Unravelling Succolarity to Quantify Multiscale Petrophysical Properties

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1. Introduction

Using **nonlinear approaches** to characterise spatial patterns and their underlying physics is becoming increasingly prevalent across multiple fields. Geomaterials often exhibit **scaling behaviour**, and their properties can be characterised by **fractal theory**.

Fractal dimension, a ratio that compares the level of detail in a structure with its size, measures its **complexity**. **Lacunarity**, derived from the Latin word "lacuna," meaning "gap," quantifies the **heterogeneity** of a texture. However, neither of these measures can fully capture a fractal's percolating properties.

Mandelbrot[1] coined the concept of **succolarity**. Given that "**percolare**" in Latin translates to "**to flow through**," the term "succolare" (sub-colare) aptly conveys the concept of "**to nearly flow through**" in neo-Latin. A **succolating fractal** almost contains the connecting paths that permit percolation.

This study presents a practical and efficient **3D succolarity computation scheme**, building upon the 2D algorithm[2]. The scheme is then put to the test by re-evaluating a synthetic volume and open-source three-dimensional digital rock samples from published literature[3,4]. The research delves into the correlations between 3D succolarity and other physical measures, providing valuable insights.

2. 3D Succolarity Computation and Samples

$$Su(BS(k),dir) = \frac{\sum_{k=1}^{n} \varphi(BS(k)) \times D(BS(k),pc)}{\sum_{k=1}^{n} D(BS(k),pc)}$$

