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# Optimizing parameterization schemes with ensemble-based parameter estimation

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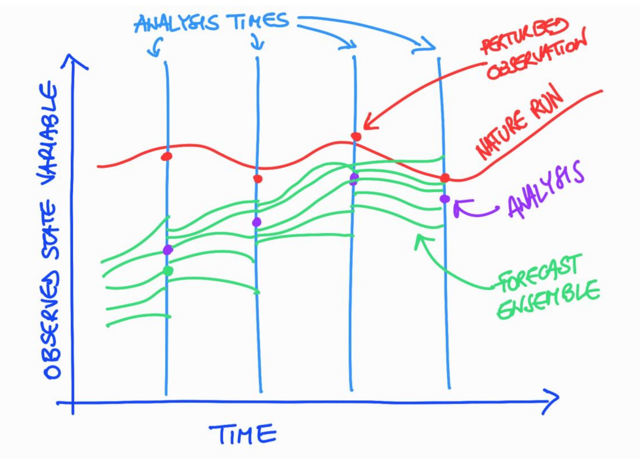
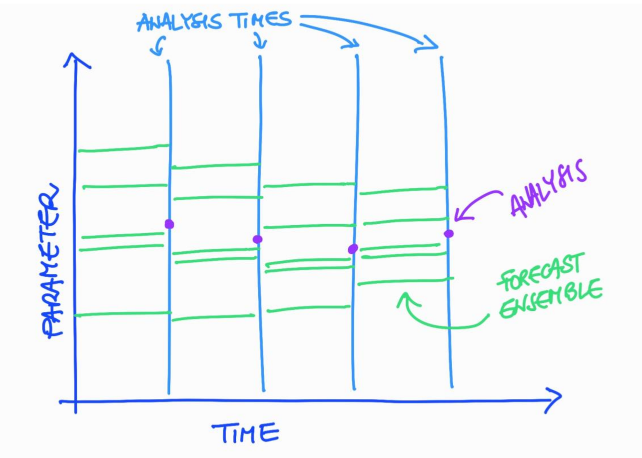
## 1. Introduction

Parameterization schemes rely on semi-empirical relationships that combine a theoretically justified functional form and empirically set parameters.

Parameterizations are a source of systematic model error (bias), which could be reduced through objective tuning of empirical parameters.

In an ensemble data assimilation (DA) cycle (left), any state variable is updated using observational information. Empirical parameters can be treated as state variables and updated in the same way (right). However, they are not observable and do not evolve during forecasts.




Parameter updates are determined by the ensemble correlations between parameters and model equivalents of the observations. For instance, using the Ensemble Adjustment Kalman Filter (EAKF):

$$\delta\bar{y} = \frac{\sigma^2_{y^b}}{\sigma^2_{y^b} + \sigma^2_{y^o}} (y^o - \bar{y}^b) \qquad \delta\bar{p}'' = \rho(p'', y^b) \frac{\sigma_{p''}}{\sigma_{y^b}} \delta\bar{y}$$

The estimated parameters values are optimal in a statistical sense, if all DA assumptions hold true. Ideally, these optimal values should also be stable and physically interpretable.

**Does ensemble-based parameter estimation converge to stable and physically interpretable optimal values?**

## 2. Model

We consider a simple model of the PBL in free convection (not subject to any mean wind forcing) over a horizontally homogeneous surface.

$$\frac{\partial\bar{\theta}}{\partial t} = \frac{\partial}{\partial z} K_h \left( \frac{\partial\bar{\theta}}{\partial z} - \gamma_c \right) \qquad K_h = \kappa z w_s \left(1 - \frac{z}{h}\right)^p$$

Here, $h$ is the PBL depth, and the eddy heat transfer coefficient $K_h$ is parameterized with the $K$-profile method. The vertical profile of $K_h$ is defined by a polynomial function dependent on the parameter $p$.

Our idealized experiments comprise:

- A 6-hour nature run initialized at sunrise, providing the assimilated observations. Here, we set $p$ = 1.5.
- A 200-member forecast ensemble providing the background states. Here, initial $p$ values are randomly drawn from $0.5 < p < 4.5$.
- Assimilation steps every 5 minutes with an EAKF. The assimilations iteratively adjust the population of $p$ values.

