Optimizing Multi-GNSS Orbit Combination:

A Comprehensive Study on Weighting Strategies and Outlier Detection

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Portfolio of actions towards multi-GNSS orbit combination

Legacy IGS GPS and GLONASS orbit and clock combination activity
Beutler et al. (1995)

IGS ACC moves towards multi-GNSS orbit combination
Masoumi et al. 2020; Sośnica et al. 2020

Independent multi-GNSS orbit combination activity at GFZ
Sakic et al. 2018
Mansur et al. 2020

Independent multi-GNSS orbit combination activity at WHU
Chen et al. (2023)

Dedicated IGS Combination Task force

In July 2022, IGS called for experts to join a new task force aimed at enhancing IGS product combinations within a multi-GNSS framework. The goal is to develop a workflow for delivering official and final, fully consistent multi-GNSS orbit and clock products.

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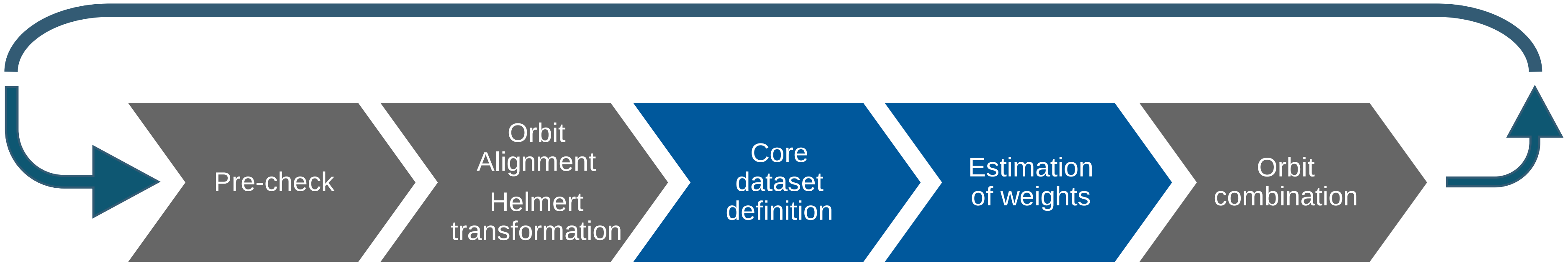
Key Insights from Previous Activities



Pre-check

Orbit Alignment

Helmert transformation

- Transformation between mean orbit and the ACs' solutions

Core dataset definition

- Excluding outliers
- Defining satellites to be used in the estimation of weights

Estimation of weights

- Estimation of weights

Orbit combination

- Weighted average

- Different handling of **outliers**

- Legacy IGS, GFZ and WHU – estimation of **AC-constellation** specific weights
- IGS ACC –estimation of **AC-satellite specific** weights

- IGS and WHU - Squared inverse of the **mean absolute deviation**
- GFZ team – Least-Squares Variance Component Estimation (**LSVCE**)
  - VCE is considered the most mathematically justified approach

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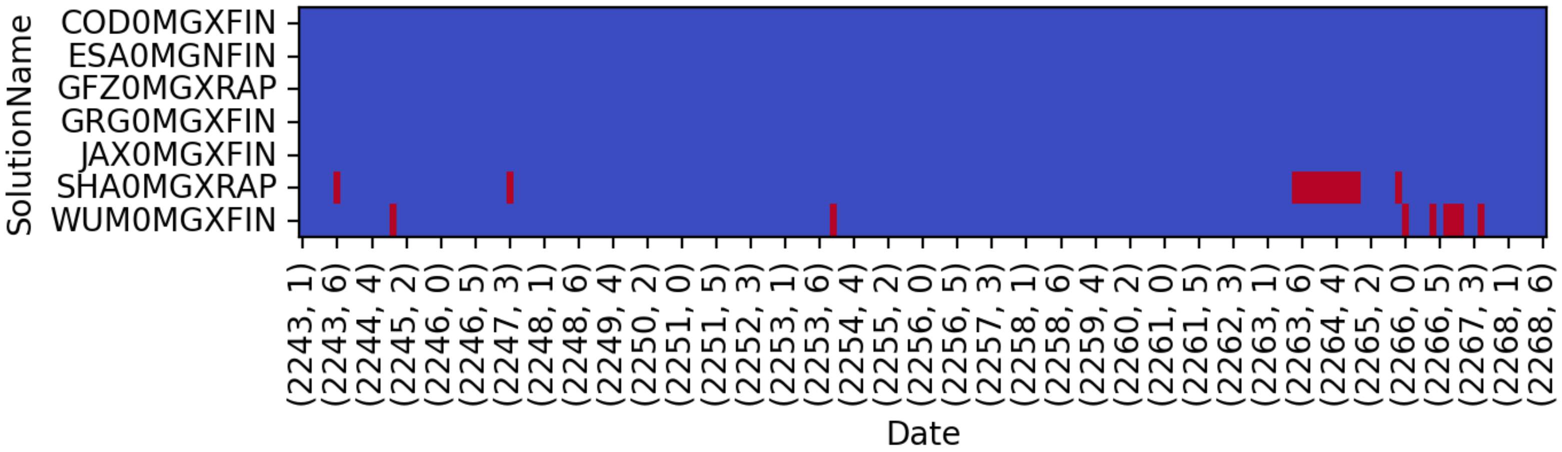
Processing Details

- Software: Software for Precise Orbit and Clock Combination (SPOCC)

Find more about SPOCC → Poster EGU24-10249 | Thursday, 18 Apr, 16:15–18:00 (CEST), Hall X2, X2.9

- Test Period: Jan – Jul 2023 (181 days)
- Dataset of multi-GNSS orbit solutions delivered by COD (GRECJ), ESA (GRECJ), GFZ (GRECJ), GRG (GRE), JAX (GRJ), SHA (GREC) and WUM (GRECJ)