$$\sigma(BS(k),dir) = \varphi(BS(k)) \times D(BS(k),pc)$$

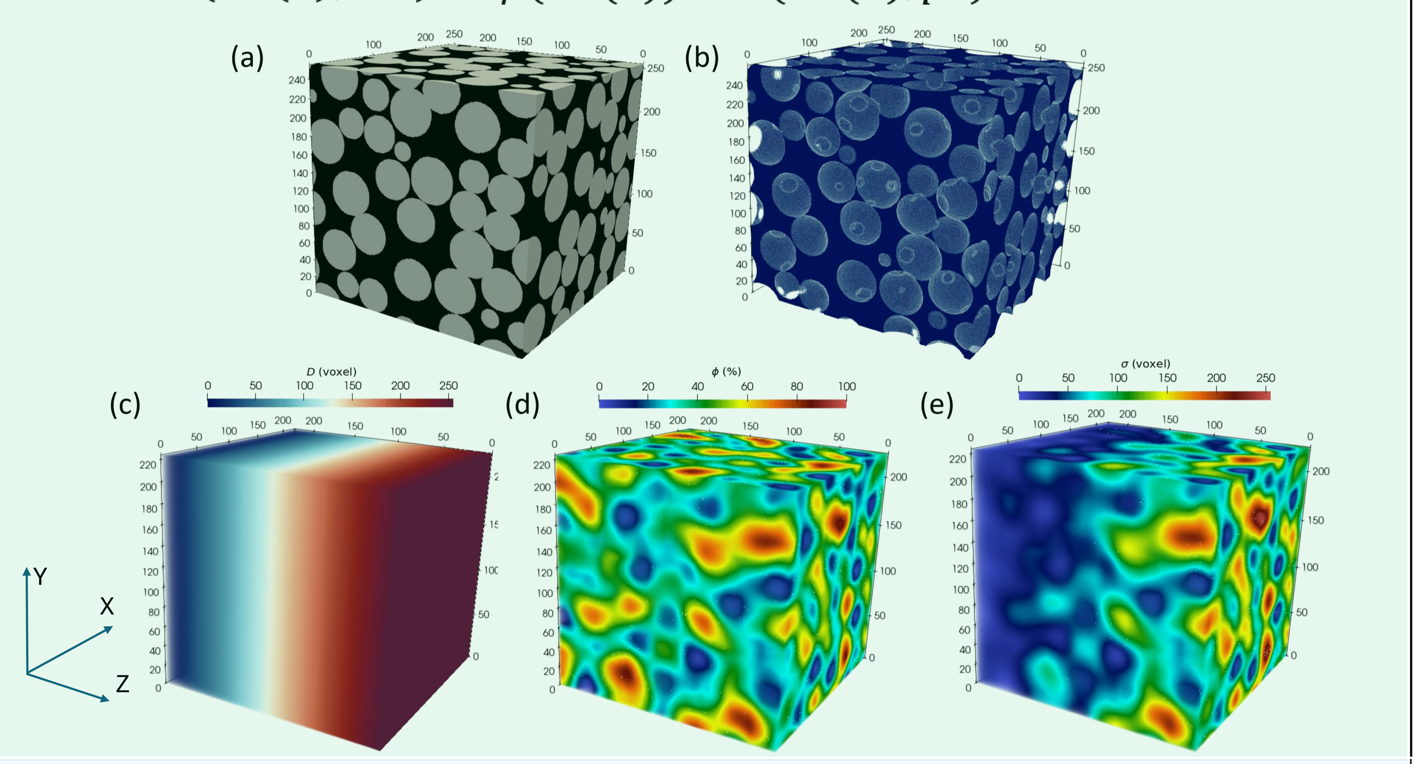

Figure 1. Succolarity computation process. (a) A random sphere packing structure. (b) Connected pore space to the front xy inlet. (c) Distance distribution to the front xy inlet. (d) Occupancy distribution. (e) The unnormalised succolarity distribution.
Note: This illustration shows the calculation process in one direction. The complete procedure contains all six directions.

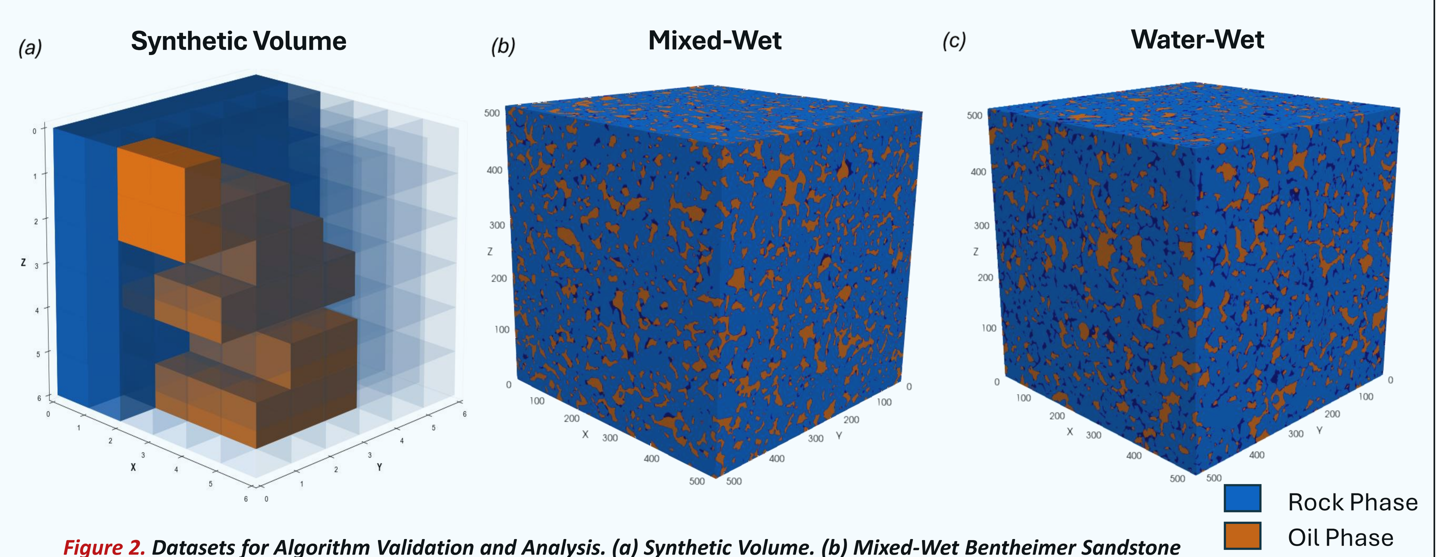

Figure 2. Datasets for Algorithm Validation and Analysis. (a) Synthetic Volume. (b) Mixed-Wet Bentheimer Sandstone with Water Fractional Flow (fw=0.02). (c)Water-Wet Bentheimer Sandstone with fw=0.05.

