Sensitivity of grounding-line migration to ice-shelf pinning

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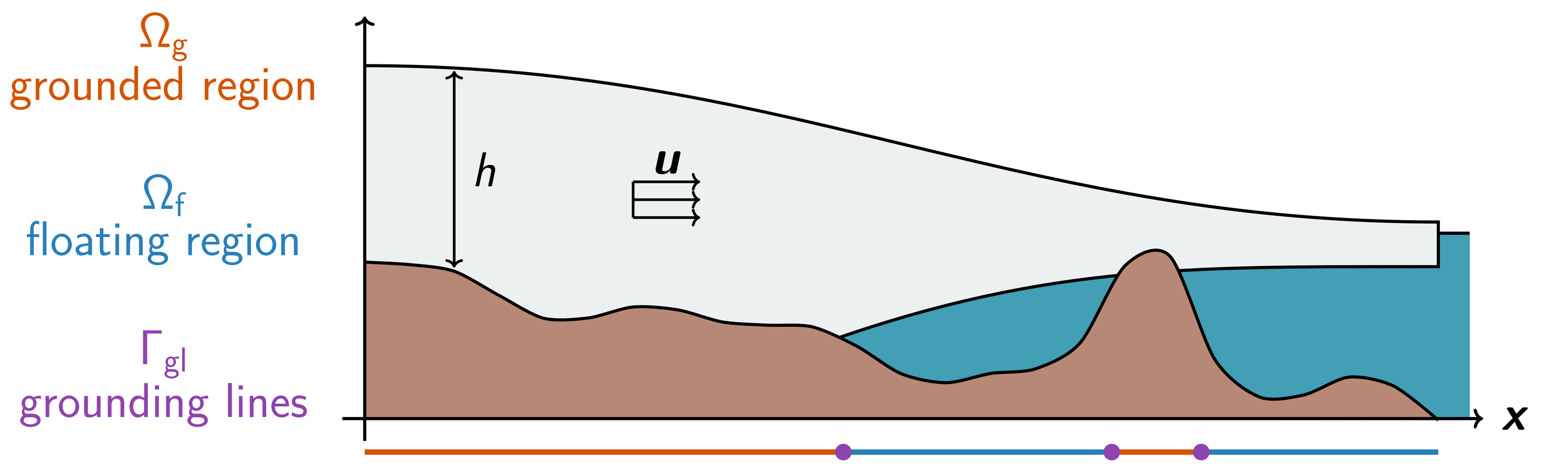
Motivation

Pinning points –**bedrock features** that can **locally ground the floating ice**– can **stabilize ice shelves** by providing resistance to flow. However, recent observations indicate a progressive unpinning of Antarctic ice shelves, which may accelerate ice loss (Miles and Bingham, 2024). While **numerical** studies have explored their impact (e.g., Favier and Pattyn, 2015; Rydt and Gudmundsson, 2016; Henry et al., 2022), **theoretical** frameworks often neglect pinning dynamics (e.g., Schoof, 2007; Pegler, 2018). Here, we examine their effect on ice-sheet dynamics to **bridge this gap** and in particular assess the **well-posedness of models**.

Ice-flow model

Shallow-shelf approximation for isothermal marine ice sheets (Morland, 1987; MacAyeal, 1989): find $(h, \boldsymbol{u})$ that obey the following equations:



Mass balance:

mass accumulation rate

$$\partial_t h + \nabla\cdot(h\,\boldsymbol{u}) = a \quad \text{in } \Omega \qquad (1)$$

Momentum balance:

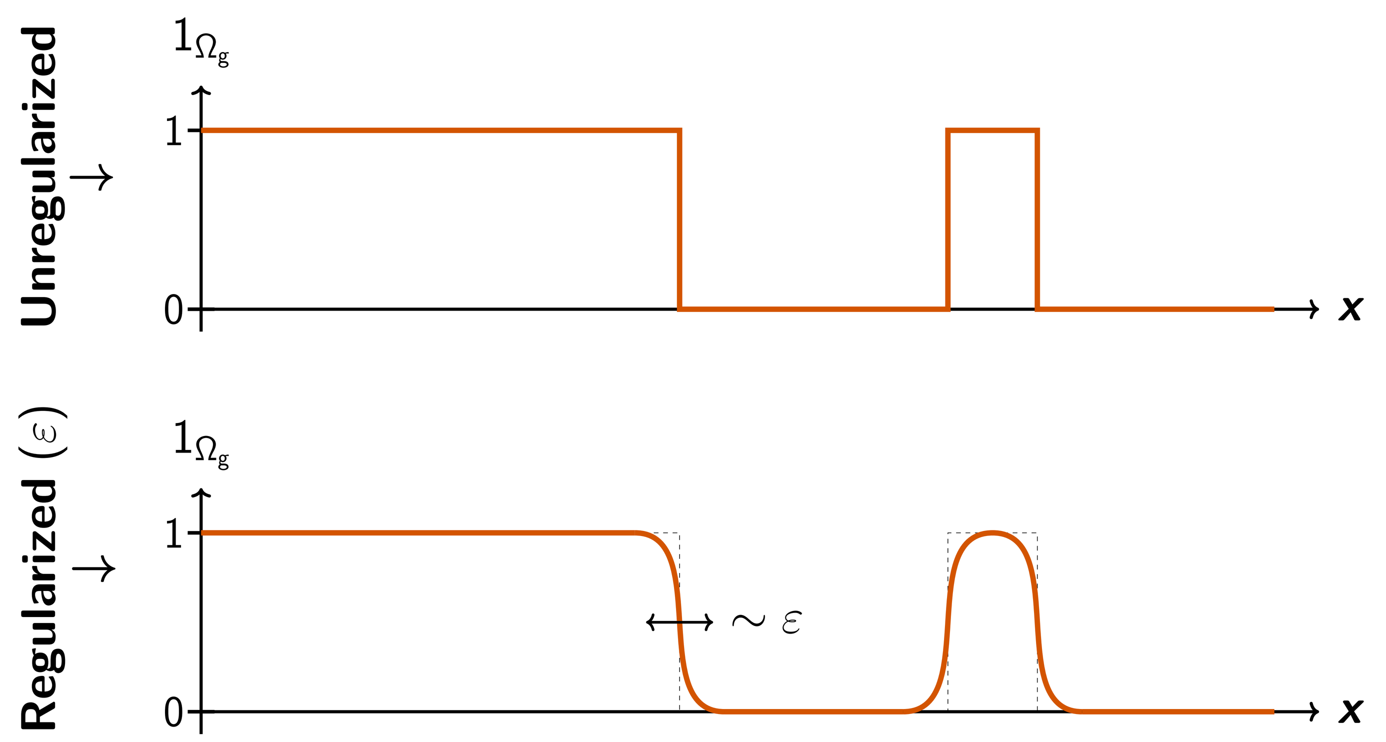
viscous stress — friction stress (regularized Coulomb) — driving stress

$$\nabla\cdot[2h\,\eta(\boldsymbol{u})\,\Sigma(\boldsymbol{u})] + \tau_b = \tau_g \quad \text{in } \Omega_g \qquad (2a)$$

$$\nabla\cdot[2h\,\eta(\boldsymbol{u})\,\Sigma(\boldsymbol{u})] = \tau_f \quad \text{in } \Omega_f \qquad (2b)$$

