Session : **Earth's and planetary cores: structure, dynamics and evolution** (EGU24-3475)

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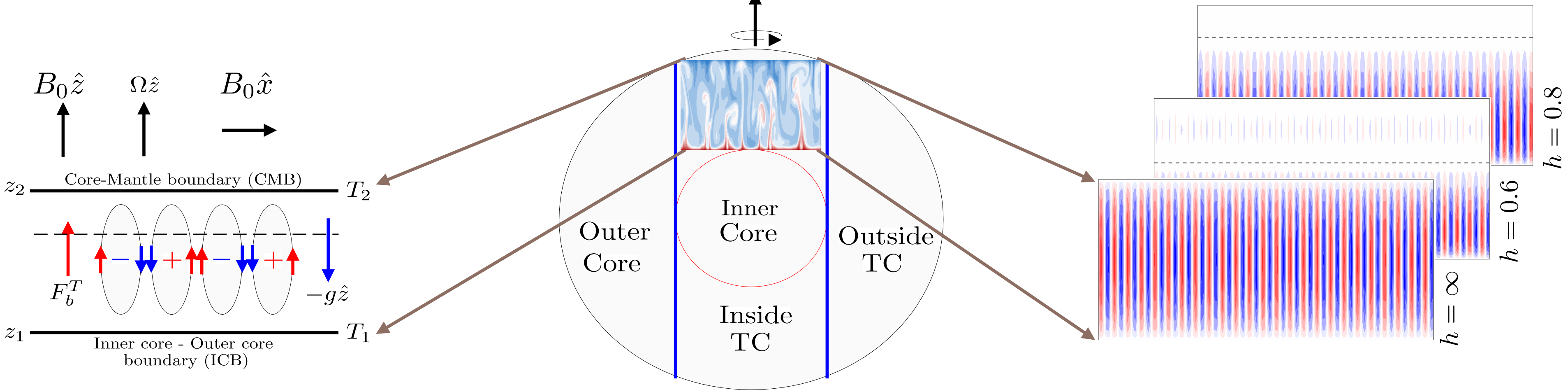

Title : **Back reaction of magnetic field on rotating penetrative convection**

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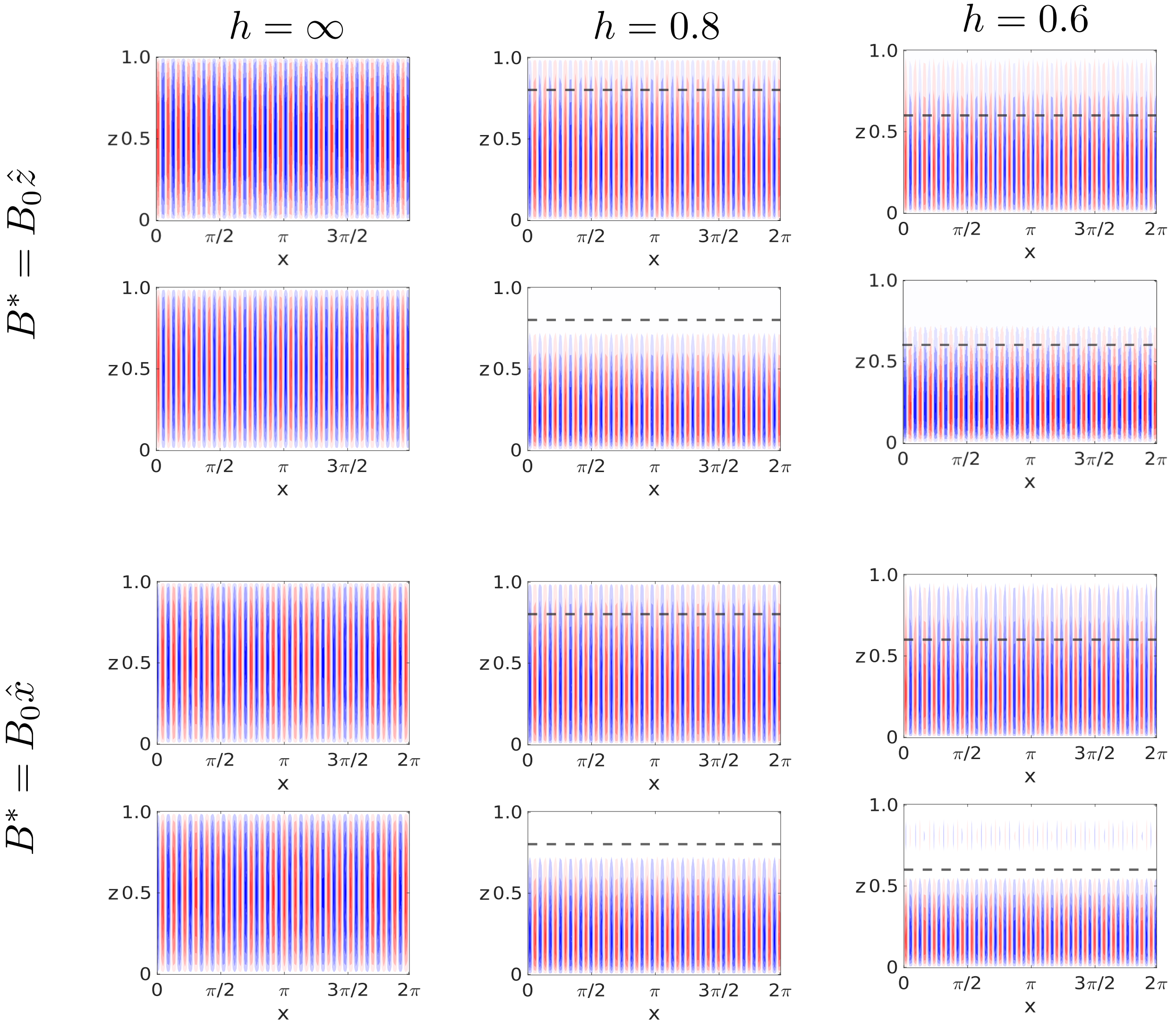
Results : $\Lambda = 0.001, E = 10^{-3}, Pr = Pm = 1$