G – GPS,
R – GLONASS
E – Galileo
C – BeiDou
J – QZSS



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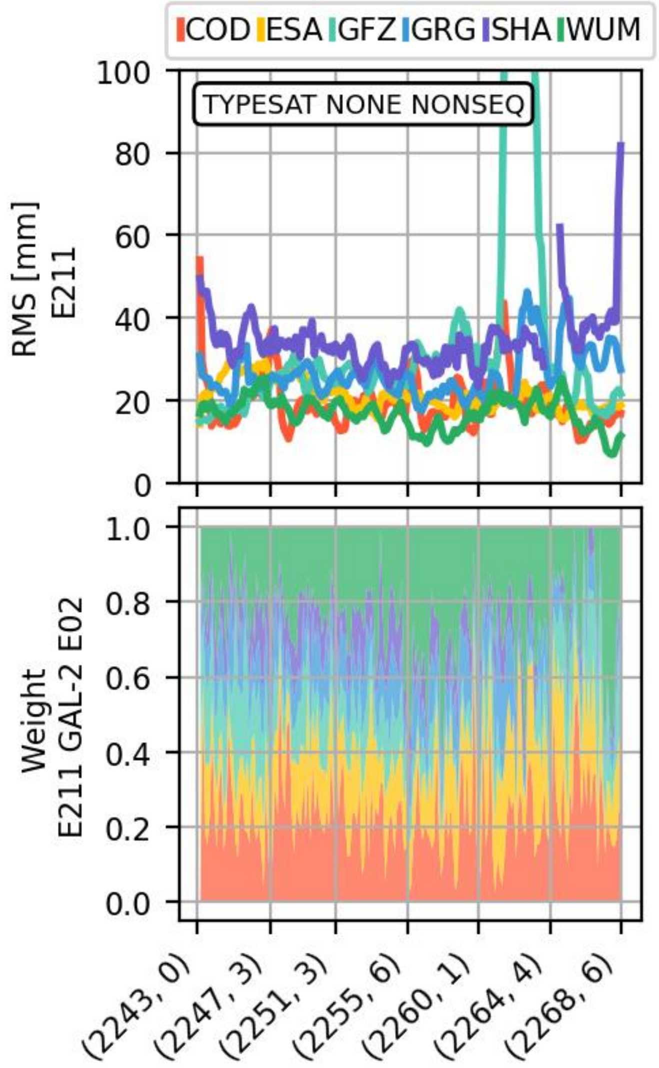
CONSTELLATION vs. SATELLITE SPECIFIC

AC – SATELLITE – SPECIFIC



- **Low redundancy issues**, like daily satellite weighting, cause significant variance day-to-day.

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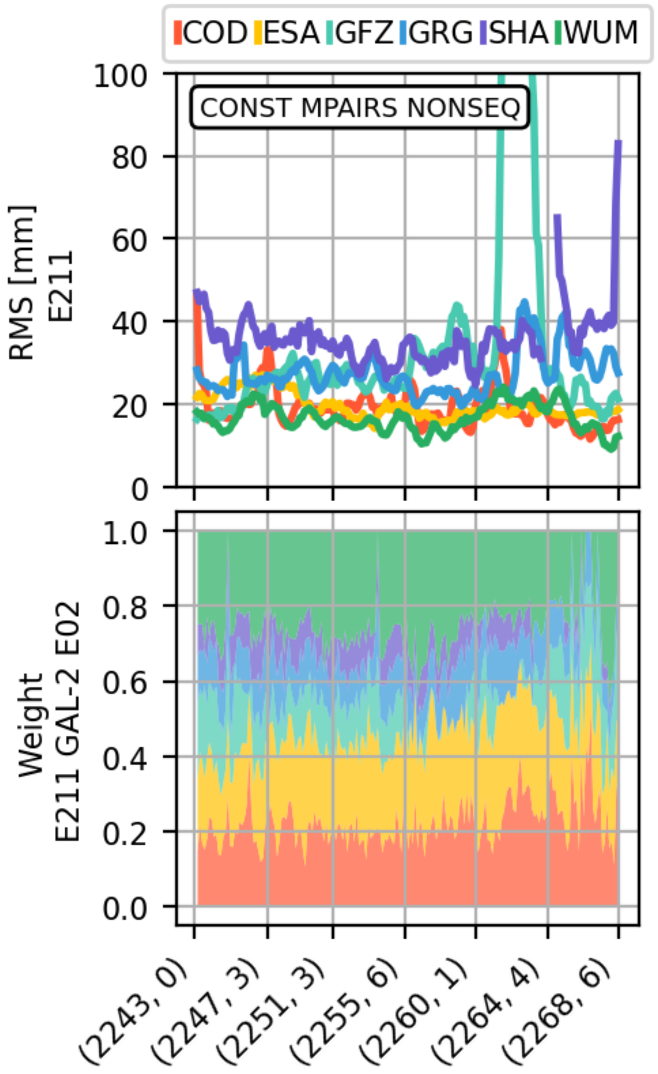
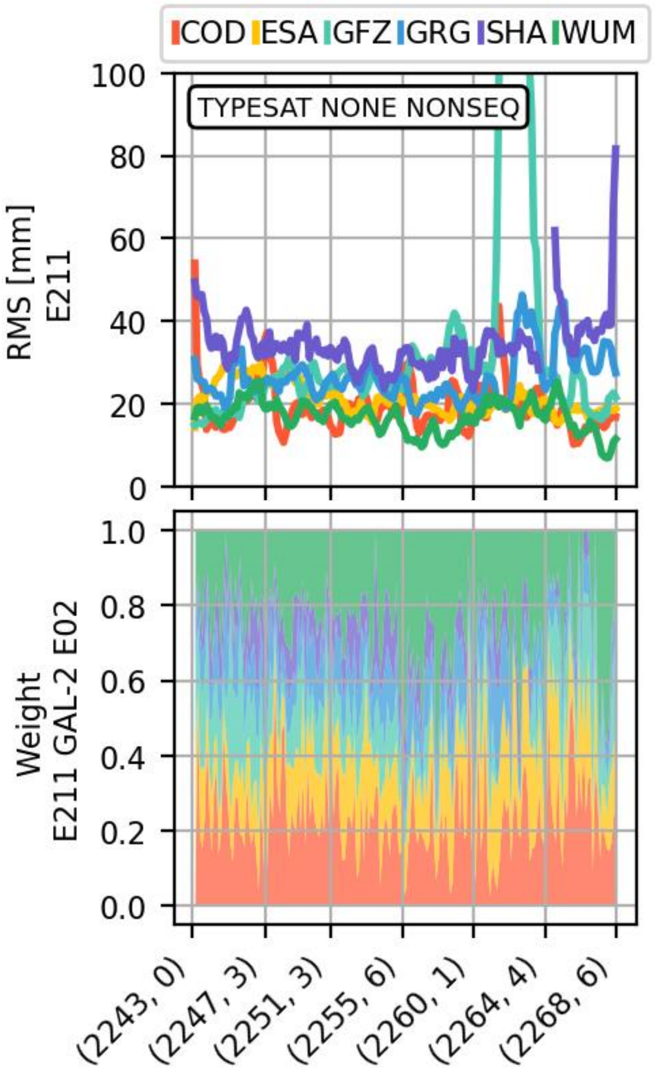
CONSTELLATION vs. SATELLITE SPECIFIC



AC – SATELLITE – SPECIFIC

AC – CONSTELLATION – SPECIFIC



- **Low redundancy issues**, like daily satellite weighting, cause significant variance day-to-day.

- Grouping satellites into constellations **enhances redundancy and stabilizes weight variability.**

- The constellation-specific weight estimation necessitates outlier management.
  - Employed modified z-score method to filter satellites in VCE algorithm, with a cutoff at 3.5.

