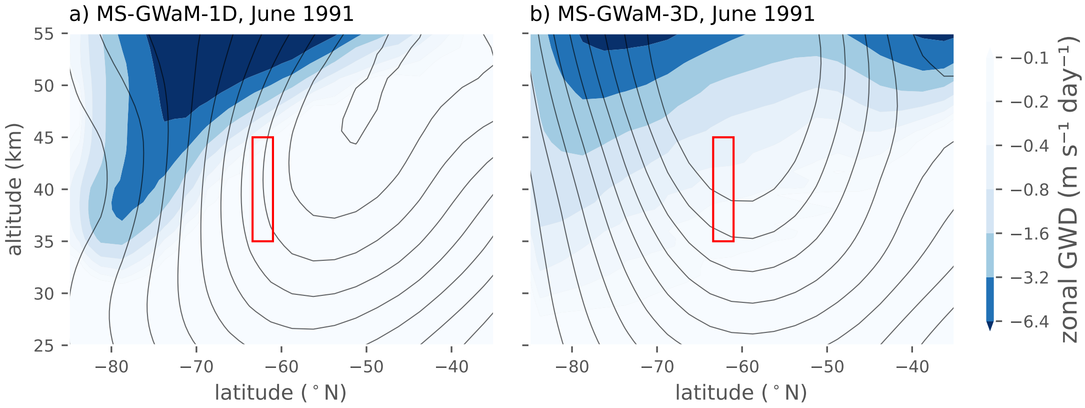


Holt et al. 2023

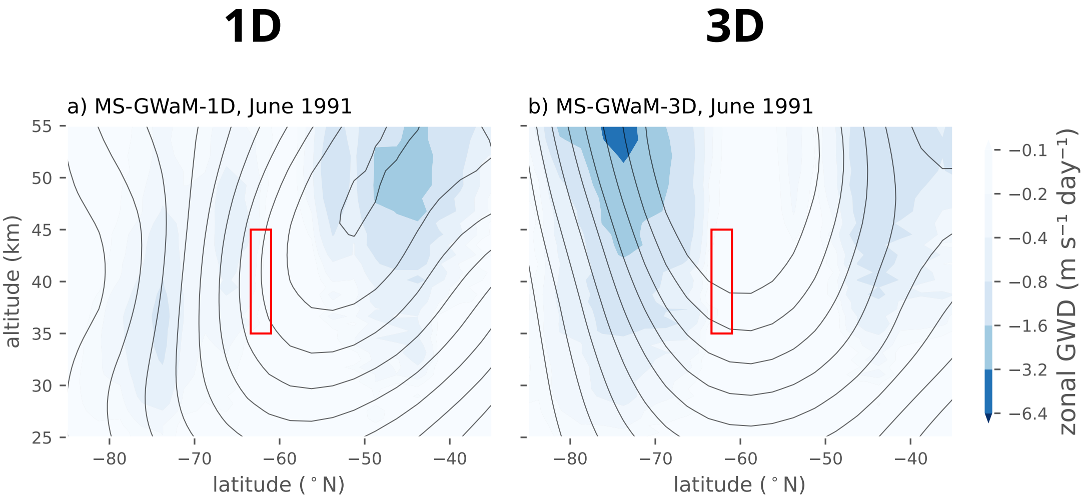
The effect of 3D transient non-orographic waves near 60°S (June 1991)

