Formation of thermal vortex rings in high Re limit

Paweł Jędrejko[1], Jun-Ichi Yano[2], Marta Wacławczyk[1]

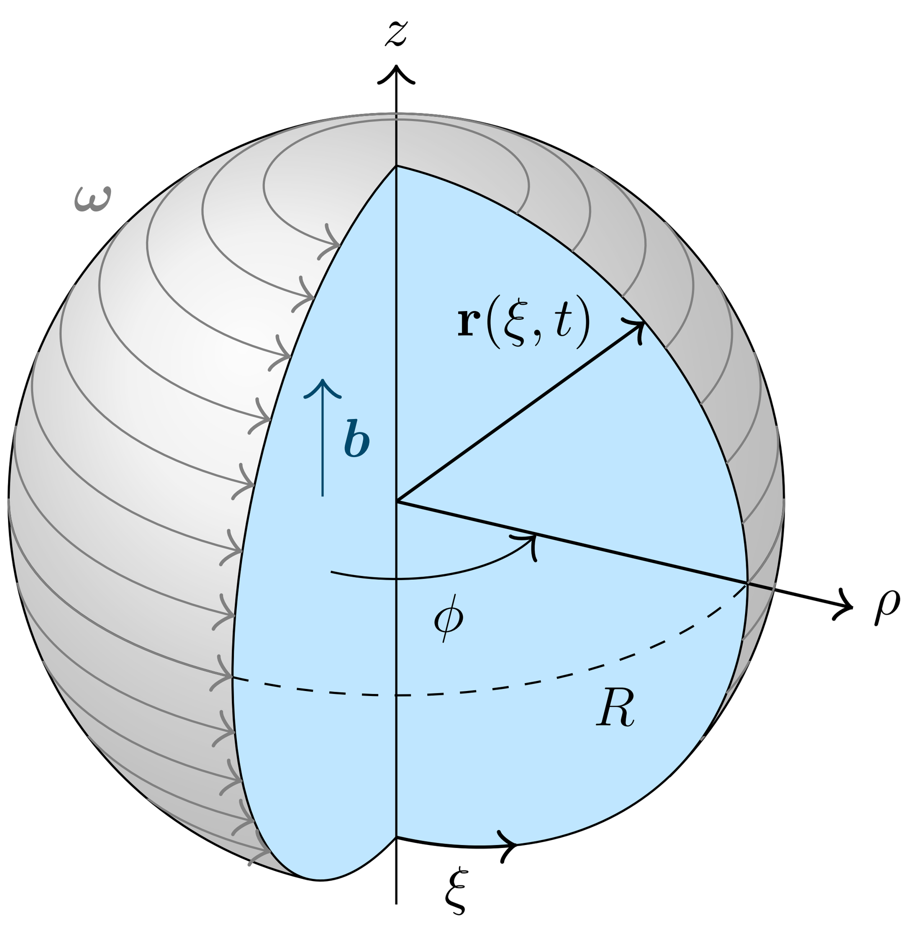
[1]University of Warsaw, Faculty of Physics, Institute of Geophysics, Pasteura 5, 02-093 Warszawa, Poland
[2]CNRM UMR3589 (CNRS), Météo-France, 31057 Toulouse Cedex, France
Correspondence: p.jedrejko@uw.edu.pl

Motivation

Thermal vortex rings are considered candidates for the building blocks of atmospheric convection. They are unresolved by most of the weather and climate models and as subgrid-scale phenomena, have to be parameterized. Of special interest are their rising speed and the features of mixing processes they drive. The latter can be assumed purely advective due to the huge Reynolds number ($\approx 10^9$)[1] which characterizes atmospheric thermals. For that reason, the qualitative insight provided by grid-based numerical methods is limited and we turn to the Lagrangian, vorticity-based approach.