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CONSTELLATION SPECTRA

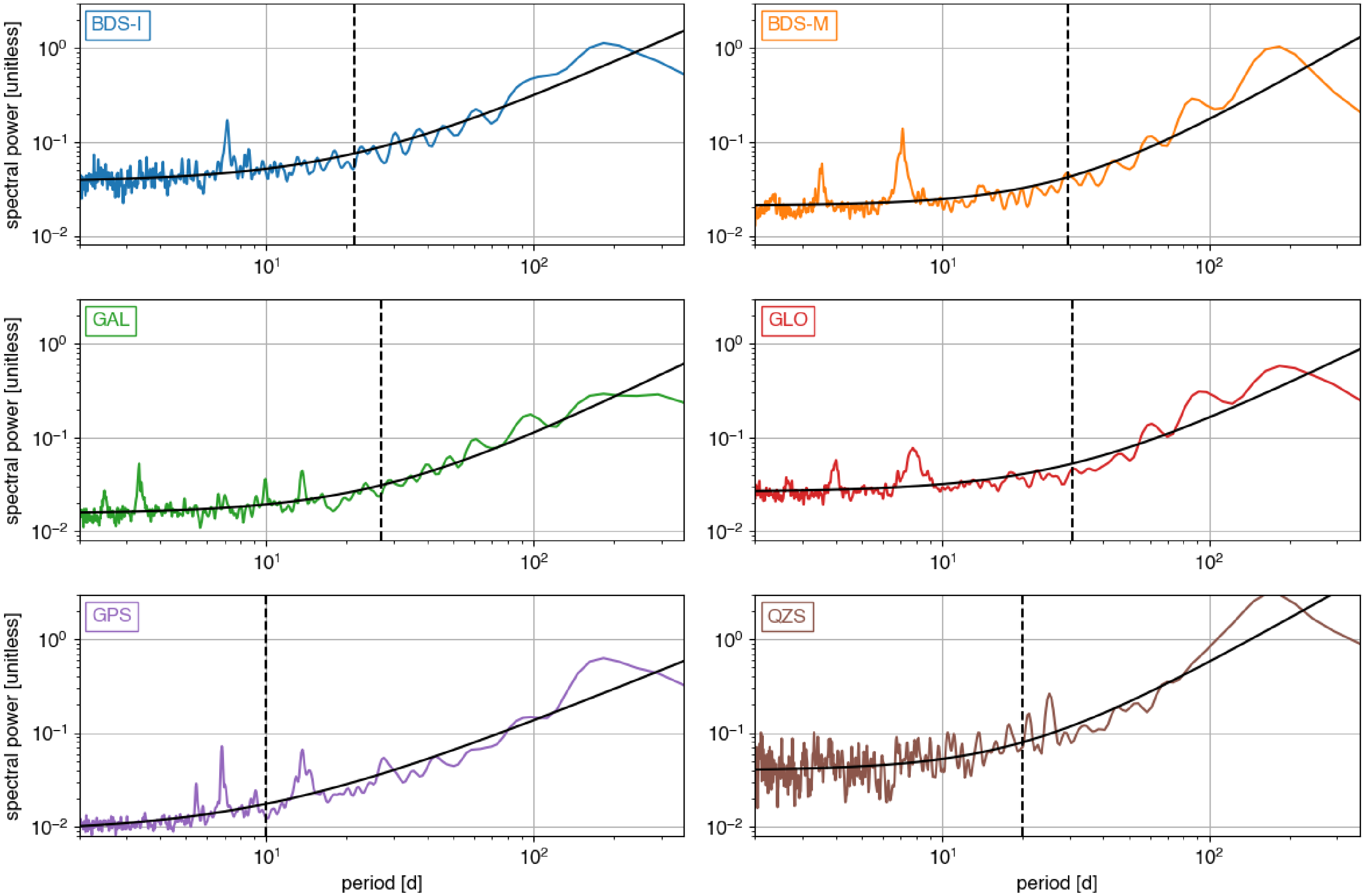
Block-specific average periodograms of satellite-specific weights.

The overall shapes are well approximated by the power spectra of white noise + power-law noise processes (black solid line).

The most prominent system-specific artifacts are at periods of about:

- 7.1 d for the BDS MEOs and IGSOs,
- 3.5 d for the BDS MEOs,
- 10.0 d, 3.3 d, 2.5 d for Galileo,
- 8 d, 4 d for GLONASS

all the peaks listed above directly have origin in the orbit modeling Zajdel et al. (2022)