1D

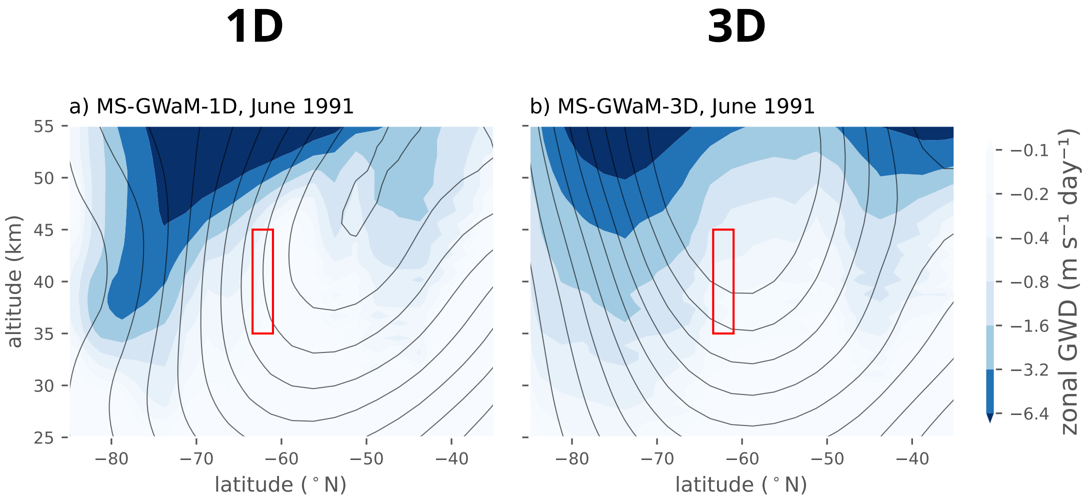
3D



- redistribution of non-orographic waves

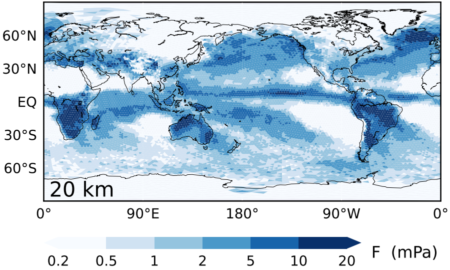


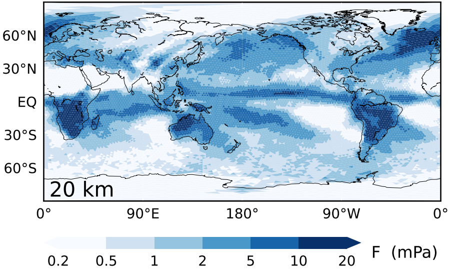
- modified wind and SSO wave propagation



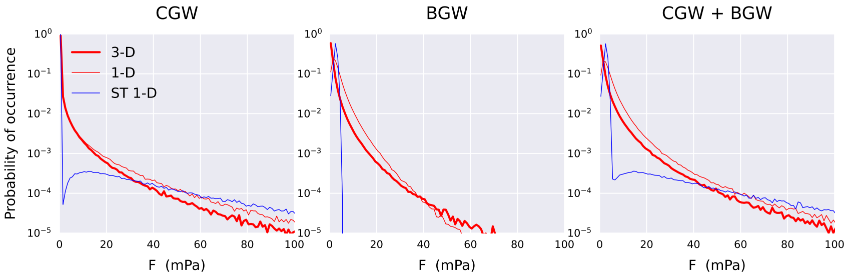
- 3D propagation of non-orographic waves alone does **not** solve it

Lateral propagation and IGW intermittency

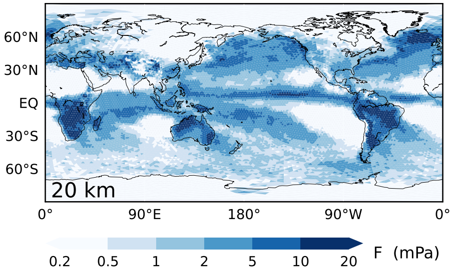

60°N
30°N
EQ
30°S
60°S
20 km
0°
90°E
180°
90°W
0°
0.2
0.5
1
2
5
10
20
F (mPa)

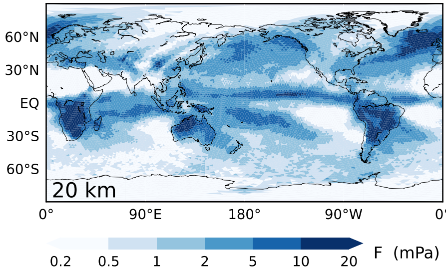

60°N
30°N
EQ
30°S
60°S
20 km
0°
90°E
180°
90°W
0°
0.2
0.5
1
2
5
10
20
F (mPa)

