Using information-theory metrics to detect regime changes in dynamical systems

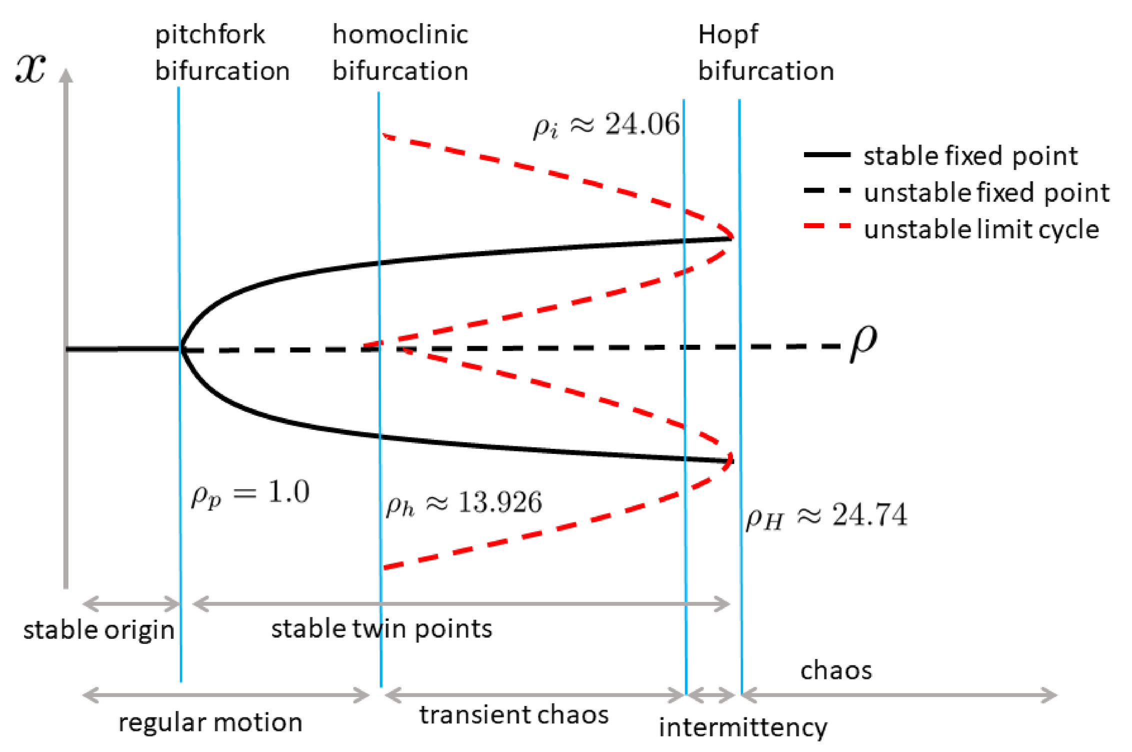
Javier Amezcua, School of Science and Engineering, Tecnológico de Monterrey
Department of Meteorology, University of Reading
Nachiketa Chakraborty, Centre for Computational Science and Mathematical Modelling, Coventry University

Abstract

Dynamical systems display a range of **dynamical regimes** depending on the **values of parameters** in the system. Here, we demonstrate how **non-parametric entropy estimation** codes based on the **Kraskov method** can be applied to **find regime transitions** in the Lorenz 1963 system when varying the values of the parameters. These **information-theory-based methods** are simpler and cheaper to apply than more traditional metrics from dynamical systems. The **non-parametric nature of the method allows for handling long time series without a prohibitive computational burden**.

Regime changes in dynamical systems

The values in the parameters of a dynamical system determine its **long-term evolution**. When parameter values are changed, this behaviour can be considerably altered, leading to a **regime change**.



The **Lorenz 1963** is an archetypical simple (3 **variables**) continuous-time model with 3 **parameters**. Varying one of them can lead to **regular motion**, **transient chaos**, and **chaos**.

Figure 1: (adapted from Ott, 2008). A simplified bifurcation diagram for the Lorenz 1963 system showing known regimes.

Analytic methods to study regime transitions can be limited. Techniques like **Finite Time Lyapunov Exponents** study the evolution of trajectories evolving from an initially tight set of points. The **calculations involved can be burdensome**.

Causal information metrics

Information-theory-based metrics are based on the **marginal and joint distributions** of a set of variables, and can readily **detect relationships amongst them.** When including **time lags** between the variables, **causal information** can be deduced.

