Towards More Reliable Surface Geometry Inversion Methods for Mineral Exploration

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Motivation

Surface geometry inversion (SGI) methods attempt to estimate positions of interfaces between rock units. While successfully applied in numerous studies, none rigorously investigate whether such highly non-linear inverse problems are well-posed. This study explores these ideas, focusing on potential field data.

Voxel inversion versus SGI

Voxel inversion: The subsurface is discretized on a mesh. Piecewise physical properties are estimated by solving an underdetermined problem. Typically, a local optimization algorithm minimizes an objective function combining data misfit, Φ_d, and model regularization, Φ_m, with tradeoff parameter, β:

$$\min_{\mathbf{m}} \Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta\Phi_m(\mathbf{m}) \qquad \text{s.t.} \;\; \Phi_d \simeq \Phi_d^*.$$

This minimum-structure approach typically produces smoothed models inconsistent with geological models, e.g. Fig. 1.

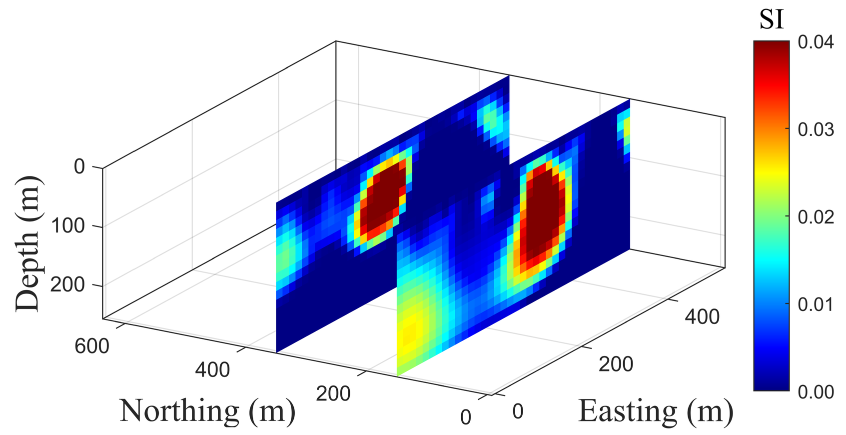

Figure 1: Voxel inversion of magnetic data to model two kimberlite pipes [8].

Surface geometry inversion: Interfaces between rock units are parameterized and their locations estimated. Resulting models are more consistent with geological models, e.g. Fig. 2b shows the SGI result for the same data inverted in Fig. 1. Our approach represents interfaces using triangulated surfaces (3D) or line elements (2D), and inversion parameters are node positions. SGI problems are often formulated as overdetermined problems where only a misfit term is minimized subject to some constraints, $\mathbf{c}$:

$$\min_{\mathbf{m}} \Phi(\mathbf{m}) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{d_i^{pred} - d_i^{obs}}{\sigma_i}\right)^2 \qquad \text{s.t.} \;\; \mathbf{c} = 0.$$

Our constraints prohibit non-geological surface intersections and we minimize using a genetic algorithm (GA).