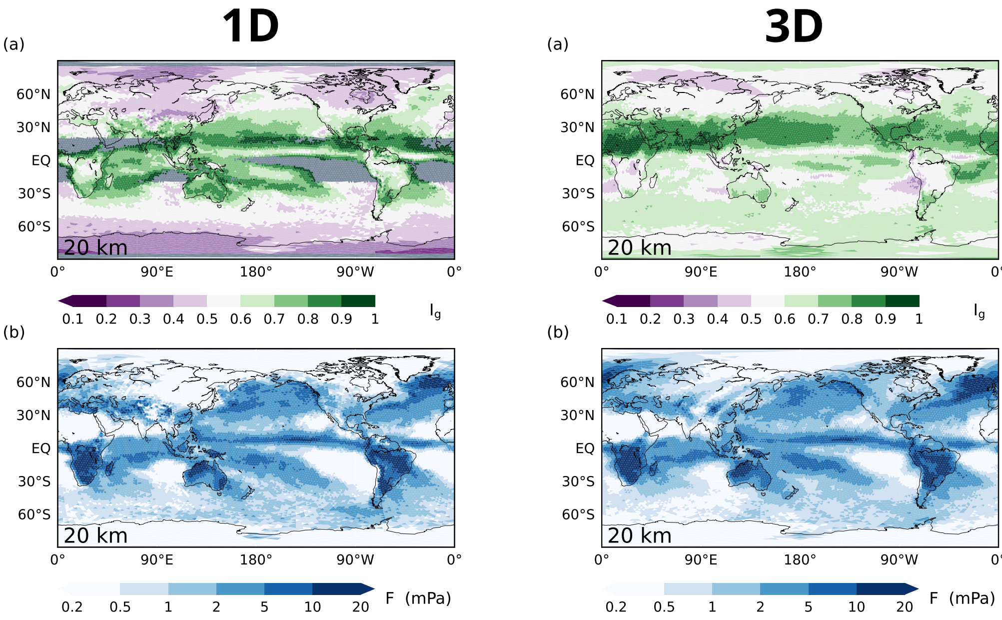
Gravity wave intermittency (PMF, Dec., z=20km)



(statistics for 50-65°S)

