Validation of effective subglacial hydrology models

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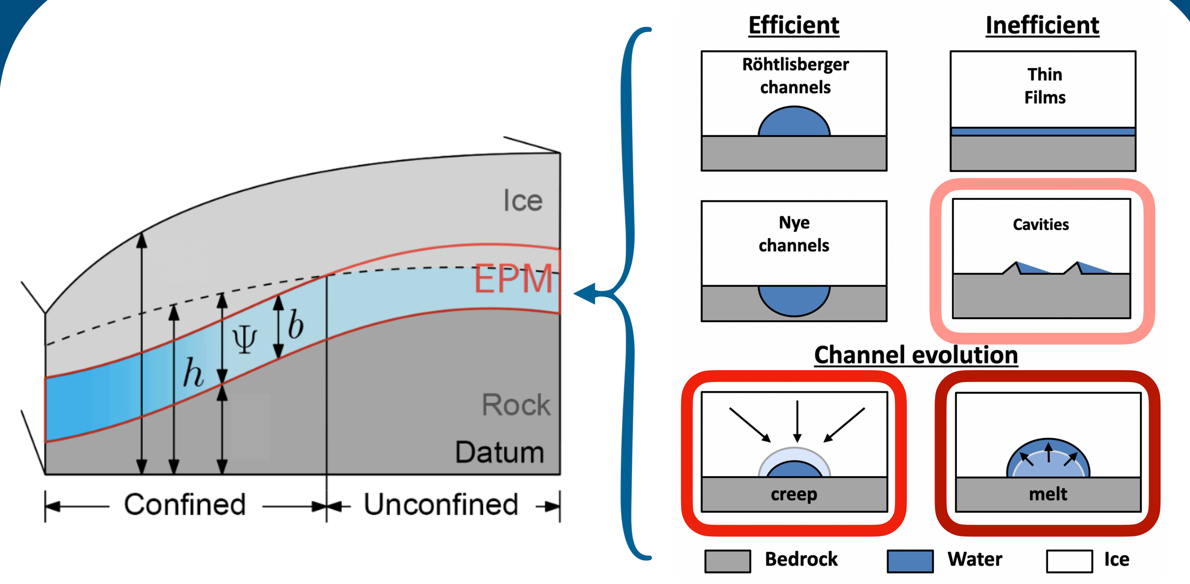
Abstract

- **What?**
  We simulate fluid flow for a variety of cases with a generalized diffusion type equation and validate a subglacial hydrology model with it.
- **Why multiple cases?**
  By exploring a range of cases that have analytical solutions we can validate more aspects of the numerical simulation, thereby ascertain its credibility.
- **Benefits?**
  Computationally lightweight and easily conductible tests to implement into the code as verification tools.
- **Novelty**
  We extend the general diffusion equation for a time evolving diffusivity (akin to creep and cavity terms in subglacial hydrology models) and find analytical solutions for this special case.

Online Abstract

Subglacial Hydrology Model

- Model: The parallel implementation of the Confined–Unconfined Aquifer System model (CUAS-MPI) [1]
- Uses an Effective porous medium (EPM) approach [1,2] (No individual channels)
- Solves for the hydraulic head h on 2 spatial dimensions
- We extended the model to solve higher order non-linear flows and make use of the internal transmissivity evolution terms



Flow equation

$$S_e \frac{\partial h}{\partial t} = \nabla \cdot \left(T \nabla h \right) + Q$$

Transmissivity evolution

$$\frac{dT}{dt} = \text{melt} - \text{creep} + \text{cavities}$$

h hydraulic head
T effective transmissivity
Q water source
S_e effective storage

Axisymmetric solutions

A Generalization of the flow equations through α and β

$$S_e \frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r T \frac{\partial h}{\partial r} \right)$$

$$2\pi \int_0^{r_N(t)} h r dr = Q t^{\alpha}$$

$$T = D h^{\beta}$$

B Time-dependent and spatially uniform transmissivity $D = D(t)$

$$\frac{dD}{dt} = -aD + b$$

creep cavities

r_N front extent, D conductivity, h_0 initial h, σ curvature

Mathematical Solutions

A Similarity solution for any α and β (constraint: D is const.) [3]