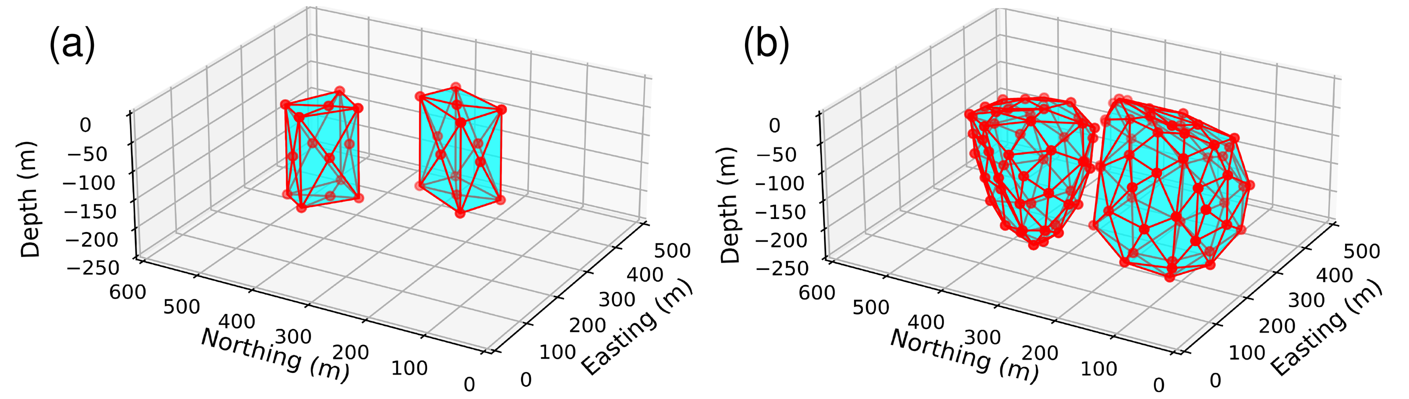

Figure 2: SGI of magnetic data to model two kimberlite pipes [8]: (a) initial model, (b) inversion result.

Sponsors

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Classification of SGI approaches

SGI approaches generally fall into five categories, illustrated in Fig. 3.