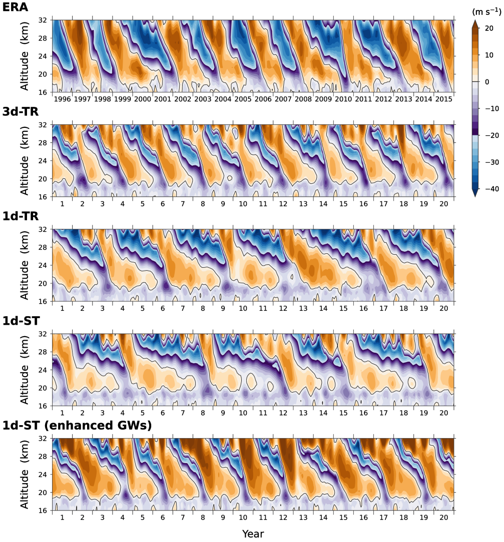


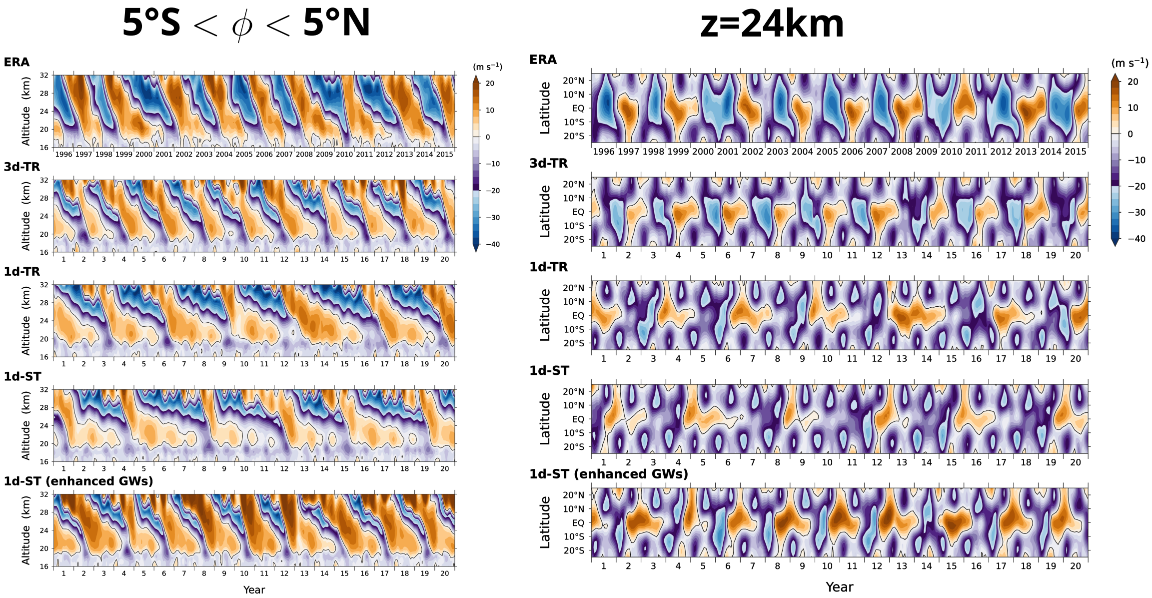



1D
3D
(a)
60°N
30°N
EQ
30°S
60°S
20 km
0°
90°E
180°
90°W
0°
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
I_g
(b)
0.2
0.5
1
2
5
10
20
F (mPa)

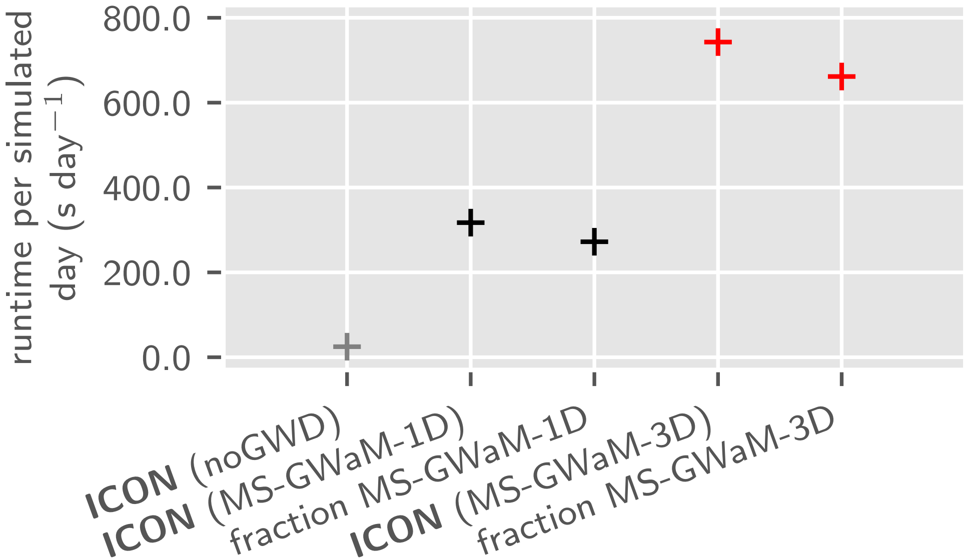
Simulating the QBO with MS-GWaM (details)