Source: IGS CTF meeting; made by P. Rebischung

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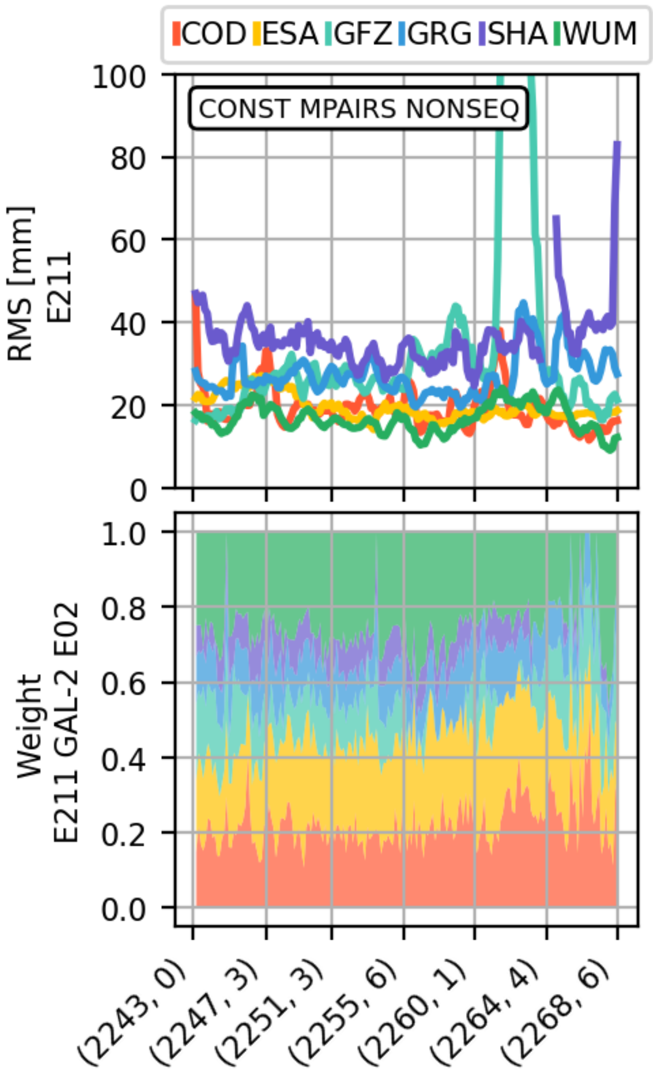
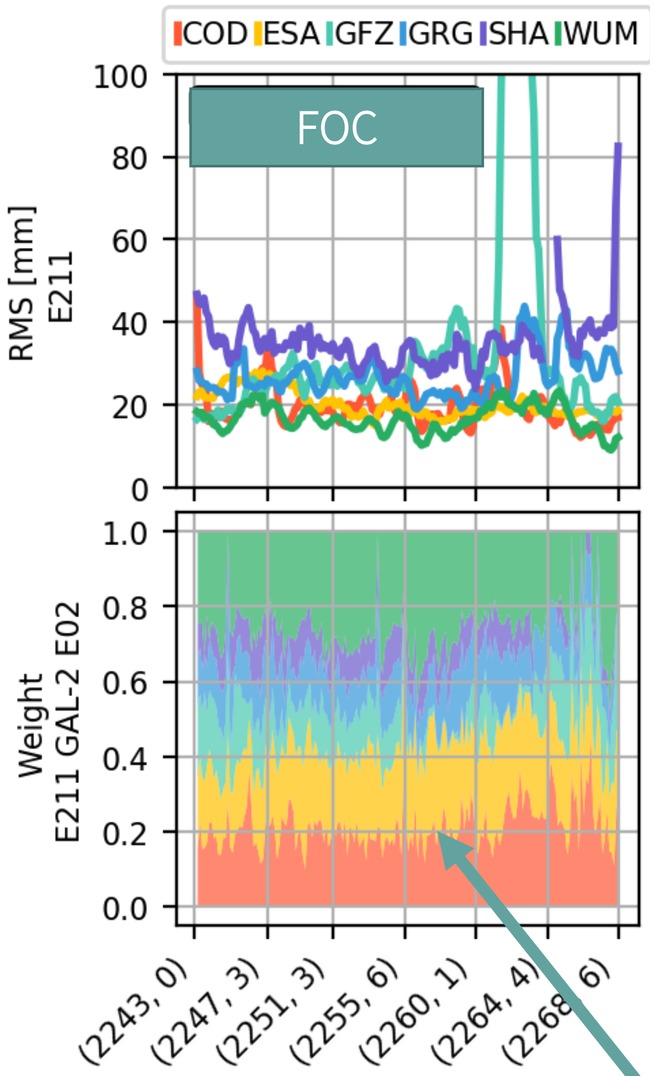
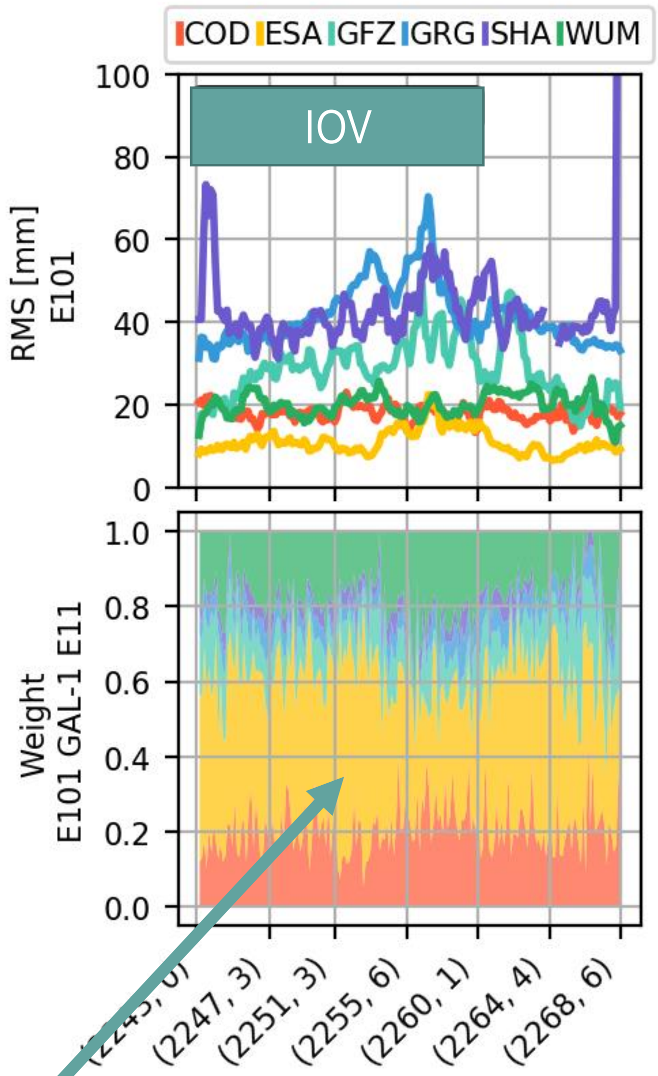
TYPE SPECIFIC



- Whole constellation weight estimation increases redundancy but requires internal consistency.
- Studies by Zajdel et al. (2023, 2024) reveal orbit modeling quality varies by AC and satellite type within constellations, notably in Galileo and BeiDou.