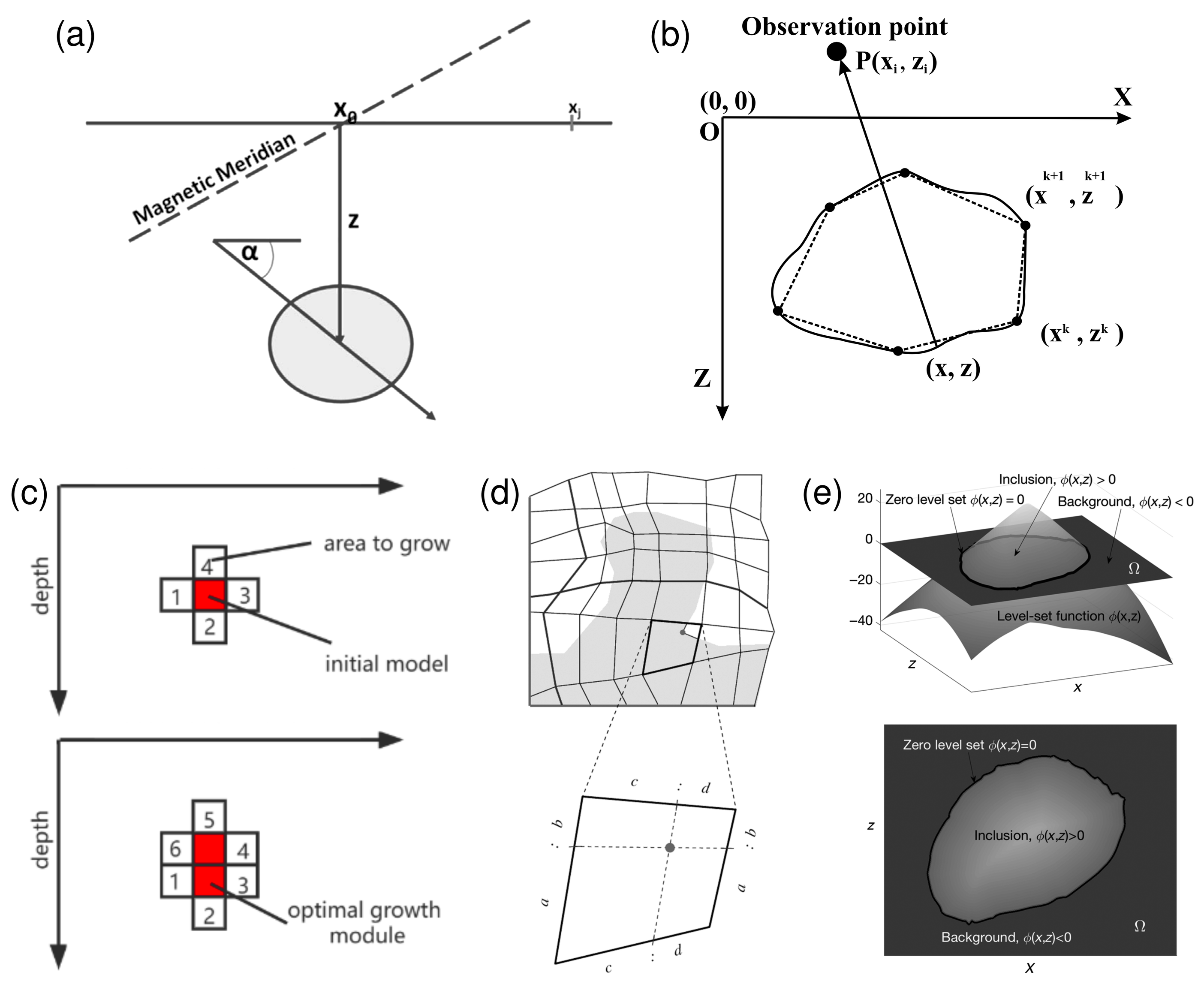

Figure 3: Examples from different SGI categories: (a) parametric shapes [2]; (b) explicit surfaces [7]; (c) growing/shrinking structures [4]; (d) mesh warping [1]; (e) implicit surfaces [9].

Prior studies using our SGI approach

Our SGI refines an initial surface-based model by adjusting node positions to better fit geophysical data. The kimberlite example of [8], Fig. 2, provides one example. Other examples are below.

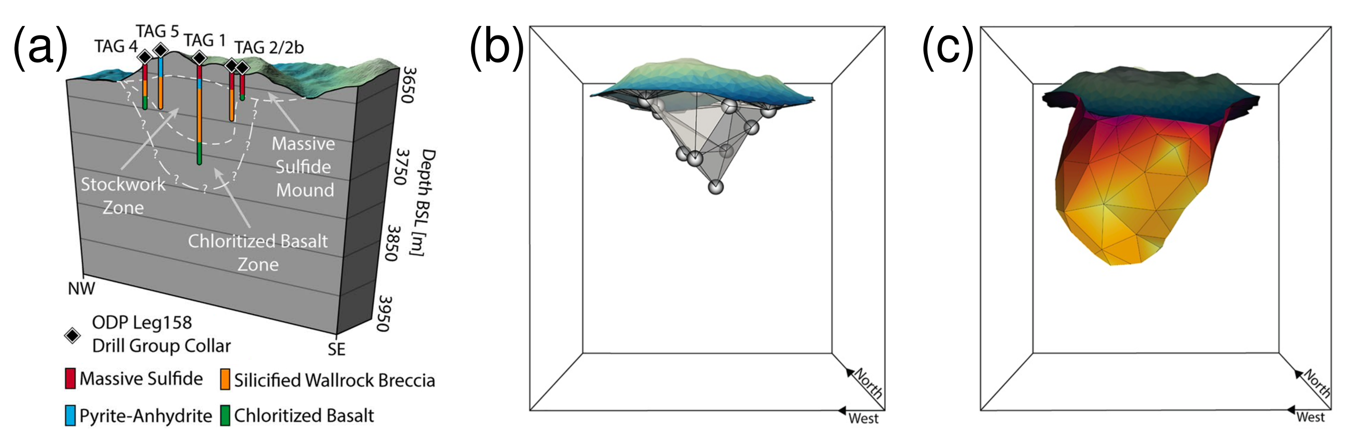

Figure 4: SGI of magnetic data to model a seafloor massive sulfide deposit [5]: (a) geological interpretation, (b) initial model, (c) inversion result.

