Rebeca Ursu[1], Mark Naylor[1], Hui Tang[2], Jens Turowski[2]
[1]University of Edinburgh, Edinburgh, United Kingdom; [2]GFZ German Research Centre for Geosciences, Potsdam, Germany.

Probabilistic Optimal Transport-Driven Inversion of the 2012 Palisades Rockfall Seismic Source

I. Aims of this study

- Investigate the **robustness of optimal transport-based misfit functions** in reconstructing the 2012 Palisades rockfall seismic source with respect to the classical L_2 norm;

  Compare the conditional posterior probabilities of model parameters generated after using L_2 norm- and quadratic Wasserstein distance-based likelihood functions.

  Qualitatively quantify if using the Wasserstein distance-based likelihood function improves the fit of the model predictions to observations.

- Assess whether the **prior transformation of the seismic signals** into probabilistic objects **introduces artifacts** in the inversion results.

  Apply likelihood functions based on two implementations of the Wasserstein distance and compare the inversion results.

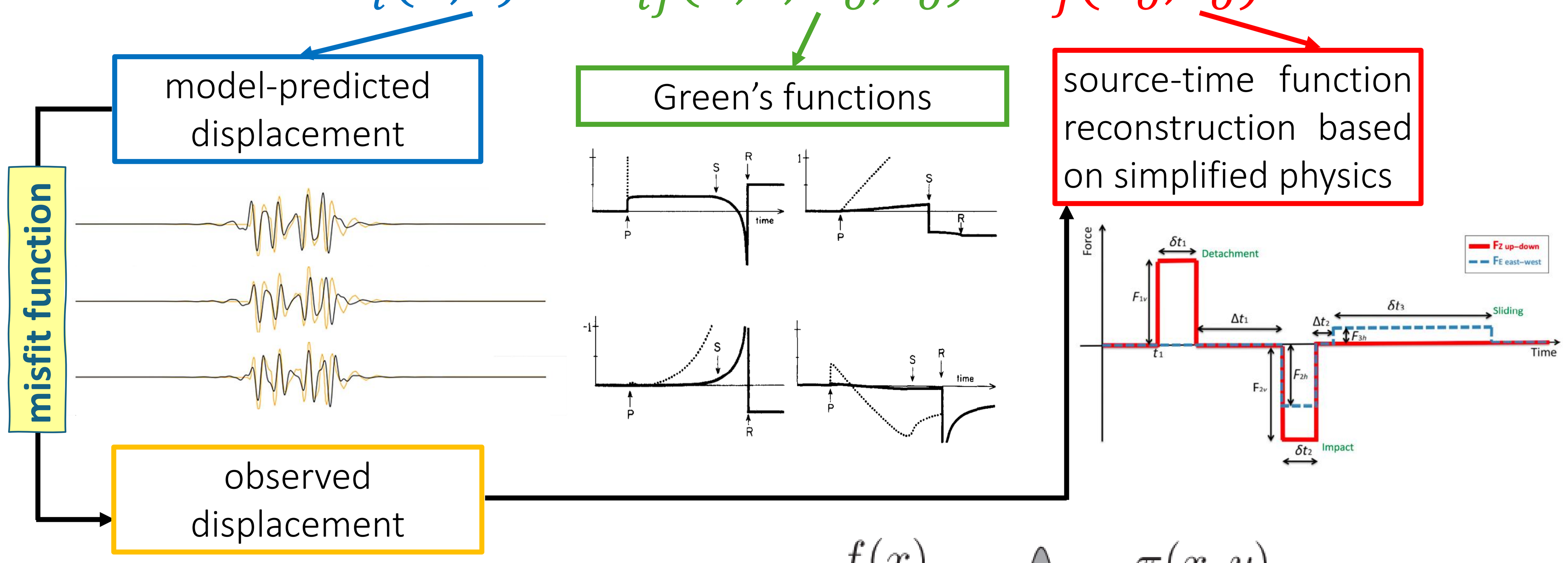
(1) quadratic Wasserstein distance [1] → seismic traces into 2-D density functions across the time-amplitude plane