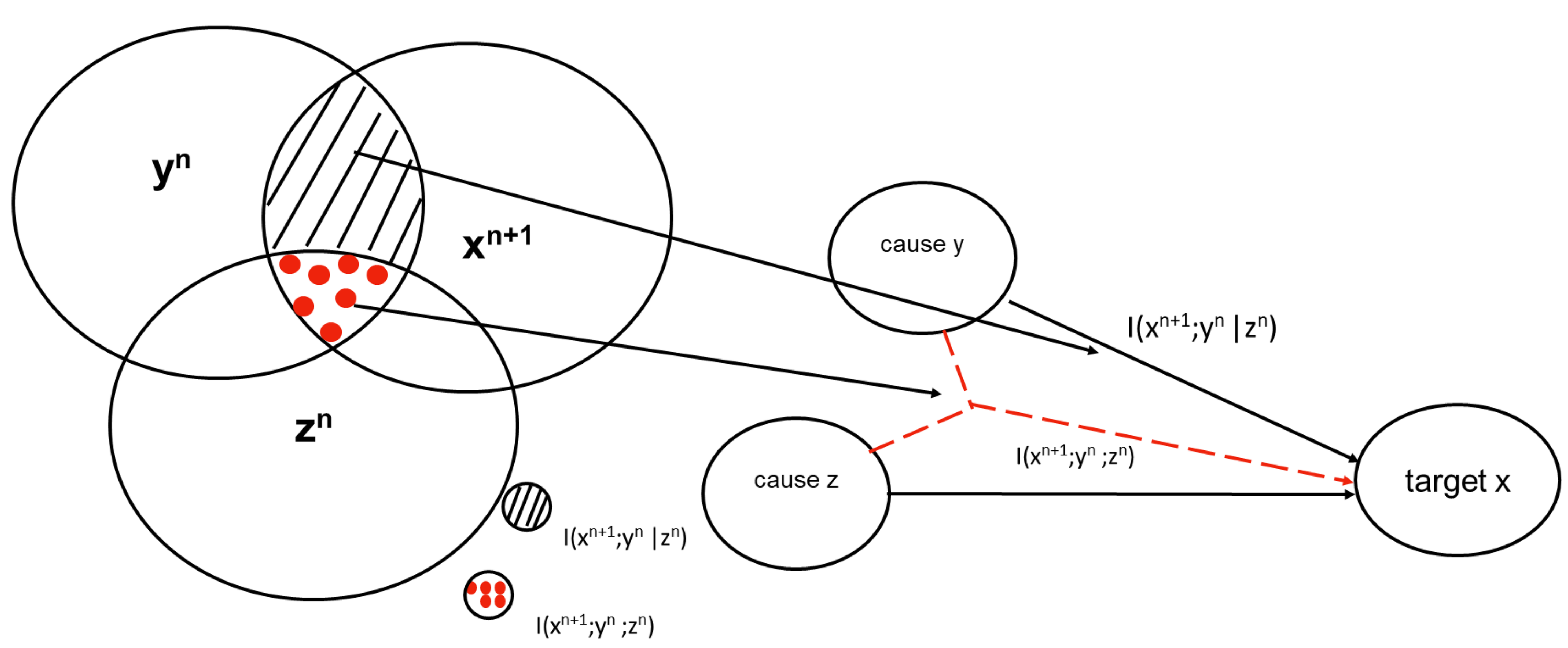


Figure 2: Simple diagram showing the information-content metrics for a 3-variable system. In this case two variables are contemporaneous (y and z), while one variable is one step ahead (x).

Computing the metrics using their definitions is very expensive. Thankfully, there are available codes like **NPEET** which use **nearest neighbor non-parametric estimators** based on the Kraskov method.

Time-lagged results

We compute the **entropies** (for each variable), **mutual information** (for each pair of variables), **conditional mutual information** (for each conditioned pair), and **interaction information** (for the triplet of variables).

Figure 3 shows results without time lagging. The **vertical lines** separate the **known regimes**. The **entropy** of the variables **grows as the parameter increases** and is **maximal in the chaotic regime**. The **interaction information decreases** steadily **as the parameter increases** and is **minimal (and negative) in the chaotic regime.** The **mutual and conditional mutual information flip their relationship when transient chaos initiates**. They reach steady values in the chaotic regime.