Discussions

- The Ra_c reduces $2-3$ % as the configuration of imposed magnetic field changes from vertical to horizontal direction for a particular Λ values. The relative drops in Ra_c are $Ra_c^{h=0.8}/Ra_c^{h=\infty} = 0.86$ and $Ra_c^{h=0.6}/Ra_c^{h=\infty} = 0.53$ for strong and weak stratifications, respectively. The critical Rayleigh number reduces more rapidly for the horizontal field ($\approx 3\%$ reduction) than imposed vertical magnetic field ($\approx 1\%$ reduction) by varying strength of magnetic field as $\Lambda = 0, 0.001, 0.01$.

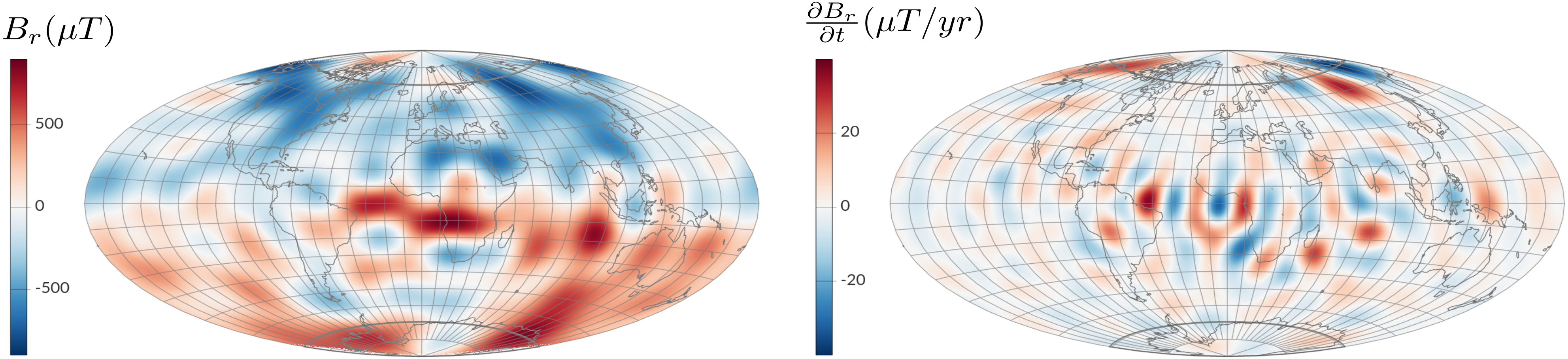
- The k_c remains unchanged as the configuration of imposed magnetic field changes from vertical to horizontal direction for a particular Λ values. Due to rapid rotation at $E = 10^{-4}$ thin columnar rolls are developed roubstly. At lower strength of magnetic field regime ($\Lambda << 1$), the flow develops geostrophic only as observed outside tangent cylinder for fully 3-D numerical simulaion in spherical shell (Olson & Glatzmaier, 1995).