3. Validation Results

De Melo manually calculated the synthetic volume's succolarity value using standard box counting, the same as our sliding box counting, as the box size is 1 voxel. The identical results in Table 1 give our program initial credibility.

d	Box Size	Su(T2B)-Standard	Su(T2B)-Sliding
6	(1*1*1)	0.2029	0.2029
3	(2*2*2)	0.1929	0.28
2	(3*3*3)	0.2012	0.353009

Table 1. Initial Validation from the Synthetic Volume

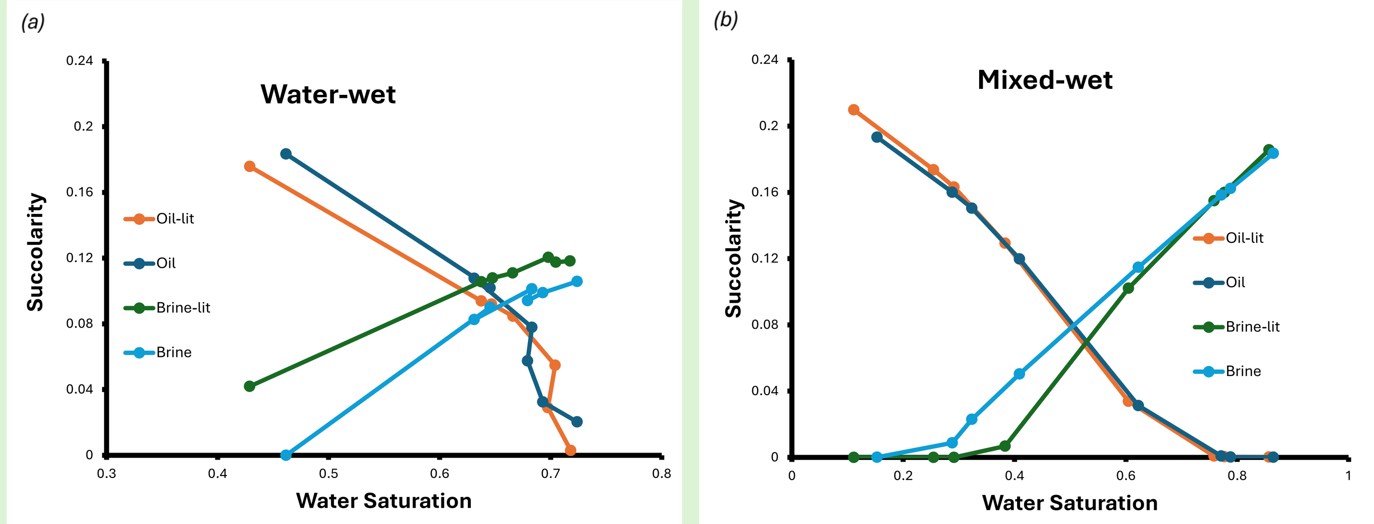

Figure 3. Validation using succolarity versus water saturation along the flow direction from Bentheimer Sandstone. (a) Water-Wet. (b) Mixed-Wet.
Note: The trends of both results in the two plots are consistent with each other at various saturation. The reasons for differences may include distinct resampling methods and algorithm implementation schemes. Oil-lit and Brine-lit are data points from the literature on the oil and brine phases.