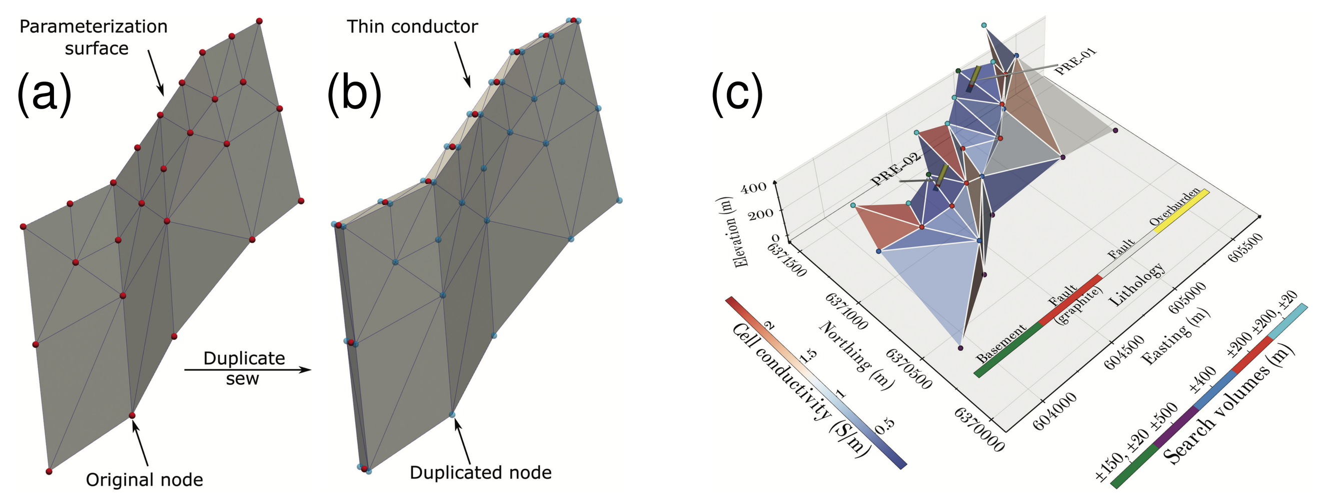

Figure 5: SGI of transient electromagnetic data to model a thin conductor [6]: (a) initial model in inversion parameterization, (b) volumetric expansion for data calculation, (c) inversion result.

Assessing the well-posedness of SGI

We need to show that the solution is both stable and unique.

- **Stability:** The algorithm's behaviour in the presence of noise.
- **Uniqueness:** Are the resulting models the same from a geological perspective despite being mathematically different?

Here we perform some preliminary tests on a simple 2D model, Fig. 6. We show the results of running our SGI approach 10 different times, each using a different random seed for the genetic algorithm.

