Toy model to study convective organization mediated by atmospheric gravity waves

Ashly Wilson[1], Jan Härter[1,2]

[1] Department of Physics and Astronomy, University of Potsdam
[2] Complexity and Climate, Leibniz ZMT, Bremen

Introduction

Organized convection leads to phenomenon like thunderstorms and tropical cyclones and thus have a huge impact on lives and livelihoods. In this project we attempt to study the role of atmospheric oscillations like gravity waves in organizing convection. Dry geophysical flows are very well understood and theoretically backed however, their interaction with convection is usually nonlinear and an active area of research. In our project we model the latent heat release from moist convergence using the Dirac delta function as a momentum kick.

The model

We propose a model for convective aggregation by formulating a feedback mechanism with convection and interacting atmospheric oscillation. A convective kick (momentum or energy) produces atmospheric oscillations. These oscillations interact with each other such that when they constructively add up and reach a maxima an additional convective boost is supplied. The convective boost results in new oscillations which interact with the existing waves and repeat the cycle.

Governing equations

- Starting from the equations of motion for a hydrostatic Boussinesq inviscid non-rotating fluid and separating the linear and nonlinear terms. One gets to the following set of equations (Raymond 1983)

$$\frac{\partial v}{\partial t} + \frac{\partial \pi}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \pi}{\partial t} - \eta = 0 \quad (2)$$

$$\frac{\partial \eta}{\partial t} + w_a = 0 \quad (3)$$

$$\frac{\partial v}{\partial x} + \frac{\partial w_a}{\partial z} = \delta(x)\delta(z)\delta(t) \quad (4)$$

The variables involved are, horizontal velocity(v) vertical velocity of adiabat (w_a), perturbation to pressure divided by mean density (π) and bouyancy η which is the fractional potential temperature perturbation times acceleration due to gravity.
Now substituting appropriate stream functions into equations (1)-(4), we get (Raymond 1983):

$$\frac{\partial^4 \phi}{\partial t^2 \partial z^2} + \frac{\partial^2 \phi}{\partial x^2} = \delta(x_0)\delta(z_0)\delta(t_0) \quad (5)$$

Where ϕ is a stream function of the form:

$$\eta = \frac{\partial^2 \phi}{\partial z \partial t}; v = \frac{\partial \phi}{\partial x}$$

$$\pi = \frac{\partial \phi}{\partial t}; w_a = \frac{\partial^3 \phi}{\partial t^2}$$

Now assuming radial symmetry and taking an ansatz that is variable separable (i.e., assuming independent dynamics in z) we get the 2D form:

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial x^2} = \delta(x_0)\delta(y_0)\delta(t_0) \quad (6)$$

Earth's rotation imposes anisotropy in the meridional direction. Thus, on averaging out dynamics in the vertical direction, the equatorial region can conceptually be argued to have a dimensionality between 1 and 2 .Therefore, we examine a one dimensional and then a 2 dimensional system in space to understand their qualitative features

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One Dimensional model

By assuming averaged out independent dynamics in the z direction, equation (6) becomes the 1D forced wave equation given as:

$$\frac{\partial^2 g}{\partial t^2} - c^2 \frac{\partial^2 g}{\partial x^2} = \delta(x-\xi)\delta(t-\tau) \quad (7)$$

The solution to this DE in a limited periodic domain is given by Green's function of the form:

$$g = \left[\sum \phi_n(\xi)\phi_n(x) \frac{\sin(k_n c(t-\tau))}{k_n c} \right] H(t-\tau) \quad (8)$$

where

$$\phi_n = \sum_{n=1}^{\infty} \sqrt{L} e^{i k_n x}$$

The parameters involved in the 1D model of interacting waves are therefore c (speed of the gravity waves) and t_{start} (which is the time it takes for a new event to start after a convective threshold is crossed).