$$h(r,t) \propto t^{\delta} \qquad \delta = \frac{\alpha - 1}{\beta + 1}$$

$$r_N \propto t^{\gamma} \qquad \gamma = \frac{\alpha\beta + 1}{2(\beta + 1)}$$

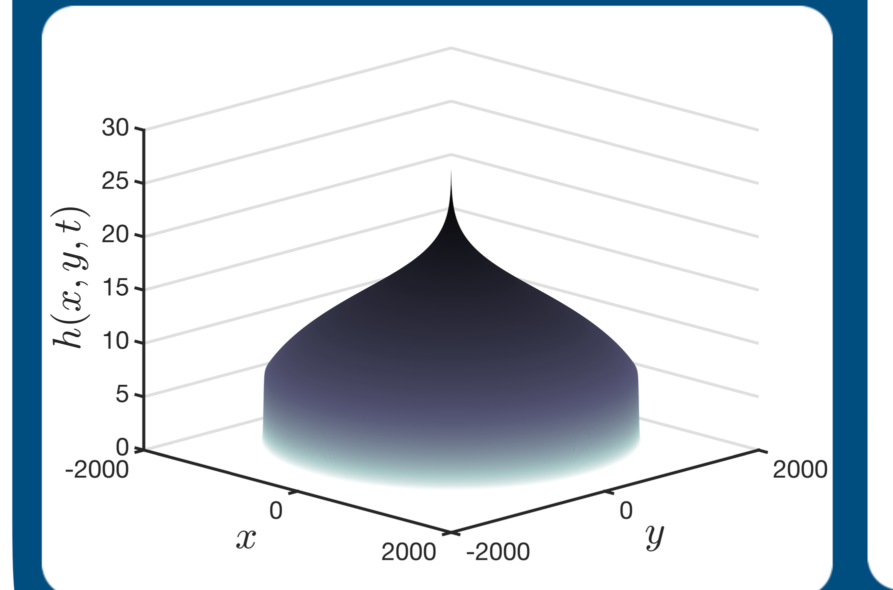
B Time-dependent transmissivity solution (constraint: $\alpha = \beta = 0$)

$$h(r,t) = \frac{h_0}{G(t)} e^{-\frac{\sigma r^2}{G(t)}}$$

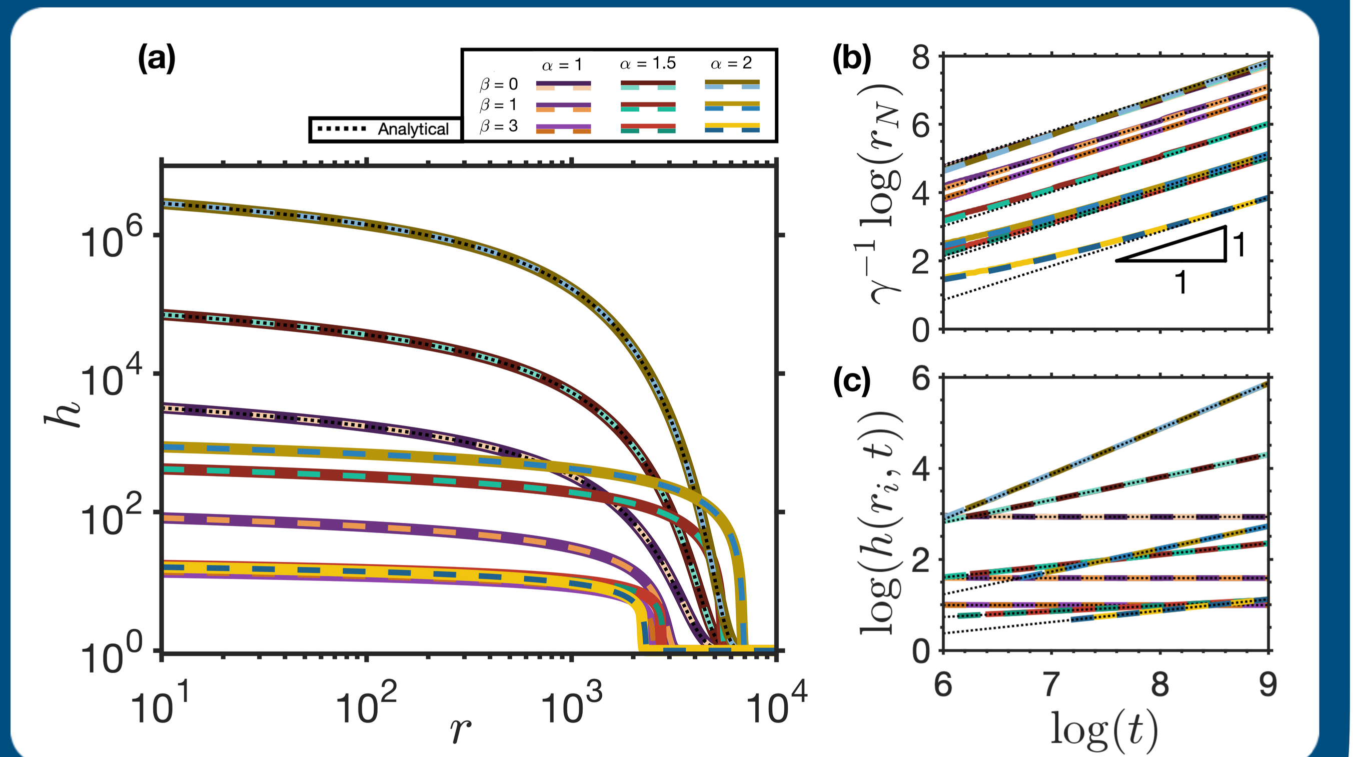
$$G(t) = 1 + \frac{4\sigma}{aS}\left(c_0(1 - e^{-at})\right) + b\,t$$