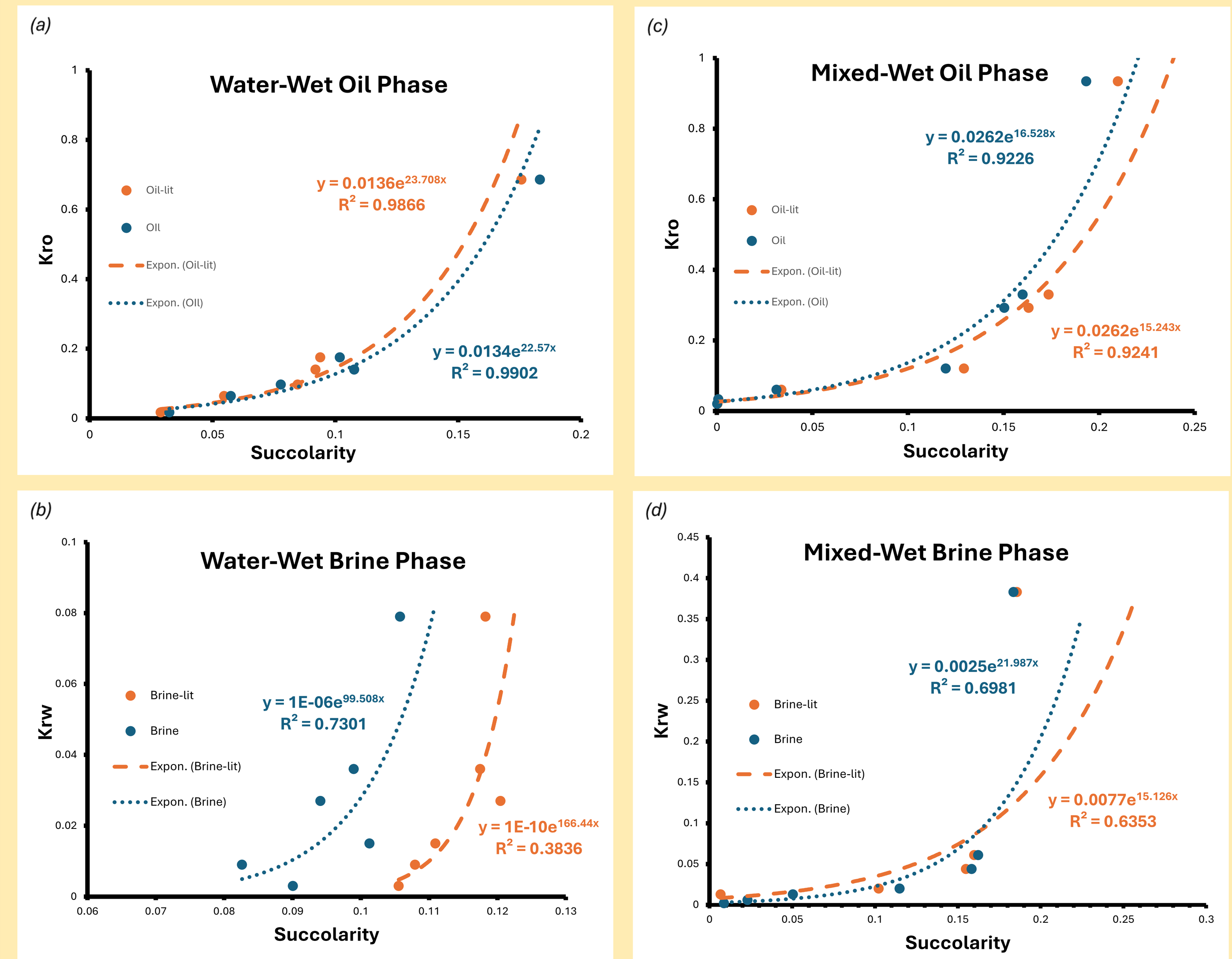

Figure 4. Validation and Analysis of Succolarity with Relative Permeability along the Z direction in Figure 2. (a) and(c) Water-Wet Bentheimer Sandstone. (b) and (d) Mixed-Wet Bentheimer Sandstone
Note: Succolarity and Oil Relative Permeability are well-fitted to similar exponential models in our results and the published ones. For Water Relative Permeability, our succolarity program produces better-fitting results.