Different decomposition of weights for different types

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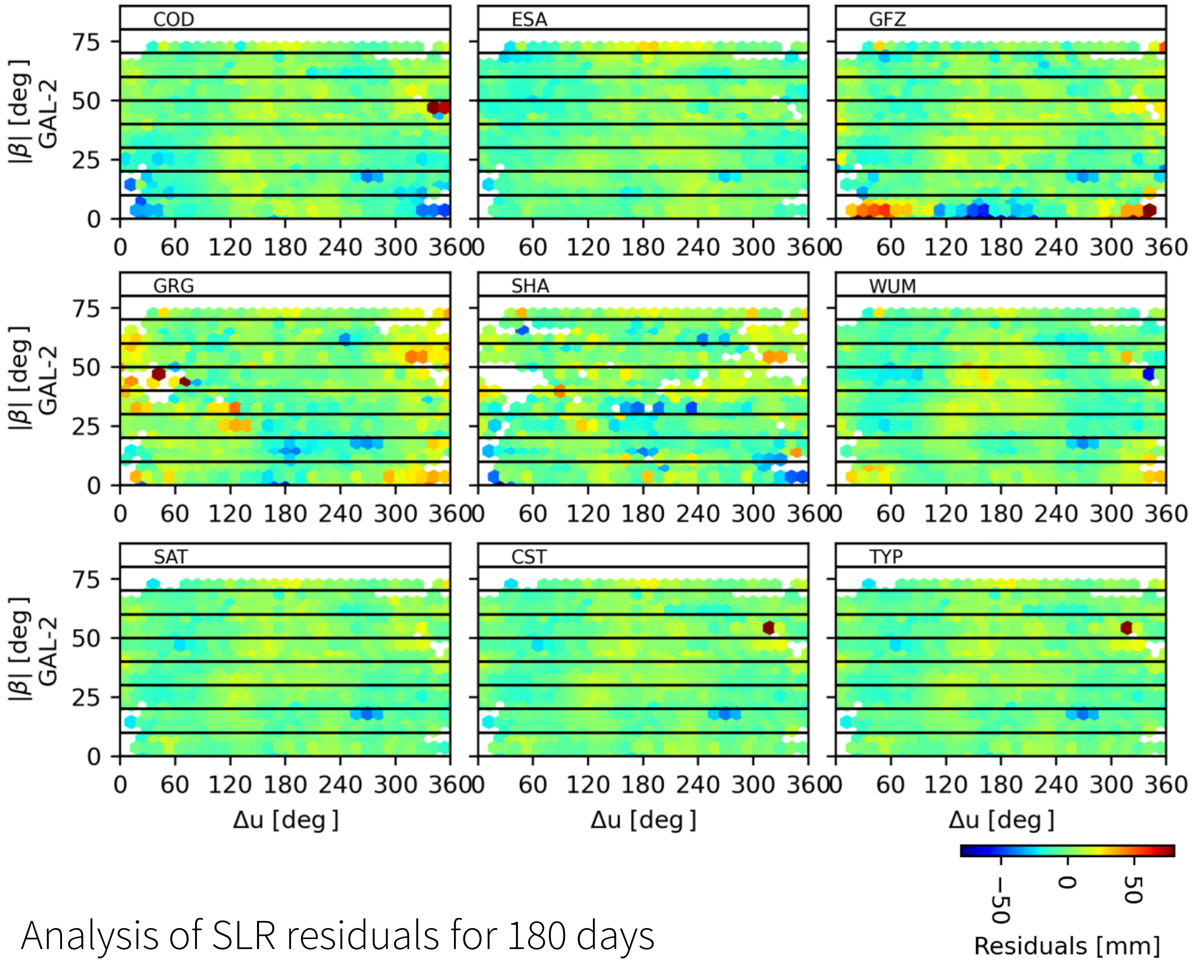
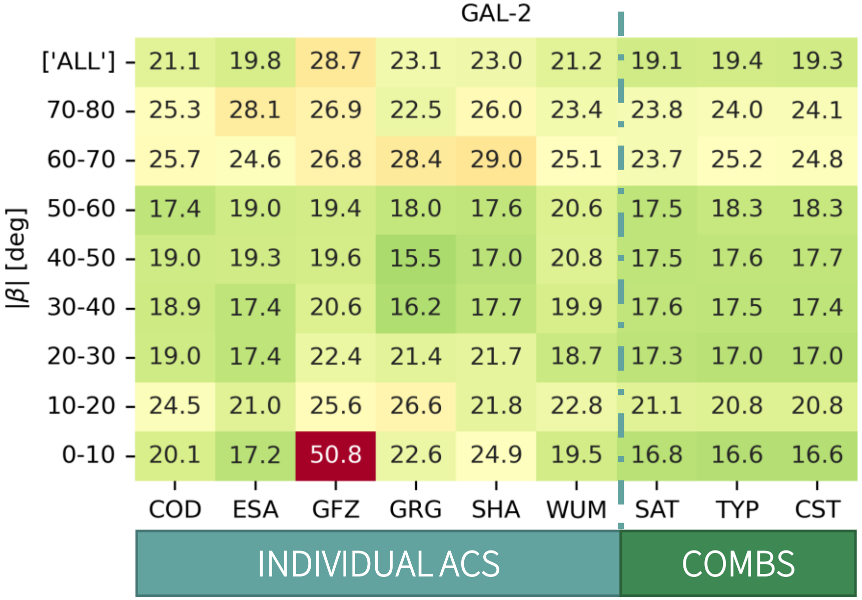
SLR Validation – Galileo FOC



Analysis of SLR residuals for 180 days
Methodology consistent with Zajdel et al. (2023)

Standard deviation of SLR residuals for different ranges of Sun elevation angle above the orbital plane (|β|)



GAL-2

|β| [deg]	COD	ESA	GFZ	GRG	SHA	WUM	SAT	TYP	CST
['ALL']	21.1	19.8	28.7	23.1	23.0	21.2	19.1	19.4	19.3
70-80	25.3	28.1	26.9	22.5	26.0	23.4	23.8	24.0	24.1
60-70	25.7	24.6	26.8	28.4	29.0	25.1	23.7	25.2	24.8
50-60	17.4	19.0	19.4	18.0	17.6	20.6	17.5	18.3	18.3
40-50	19.0	19.3	19.6	15.5	17.0	20.8	17.5	17.6	17.7
30-40	18.9	17.4	20.6	16.2	17.7	19.9	17.6	17.5	17.4
20-30	19.0	17.4	22.4	21.4	21.7	18.7	17.3	17.0	17.0
10-20	24.5	21.0	25.6	26.6	21.8	22.8	21.1	20.8	20.8
0-10	20.1	17.2	50.8	22.6	24.9	19.5	16.8	16.6	16.6
	INDIVIDUAL ACS						COMBS		

- Galileo-FOC combined solutions outperform all other validated solutions. **3.5 % better than ESA.**