5°S $< \phi <$ 5°N

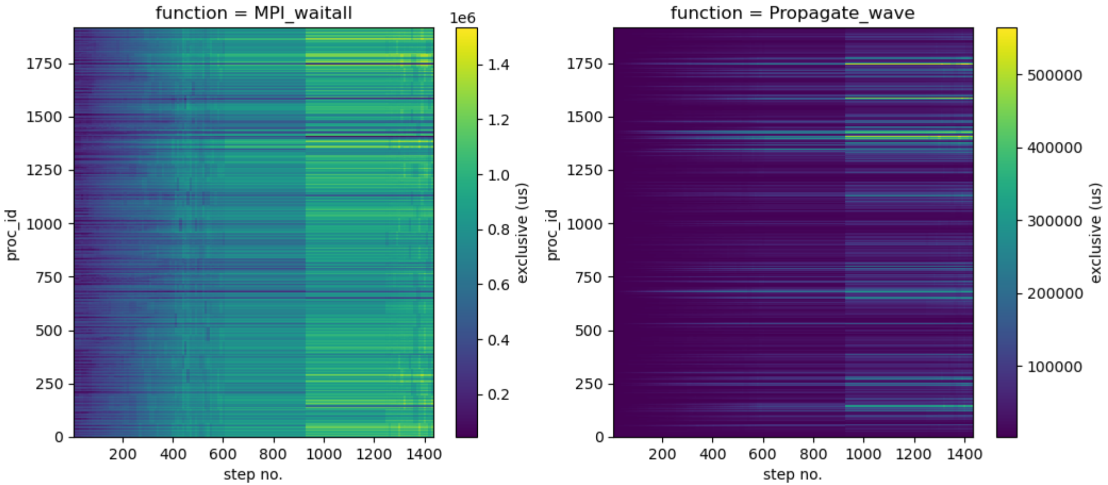

ERA
3d-TR
1d-TR
1d-ST
1d-ST (enhanced GWs)
Altitude (km)
Year
(m s⁻¹)
1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015


5°S $< \phi <$ 5°N
z=24km
ERA
3d-TR
1d-TR
1d-ST
1d-ST (enhanced GWs)
Altitude (km)
Latitude
20°N
10°N
EQ
10°S
20°S
1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Year
(m s^{-1})
20
10
0
−10
−20
−30
−40

Notes on the model performance


runtime per simulated day (s day^{-1})
800.0
600.0
400.0
200.0
0.0
ICON (noGWD)
ICON (MS-GWaM-1D)
fraction MS-GWaM-1D
ICON (MS-GWaM-3D)
fraction MS-GWaM-3D

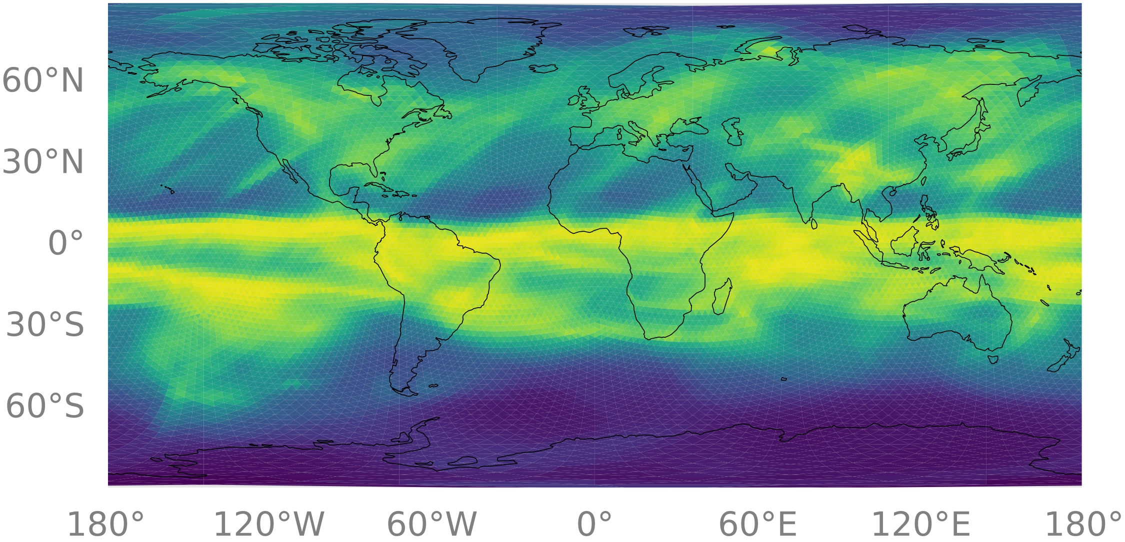
Load imbalance (for physical reasons)



