GRAVHEDRAL: a novel gravity inversion method to unravel the interior of planetary bodies

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GRAVHEDRAL

Introduction

- The gravity signature of a planet reflects its internal density distribution, which in turn informs about its architecture shaped by millions of years of geological processes.
- Gravity anomaly data often represent one of the few global-scale geophysical datasets available for planetary bodies other than Earth.
- The capability to properly model these datasets represents a key chance to reconstruct the internal framework of such bodies and better elucidate their geological history.

1. *GRAVHEDRAL*: methodology overview

- Novel tool to perform 3D inverse gravity modelling specifically designed for planetary-scale applications.
- Planets' interior parameterized by polyhedra, suitable to faithfully approximate shapes of topography/internal layers.
- Densities of polyhedra defined by polynomial functions[1] able to deal with the complexity of actual density distributions.
- Model parameters < polynomial coefficients *or* polyhedral node positions
- Linear inverse problem when model parameters are polynomial coefficients (i.e., *Least-square* inversion).

Work in progress

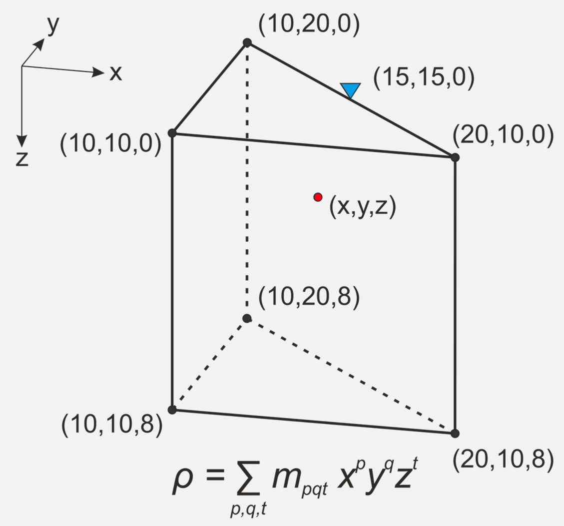
- Non-linear inverse problem when model parameters are polyhedral node positions.

Hamiltonian Monte Carlo probabilistic inversion scheme[2,3]

HMCLab

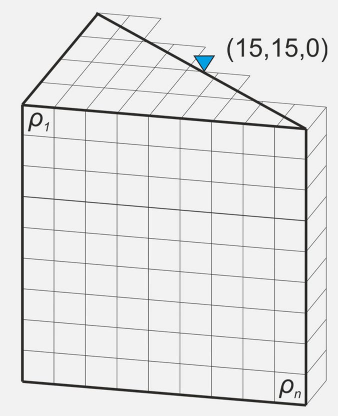
2. *GRAVHEDRAL* vs prism-based[4] approach:

Parameterization strategy



$\rho = \sum_{p,q,t} m_{pqt}\, x^p y^q z^t$

GRAVHEDRAL	Prism-based approach
Nodes, edges & facets	*n* prisms
Density inside target body defined on every site *(x,y,z)* (**not discretized**)	Density inside target body **discretized**



ρ_1 = const. 1
⋮
ρ_n = const. n

Performance

(**Chipset**: Intel i7-11390H @ 3.40 GHz, 32.0 GB RAM)

Unit of length: km | **Observation point:** (15,15,0) | **Density function:** $\rho = 10000.0\, x^2yz$

	GRAVHEDRAL	# prisms: 360 (dx=dy=dz=1km)	# prisms: 396,000 (dx=dy=dz=0.1km)	# prisms: 399,606,400 (dx=dy=dz=0.01km)
g_z [mGal]	5.9015786726e6	4.9273082592e6	5.7950123257e6	5.8908836669e6
Time [s]	6.8e-1	0.3e-2	3.5	3.3e3
Memory [MiB]	177.7	2.4	2.3e3	2.3e6

Sponsors

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References

1. Ren, Z., Chen, C., Zhong, Y., Chen, H., Kalscheuer, T., Maurer, H., Tang, J., & Hu, X. (2020). Recursive analytical formulae of gravitational fields and gradient tensors for polyhedral bodies with polynomial density contrasts of arbitrary non-negative integer orders. *Surveys in Geophysics*, *41*, 695-722.
2. Zunino, A., Gebraad, L., Ghirotto, A., & Fichtner, A. (2023). HMCLab: a framework for solving diverse geophysical inverse problems using the Hamiltonian Monte Carlo method. *Geophysical Journal International*, *235*(3), 2979-2991.
3. Zunino, A., Ghirotto, A., Armadillo, E., & Fichtner, A. (2022). Hamiltonian Monte Carlo probabilistic joint inversion of 2D (2.75 D) gravity and magnetic data. *Geophysical Research Letters*, *49*(20), e2022GL099789.
4. Nagy, D. (1966). The gravitational attraction of a right rectangular prism. *Geophysics*, *31*(2), 362-371.
5. Nolan, M. C., Magri, C., Howell, E. S., Benner, L., Giorgini, J., Hergenrother, C., Hudson, R., Lauretta, D., Margot, J., Ostro, S., & Scheeres, D. J. (2013). Asteroid (101955) Bennu Shape Model V1.0. EAR-A-I0037-5-BENNUSHAPE-V1.0. *NASA Planetary Data System*, https://sbn.psi.edu/pds/resource/bennushape.html.
6. Tarantola, A. (2005). *Inverse problem theory and methods for model parameter estimation*. Society for industrial and applied mathematics.
7. Ren, Z., Zhang, Y., Zhong, Y., Pan, K., Wu, Q., & Tang, J. (2022). Magnetic anomalies caused by 3D polyhedral structures with arbitrary polynomial magnetization. *Geophysical Research Letters*, *49*(18), e2022GL099209.

