High-resolution zenith delay and tropospheric gradient fields track precipitation during heavy local-scale rainfall events

Supplementary Materials

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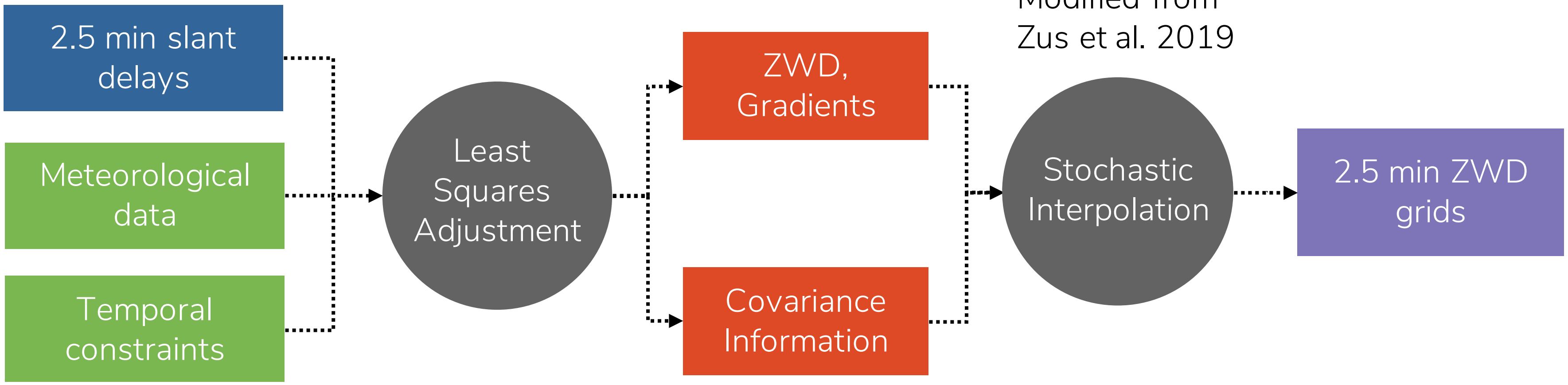
01

Derivation of high-resolution gridded ZWD and tropospheric gradient fields

Data and processing flow

Modified from Zus et al. 2019



slant delays processed by
GFZ Helmholtz Centre for Geosciences

Assumption: $\nabla ZWD \propto \begin{bmatrix} G_E \\ G_N \end{bmatrix}$

(Elósegui et al. 1999)

Least squares setup

Standard formulation of slant total delay:

$$STD(\epsilon,\alpha) - m_H(\epsilon)ZHD_0 - m_{GH}(\epsilon)[\cos\alpha\, G_{NH} + \sin\alpha\, G_{EH}] = m_W(\epsilon)ZWD + m_{GW}(\epsilon)[\cos\alpha\, G_N + \sin\alpha\, G_E] + e_k$$

STD... slant total delay
ϵ, α ... elevation, azimuth
m_H, m_W ... hydrostatic (H) and wet (W) mapping function (GMF, Böhm et al. 2006)
ZHD_0 ... apriori zenith hydrostatic delay (Saastamoinen 1976, Davis et al. 1985)
m_{GH}, m_{GW} ... hydrostatic (H) and wet (W) gradient mapping function (Chen and Herring 1997)
G_{NH}, G_{EH} ... apriori hydrostatic gradients (north, east)

ZWD ... zenith wet delay
G_N, G_E ... wet gradients (north, east)

Least squares setup

Standard formulation of slant total delay:

$$STD(\epsilon,\alpha) - m_H(\epsilon)ZHD_0 - m_{GH}(\epsilon)[\cos\alpha\, G_{NH} + \sin\alpha\, G_{EH}] = m_W(\epsilon)ZWD + m_{GW}(\epsilon)[\cos\alpha\, G_N + \sin\alpha\, G_E] + e_k$$

Applied Constraints:

$$0 = ZWD(t_k) - ZWD(t_{k-1}) + u_k$$

$$0 = G_N(t_k) - G_N(t_{k-1}) + v_k$$

$$0 = G_E(t_k) - G_E(t_{k-1}) + w_k$$

Differences between epochs are constraint for all unknown parameters – relative weights are determined by variance component estimation.