Problem statement

The system is initialized as a spherical buoyancy anomaly assuming:



- axial symmetry
- Boussinesq approximation
- discontinuous buoyancy
- negligible molecular diffusion

these reduce the system to the self-advection of the interfacial vortex sheet, i.e. one-dimensional, singular distribution of vorticity. It is described parametrically in cylindrical coordinates $\{\rho, \phi, z\}$:

$$\xi \in [0,\,1), \qquad \gamma = \gamma(\xi,t), \qquad \boldsymbol{r} = \Big(\rho(\xi,t),\, z(\xi,t)\Big)$$

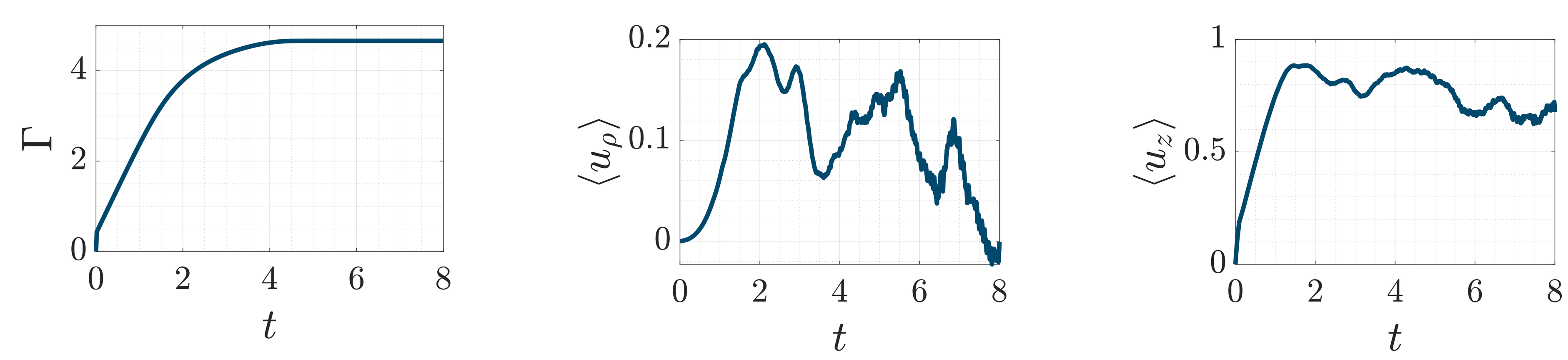
where γ is the sheet's strength related to vorticity by $\gamma\, d\xi = \omega\, d\rho dz$. Nondimensionalization is done with b and R. The evolution is governed by:

$$\frac{\partial \gamma}{\partial t} = \frac{\partial z}{\partial \xi}, \qquad \frac{d\boldsymbol{r}_0}{dt} = \int_0^1\int_0^{2\pi} \frac{\gamma\hat{\phi}\times(\boldsymbol{r}_0-\boldsymbol{r})}{(|\boldsymbol{r}_0-\boldsymbol{r}|^2+\delta^2)^{3/2}}\,\rho d\phi d\xi$$