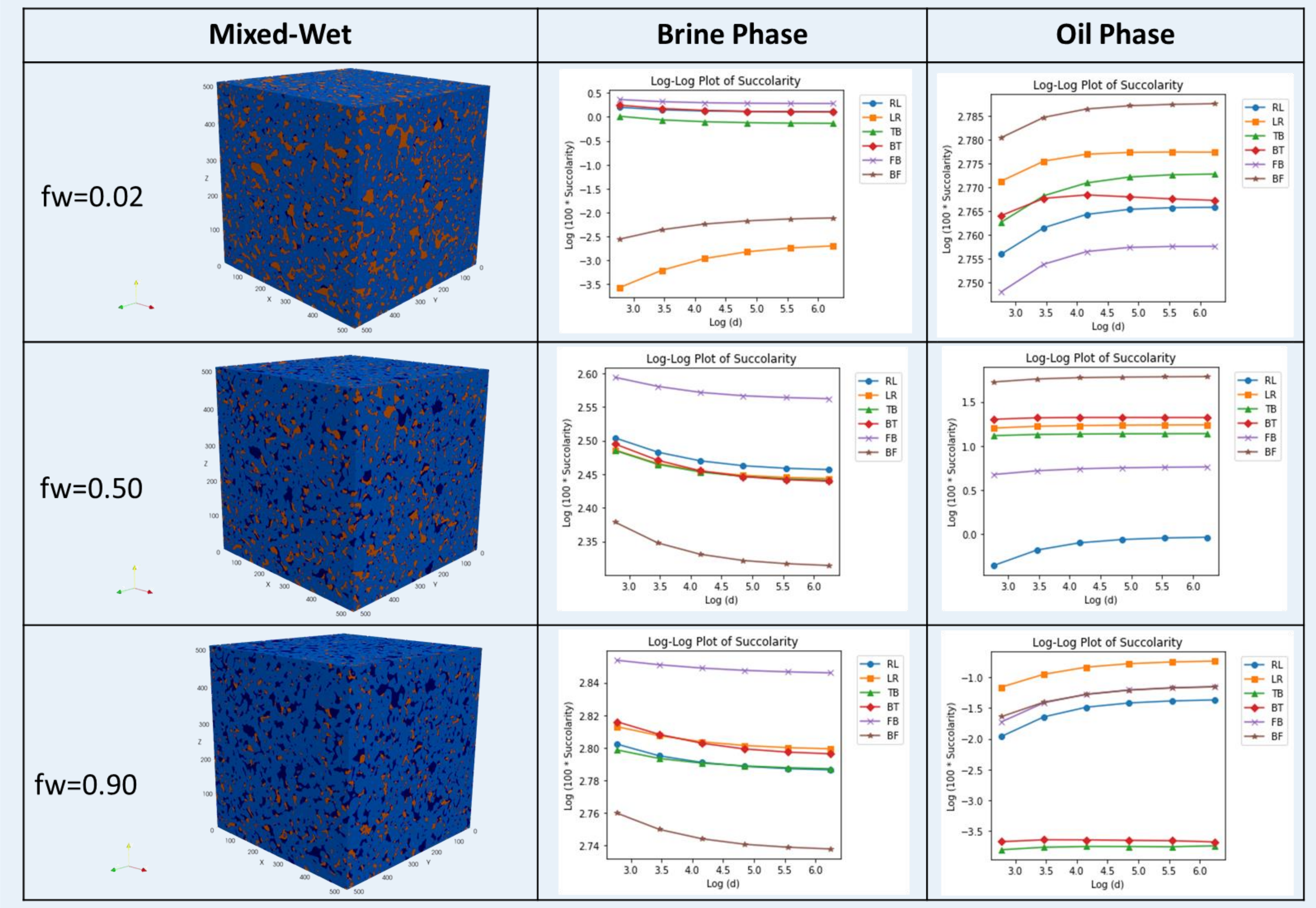

Table 2. Log-Log plots of succolarity versus scan box sizes for both phases under the mixed-wet condition and different water fractions.

- Succolarity varies depending on phase fractions and connectivity, and directional differences were observed.
- Succolarity becomes stable or scale-invariant below certain box size thresholds.

4. Anisotropy and Connectivity Functions based on Succolarity

★ A New Anisotropy Measure

$$\xi(dir) = |Su(dir) - \phi|$$

Anisotropy is denoted as ξ; Su is the succolarity of each direction; ϕ is the porosity.