References

- Olson, Peter, and Gary A. Glatzmaier. "Magnetoconvection in a rotating spherical shell: structure of flow in the outer core." Physics of the Earth and Planetary Interiors 92.1-2 (1995): 109-118.

- Sreenivasan, Binod, and Venkatesh Gopinath. "Confinement of rotating convection by a laterally varying magnetic field." Journal of Fluid Mechanics 822 (2017): 590-616.

Introduction



$B_r(\mu T)$

$\frac{\partial B_r}{\partial t}(\mu T/yr)$

Figure 1 (a) Magnetic field of the Earth at CMB (b) Secular variation of geomagnetic field of the Earth at CMB

Source : Reproduced from CHAOS-7 geomagnetic field model , epoch 2020.10 yr
https://geodyn.univ-grenoble-alpes.fr/

- Earth and other planetary bodies sustain magnetic field by dynamo action. The structure of the magnetic fields are not invariant for different planets. The origin of this anamoly is profoundly due to the differences in the internal structures of various planets.

- The structure of the magnetic field and secular variation of the Earth are shown in figure 1 (a) and 1 (b), respectively, at the depth of core-mantle boundary (after downward continuation of surface magnetic field) in epoch 2020.10 from CHAOS-7 geomagnetic field model.

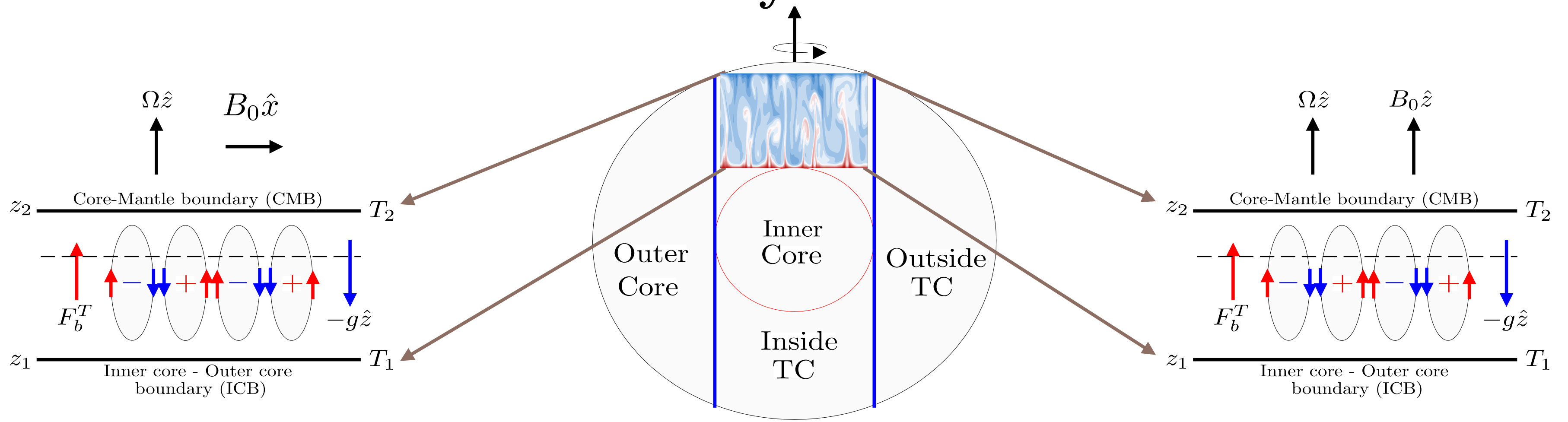
- It has been observed that the transition zones (like CMB, ICB, ULVZ) have immense impact on the generation of the geomagnetic field, derived from various satellite observation (like Swarm, Magsat, CHAMP). Likewise, the core-mantle boundary which is beleived to be thermally or compositionally heterogeneous may affect the dynamics of the core convection for self-sustaining dynamo action.