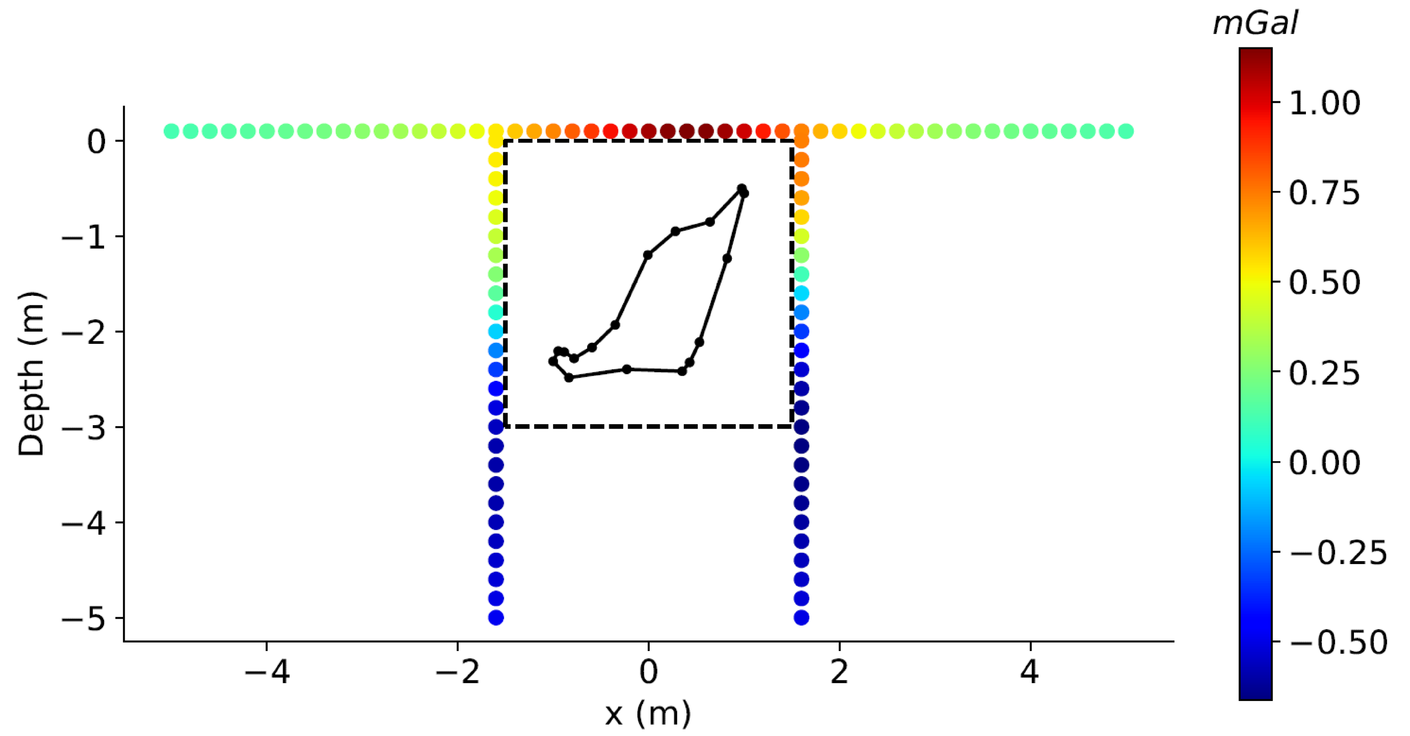

Figure 6: Our 2D test model (solid black lines), surface and borehole gravity data (coloured points) and node position search bounds for SGI (black dashed lines).

Inversions with noise-free data

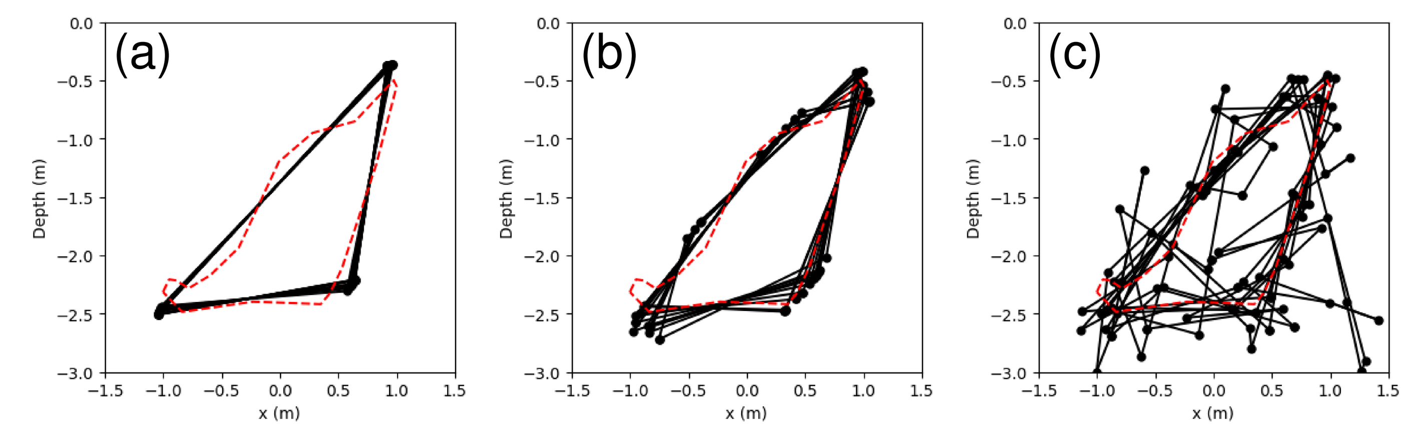

Figure 7: SGI results with noise-free data: (a) 3 nodes, (b) 4 nodes, (c) 8 nodes.