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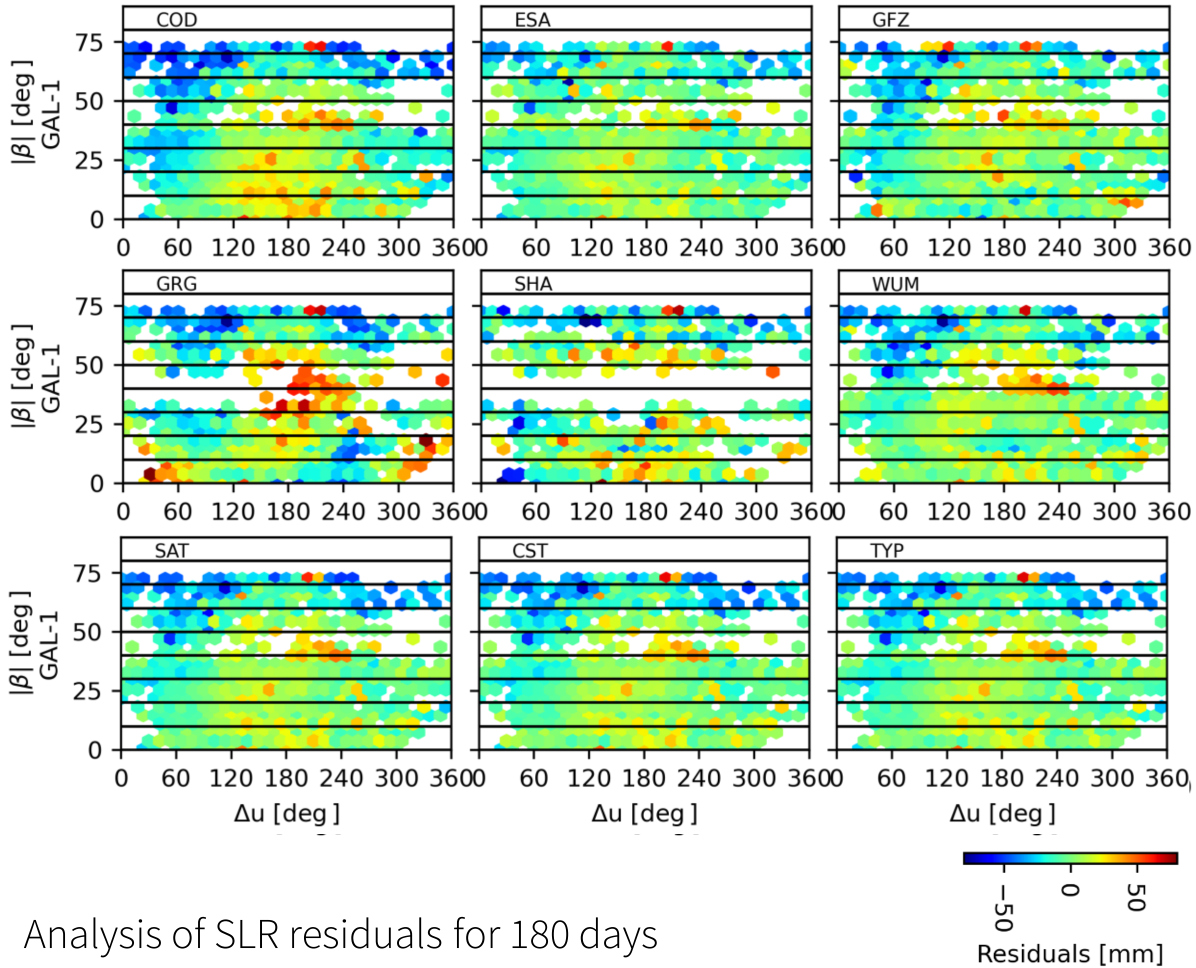
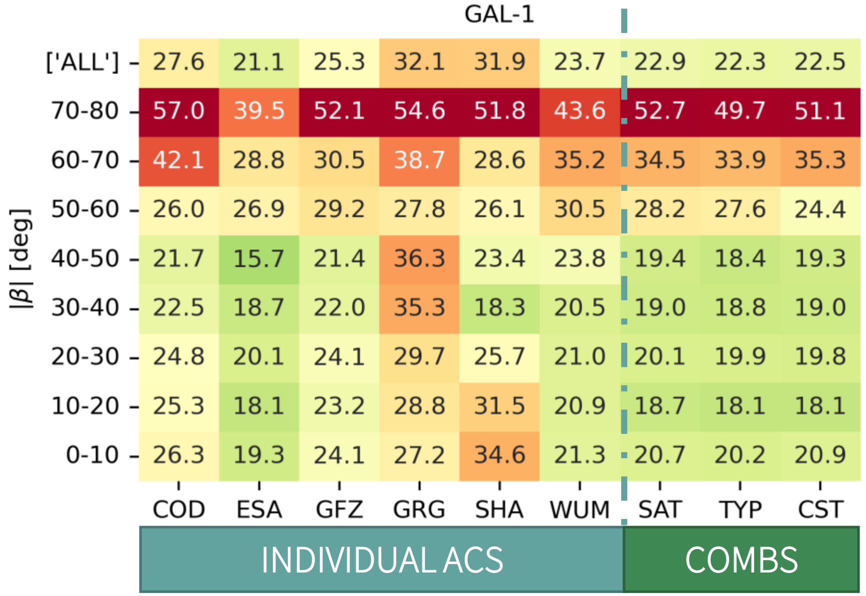
SLR Validation – Galileo IOV



Analysis of SLR residuals for 180 days
Methodology consistent with Zajdel et al. (2023)

Standard deviation of SLR residuals for different ranges of Sun elevation angle above the orbital plane (|β|)



GAL-1

|β| [deg]	COD	ESA	GFZ	GRG	SHA	WUM	SAT	TYP	CST
['ALL']	27.6	21.1	25.3	32.1	31.9	23.7	22.9	22.3	22.5
70-80	57.0	39.5	52.1	54.6	51.8	43.6	52.7	49.7	51.1
60-70	42.1	28.8	30.5	38.7	28.6	35.2	34.5	33.9	35.3
50-60	26.0	26.9	29.2	27.8	26.1	30.5	28.2	27.6	24.4
40-50	21.7	15.7	21.4	36.3	23.4	23.8	19.4	18.4	19.3
30-40	22.5	18.7	22.0	35.3	18.3	20.5	19.0	18.8	19.0
20-30	24.8	20.1	24.1	29.7	25.7	21.0	20.1	19.9	19.8
10-20	25.3	18.1	23.2	28.8	31.5	20.9	18.7	18.1	18.1
0-10	26.3	19.3	24.1	27.2	34.6	21.3	20.7	20.2	20.9
	INDIVIDUAL ACS						COMBS		

- For Galileo-IOV all the combined solutions are almost the best among the validated solutions.

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Incorporation of a priori information about variances

- Förstner (1979) introduced the following efficient iterative scheme of finding optimal variance factors.

$$\left(\sigma_i^2\right)^{(k+1)} = \frac{\left(v_i^{(k)}\right)^T v_i^{(k)}}{n_i\left(1 - \frac{1/\left(\sigma_i^2\right)^{(k)}}{\Sigma\, 1/\left(\sigma_j^2\right)^{(k)}}\right)}$$

Least Squere VCE, as formalized by Amiri-Simkooei (2007) and Teunissen and Amiri-Simkooei (2008), and Förstner's schemes can be seen as two different iterative optimization methods to find the same optimal variance factor estimates.

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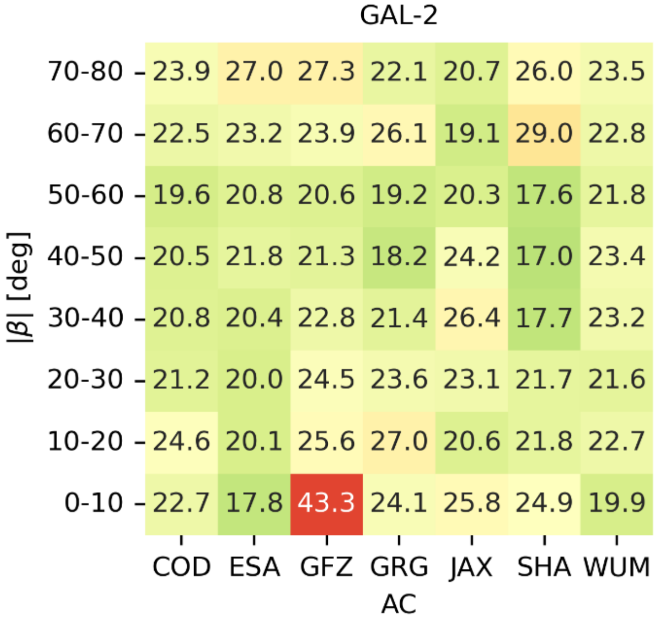
Least Squere VCE, as formalized by Amiri-Simkooei (2007) and Teunissen and Amiri-Simkooei (2008), and Förstner's schemes can be seen as two different iterative optimization methods to find the same optimal variance factor estimates.