- The dynamo action maintains the magnetic field by the thermal and compositional convection at the Earth's core. The outer core derives energy from the inner core solidification for convective heat transfer. The core-mantle boundary thermal or compositional structure is highly heterogeneous which can affect the convection process and also the generation of magnetic field.

- It is beleived that a thermal or compositional stable stratification can exist near the CMB. Hence, it can significantly affect the interior dynamics, and interplay between fluid flow and magnetic field to generate planetary magnetic fields. The thickness of stable layer is not correctly constrained by the numerical simulation or geomagnetic field data.

- The interaction of the magnetic field with the fluid flow is one of the complex physical phenomena which has been studied for so long. Hence, the presence of rotation makes this phenomena more complex. Consequently, the study of generation of magnetic field by geo-dynamo in the Earth or other planetary bodies is full of mystery and simultaneously challenging.

Study Area



3 (c) Configuration : II ($B_0\hat{x}$)

Figure. 3 (a) Schematic diagram of Earth interior

Source : Modified from T. Barman, IIT(ISM) Dhanbad, Master thesis, 2022

3 (b) Configuration : I ($B_0\hat{z}$)

- In present study, we focus on the outer core convection which derives energy through inner core solidification. This provides energy to the self-sustaining geodynamo to produce and maintain magnetic field of the Earth.

- Currently, we focus on the polar region of the Earth (indicated by black rectangle in figure (3.a)), inside of the tangent cyliner (TC) which is a mathematically developed region comprising inner core volume and axis is tangential to inner core surface at the equator region, to keep the investigation simple. The core flow structure, heat transfer characteristics , magnetic field morphologies are completely different at outside of the tangent cylinder than inside of the tangent cylinder.

- In the present investigation the magnetoconvection is studied. The inherent difference between the dynamo action and magnetoconvection is that former produces magnetic field by strongly driven convective process while later provides insight about how the generated magnetic field interact with fluid flow.

- We develop a plane layer magneto-convection model, which is inherently different from dynamo action as it can not produce any magnetic field, (as shown in figure (3.b.)) to study the onset features of convective flows by implementing core-mantle thermal interaction in the presence of rotation ($\Omega\hat{z}$) and a uniform magnetic field ($B_0\hat{z}$) along vertical direction (z). Where, at bottom plate($z_1 = 0$ acts as hotter ICB), $T_1 = T + \Delta T$ and at top plate ($z_2 = D$ act as relatively colder CMB), $T_2 = T$. Also, $-g\hat{z}$ is the gravity and F_b^T is the thermal buoyancy force acting in opposite direction. The prolate ellipses represent convection rolls with upwelling flows by red arrows and downwelling flows by blue arrows.

- The onset features of penetrative magnetoconvection is studied by using thermal stable stratification of various strength. As results, we have reported thermal and axial velocity structures with critical Rayleigh values and corresponding convective flow length scales (horizontal).

Methodology

- Governing equation (dimensionless)

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\begin{aligned}\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + PmE^{-1}(\hat{z}\times\mathbf{u}) &= -\nabla\mathbf{P} + \Lambda PmE^{-1}((\nabla\times\mathbf{B})\times\mathbf{B}) \\ &+ qRaPmT\hat{z} + \Lambda PmE^{-1}((\nabla\times\mathbf{B})\times\mathbf{B}^*) + (\nabla\times\mathbf{B}^*)\times\mathbf{B})) + Pm\nabla^{\mathbf{2}}\mathbf{B}\end{aligned} \tag{3}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T + (\mathbf{u}\cdot\nabla)T^* = PmPr^{-1}\nabla^{\mathbf{2}}T \; + Q \tag{4}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla\times(\mathbf{u}\times\mathbf{B}^*) + \nabla^{\mathbf{2}}\mathbf{B} \tag{5}$$

- Initial conditions

$$\mathrm{u}^* = 0; \quad \frac{\partial T^*}{\partial t} = 0; \quad B^* = B_0\hat{z} \tag{6}$$