(2) transport-Lagrangian distance [2] → applied directly to seismic traces

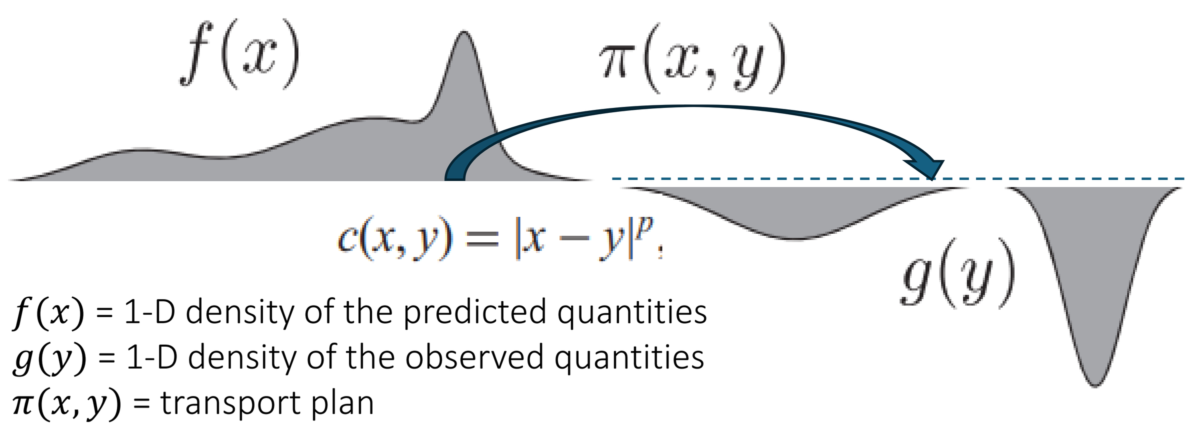
II. Methodology: Part 1

The forward seismic source inversion involves adjusting the force parameters to minimize the misfit between the observed and model-predicted displacement.

$$u_i(\boldsymbol{x},t) = G_{ij}(\boldsymbol{x},t;\boldsymbol{x}_o,t_o) * F_j(\boldsymbol{x}_o,t_o)$$



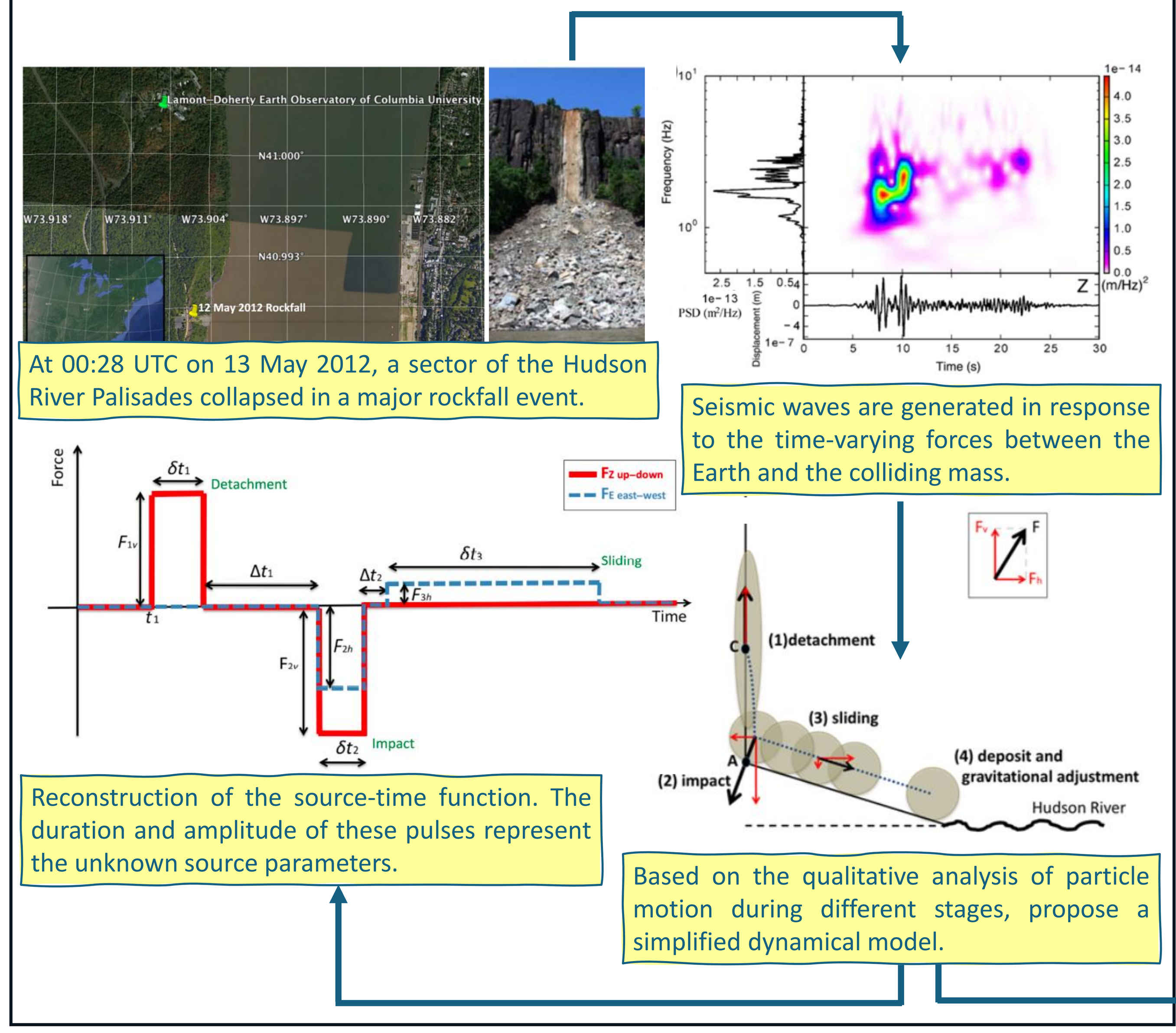
Optimal transport-based misfit functions are sensitive to the inherent temporal structure of the discretized signals and, hence, are proposed as alternatives to the L_2 norm.