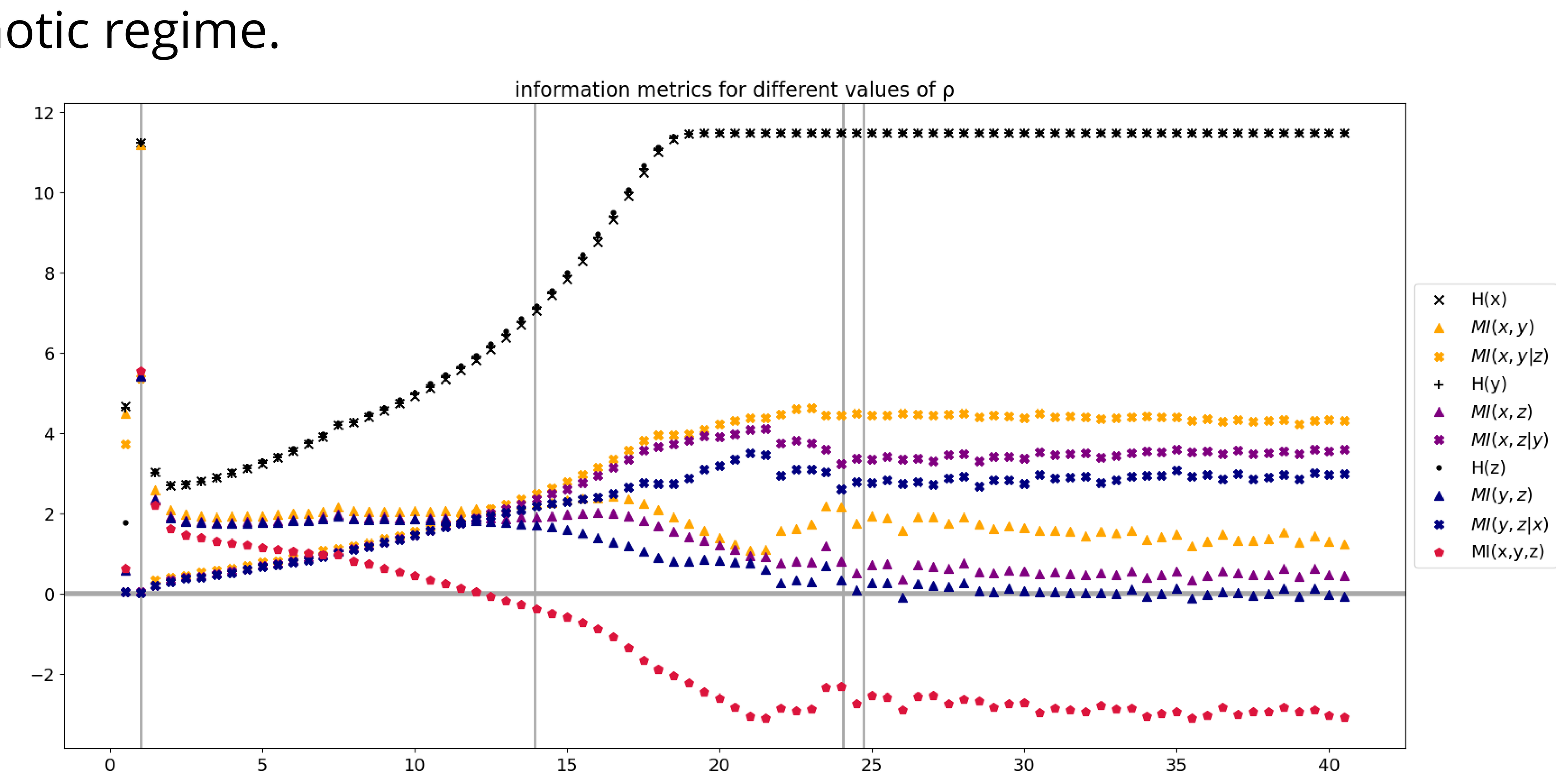


Figure 3: Computation of information-based metrics without time lags.