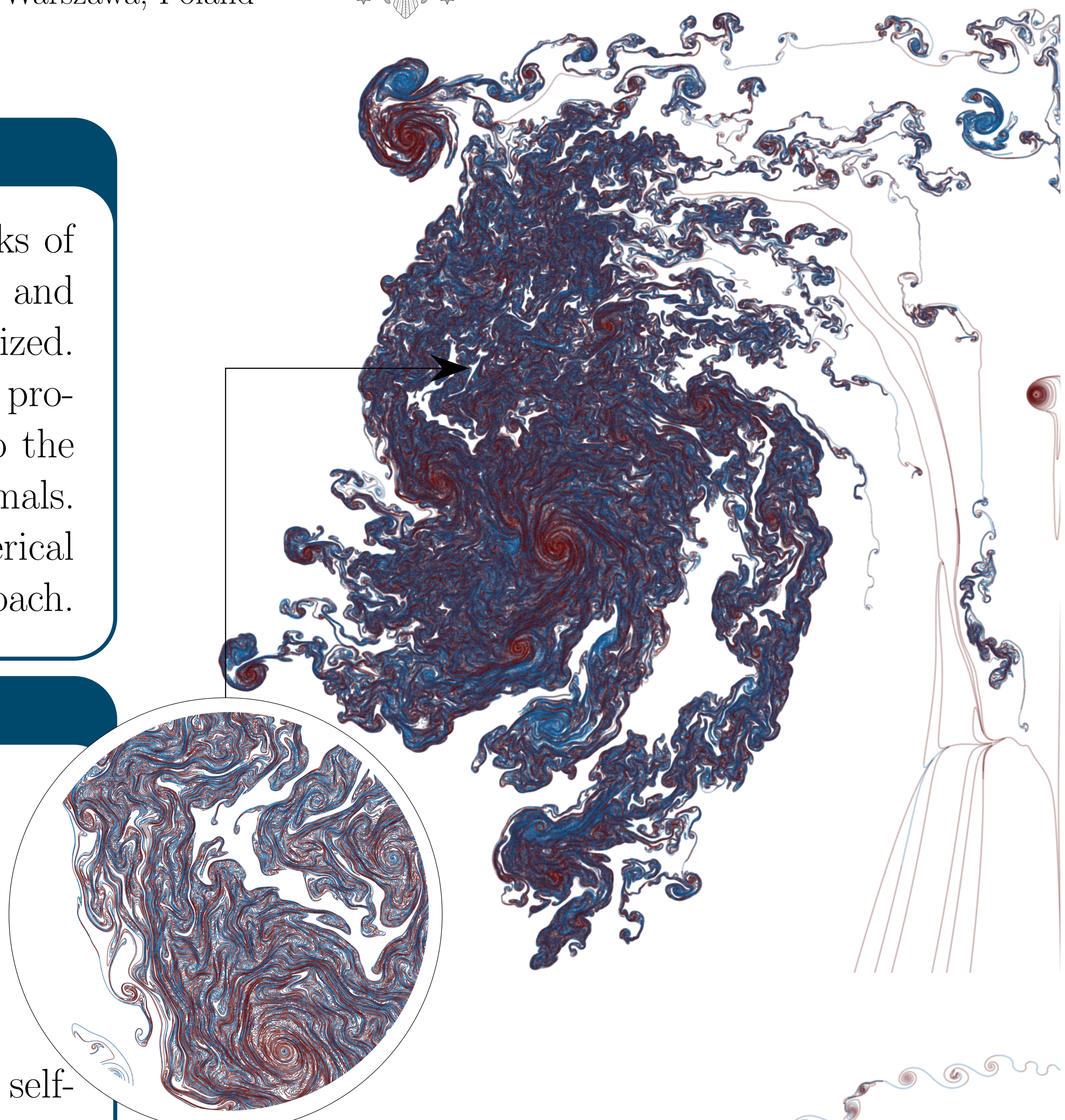
where δ is the Biot-Savart regularization parameter[2] and the second equation captures the buoyant generation of interfacial vorticity.