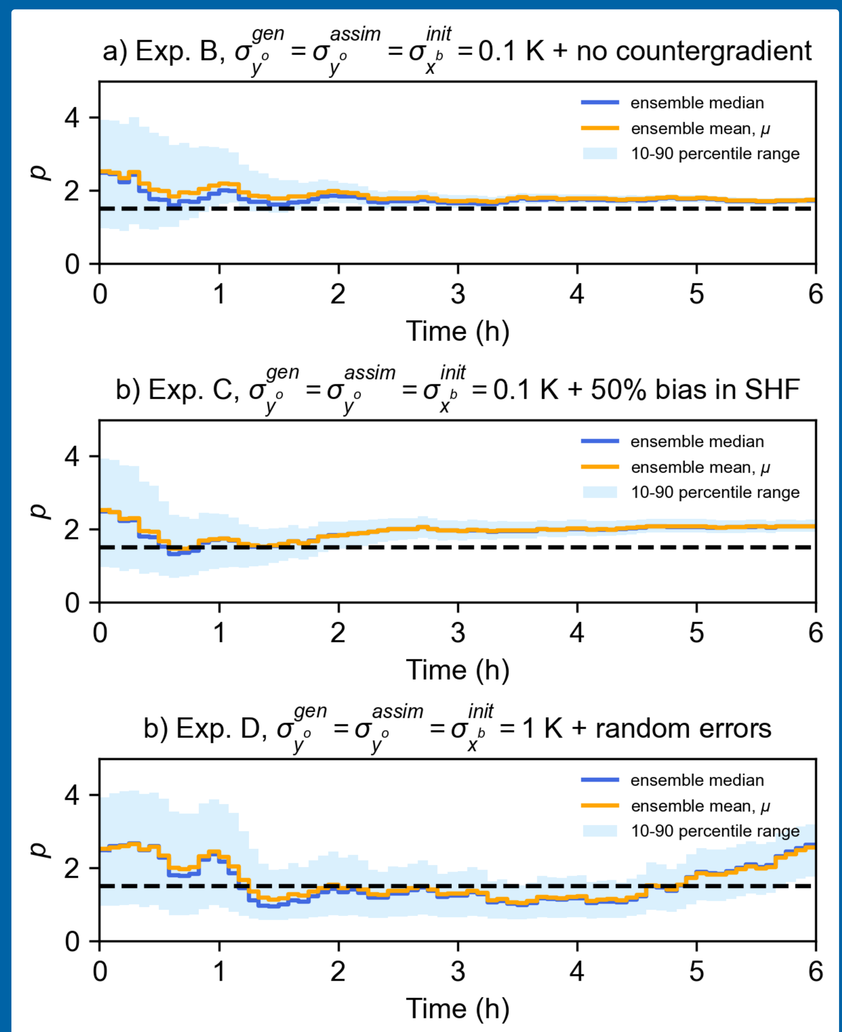
Ideally, in few iterations the filter should identify the true value $p$=1.5.

**Ensemble-based parameter estimation converges to stable and correct optimal parameter values if: (i) the estimated parameters are the only error source in the ensemble; (ii) the assimilation system is properly tuned. See the identical-twin Experiment A in Box 4.**

Experiments **B**, **C** and **D** (here) are not based on identical twins.

**B** and **C**: If parametric uncertainty is combined with a source of systematic error in a properly-tuned system, ensemble-based parameter estimation compensates the bias by converging to "wrong" values. This is a useful property: objectively adjusted parameters compensate structural model errors.

**D**: If parametric uncertainty is combined with random error sources in an under-dispersive ensemble (as frequently found in operations), random errors project on parameter updates and the estimation does not converge.