$f(x)$ = 1-D density of the predicted quantities
$g(y)$ = 1-D density of the observed quantities
$\pi(x,y)$ = transport plan
$c(x,y)$ = cost of transporting a unit volume of material from point x to point y

III. Methodology: Part 2

1. Setting up the forward model



At 00:28 UTC on 13 May 2012, a sector of the Hudson River Palisades collapsed in a major rockfall event.

Seismic waves are generated in response to the time-varying forces between the Earth and the colliding mass.

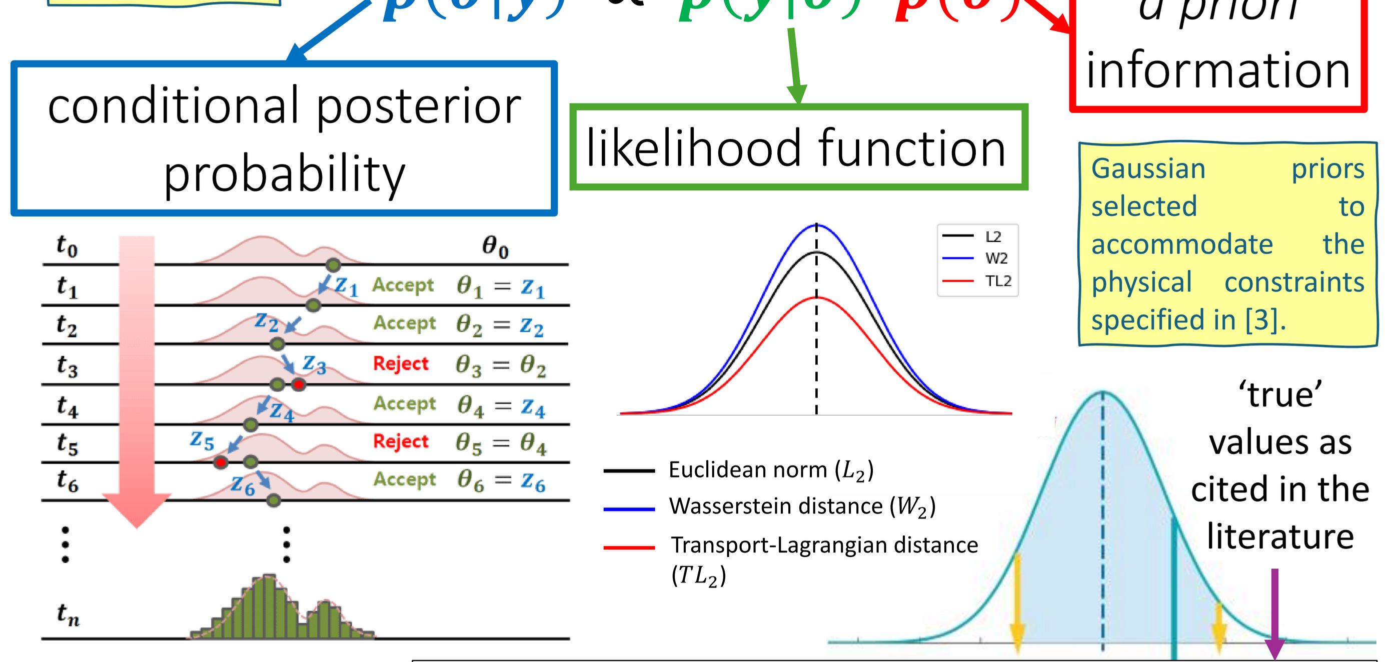
Reconstruction of the source-time function. The duration and amplitude of these pulses represent the unknown source parameters.