- The following modification of Förstner's iterative scheme can be used to introduce the a priori variance factors:

$$\left(\sigma_i^2\right)^{(k+1)} = \frac{\left(v_i^{(k)}\right)^T v_i^{(k)}\boxed{+\phi\times\nu_{i,0}\sigma_{i,0}^2}}{n_i\left(1-\frac{1/\left(\sigma_i^2\right)^{(k)}}{\Sigma 1/\left(\sigma_j^2\right)^{(k)}}\right)+\boxed{\phi\times\nu_{i,0}}}$$, with the a priori variance factors $\boldsymbol{\sigma_{i,0}^2}$ with weights $\boldsymbol{\nu_{i,0}}$

To allow for variability of the variance factors, the weight of a priori information can be damped by a factor $0 \leq \phi \leq 1$.

SLR-based a priori information about weights

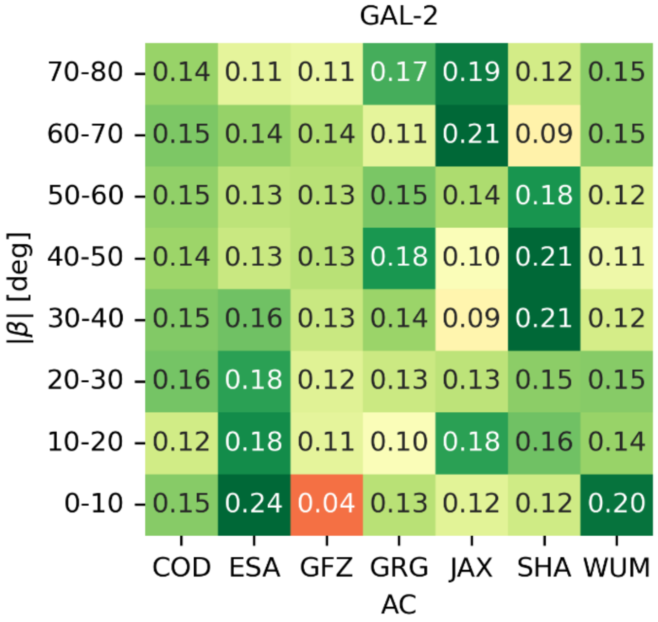


GAL-2

|β| [deg]	COD	ESA	GFZ	GRG	JAX	SHA	WUM
70-80	23.9	27.0	27.3	22.1	20.7	26.0	23.5
60-70	22.5	23.2	23.9	26.1	19.1	29.0	22.8
50-60	19.6	20.8	20.6	19.2	20.3	17.6	21.8
40-50	20.5	21.8	21.3	18.2	24.2	17.0	23.4
30-40	20.8	20.4	22.8	21.4	26.4	17.7	23.2
20-30	21.2	20.0	24.5	23.6	23.1	21.7	21.6
10-20	24.6	20.1	25.6	27.0	20.6	21.8	22.7
0-10	22.7	17.8	43.3	24.1	25.8	24.9	19.9

AC

$$w_{k,j,\beta} = \left(\frac{1}{RMS_{k,j,\beta}}\right)^2$$



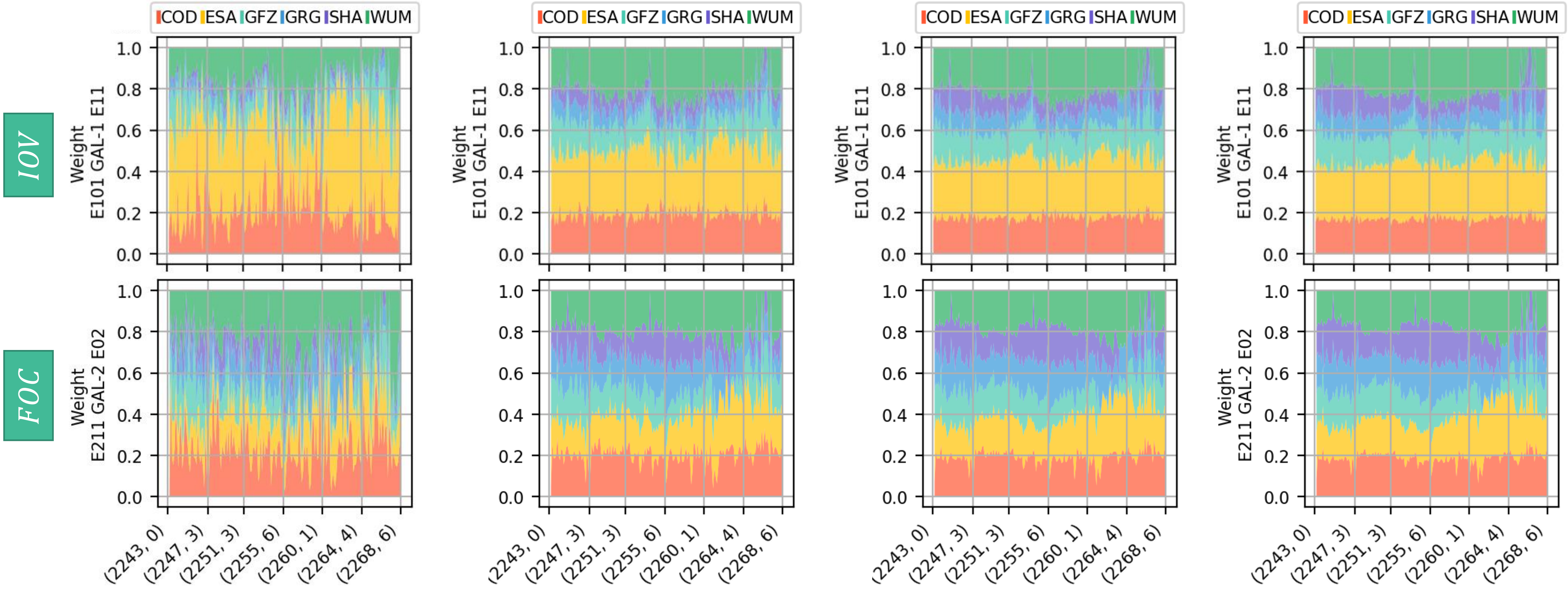
GAL-2

|β| [deg]	COD	ESA	GFZ	GRG	JAX	SHA	WUM
70-80	0.14	0.11	0.11	0.17	0.19	0.12	0.15
60-70	0.15	0.14	0.14	0.11	0.21	0.09	0.15
50-60	0.15	0.13	0.13	0.15	0.14	0.18	0.12
40-50	0.14	0.13	0.13	0.18	0.10	0.21	0.11
30-40	0.15	0.16	0.13	0.14	0.09	0.21	0.12
20-30	0.16	0.18	0.12	0.13	0.13	0.15	0.15
10-20	0.12	0.18	0.11	0.10	0.18	0.16	0.14
0-10	0.15	0.24	0.04	0.13	0.12	0.12	0.20

AC

1. Analysis of SLR residuals as a function of the Sun elevation angle above the orbital plane (|β|).
2. Segmentation based on the |β|, followed by computing the root mean square (RMS) value of the SLR residuals within each segment.
3. Estimation of the SLR-based a priori weight for AC (k), satellite type (j) in the specific β segment.