If the pore system is evenly distributed and well connected in all six directions, ξ should be zero. The bigger the variations in each direction, the stronger the sample's anisotropy.

The mix-wet brine phase here exhibits various anisotropic features under different flow fractions.

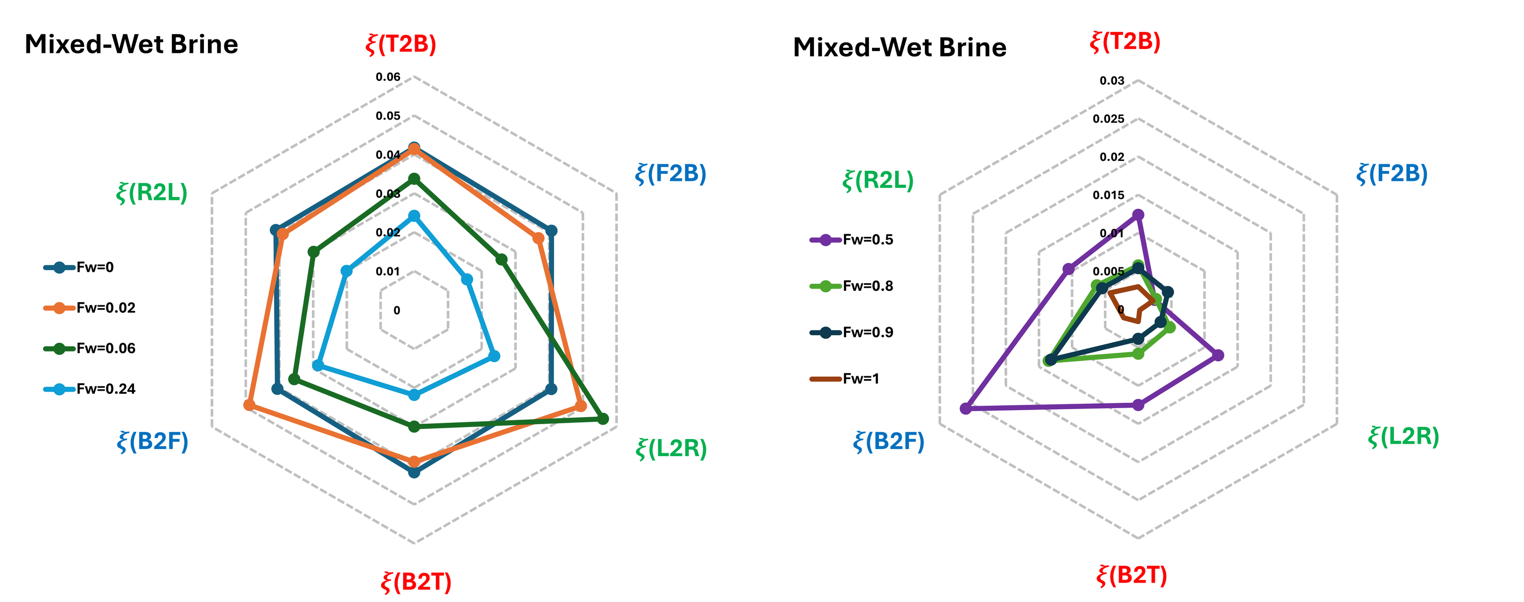

Figure 5. Anisotropy of Mixed-Wet Bentheimer Sandstone under different water flow fractions. (a) Fw=0,0.02,0.06,0.24. (b) Fw=0.5,0.8,0.9,1.

★ Connectivity Functions

- The specific Euler number is a topological index used to build the connectivity function and serves as the reference for succolarity's ability to quantify connectivity[5].
- $\chi_V(P) = \frac{N(P) - C(P) + H(P)}{V}$
- Succolarity and the specific Euler number were sequentially calculated for the pore spaces (containing brine or oil phases), with the pore diameter threshold increasing step by step from a lower limit to a maximum threshold.

