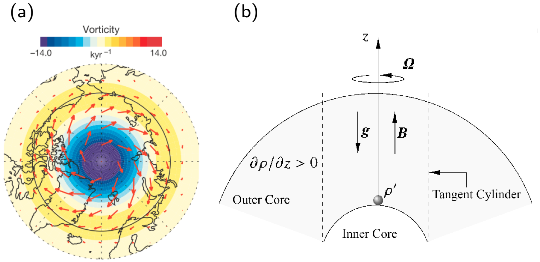
A criterion for the existence of polar vortices in the Earth's core

Debarshi Majumder, Binod Sreenivasan

Centre for Earth Sciences,
Indian Institute of Science, Bangalore
India

Tangent cylinder and polar vortices



(a) Time-averaged polar vortex in the northern hemisphere. Arrows indicate tangential velocities and colour contours indicate vorticity (taken from Olson and Aurnou (1999)). (b) Tangent cylinder inside Earth's core. The tangent cylinder is an imaginary cylinder tangent to the inner core boundary (ICB) and parallel to the Earth's rotation axis z. It cuts the core–mantle boundary (CMB) at approximately latitude 70°.

- The secular variation of the geomagnetic field suggests the presence of anticyclonic polar vortices in the Earth's core. Using the frozen-flux approximation, the inferred peak azimuthal velocity of the polar vortex is calculated to be 0.6 to 0.9° yr^{-1} (Olson and Aurnou, 1999; Hulot et al., 2002).
- The polar vortex is an anticyclonic flow located in the polar region of the outer core. Under the influence of magnetic field, the polar azimuthal flow is thought to be produced by one or more coherent upwellings within the tangent cylinder, offset from the rotation axis.
- **Aim of the study:**
  Our objective is to define the parameter space within which the geodynamo operates, ensuring the emergence of a polar vortex that aligns with the observed speed of the anticyclonic flow.

Governing equations of the dynamo model

The non-dimensional equations for velocity, magnetic field, and temperature in MHD under the Boussinesq approximation for our dynamo model are as follows:

$$E\ Pm^{-1}\left(\frac{\partial \boldsymbol{u}}{\partial t}+(\nabla\times\boldsymbol{u})\times\boldsymbol{u}\right)+\hat{\boldsymbol{z}}\times\boldsymbol{u}=-\nabla p^{*}+PmPr^{-1}Ra\,T\,\boldsymbol{r}+(\nabla\times\boldsymbol{B})\times\boldsymbol{B}+E\nabla^{2}\boldsymbol{u}, \quad (1)$$

$$\frac{\partial T}{\partial t}+\boldsymbol{u}.\nabla T=PmPr^{-1}\nabla^{2}T, \quad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{u}\times\boldsymbol{B})+\nabla^{2}\boldsymbol{B}, \quad (3)$$

$$\nabla.\boldsymbol{u}=\nabla.\boldsymbol{B}=0, \quad (4)$$

The modified pressure p^{*} is given by $p+\frac{1}{2}E\,Pm^{-1}\,|\boldsymbol{u}|^{2}$. The dimensionless parameters in the above equations are the Ekman number, $E=\nu/2\Omega L^{2}$; the Prandtl number, $Pr=\nu/\kappa$; the magnetic Prandtl number, $Pm=\nu/\eta$; and the modified Rayleigh number, $Ra=g\alpha\beta L^{2}/2\Omega\kappa$. The parameters g, ν, κ, and α denote the gravitational acceleration, kinematic viscosity, thermal diffusivity, and coefficient of thermal expansion, respectively.

Magnetic-Archimedean-Coriolis (MAC) waves

- We study the evolution of a density perturbation ρ' that evolves in an unstably stratified fluid layer under a uniform axial magnetic field $\boldsymbol{B}$ and background rotation $\boldsymbol{\Omega}$.
- In the diffusionless limit ($\nu = \kappa \to 0$), simplifying the linearised governing equations through algebraic operations and considering plane wave solutions for the stream function ψ in the form $\hat{\psi} \sim e^{i\lambda t}$ leads to the following dispersion relation (Sreenivasan and Maurya, 2021).

$$\lambda^5 - 2i\omega_\eta \lambda^4 - \left(\omega_A^2 + \omega_\eta^2 + 2\omega_M^2 + \omega_C^2\right)\lambda^3 + 2i\omega_\eta\left(\omega_A^2 + \omega_M^2 + \omega_C^2\right)\lambda^2 + \left(\omega_A^2\omega_\eta^2 + \omega_A^2\omega_M^2 + \omega_M^4 + \omega_\eta^2\omega_C^2\right)\lambda - i\omega_A^2\omega_\eta\omega_M^2 = 0. \quad (5)$$