- Boundary conditions

$$(u_x, u_z) = (0,0);\ T = 0;\ (B_x, \frac{\partial B_z}{\partial z}) = (0,0)\ at\, z = 1 \tag{7a}$$

$$(u_x, u_z) = (0,0);\ T = 1;\ (B_x, \frac{\partial B_z}{\partial z}) = (0,0)\ at\, z = 0 \tag{7b}$$

- Basic state temperature profiles

$$T^* = 1 - z \tag{8a}$$

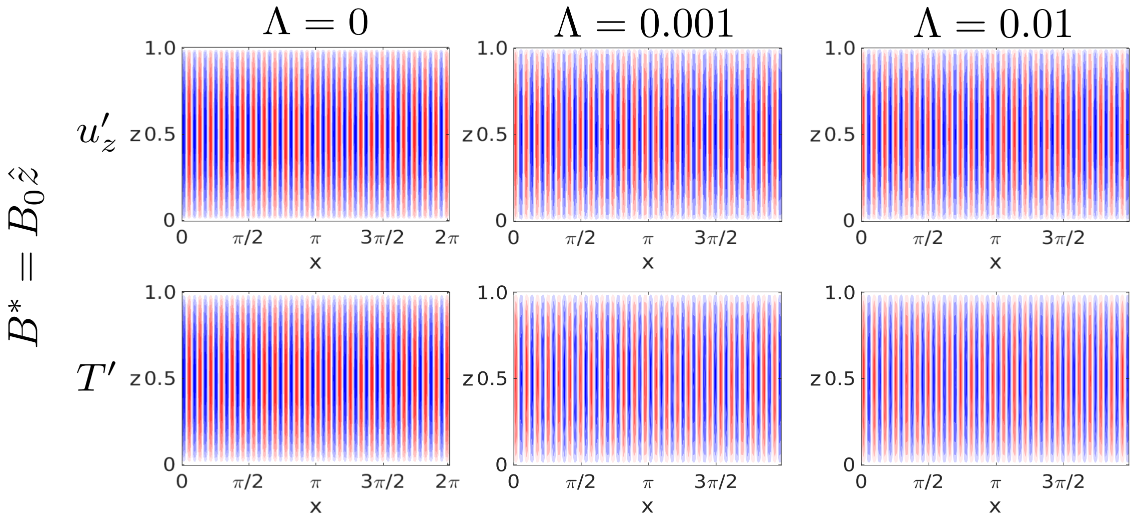
$$T^* = -\frac{Qz^2}{2q} + \frac{Qhz}{q} + 1 \tag{8b}$$

- Non-dimensional numbers

$$\begin{aligned}&Ra = \frac{g\alpha\Delta T D^3}{\nu\kappa};\quad Pr = \frac{\nu}{\kappa};\quad Pm = \frac{\nu}{\eta};\\ &E = \frac{\nu}{2\Omega D^2};\quad \Lambda = \frac{{B_0}^2}{2\Omega\rho\mu\eta};\ q = \frac{Pm}{Pr} = \frac{\kappa}{\eta}\end{aligned} \tag{9}$$