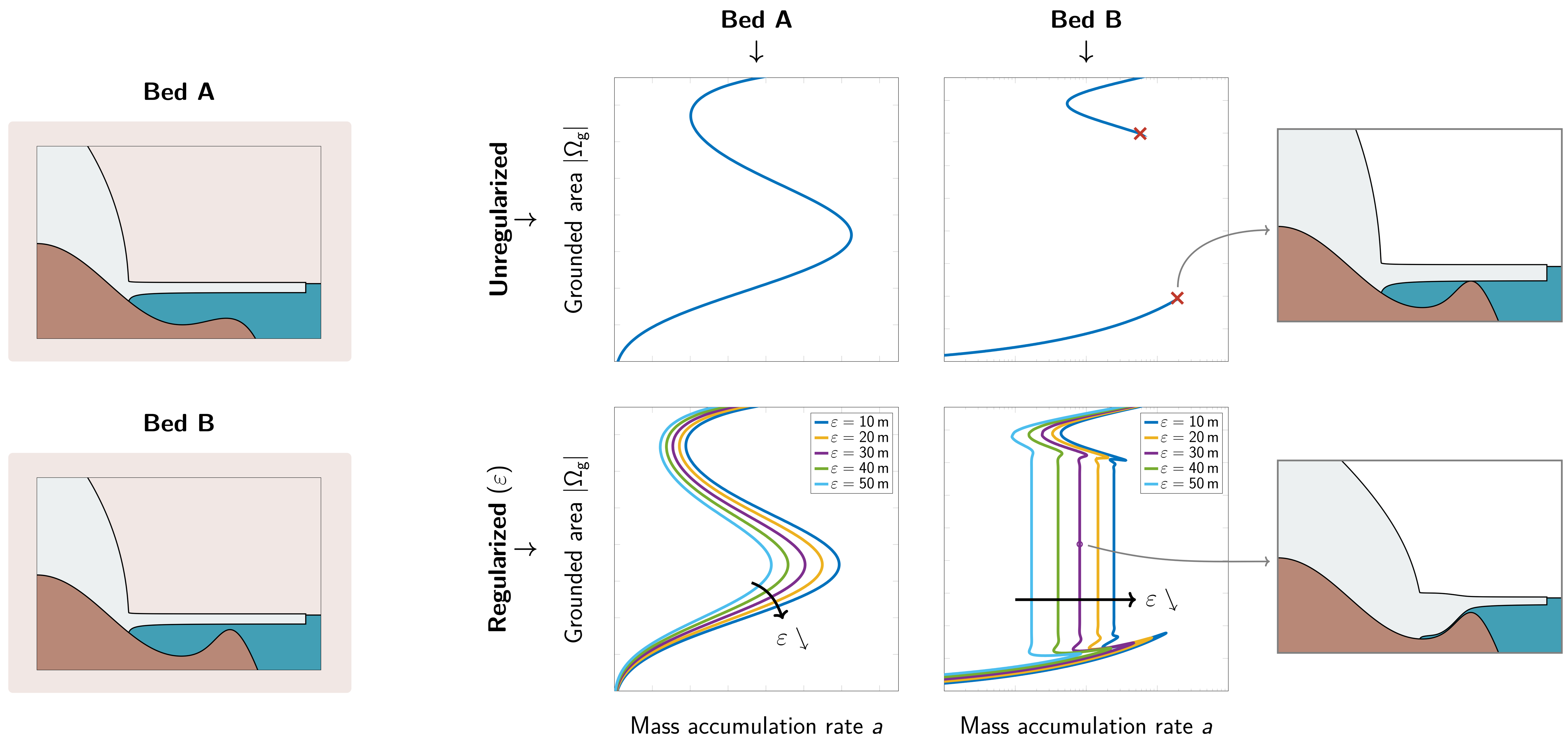
unify

$$\nabla\cdot[2h\,\eta(\boldsymbol{u})\,\Sigma(\boldsymbol{u})] + 1_{\Omega_g}\tau_b = 1_{\Omega_g}\tau_g + 1_{\Omega_f}\tau_f \quad \text{in } \Omega \qquad (2^*)$$



The equations (1)–(2) are written in a weak form and then discretized with the **finite-element method**. An appropriate quadrature rule is performed for the elements that contain both grounded and floating areas (Seroussi et al., 2014).

Numerical experiments: bifurcation plots



To better understand the impact of pinning points, we perform a series of numerical experiments on idealized set-ups. We consider two synthetic 2D bed geometries (**bed A**/**bed B**) and compute the associated bifurcation plots, which represent the set of **steady states** as a function of a. These plots are obtained for the two formulations (**unregularized**/**regularized**). To generate the bifurcation plots, we employ **numerical continuation** methods (e.g., Keller, 1977; Mulder et al., 2018).

Anatomy of a singularity

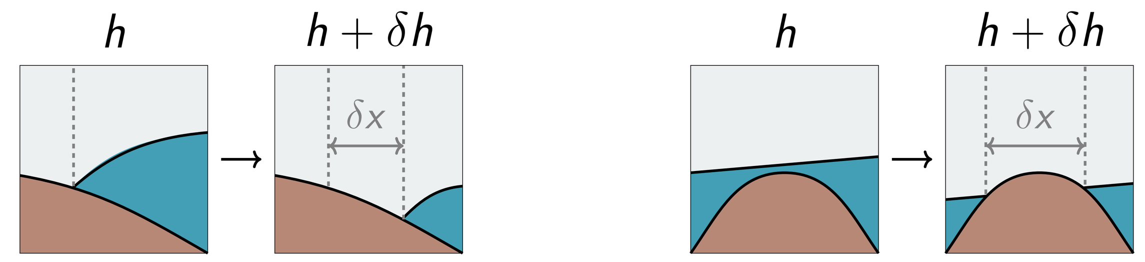
The weak form involves integrals over Ω_g and Ω_f.

Reynolds' theorem:

$$F(h) = \int_{\Omega_g(h)} f(h)\,\mathrm{d}\Omega \;\Rightarrow\; \delta F = \underbrace{\int_{\Omega_g(h)} f(h+\delta h)\,\mathrm{d}\Omega}_{\text{fixed domain}} + \underbrace{\int_{\Omega_g(h+\delta h)} f(h)\,\mathrm{d}\Omega}_{\text{moving domain}}$$

$$\delta F \sim f\,\delta x$$

$$\frac{\delta F}{\delta h} \sim f\,\frac{\delta x}{\delta h}$$



$\delta x \sim \delta h$ — linearization ✓

$\delta x \sim (\delta h)^{1/2}$ — linearization ✗

Ingredient for singularity: $f \neq 0 \Rightarrow \tau_b \neq 0$ or $\tau_g \neq \tau_f$ at Γ_{gl}.

Here, the inequality $\tau_g \neq \tau_f$ is associated with a discontinuous upper-surface slope. Essentially, this stems from the assumption of a vertical cryostatic equilibrium in the momentum balance (see Schoof, 2011).

Take-home messages

Singularity:
- associated with pinning points
- due to geometry/momentum-balance coupling
- persists for smooth beds and vanishing friction

Practically:
- increased numerical sensitivity (Δx and Δt)
- regularization → nonphysical solutions
- regularization → strong dependency on ε

Perspectives:
- non-smooth method (e.g., contact approach)
- novel model (e.g., 'grounding zone')
- higher-order ice flow (full Stokes)

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