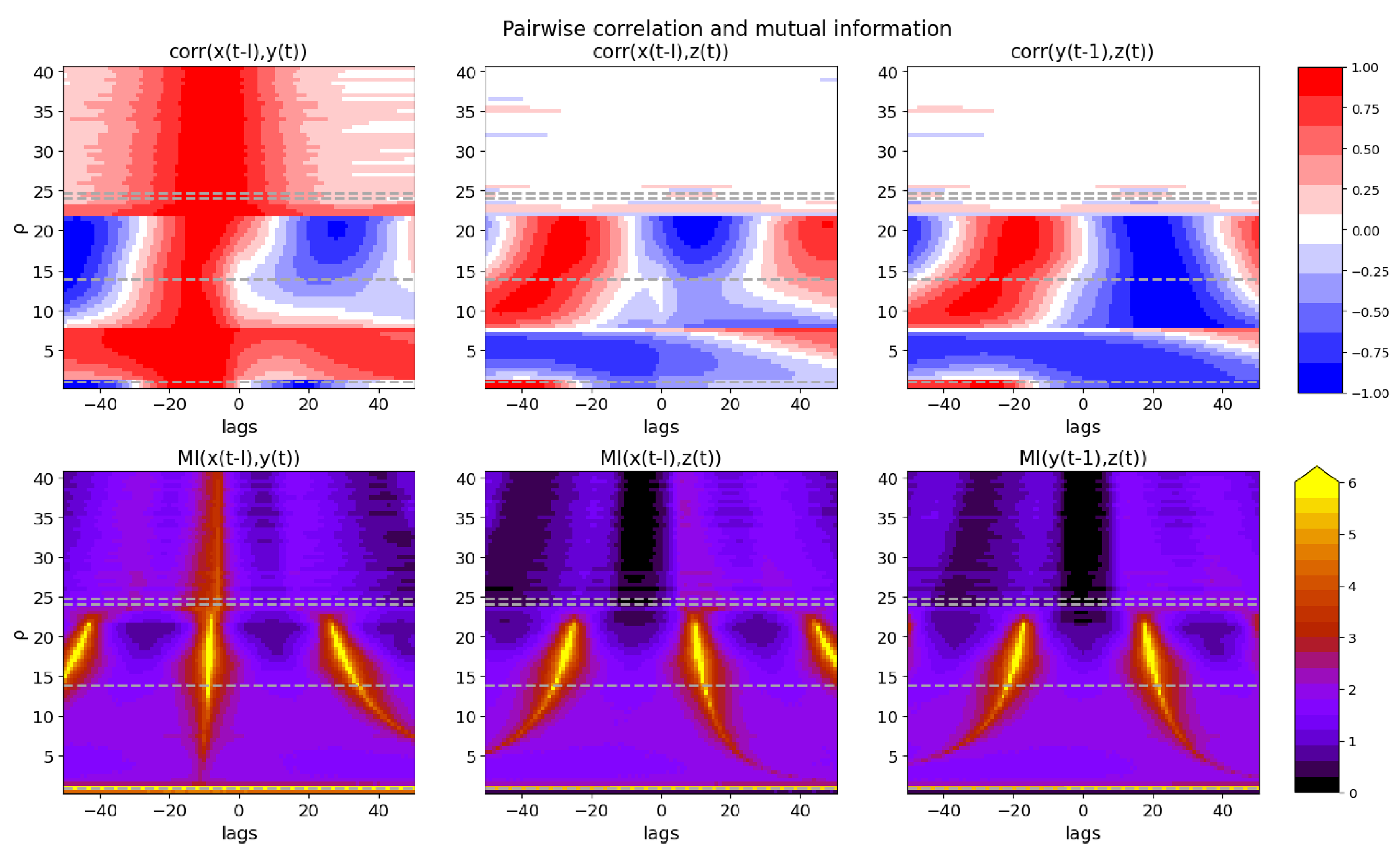


Figure 4: Computation of correlation (top row) and mutual information (bottom row) for pairs of variables when including time lags.

Figure 4 computes pairwise metrics with time lags introduced. There is an **abrupt change in the pairwise correlation** (computed in the top-row for comparison) and the **mutual information** (bottom row) **just before the beginning of intermitency**.

The advantage of using **mutual information** is clear with the **maximum values corresponding to the periods of the unstable lymit cycle**. This information is **not detected by the correlation**.

Conclusions

We have used the Lorenz 1963 model to show that **information-theory-based metrics** can be used to **detect regime changes** when varying a parameter.
The **metrics** can be **applied with or without time lagging**. In both cases, we have shown **distinct behaviours of the metrics for different regimes.**
We have **yet to study and decript the causality information** of our results.

References

- J. Amezcua and N. Chakraborti (2024): Using information-theory metrics to detect regime changes in dynamical systems. *In preparation*.
- N. Chakraborty and P. J. van Leeuwen (2022): Using mutual information to measure time lags from nonlinear processes in astronomy. *Physical Review Research* 4, 013036 (2022)
- Non-Parametric Entropy Estimation Toolbox (NPEET): https://github.com/gregversteeg/NPEET
- E. Ott. Chaos in Dynamical Systems. (2008). Second Edition, *Cambridge Press*.
- A. Kraskov, H. Stögbauer, and P. Grassberger, Estimating mutual information, Phys. Rev. E 69, 066138 (2004).