Inversions with noisy data (8%)

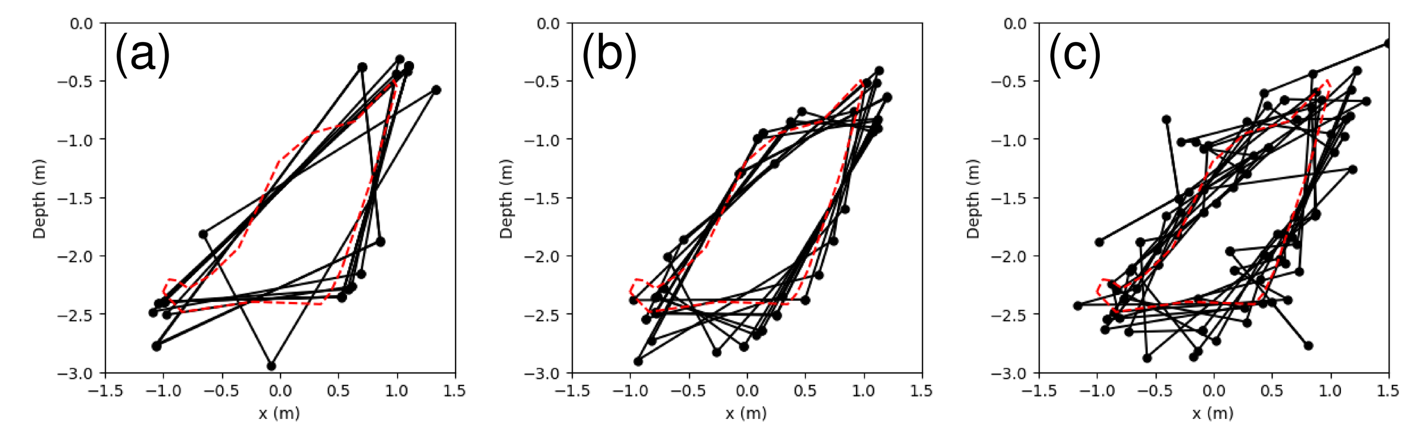

Figure 8: As in Fig. 7 but with 8% noise added.

Observations

- Adding less noise to the data results in smaller changes in the recovered model, suggesting the solutions are stable.
- With a small number of nodes and low noise, the solutions seem unique, suggesting a simple practical approach to form a uniquely determined problem: reduce the number of parameters.
- With an increasing number of nodes, the uniqueness suffers and there is the suggestion of multiple local minima.

Surface subdivision

The non-uniqueness of the problem can be mitigated by reducing the number of parameters. The inversion can work with a set of "control nodes" on a coarse surface and use surface subdivision to generate smooth models with which to calculate the data response, Fig. 9.

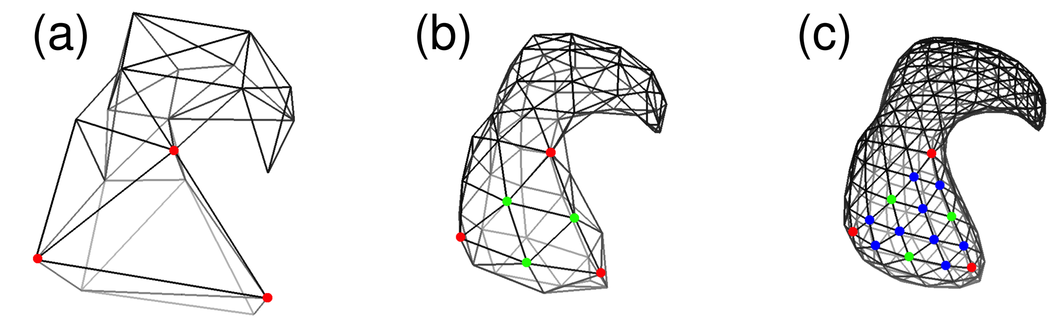

Figure 9: (a) A coarse control surface, (b) subdivided once, (c) subdivided twice.