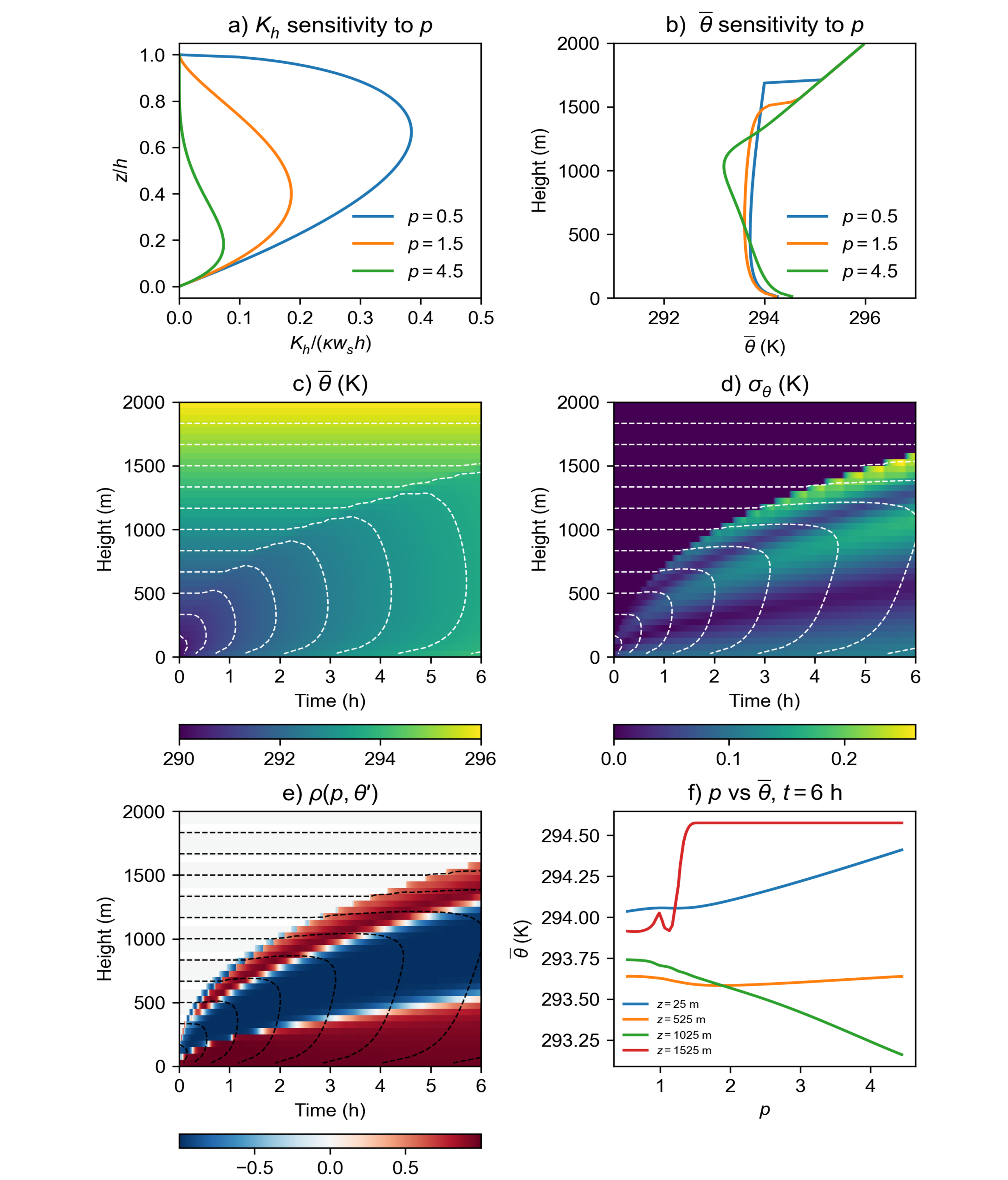



## Outlook

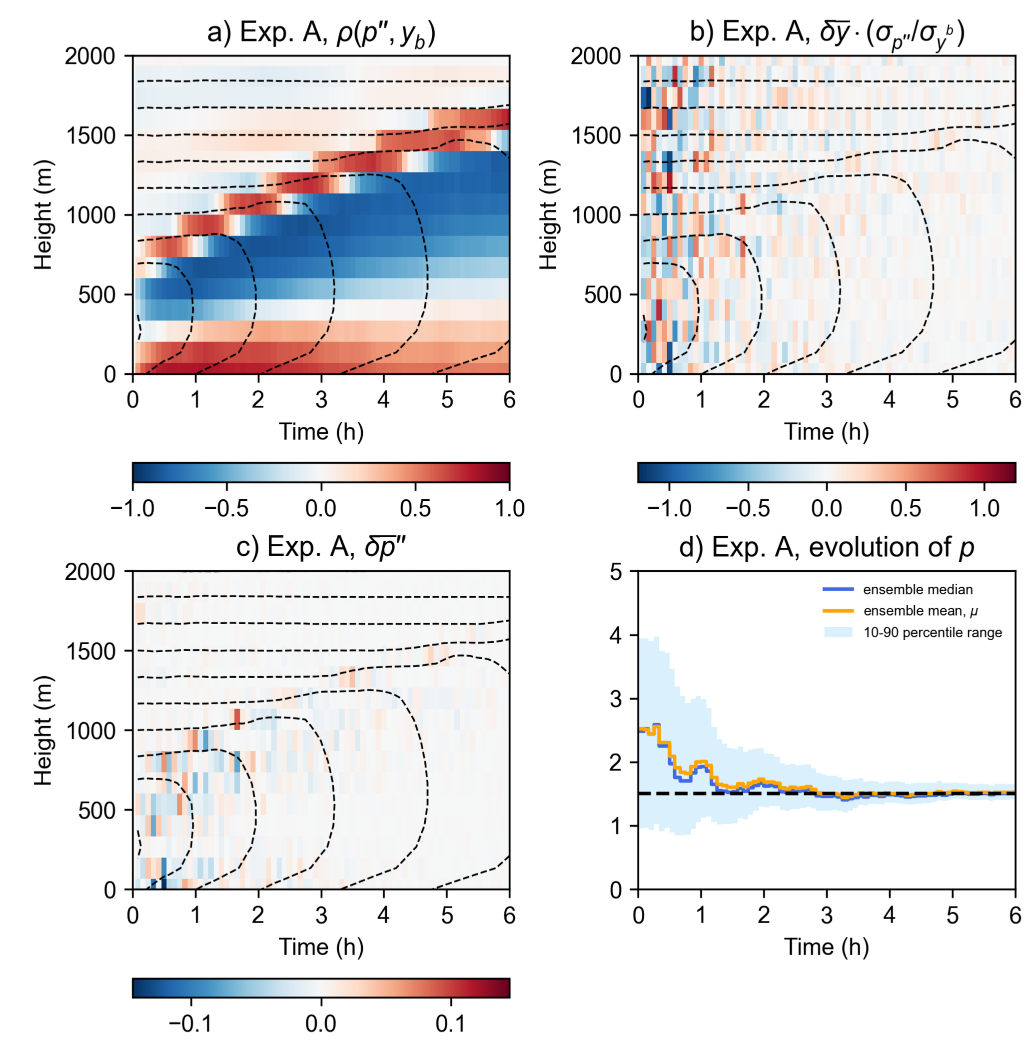
We are working on ***adaptive*** turbulence parameterizations. We replace traditionally fixed global parameters (such as $p$ in this test case) with parameters that adapt to the atmospheric state. Adaptive parameter values are drawn from lookup tables, populated offline with the results of ***idealized*** ensemble-based parameter estimation experiments similar to B and C above.

Online parameter estimation with ***real*** observations is possible, but often the estimated parameters will be optimal for the wrong reason: random error compensation, similar to experiment D.

## 3. Parameter-state correlations in a free ensemble




## 4. A successful parameter estimaton experiment




Parameter convergence depends on the statistical properties of the background and observation errors. In Experiment A (above) the observations have small error ($\sigma_{yo}^{gen}$ = 0.1 K) and are assimilated with the correct weight ($\sigma_{yo}^{assim}$ = 0.1 K). The initial ensemble spread is also small ($\sigma_{xb}^{init}$ = 0.1 K). The nature run and forecast ensemble models are identical twins except for $p$. The algorithm works nicely.