Results: A with $D =$ const. for different α, β

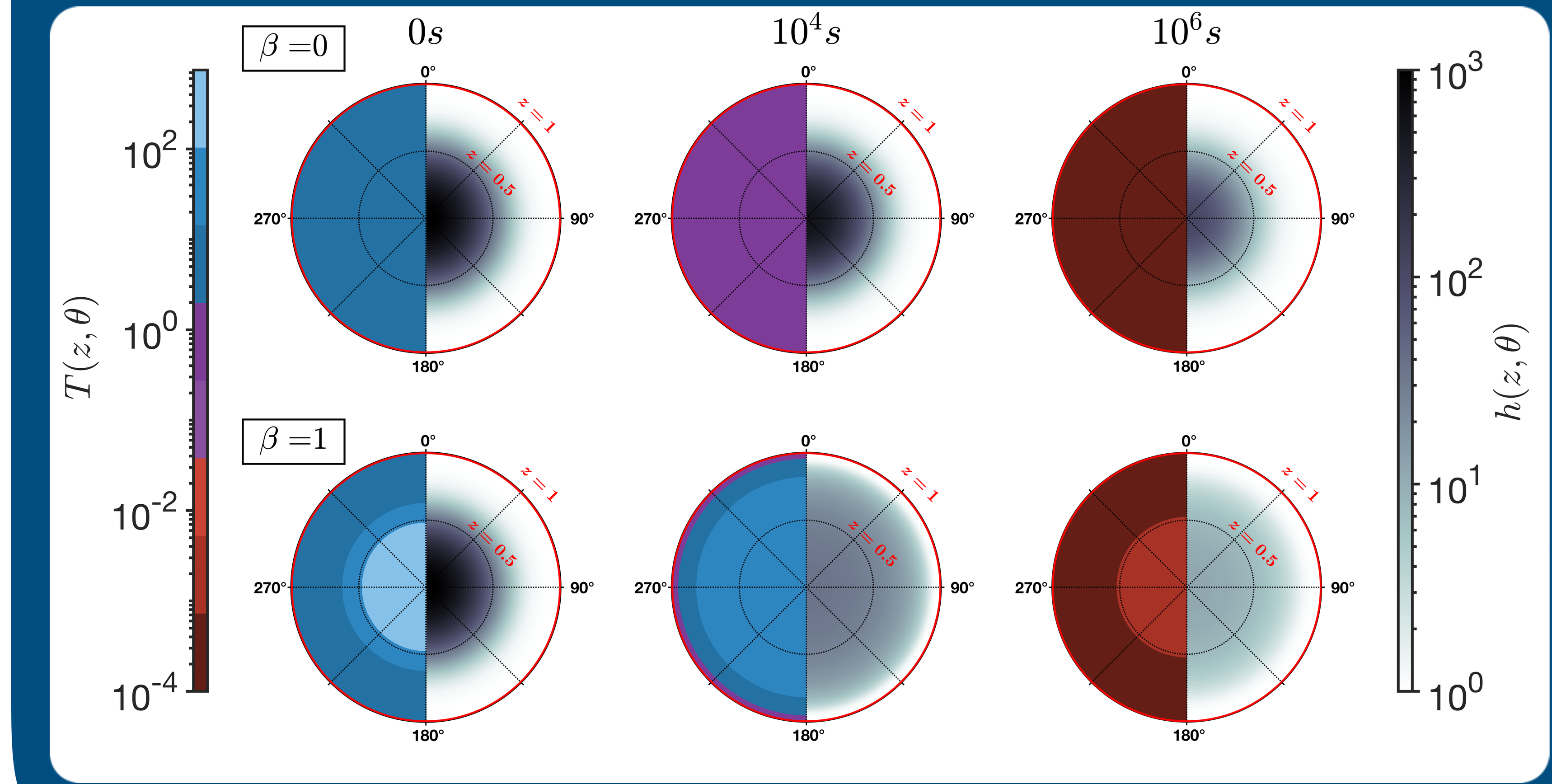
3D field CUAS-MPI

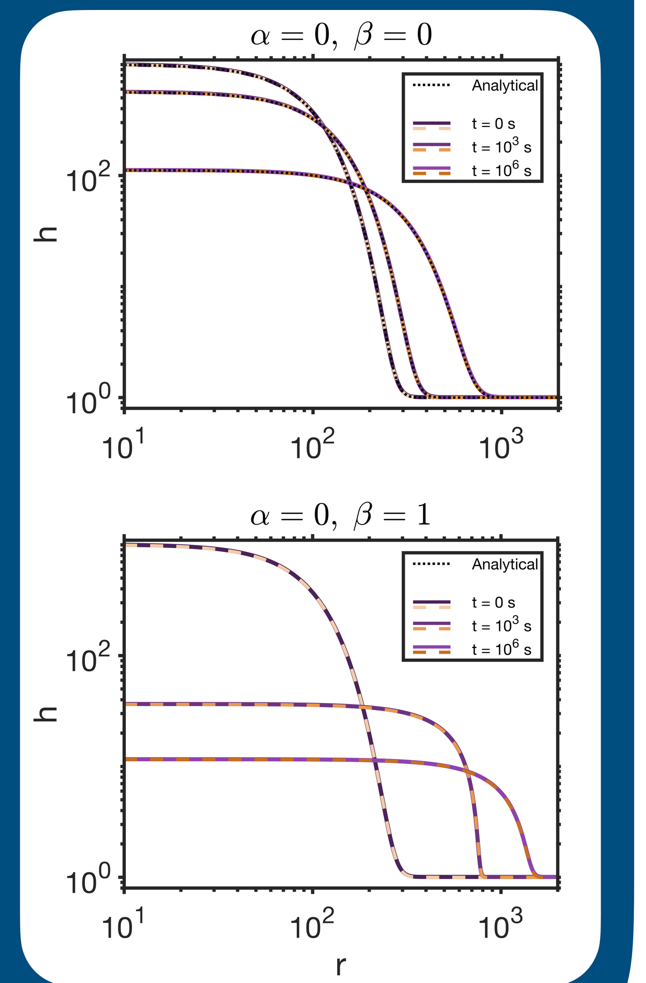


Similarity solution



B for time-dependent transmissivity $D = D(t)$





Conclusion

- We show that the non-axisymmetric solver can simulate axisymmetric problems consistently with our analytical predictions
- Numerical results are consistent with similarity solutions when D is constant, and with non-selfsimilar solutions when D evolves in time.
- Our solutions are readily applicable to other subglacial hydrology models using an EMP approach

References

[1] Fischler, Y. et al. (2023) A parallel implementation of the confined–unconfined aquifer systemmodel for subglacial hydrology: Design, verification, and performance analysis (cuas-mpi v0.1.0). Geoscientific Model Development, 16(18), 5305–5322. DOI: 10.5194/gmd-16-5305-2023

[2] Beyer, S. et al. (2018) "A confined–unconfined aquifer model for subglacial hydrology and its application to the Northeast Greenland Ice Stream" In: *The Cryosphere 12.12, pp. 3931–3947*. DOI: 10.5194/tc-12-3931-2018

[3] Barenblatt, G.I., 1952. On some unsteady-state movements of liquid and gas in porous medium. Prikl. Mat. I Mekh. 16 (1), 67–78, [In Russian].

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