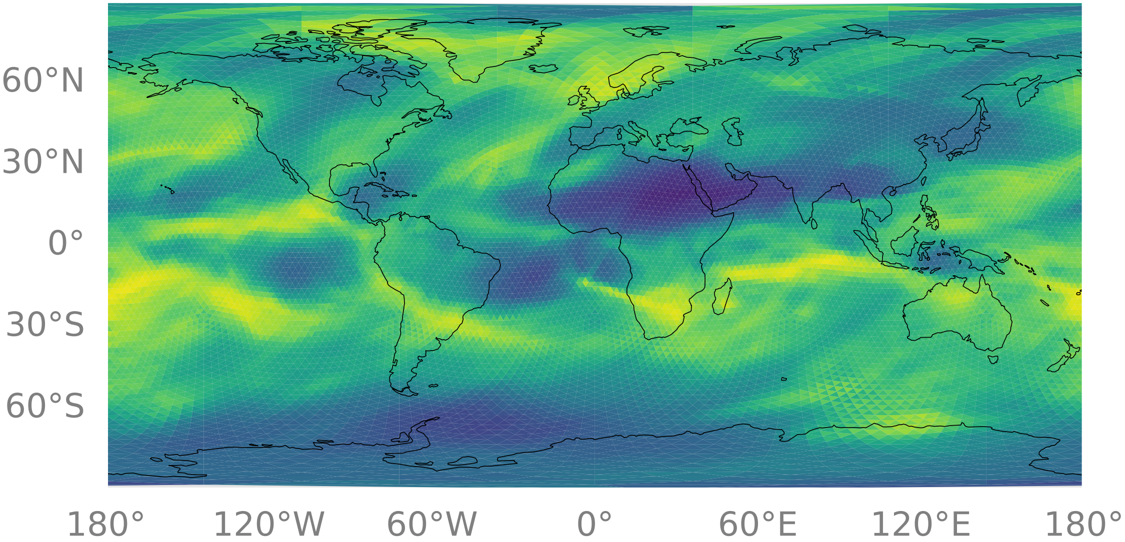
MPI barrier

computation

Ray volume counts (June)



Ray volume counts (December)



Towards a community-available model

MS-GWaM in ICON - From deep integration to a modular framework

+ convection
+ ice microphysics
+ turbulence
(planned)

MS-GWaM

MS-GWaM in ICON - From deep integration to a modular framework

+ convection
+ ice microphysics
+ turbulence
(planned)

MS-GWaM

ComIn

Modular Earth Submodel System

ice microphysics (current work by Alena Kosareva)

MS-GWaM

turbulence (planned)

Important references

Achatz, U., B. Ribstein, F. Senf, and R. Klein (2017). "The Interaction between Synoptic-Scale Balanced Flow and a Finite-Amplitude Mesoscale Wave Field throughout All Atmospheric Layers: Weak and Moderately Strong Stratification". *Quarterly Journal of the Royal Meteorological Society* 143.702, pp. 342–361. DOI: 10.1002/qj.2926.

Achatz, U., Y.-H. Kim, and G. S. Voelker (2023). *Multi-Scale Dynamics of the Interaction Between Waves and Mean Flows: From Nonlinear WKB Theory to Gravity-Wave Parameterizations in Weather and Climate Models*. DOI: 10.48550/arXiv.2310.07334. Journal of Mathematical Physics, accepted: arXiv:2310.07334.

Achatz, U., R. Klein, and F. Senf (2010). "Gravity Waves, Scale Asymptotics and the Pseudo-Incompressible Equations". *Journal of Fluid Mechanics* 663, pp. 120–147. DOI: 10.1017/S0022112010003411.

Bölöni, G., Y.-H. Kim, S. Borchert, and U. Achatz (2021). "Toward Transient Subgrid-Scale Gravity Wave Representation in Atmospheric Models. Part I: Propagation Model Including Nondissipative Wave–Mean-Flow Interactions". *Journal of the Atmospheric Sciences* 78.4, pp. 1317–1338. DOI: 10.1175/JAS-D-20-0065.1.

Drob, D. P. et al. (2015). "An Update to the Horizontal Wind Model (HWM): The Quiet Time Thermosphere". *Earth and Space Science* 2.7, pp. 301–319. DOI: 10.1002/2014EA000089.

Emmert, J. T. et al. (2022). "NRLMSIS 2.1: An Empirical Model of Nitric Oxide Incorporated Into MSIS". *Journal of Geophysical Research: Space Physics* 127.10. DOI: 10.1029/2022JA030896.