Incorporation of SLR-based a priori information about weights



$SAT - SPECIFIC \mid \phi = 0$

$\phi = 0.2$

$\phi = 0.5$

$\phi = 0.8$

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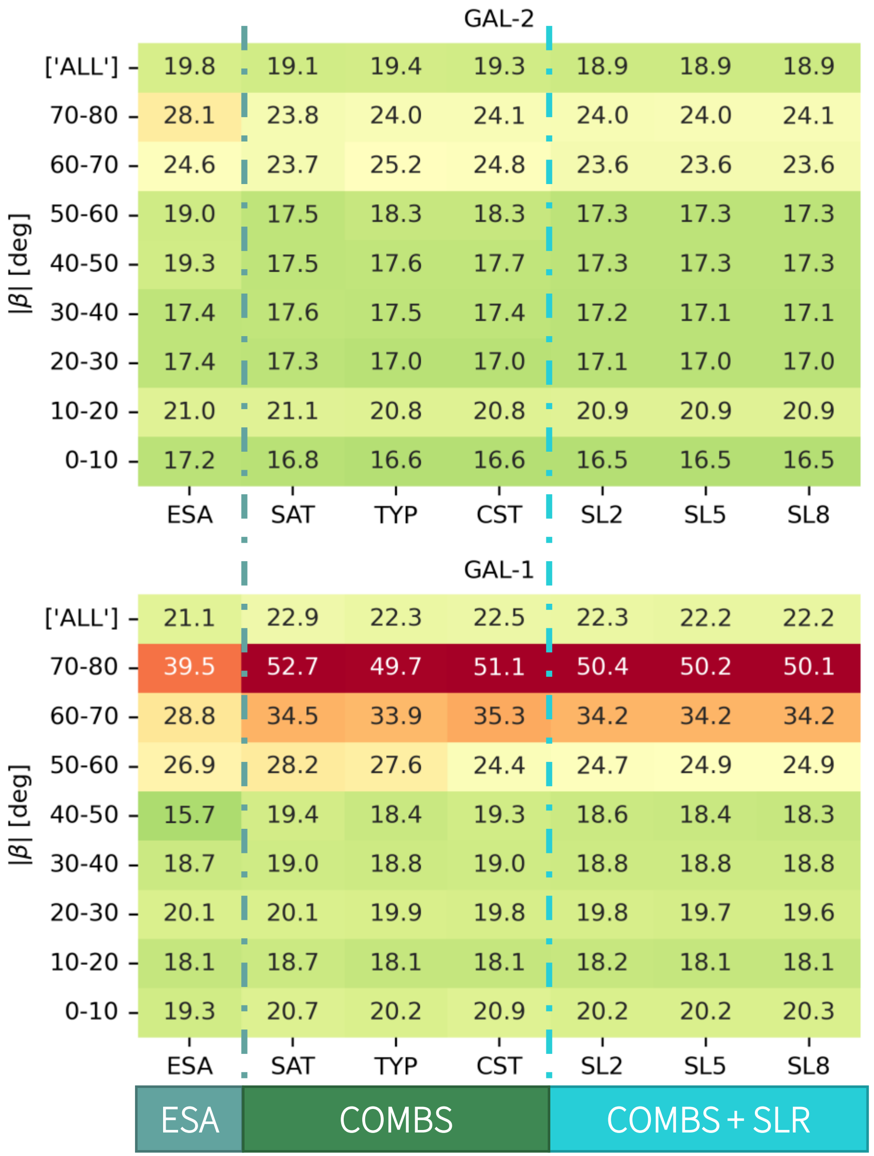
Incorporation of SLR-based a priori information about weights

- Incorporation of SLR-based a priori information about weights improves the solution, however, the improvement is rather marginal.



GAL-2

|β| [deg]	ESA	SAT	TYP	CST	SL2	SL5	SL8
['ALL']	19.8	19.1	19.4	19.3	18.9	18.9	18.9
70-80	28.1	23.8	24.0	24.1	24.0	24.0	24.1
60-70	24.6	23.7	25.2	24.8	23.6	23.6	23.6
50-60	19.0	17.5	18.3	18.3	17.3	17.3	17.3
40-50	19.3	17.5	17.6	17.7	17.3	17.3	17.3
30-40	17.4	17.6	17.5	17.4	17.2	17.1	17.1
20-30	17.4	17.3	17.0	17.0	17.1	17.0	17.0
10-20	21.0	21.1	20.8	20.8	20.9	20.9	20.9
0-10	17.2	16.8	16.6	16.6	16.5	16.5	16.5

GAL-1

|β| [deg]	ESA	SAT	TYP	CST	SL2	SL5	SL8
['ALL']	21.1	22.9	22.3	22.5	22.3	22.2	22.2
70-80	39.5	52.7	49.7	51.1	50.4	50.2	50.1
60-70	28.8	34.5	33.9	35.3	34.2	34.2	34.2
50-60	26.9	28.2	27.6	24.4	24.7	24.9	24.9
40-50	15.7	19.4	18.4	19.3	18.6	18.4	18.3
30-40	18.7	19.0	18.8	19.0	18.8	18.8	18.8
20-30	20.1	20.1	19.9	19.8	19.8	19.7	19.6
10-20	18.1	18.7	18.1	18.1	18.2	18.1	18.1
0-10	19.3	20.7	20.2	20.9	20.2	20.2	20.3

ESA	COMBS			COMBS + SLR		
ESA	SAT	TYP	CST	SL2	SL5	SL8

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Conclusions

- **The SPOCC software provides multi-GNSS orbit solutions of outstanding quality**, matching or surpassing those from individual Analysis Centers (ACs).

- Weighting methods used in SPOCC software (SAT/TYPE/CONST) offer similar quality levels, as determined by Satellite Laser Ranging (SLR) comparisons.

  - SLR validation indicates that ESA's orbit products for Galileo satellites are the most precise compared to other individual solutions. Nonetheless, the combined solution from SPOCC matches the quality of ESA's products.

- **Integrating SLR-based a priori information on weights reduces the day-to-day fluctuations in weight estimates and marginally enhances SLR validation outcomes.** This improvement is currently applicable only to Galileo and BeiDou satellites due to the absence of Laser Retroreflector Arrays (LRAs) on GPS satellites and the suspension of GLONASS data sharing by global datacenters, a consequence of the Russian invasion of Ukraine.

  - To achieve more stable weightings for specific satellites, one method involves constraining the weights to those estimated on the preceding day, for example, using sequential VCE.

- SPOCC software enables the consistent determination of combined GNSS clock products. Evaluating the combined orbit and clock solutions through Precise Point Positioning is a forthcoming research phase to validate their practical applicability.

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GFZ Helmholtz Centre POTSDAM

IPGP

IAG International Association of Geodesy

IGS INTERNATIONAL GNSS SERVICE

Co-funded by the European Union

Central Bohemia Region

MERIT ...brains in motion

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Thank you

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