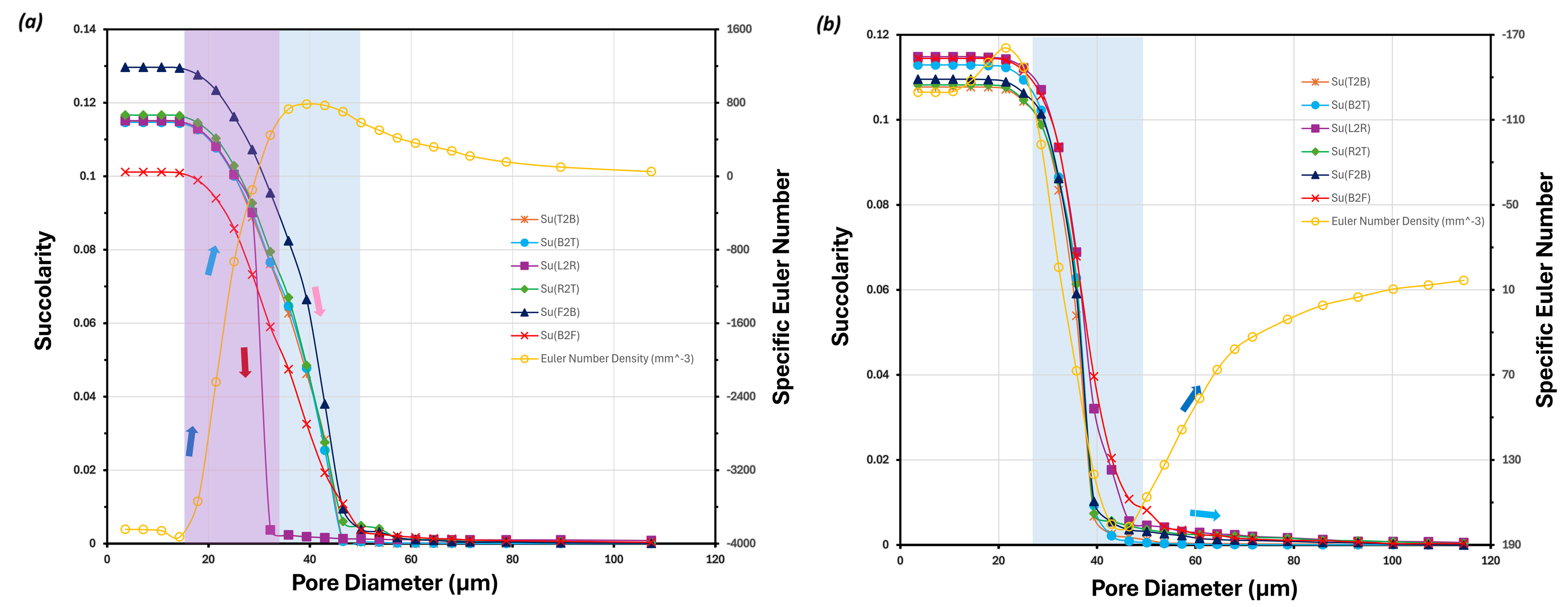

Figure 6. Connectivity function curves for two Bentheimer Sandstones under different water flow fractions. (a) Mixed-Wet Brine phase, fw=0.5. (b) Water-Wet Oil Phase, fw=0.15.
Note: In (b), the connectivity curve of the specific Euler number is placed reversely for comparison.

5. Connectivity vs Heterogeneity

- Lacunarity as a measure of heterogeneity
- Succolarity bears the percolation degree and connectivity information.