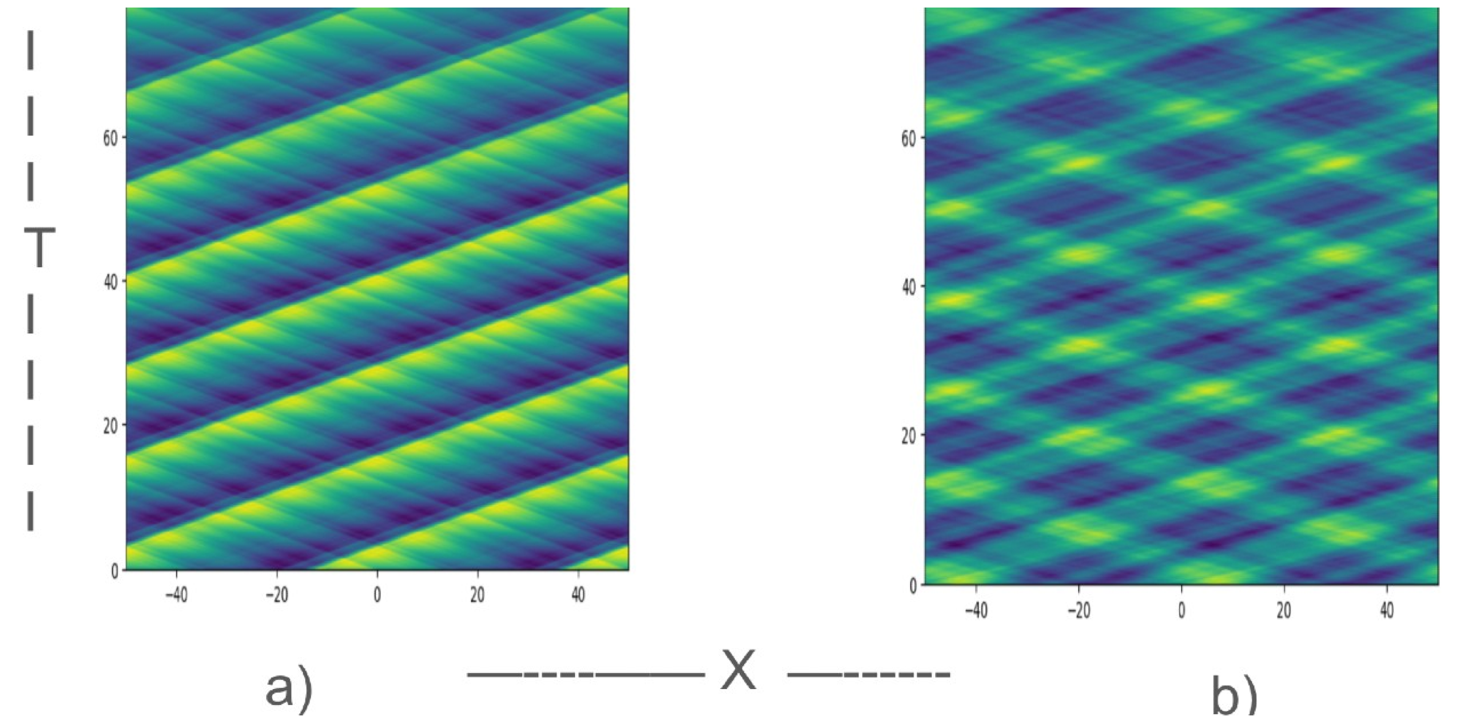

Fig 1: Evolution of interacting 1D kicked gravity waves: a) without dispersion, b) with dispersion.

To accommodate the qualitative features in the wave characteristics brought about by the dynamics in the Z direction , we looked at kicked dispersed waves. A differential equation of the form eq. (5), such that a 4th order differential term accounts for dispersion, was looked at. The increased stability of the atmosphere after each convective kick was accommodated by having a dynamic threshold value whose value changed depending the average energy of the system:

$$\frac{d^2 \phi}{dx^2} - \frac{d^4 \phi}{dx^4} = \frac{d^2 \phi}{dt^2} \quad (9)$$

The solution to this DE in a limited periodic domain is given by:

$$g = \left[\sum \phi_n(\xi)\phi_n(x) \frac{\sin\left(\sqrt{(k_n^4 + k_n^2)c^2}(t-\tau)\right)}{\sqrt{(k_n^4 + k_n^2)c^2}} \right] H(t-\tau) \quad (10)$$

Discussion

It was observed that without dispersion, the kicked gravity waves quickly organize into a slow-moving traveling wave. With dispersion, the system organizes into standing waves of a characteristic wavelength. On running the simulations over different length domains it was observed that the system exhibits a form of frustration for some domain sizes and show neat standing wave like pattern for certain other domain sizes. Analysis of this emergent wavelength of the system would be explored further. The order of dependence of the variables involved would be analyzed using the Buckingham Pi theory.

Two dimensional Model

Following equation (9), we see that in the case of radial symmetry, the governing equation reduces to the 2-dimensional forced wave equation. Accommodating the effect of damping with a damping coefficient b, we have:

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - b\frac{\partial \phi}{\partial t} = \delta(x_0)\delta(y_0)\delta(t_0) \quad (11)$$

The solution is of the form:

$$g(x,y,t|\xi\eta,\tau) = e^{-\epsilon c(t-\tau)} H(t-\tau-r/c) \frac{\cosh[\epsilon\sqrt{(t-\tau)^2-(r/c)^2}]}{\sqrt{|(t-\tau)^2-(r/c)^2|}} \quad (12)$$

where

$$\epsilon = \frac{bc}{2}$$

The main parameters involved in the 2D model are c (speed of the waves), b (damping coefficient), and t_{start}.The solution for an infinite domain is numerically implemented for a periodic domain by computing the functions on a large domain and wrapping up the results around a smaller domain.Like the 1D case a dynamical threshold function was implemented. We deploy a triggered convective feedback to the interacting 2D waves to observe the following results.