Grimshaw, R. (1972). "Nonlinear Internal Gravity Waves in a Slowly Varying Medium". *Journal of Fluid Mechanics* 54.2, pp. 193–207. DOI: 10.1017/S0022112072000631.

— (1974). "Internal Gravity Waves in a Slowly Varying, Dissipative Medium". *Geophysical Fluid Dynamics* 6.2, pp. 131–148. DOI: 10.1080/03091927409365792.

Hasha, A., O. Bühler, and J. Scinocca (2008). "Gravity Wave Refraction by Three-Dimensionally Varying Winds and the Global Transport of Angular Momentum". *Journal of the Atmospheric Sciences* 65, pp. 2892–2906. DOI: 10.1175/2007JAS2561.1.

Hindley, N. P., C. J. Wright, L. Hoffmann, T. Moffat-Griffin, and N. J. Mitchell (2020). "An 18-Year Climatology of Directional Stratospheric Gravity Wave Momentum Flux From 3-D Satellite Observations". *Geophysical Research Letters* 47.22. DOI: 10.1029/2020GL089557.

Holt, L. A., C. M. Brabec, and M. J. Alexander (2023). "Exploiting High-Density Zonal-Sampling of HIRDLS Profiles Near 60°S to Investigate Missing Drag in Chemistry-Climate Models". *Journal of Geophysical Research: Atmospheres* 128.8, e2022JD037398. DOI: 10.1029/2022JD037398.

Kim, Y. H., G. Bölöni, S. Borchert, H. Y. Chun, and U. Achatz (2021). "Toward Transient Subgrid-Scale Gravity Wave Representation in Atmospheric Models. Part II: Wave Intermittency Simulated with Convective Sources". *Journal of the Atmospheric Sciences* 78.4, pp. 1339–1357. DOI: 10.1175/JAS-D-20-0066.1.

Kim, Y.-H., G. S. Voelker, G. Bölöni, G. Zängl, and U. Achatz (2024). "Crucial Role of Obliquely Propagating Gravity Waves in the Quasi-Biennial Oscillation Dynamics". *Atmospheric Chemistry and Physics* 24.5, pp. 3297–3308. DOI: 10.5194/acp-24-3297-2024.

Lindzen, R. S. (1981). "Turbulence and Stress Owing to Gravity Wave and Tidal Breakdown". *Journal of Geophysical Research* 86.C10, pp. 9707–9714. DOI: 10.1029/jc086ic10p09707.

Muraschko, J., M. D. Fruman, U. Achatz, S. Hickel, and Y. Toledo (2015). "On the Application of Wentzel-Kramer-Brillouin Theory for the Simulation of the Weakly Nonlinear Dynamics of Gravity Waves". *Quarterly Journal of the Royal Meteorological Society* 141.688, pp. 676–697. DOI: 10.1002/qj.2381.

References III

Orr, A., P. Bechtold, J. Scinocca, M. Ern, and M. Janiskova (2010). "Improved Middle Atmosphere Climate and Forecasts in the ECMWF Model through a Nonorographic Gravity Wave Drag Parameterization". *Journal of Climate* 23.22, pp. 5905–5926. DOI: 10.1175/2010JCLI3490.1.

Swinbank, R. and D. A. Ortland (2003). "Compilation of Wind Data for the Upper Atmosphere Research Satellite (UARS) Reference Atmosphere Project". *Journal of Geophysical Research: Atmospheres* 108.D19, p. 4615. DOI: 10.1029/2002jd003135.

Voelker, G. S., G. Bölöni, Y.-H. Kim, G. Zängl, and U. Achatz (2023). *MS-GWaM: A 3-Dimensional Transient Gravity Wave Parametrization for Atmospheric Models*. DOI: 10.48550/arXiv.2309.11257. Journal of Atmospheric Sciences, accepted: arXiv:2309.11257.

Wei, J., G. Bölöni, and U. Achatz (2019). "Efficient Modeling of the Interaction of Mesoscale Gravity Waves with Unbalanced Large-Scale Flows: Pseudomomentum-Flux Convergence versus Direct Approach". *Journal of the Atmospheric Sciences* 76.9, pp. 2715–2738. DOI: 10.1175/JAS-D-18-0337.1.