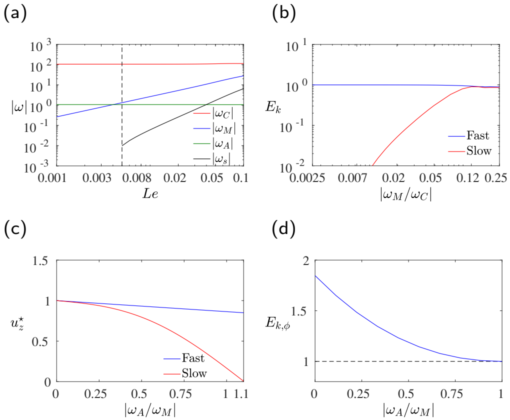
The fundamental frequencies in (5) are given by,

$$\omega_C^2 = \frac{4\Omega^2 k_z^2}{k^2}, \quad \omega_M^2 = V_M^2 k_z^2, \quad \omega_A^2 = g\alpha\beta\frac{k_s^2}{k^2}, \quad \omega_\eta^2 = \eta^2 k^4, \quad \text{(6a-d)}$$

representing square of linear inertial waves, Alfvén waves, internal gravity waves and magnetic diffusion respectively. In unstable density stratification, $\omega_A^2 < 0$. Here, $k^2 = k_s^2 + k_z^2$.

- This characteristic equation has five roots. First two roots represent oppositely travelling fast Magnetic-Archimedean-Coriolis (MAC) waves, two other roots represent oppositely travelling slow MAC waves, and the fifth root represents the overall growth of the velocity perturbation.
  - *Fast MAC waves* - These are linear inertial waves weakly modified by the magnetic field and buoyancy.
  - *Slow MAC waves* - These are magnetostrophic waves generated by the balance between the magnetic, Coriolis and buoyancy forces.

Intensity of the MAC waves



(a): Variation of absolute values of the frequencies with the Lehnert number Le.

$$Le = V_M / 2\Omega\delta$$

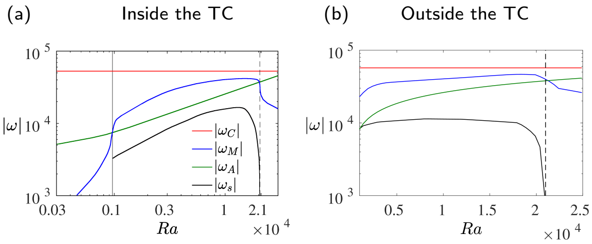
Here V_M is the Alfvén velocity and δ is the length scale of the perturbation. The slow MAC wave is present when $|\omega_M| > |\omega_A|$.

(b): Variation of the kinetic energy of the fast and slow MAC waves (normalized by the nonmagnetic kinetic energy) with $|\omega_M/\omega_C|$. The slow MAC wave intensity is comparable to that of the fast MAC wave when $|\omega_M/\omega_C| \sim 1$.

(c): Variation of the peak z velocity of fast and slow MAC waves with $|\omega_A/\omega_M|$ for $Le = 0.09$. The slow wave are attenuated at $|\omega_A/\omega_M| \sim 1$.

(d): The variation of the ϕ component of kinetic energy, normalized by its nonmagnetic value, with progressively increasing forcing for a given ω_M/ω_C.

Fundamental frequencies (dynamo model)

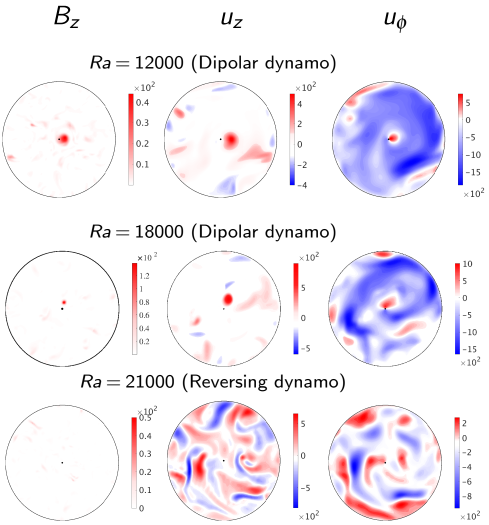


Dipolar regime
$|\omega_C| \gg |\omega_M| \gg |\omega_A| \gg |\omega_\eta|$

Reversing/Multipolar regime
$|\omega_C| \gg |\omega_M| \sim |\omega_A| \gg |\omega_\eta|$