3. Linear inverse problem: synthetic test on Bennu[5]

Synthetic forward problem (300 observations $\boldsymbol{d_{obs}}$)

- Polynomial density function used:

$$\rho_{true} = \underbrace{2670.0}_{m_1} \underbrace{- 5000.0x^2}_{m_2} \underbrace{- 5000.0y^2}_{m_3} \underbrace{- 5000.0z^2}_{m_4} + \underbrace{800.0y^2z}_{m_5} + \underbrace{8000.0xyz}_{m_6} + \underbrace{2000.0xy^2}_{m_7} + \underbrace{1000.0x^2z}_{m_8} + \underbrace{1000.0x^2y}_{m_9}$$

$\boldsymbol{m} = [m_1, \dots\dots, m_9]$ ⟹ Polynomial coefficients to estimate by *Least-square* inversion

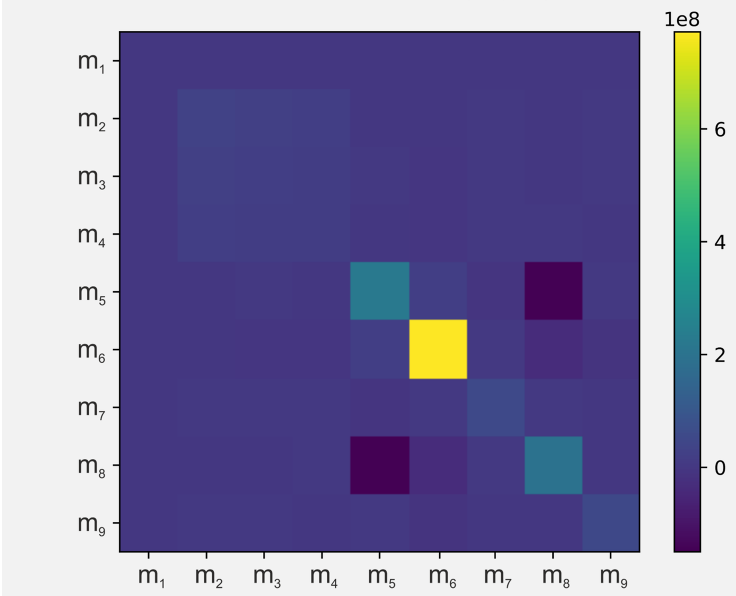
Least-square inversion[6] (with uncertainty quantification)

$$\boldsymbol{m_{inv}} = \left(\boldsymbol{G^t C_D^{-1} G}\right)^{-1}\left(\boldsymbol{G^t C_D^{-1} d_{obs}}\right) = \boldsymbol{\widetilde{C}_M}\left(\boldsymbol{G^t C_D^{-1} d_{obs}}\right)$$

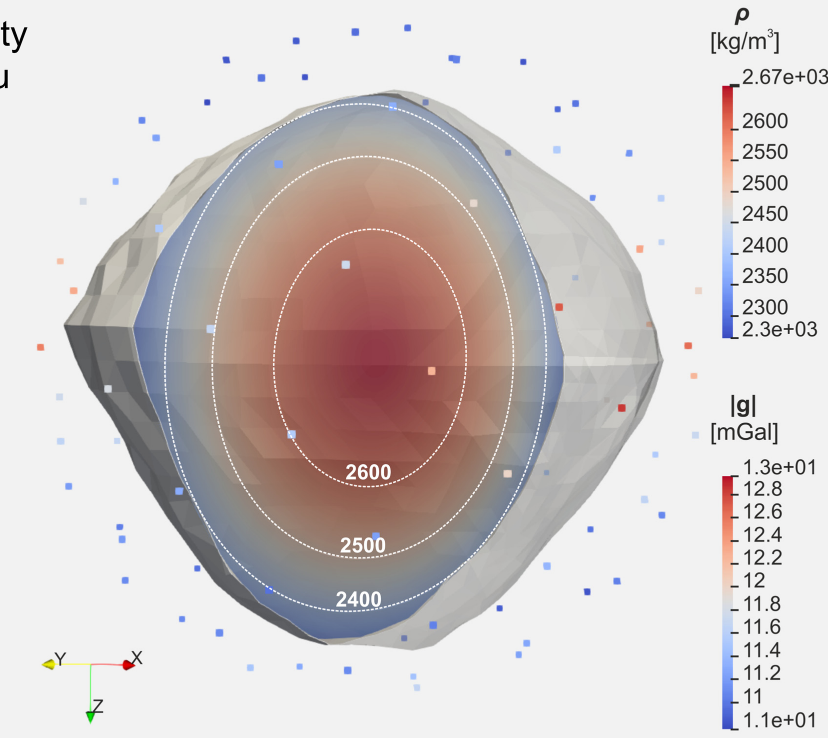
$\boldsymbol{G}$ kernel matrix: [300 rows x 9 columns] ⟹ Very small size compared to that using the prism-based approach

$$\rho_{inv} = 2670.0 - 5000.0x^2 - 5000.0y^2 - 5000.0z^2 + 800.0y^2z + 7999.99xyz + 1999.99xy^2 + 1000.0x^2z + 999.99x^2y$$

Interrogation of ρ_{inv} provides the density value at each point *(x,y,z)* inside Bennu



Posterior covariance matrix $\widetilde{C}_M$



4. Conclusions and future prospectives

- Gravity response precise and fast to calculate.
- Density information continuous inside the target body.
- Inverse problem w.r.t. polynomial coefficients becomes linear and easy to manage (i.e., few model parameters).
- Inversion results depend mainly on < polynomial basis chosen; data quality/resolution
- Methodology-related code released open-source soon.
- *GRAVHEDRAL* planned to be extended to the magnetic case[7].

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