Least squares output

ZWD, Gradients

$$\widehat{ZWD}(t_k), \hat{G}_N(t_k), \hat{G}_E(t_k)$$

Covariance Information

$$\hat{\Sigma}(t_k) = \begin{bmatrix} \hat{\sigma}^2_{\widehat{ZWD}} & \hat{\sigma}_{\widehat{ZWD},\hat{G}_N} & \hat{\sigma}_{\widehat{ZWD},\hat{G}_E} \\ . & \hat{\sigma}^2_{\hat{G}_N} & \hat{\sigma}_{\hat{G}_N,\hat{G}_E} \\ . & . & \hat{\sigma}^2_{\hat{G}_E} \end{bmatrix}$$

Least squares solution is computed independently for each station s

02

High-resolution gridded zenith delay and gradients

High-resolution gridded zenith delay and gradients

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01 Taylor series expansion of ZWD at each station position x_S

$$ZWD_s(x) = ZWD(x_s) + \nabla ZWD\Big|_{x_s} (x - x_s) + \ldots$$

02 Approximation of ∇ZWD with $\frac{1}{c}[\hat{G}_N \quad \hat{G}_E]$

$$ZWD_s(x) \approx ZWD(x_s) + \frac{1}{C}[\hat{G}_N \quad \hat{G}_E](x - x_s) + \ldots$$

03 Variance propagation and weights

$$\sigma_S^2(x) = \begin{bmatrix} 1 & \frac{\Delta x}{c}\hat{G}_N & \frac{\Delta y}{c}\hat{G}_E \end{bmatrix} \begin{bmatrix} \hat{\sigma}^2_{\widehat{ZWD}} & \hat{\sigma}_{\widehat{ZWD},\hat{G}_N} & \hat{\sigma}_{\widehat{ZWD},\hat{G}_E} \\ . & \hat{\sigma}^2_{\hat{G}_N} & \hat{\sigma}_{\hat{G}_N,\hat{G}_E} \\ . & . & \hat{\sigma}^2_{\hat{G}_E} \end{bmatrix} \begin{bmatrix} 1 \\ \frac{\Delta x}{c}\hat{G}_N \\ \frac{\Delta y}{c}\hat{G}_N \end{bmatrix} \longrightarrow w_s = \frac{1}{\sigma_S^2}$$