Formation process

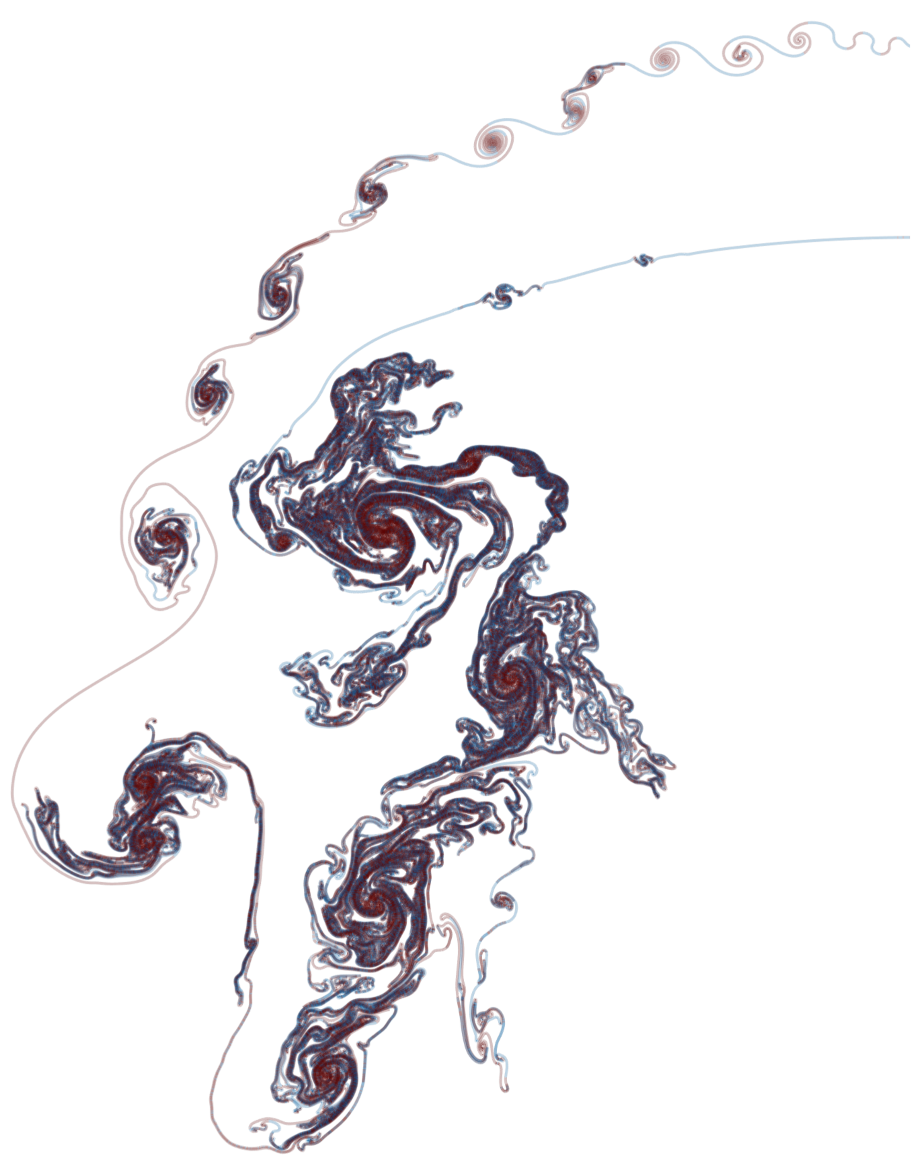
The evolution of the vortex ring is characterized by the half-plane circulation Γ and volume-averaged velocities $\langle u_\rho \rangle$, $\langle u_z \rangle$, which are presented below.



At initial stage, buoyancy force amplifies the interfacial vortex sheet. This leads to the collapse of the anomaly at the bottom and Kelvin-Helmholtz instability in weakly stretched regions. The resulting layer of "cat-eye" vortices acts like a thicker vortex sheet and is subjected to analogical instability but in a lower wavenumber. This hierarchical process repeats iteratively until the system is dominated by a few large eddies. Their chaotic interplay distributes the vortex sheet over the space, with increasing density, although high intermittency. The latter manifests by the long persistence of sheet-free "islands" surrounded by a nearly continuous distribution of vorticity.



Detail: Intermittency in the ring's body.



Detail: Remnants of a previous generation K-H vortices forming a new one.