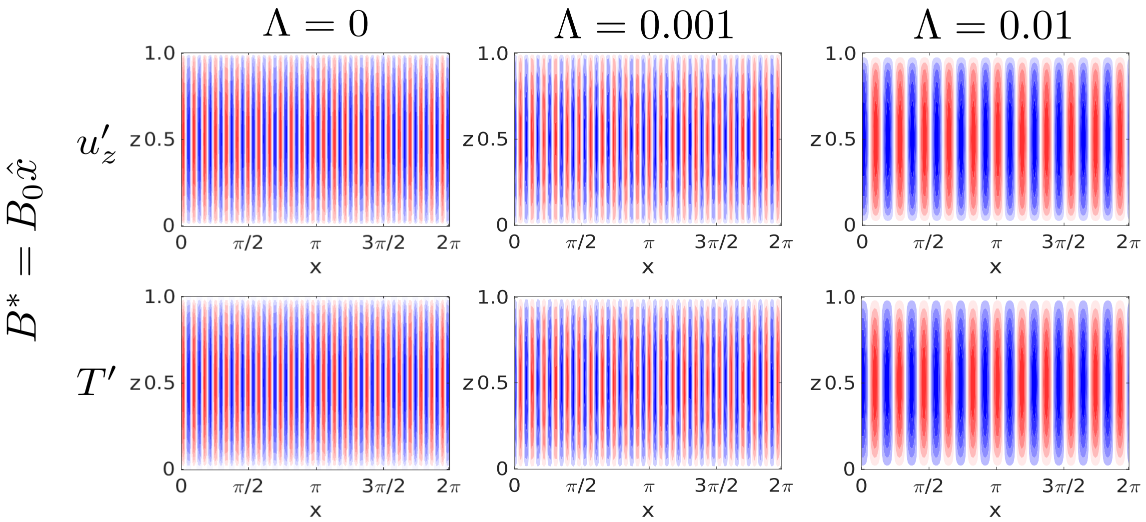
- Where, $\mathbf{u} = (u_x,\ u_z)$ is the perturbation velocity field; $\mathbf{B} = (u_x, u_z)$ is the perturbation magnetic field; $\mathbf{T}$ is perturbation temperature field. Whereas, u^*, B^*, T^* are basic state velocity, magnetic field and temperature field, respectively.
- The equations $(1) - (5)$ are solved using boundary conditions $(7a) - (7b)$ and initial conditions in (6) by implementing spectral method for DNS with various combinations of non-dimensional numbers in equation (9)
- For spatial discritization maximum 512×512 expansion spectral coefficients are used by implementing Chebyshev in axial direction and Fourier along lateral direction. 2^{nd} order R-K method is used for time integration, also the stability and convergence is tested with optimal time step of 1.25×10^{-4}.
- By using two basic state temperature profiles in equation (8), two models are studied. $(8a)$ indicates differential heating; and $(8b)$ represents the cases of partial stable stratification with various thicknesses. Wherein, $h = \infty, 0.8, 0.6$ represent fully unstable stratification, strong stable stratification, weak stbale stratification, respectively.
- Parameter regime for current study is, $E = 10^{-4}$; $\Lambda = 0, 0.001, 0.01$; $q = Pr = Pm = 1$

Results I : Fully unstable stratification ($h = \infty$)



Configuration : I ($B_0\hat{z}$)

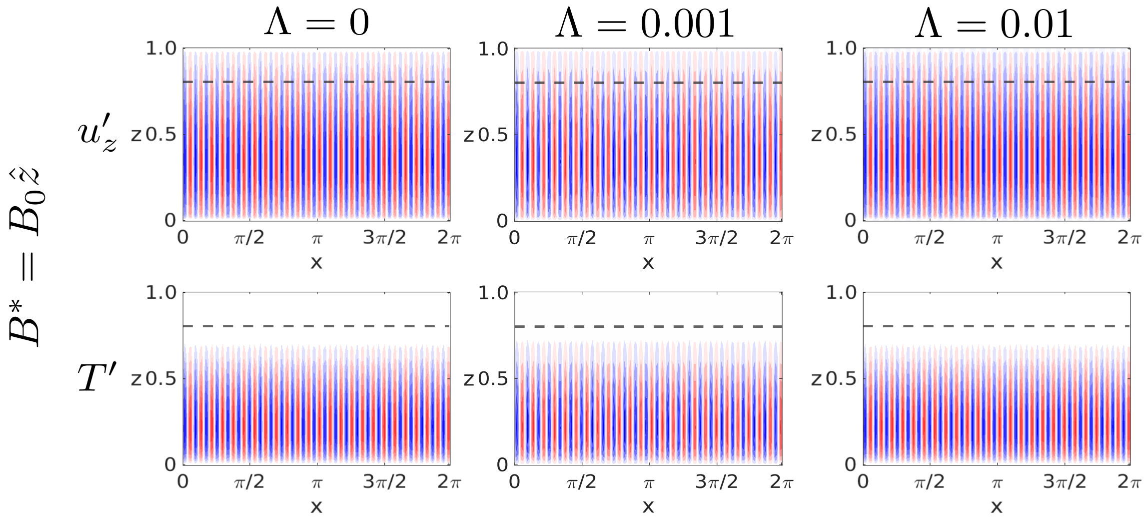
Λ	Ra_c	k_c
0	1525989	25
0.001	1525698	25
0.01	1523690	25