High-resolution gridded zenith delay and gradients

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04

Merging of individual ZWD fields

$$ZWD(x) = \frac{1}{\sum_s w_s(x)} \sum_s ZWD_s(x)\, w_s(x)$$

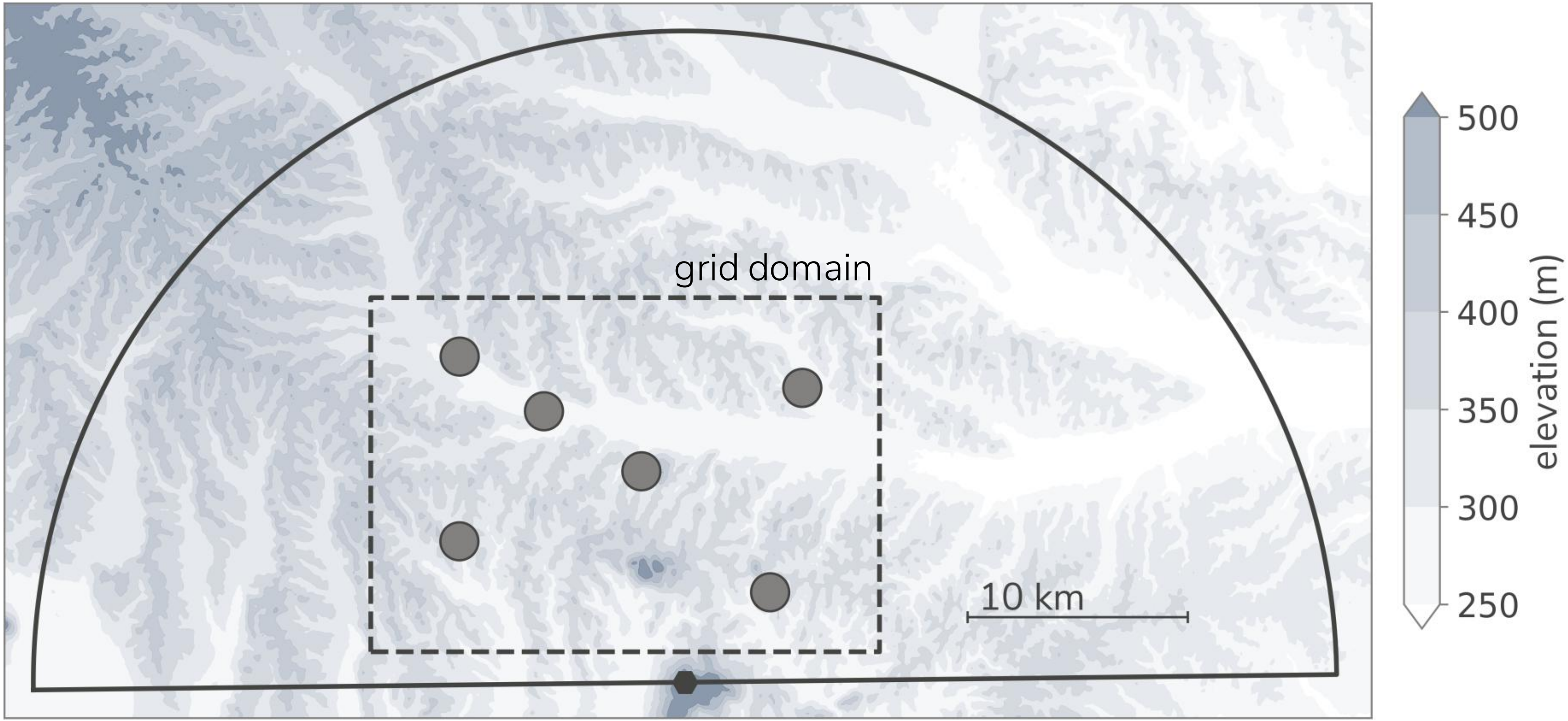
05

Derivation of $\hat{G}_N$ and $\hat{G}_E$ from ∇ZWD

$$[\hat{G}_N(x) \quad \hat{G}_E(x)] = \nabla ZWD(x) \cdot C$$

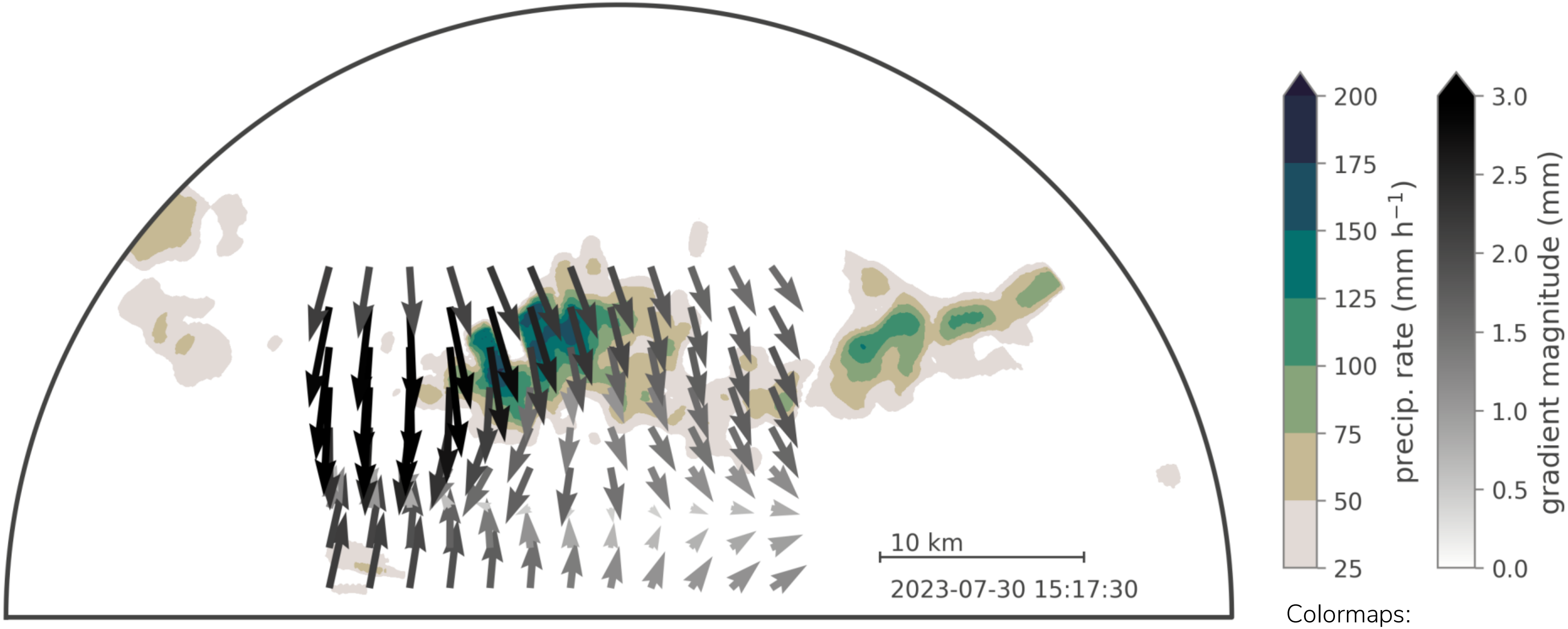
Steps 01 – 05 can be computed for arbitrary positions x