Based on the qualitative analysis of particle motion during different stages, propose a simplified dynamical model.

2. Condition the forward model on observed data

Bayes' theorem

$$p(\theta|y) \propto p(y|\theta)\ p(\theta)$$



Gaussian priors selected to accommodate the physical constraints specified in [3].

Arbitrarily selected five starting points and ran Markov chains comprised of 10,000 samples for each likelihood function.

	Parameters explored during minimization		Parameters used in forward modelling	
	Vertical	Eastward	Vertical	Eastward
δt_1 (s)	0.10 – 2.00	–	0.90	–
F_1 (N)	(0.10 – 5.00) × 10^8	–	3.00 × 108	0.00
Δt_1 (s)	0.10 – 2.00	–	1.25	–
δt_2 (s)	0.05 – 4.00	0.05 – 4.00	0.75	0.90
F_2 (N)	(0.05 – 10.00) × 10^8	(0.05 – 10.00) × 10^8	3.60 × 108	1.45 × 108
Δt_2 (s)	–	0.00 – 0.50	–	0.10
δt_3 (s)	–	1.00 – 12.0	–	9.00
F_3 (N)	–	(0.05 – 10.00) × 10^7	–	1.45 × 107

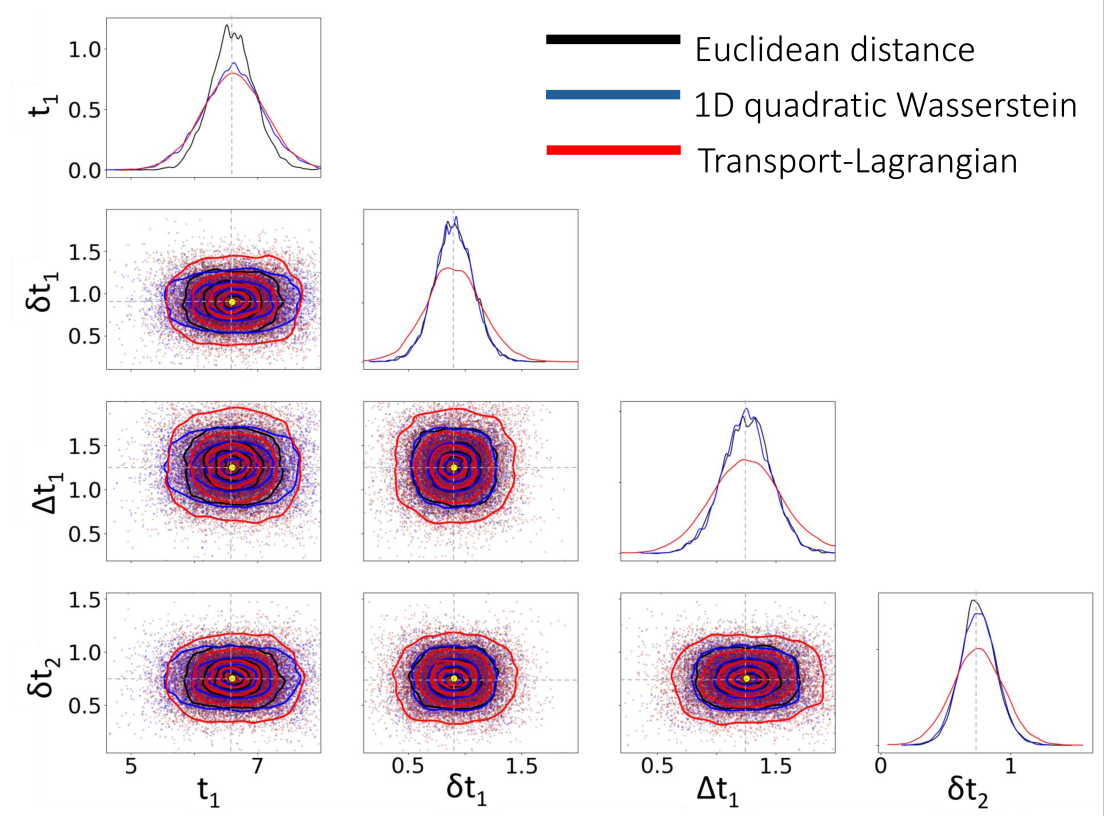
IV. Results & Discussion

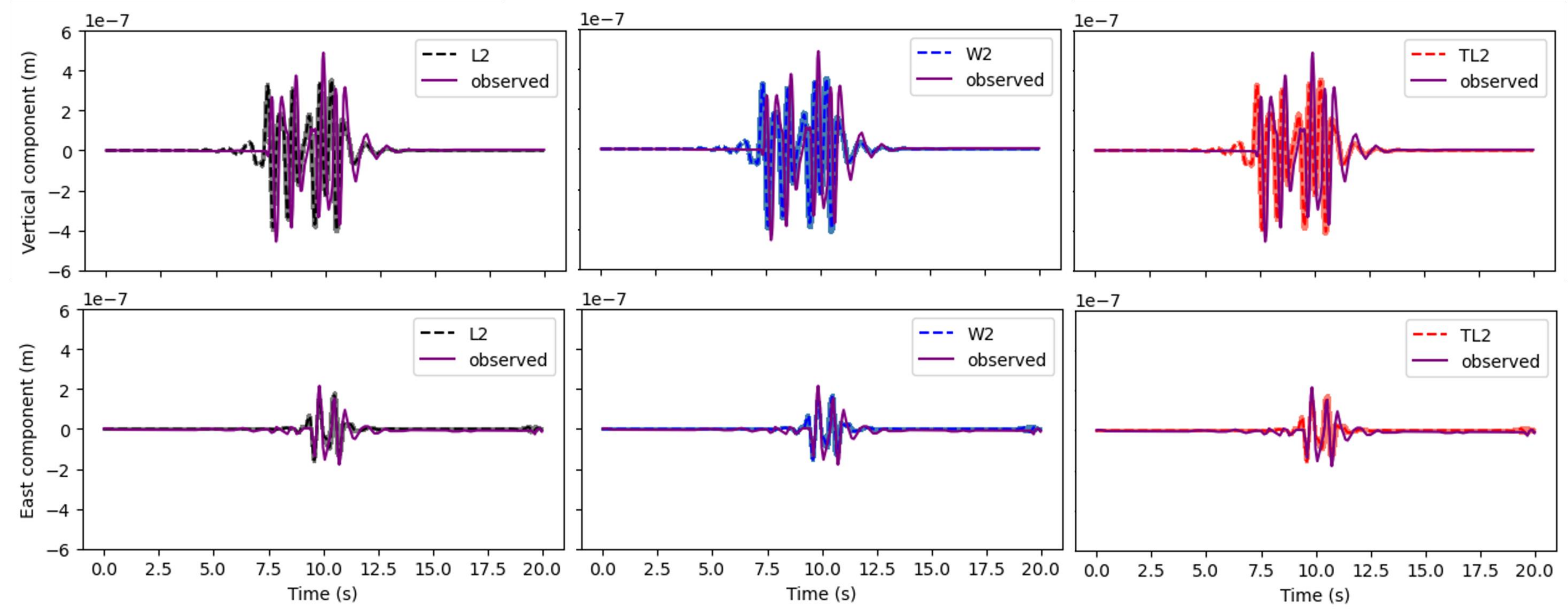
How close are the posteriors to the values cited in the literature?
Location parameter of optimal transport-based posteriors **closer to the values** cited in the literature.