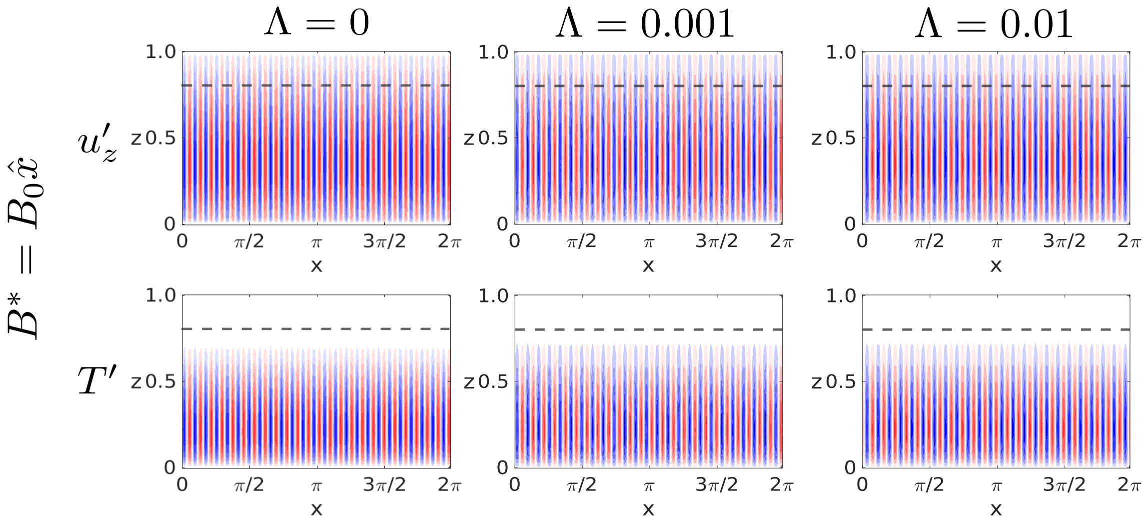


Configuration : II ($B_0\hat{x}$)

Λ	Ra_c	k_c
0	1525985	25
0.001	1522265	24
0.01	1483221	11

Results II : Thin thermal stable stratification ($h = 0.8$)





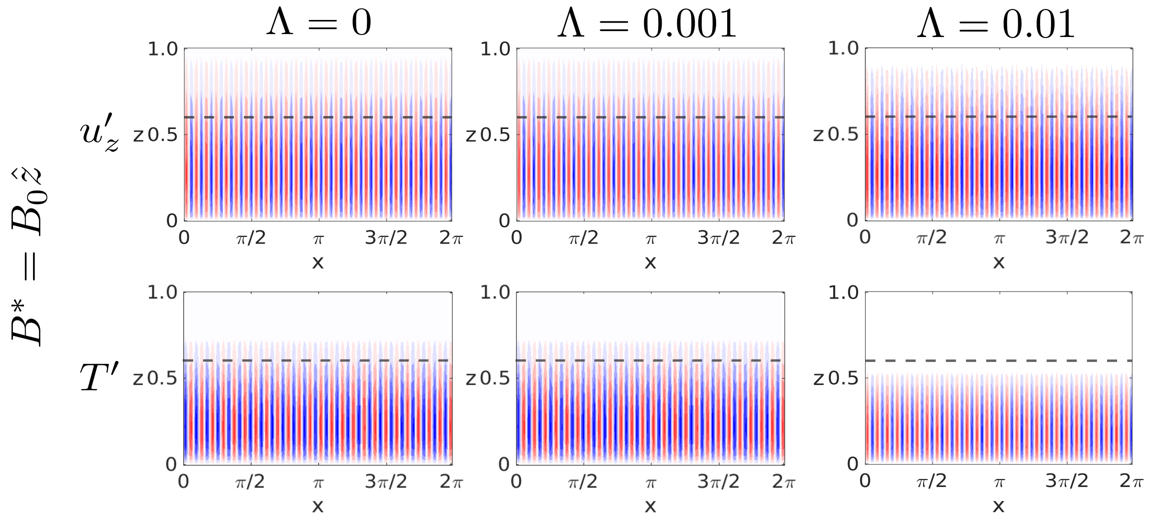
Configuration : I ($B_0\hat{z}$)