Inversions with noise-free data

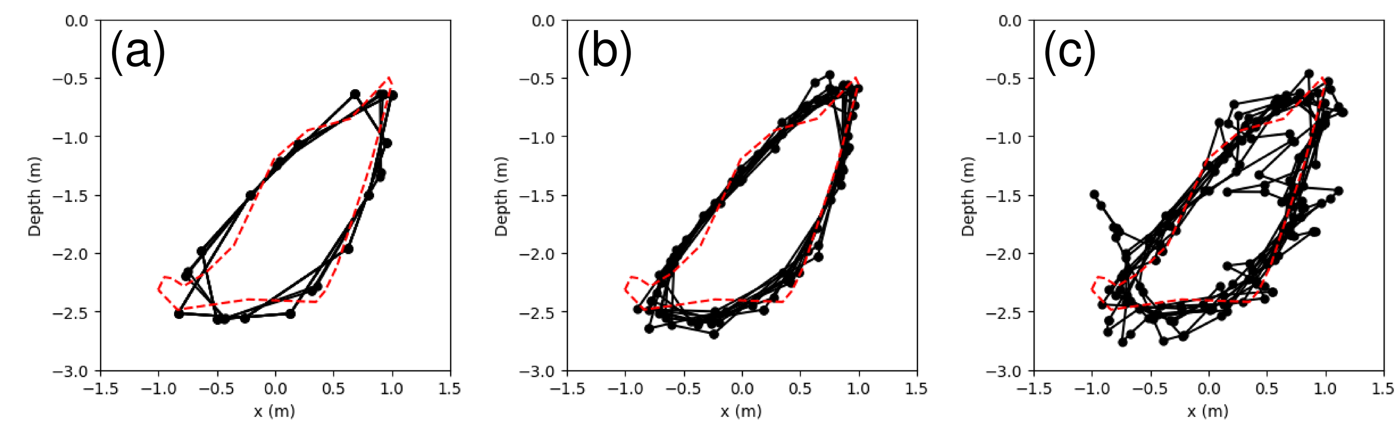

Figure 10: As in Fig. 7 (noise-free) but with one level of subdivision.

Inversions with noisy data (8%)

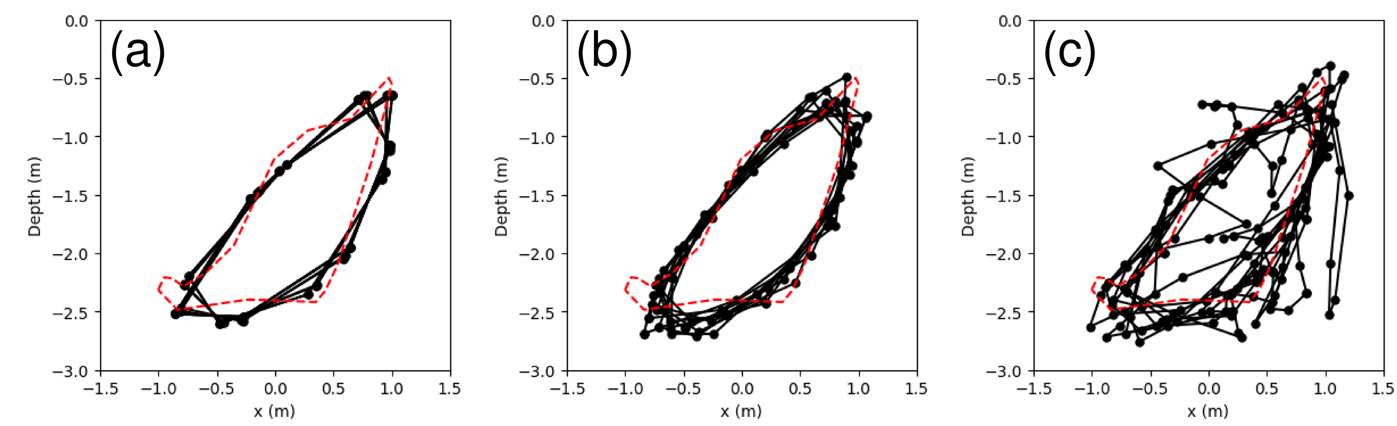

Figure 11: As in Fig. 8 (8% noise) but with one level of subdivision.