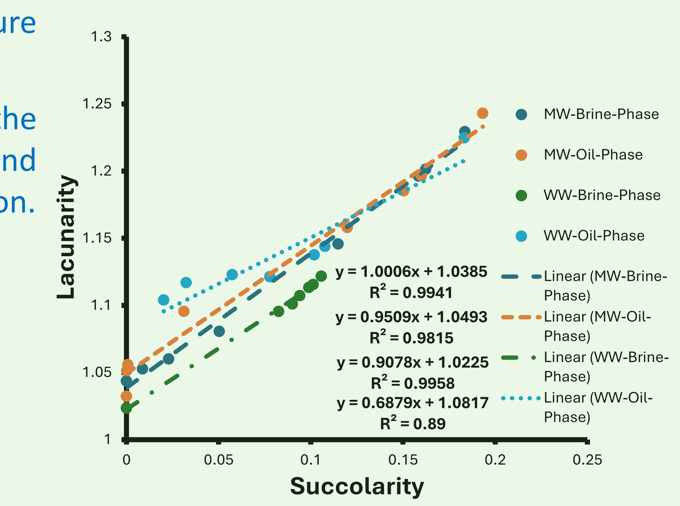

Figure 7. Lacunarity versus Succolarity for Oil and Brine phases in Mixed/Water-Wet Bentheimer Sandstone
Note: This is the lacunarity value of the solid phase

6. Computation Efficiency

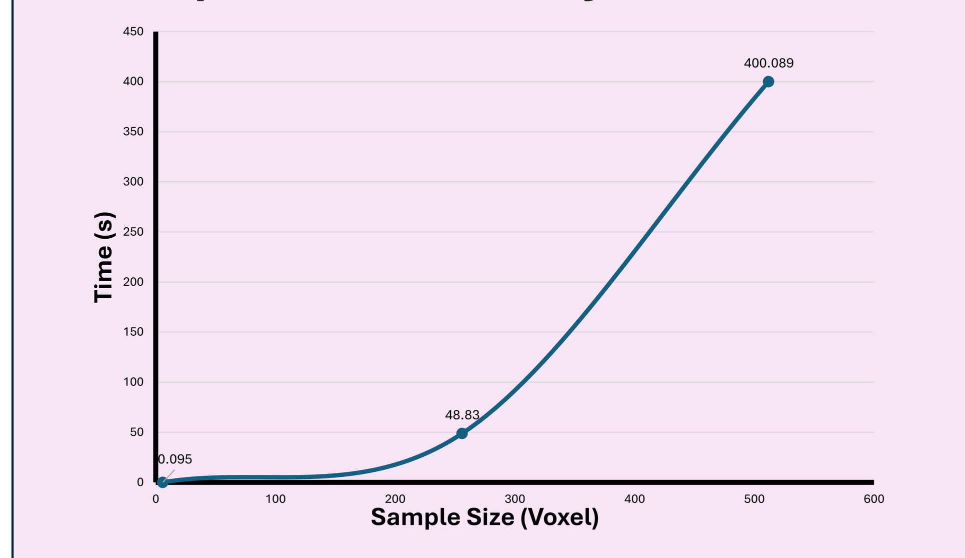

Figure 8. The computation time of 3D Succolarity for different Sample sizes

7. Summary

- Succolarity, a unique concept, encapsulates crucial information about a structure's anisotropy, phase fraction (such as porosity in the case of pore space), and percolation, setting it apart from other measures.
- It is susceptible to connectedness. As we cut out smaller pores of a structure, succolarity remains stable until a pore threshold is reached, then drops significantly within a specific pore size range.
- Permeability (k) is an exponential function of succolarity (Su): $k = ae^{bSu}$. Calculating succolarity excludes isolated pores for a given flooding direction, allowing it to reflect the flow properties better than porosity alone.
- There is a direct and positive correlation between the values of lacunarity and succolarity, suggesting possible relationships with a structure's connectivity and heterogeneity.
- Succolarity can be efficiently built into reservoir models and help manage fluid flow upscaling. The flexible calculation algorithm can be customised based on specific needs by substituting its parameters.

Main References

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(2) Melo, R.H.C.D., 2007. Using fractal characteristics such as fractal dimension, lacunarity and succolarity to characterise texture patterns on images. https://app.uff.br/riuff/handle/1/17146
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(4) Zou, S., Xu, P., Xie, C., Deng, X. and Tang, H., 2022. Characterisation of Two-Phase Flow from Pore-Scale Imaging Using Fractal Geometry under Water-Wet and Mixed-Wet Conditions. Energies 2022, 15, 2036. https://doi.org/10.3390/en15062036
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