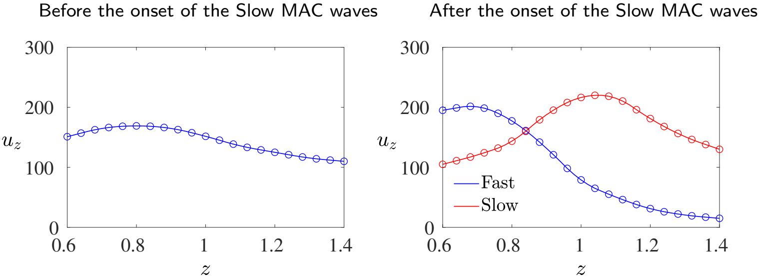
- The solid vertical line indicates the onset of the slow MAC waves inside the tangent cylinder while the dashed vertical lines mark the suppression of the slow waves. The dynamo parameters are $E = 6 \times 10^{-5}$, $Pm = Pr = 5$.
- Then slow MAC wave is present when $|\omega_M| > |\omega_A|$

Isolated plume convection



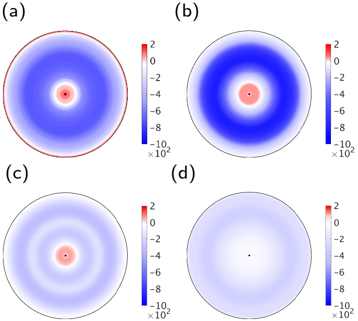
Horizontal (z) section plots within the tangent cylinder of the axial magnetic field B_z at $z = 0.9$ (left panels), u_z at $z = 1.4$ (middle panels) and u_ϕ at $z = 1.4$ (right panels)

Isolated plume convection



Variation of the time-averaged peak value of the axial (z) velocity with z within the TC for (a) $Ra = 800$ and (b) $Ra = 1000$.

Polar vortex intensities and applications to the inertia-free core



Horizontal (z) section plots at height $z = 1.4$ above the equator of the time and azimuthally averaged flow u_ϕ within the TC. (a) $Ra = 12000$, (b) $Ra = 18000$, (c) $Ra = 21000$, (d) nonmagnetic run at $Ra = 12000$.

The maximum dimensionless time and azimuthally averaged value of u_ϕ is measured within the TC, with its radial distance from the rotation axis. For example, at $Ra = 18000$, this magnitude of u_ϕ is 814, at radius 0.35. This could be scaled up to its value in the Earth's core (Sreenivasan and Jones, 2005), giving,

$$u_{\phi,\mathrm{sc}} = \frac{u_\phi \eta}{L} = 3.602 \times 10^{-4}\mathrm{ms}^{-1} \approx 0.81^\circ\mathrm{yr}^{-1},$$

where η and L have the values 1 $\mathrm{m}^2\mathrm{s}^{-1}$ and 2.26×10^6 m respectively.

To obtain the observed peak azimuthal velocity of 0.6–0.9$^\circ\mathrm{yr}^{-1}$ (Olson and Aurnou, 1999; Hulot et al., 2002), the Rayleigh number in the low-inertia geodynamo must be $\sim 10^3$ times the Rayleigh number for the onset of nonmagnetic convection.

Concluding remarks

- We have constrained a parameter space within which the geodynamo operates, ensuring the emergence of the polar vortices that aligns with the observed speed of the anticyclonic flow in the polar region. To obtain the observed peak azimuthal motions of 0.6–0.9° yr^{-1} , the Rayleigh number in the low-inertia geodynamo must be 10^3 times the Rayleigh number for the onset of nonmagnetic convection.
- For $|\omega_M/\omega_C| \sim 0.1$, the time and azimuthally averaged intensity of the polar vortex is much higher than that in nonmagnetic convection. If the forcing is so strong as to cause polarity reversals, the field within the TC decays away, resulting in much weaker circulation in the polar regions.
- The slow MAC waves generated at the length scale of convection support the isolated TC upwellings in the dipole-dominated dynamo regime, in turn producing strong anticyclonic polar vortices. In regions where the magnetic flux is relatively weak, fast MAC waves are excited, although these waves are unable to penetrate the neutrally buoyant fluid layer that lies above them.

For further reading, you can refer to Majumder and Sreenivasan (2023).

References

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