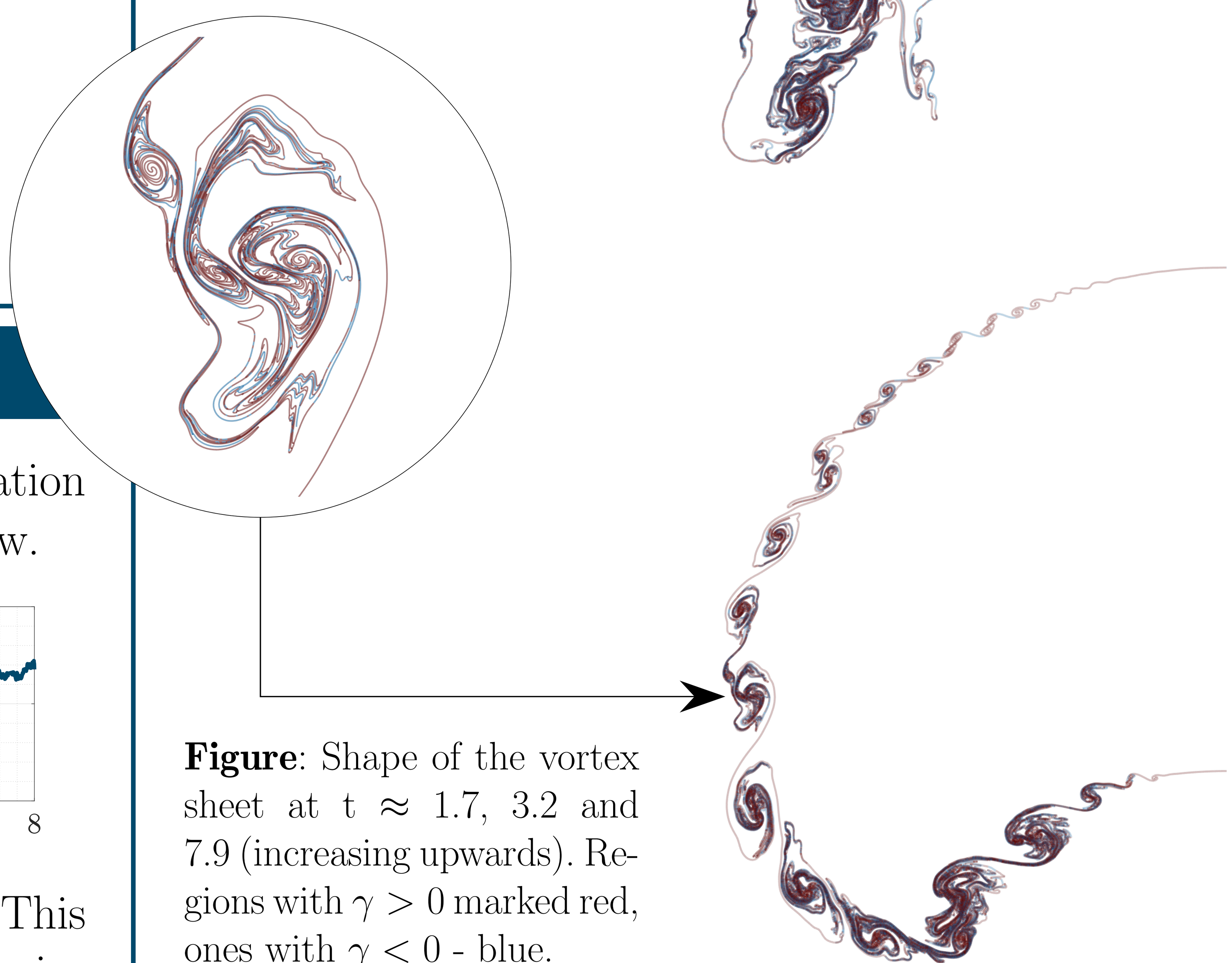


Figure: Shape of the vortex sheet at t ≈ 1.7, 3.2 and 7.9 (increasing upwards). Regions with $\gamma > 0$ marked red, ones with $\gamma < 0$ - blue.

References

[1] Morrison H. et al. "What Controls the Entrainment Rate of Dry Buoyant Thermals with Varying Initial Aspect Ratio?" *J. Comput. Phys* 80, 2711 (2023).
[2] Krasny R. "Desingularization of Periodic Vortex Sheet Roll-up". *J. Atmos. Sci.* 65, 292 (1985).

EGU General Assembly 2024, Vienna

LaTeX TikZposter