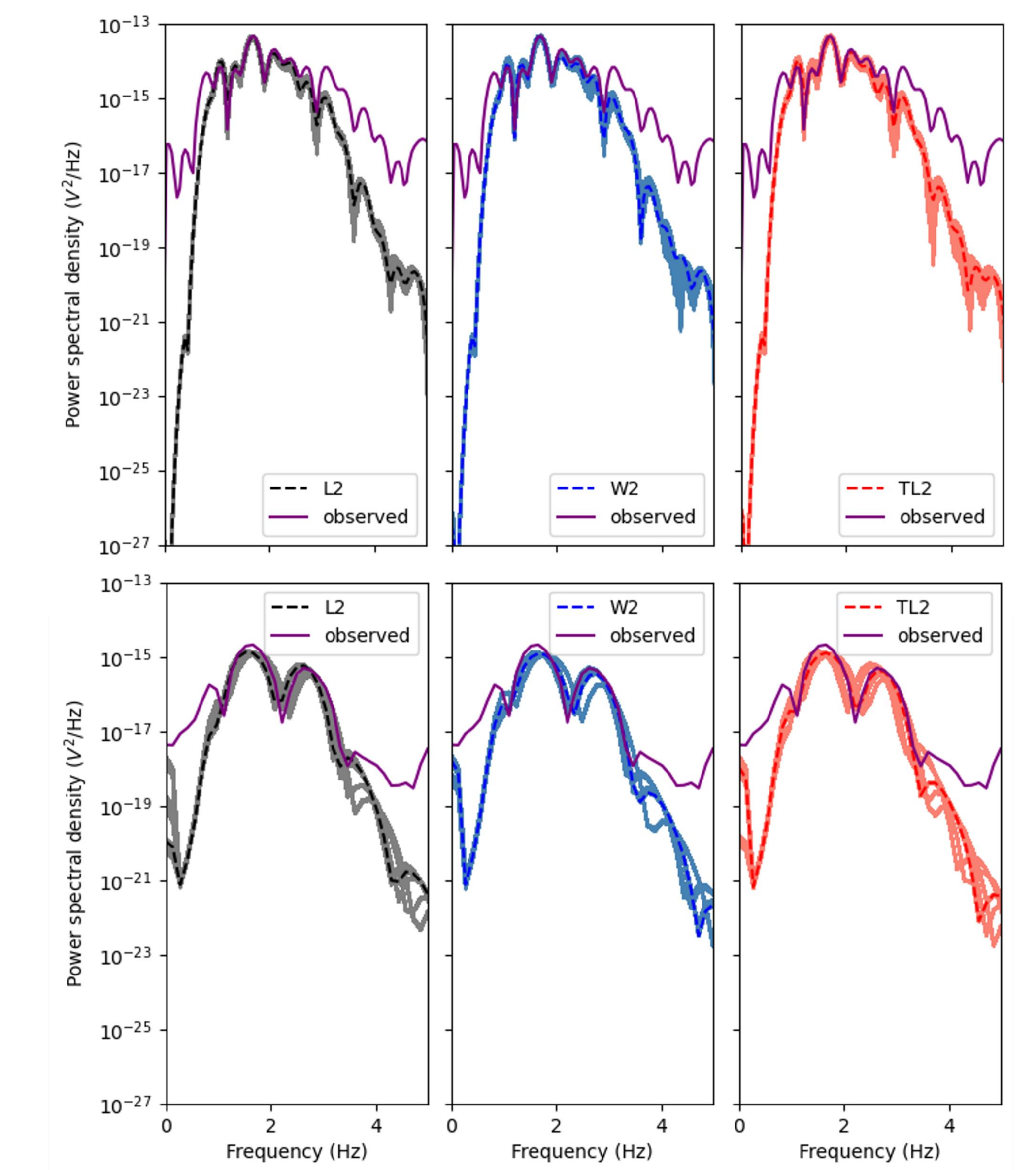
Do optimal transport distances reduce the variance in posteriors?
Optimal transport-based posteriors exhibit **larger variances** compared to those computed with the L_2 norm.

Noticeable improvements in the reconstruction of waveforms?
Using optimal transport distances leads to **negligible improvement** in the fit to the observed waveforms.

Ad-hoc transformation of the seismic waveforms introduce artifacts?
Prior transformation of the signals is **unlikely to introduce artefacts** in the Wasserstein distance-based inversion.







Related literature: [1] Sambridge, M. et al. 2022. *Geophysical Journal International 231(1)*: 172-198; [2] Scarinci, A. et al. 2023. *Geophysical Journal International 234*(2): 1169-1190; [3] Gualtieri, L. and Ekström, G. 2016. *Bulletin of the Seismological Society of America 107*(1): 63-71; [4] Jin, S.S. et al. 2019. *Structure and Infrastructure Engineering 15(11)*: 1548-1565.