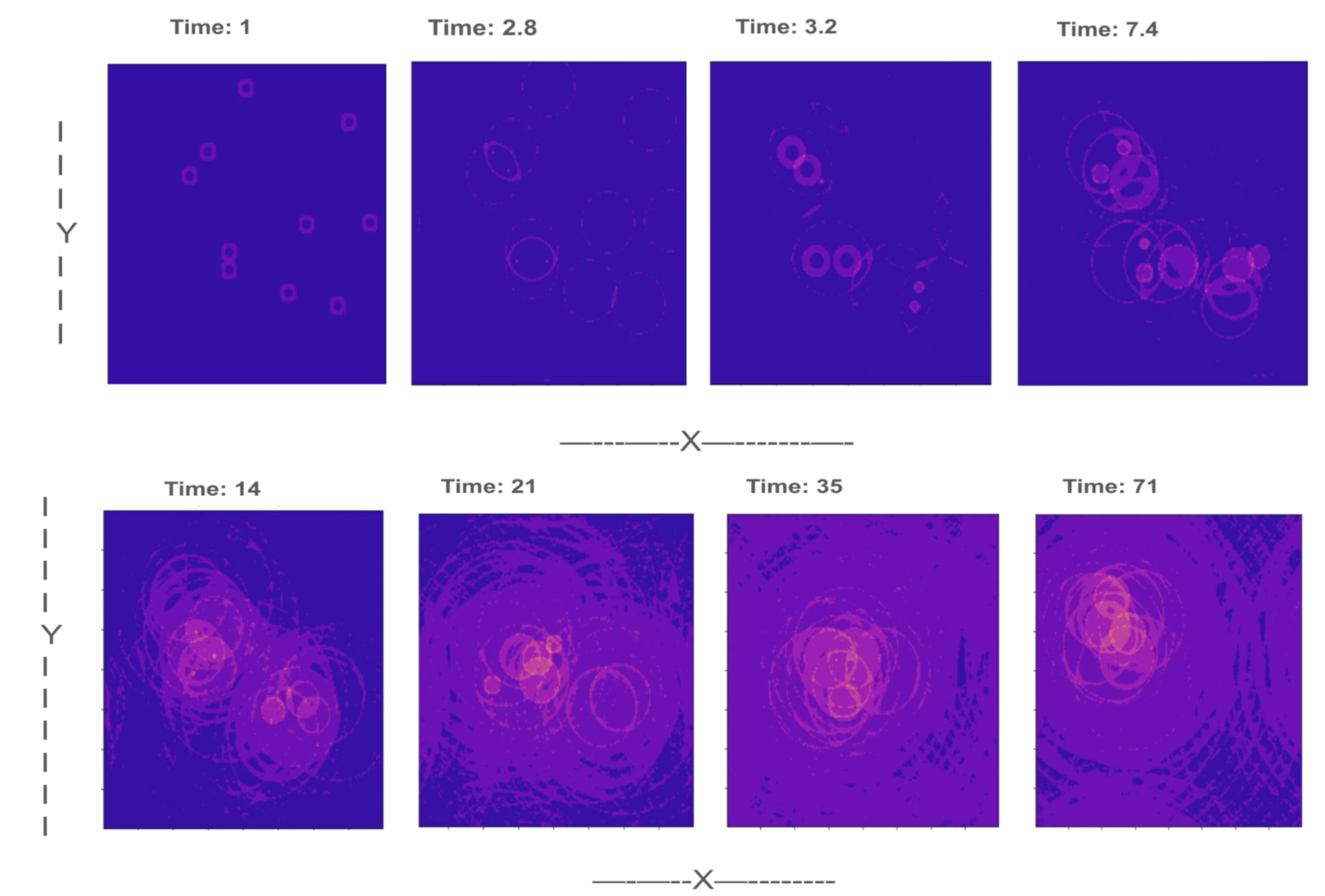

Fig 2 : Evolution of interacting 2D kicked gravity waves traveling with a speed of 3m/s. 1 time unit = 12 hours. Damping coefficient 0.0125.

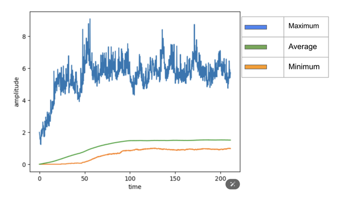

Fig 3 :Time evolution of the minimum, maximum, and average amplitude over space of the system.

Discussion

As it can be seen in fig 3, the average amplitude of the system over space settles down to a fixed value over time, while the maximum value seems to exhibit slight oscillations around a mean value. Thus, it can be claimed that starting from a random initialization, the system of dynamically forced 2D waves self-organizes into a singular epicenter of increased activity for the given set of parameters.