Λ	Ra_c	k_c
0	1313512	25
0.001	1313220	25
0.01	1310710	25

Configuration : II ($B_0\hat{x}$)

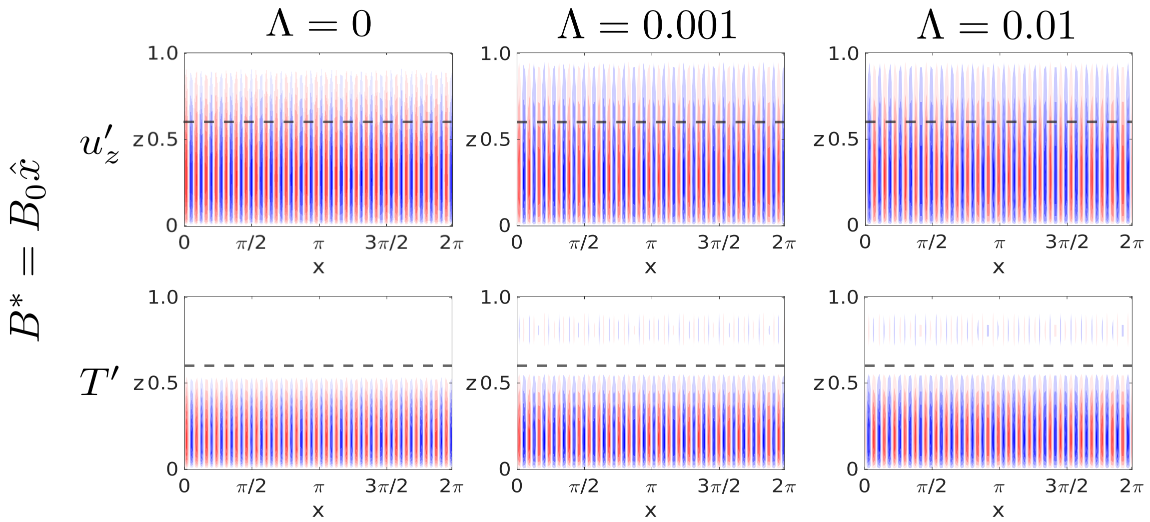
Λ	Ra_c	k_c
0	1313511	25
0.001	1309880	25
0.01	1280520	24

Results III : Thick thermal stable stratification ($h = 0.6$)



Configuration : I ($B_0\hat{z}$)

Λ	Ra_c	k_c
0	815190	27
0.001	815015	27
0.01	813348	27



Configuration : II ($B_0\hat{x}$)

Λ	Ra_c	k_c
0	815190	27
0.001	813720	27
0.01	799065	27

Discussions and Conclusions

- Three different cases ($h = \infty, 0.8, 0.6$) of no stable stratification, weak and strong stable stratifications are used to study penetrative magneto-convection by imposing uniform magnetic field as $B^* = B_0\hat{z}$ and $B_0\hat{x}$ in the axial and lateral directions. The choosen control parameter regime is $E = 10^{-4}$, $q = Pm = Pr = 1$, $\Lambda = 0, 0.001, 0.01$.

- As results, we have reported the perturbated temeperature (T) and axial velocity (u_z) for qualitative understanding. Also, we have reported, critical Rayleigh number (Ra_c) and critical horizontal inverse length scale ($k_c = \frac{\sum u_z(k_x)k_x}{\sum u_z(k_x)}$) of axial velocity.

- The Ra_c reduces $2 - 3$ % as the configuration of imposed magnetic field changes from vertical to horizontal direction for a particular Λ values. The relative drops in Ra_c are $Ra_c^{h=0.8}/Ra_c^{h=\infty} = 0.86$ and $Ra_c^{h=0.6}/Ra_c^{h=\infty} = 0.53$ for strong and weak stratifications, respectively. The critical Rayleigh number reduces more rapidly for the horizontal field ($\approx 3\%$ reduction) than imposed vertical magnetic field ($\approx 1\%$ reduction) by varying strength of magnetic field as $\Lambda = 0, 0.001, 0.01$.

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Thank You