Adding regularization

Instead of surface subdivision, we can consider an equidistance regularization term [3] that measures the variance of the distances between all pairs of nodes. We solve a multi-objective problem using a Pareto multi-objective GA:

Inversions with noisy data (8%)

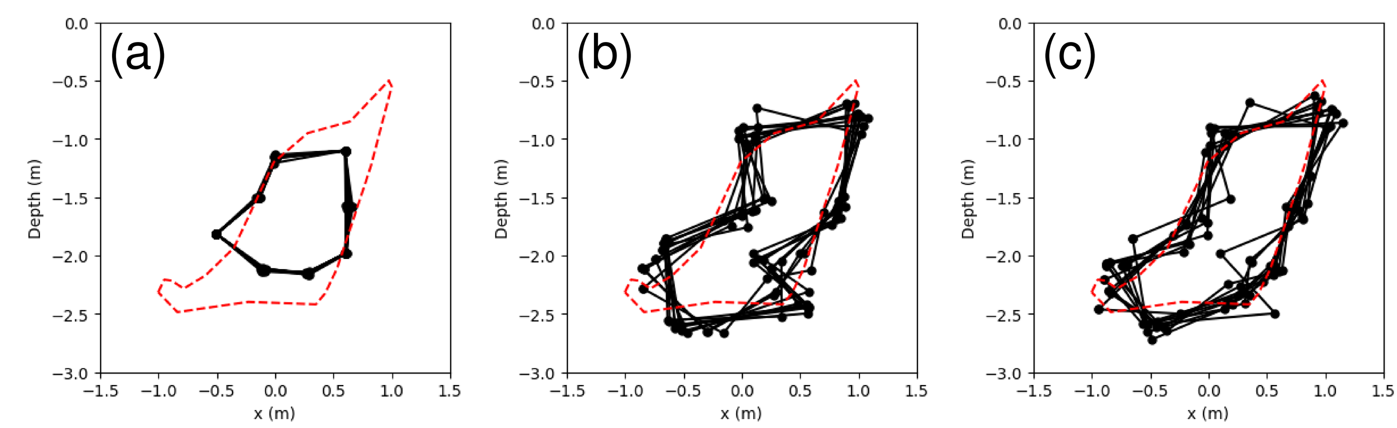

Figure 12: As in Fig. 8c (8% noise, 8 nodes) but with equidistance regularization applied: solutions chosen from the Pareto fronts with (a) lower regularization / larger misfit of 40.0, (b) misfit of 1.0 expected for this synthetic scenario, (c) lowest misfit / largest regularization. The lowest misfit achieved across all runs was 0.95.

Observation: More stable and unique solutions are obtained using either subdivision or regularization.

References: **[1]** Alvers et al., 2023, Inversion of potential fields by interactive optimization of 3D subsurface models using a spring-based space warping and evolution strategy, Geophysics, 88. **[2]** Ben et al., 2022, Interpretation of magnetic anomalies by simple geometrical structures using the manta-ray foraging optimization, Front. Earth Sci., 10. **[3]** Bijani et al., 2015, Three-dimensional gravity inversion using graph theory to delineate the skeleton of homogeneous sources, Geophysics, 80. **[4]** Cai et al., 2022, Three-dimensional inversion method for small-scale magnetic objects based on normalized magnetic source strength, Micromachines, 13. **[5]** Galley et al., 2021, Magnetic and gravity surface geometry inverse modeling of the TAG active mound, JGR: Solid Earth, 126. **[6]** Lu et al., 2024, Surface geometry inversion of transient electromagnetic data, Geophysics, 89. **[7]** Roy et al., 2021, Gravity inversion of basement relief using particle swarm optimization by automated parameter selection of Fourier coefficients, Computers & Geosciences, 156. **[8]** Vatankhah et al., 2024, Magnetic surface geometry inversion of kimberlites in Botswana, Geophysical Prospecting, 72. **[9]** Zheglova et al., 2018, Multiple level-set joint inversion of traveltime and gravity data with application to ore delineation: a synthetic study, Geophysics, 83.