Station distribution and grid domain



High-resolution gridded gradient fields

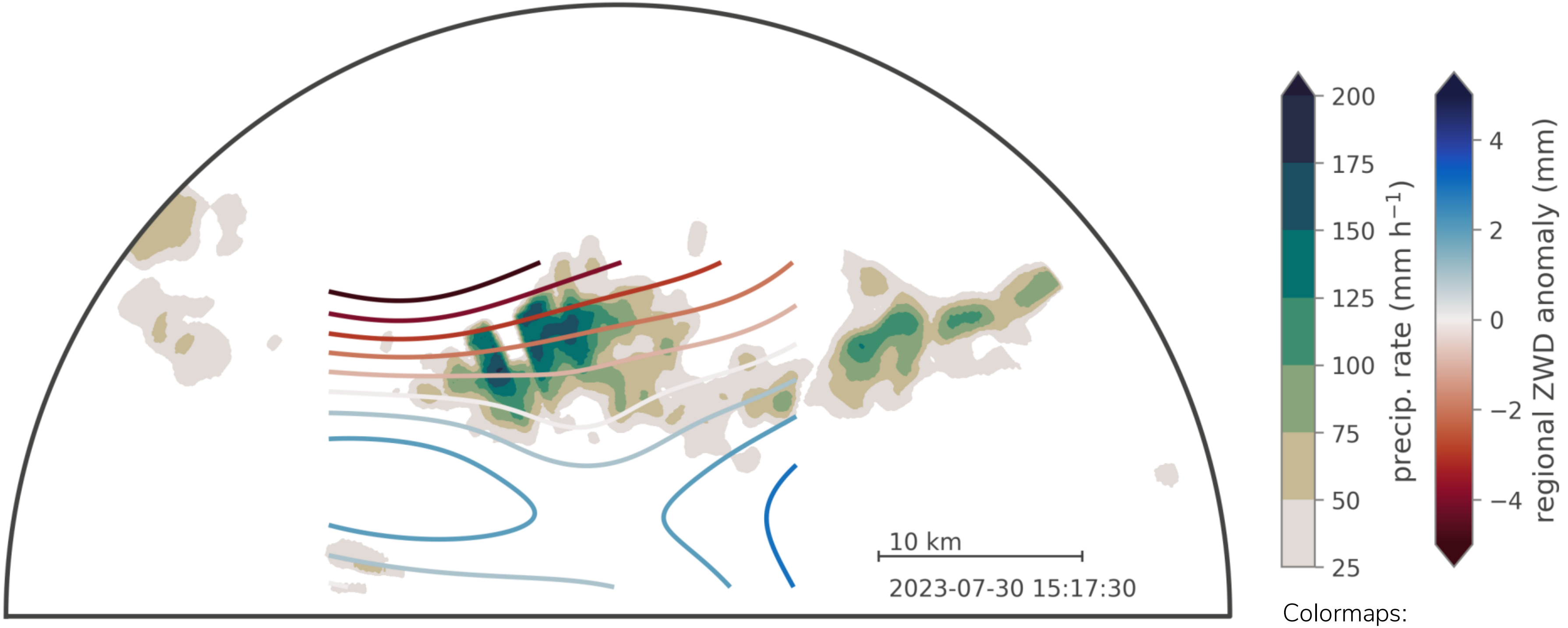
Result of stochastic interpolation - gradients



Colormaps:
Thyng et al. 2016

High-resolution gridded zenith delay fields

Result of stochastic interpolation – zenith wet delay



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Thank you!

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