


Non-reflecting cylindrical wave propagation in the ocean of changing depth

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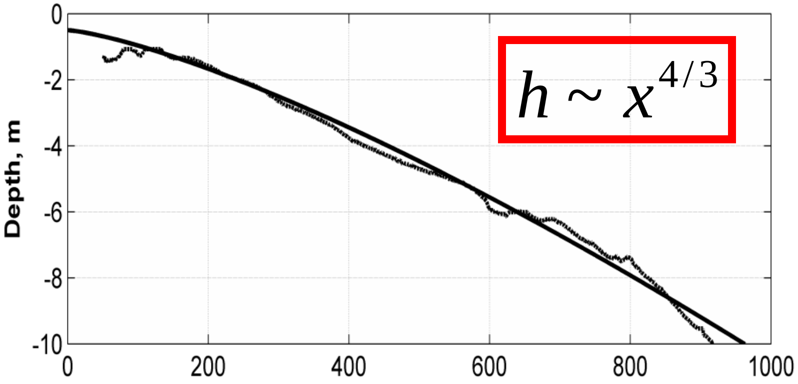
Influence of bathymetry: nonreflecting geometries



[ID et al. 2009, ID & Pelinovsky 2011a]

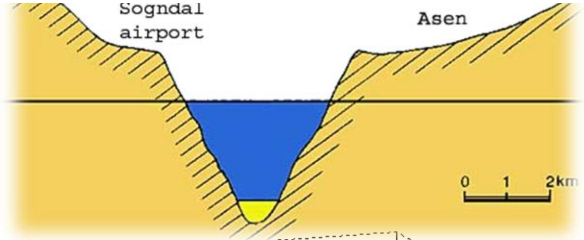
$$A(x) = A_0 \left[\frac{h_0}{h(x)} \right]^{1/4}$$

Green's law

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x}\left[c^2(x) \frac{\partial \eta}{\partial x} \right] = 0$$

$$\eta(x,t) = A(x) f\left[t - \tau(x)\right]$$

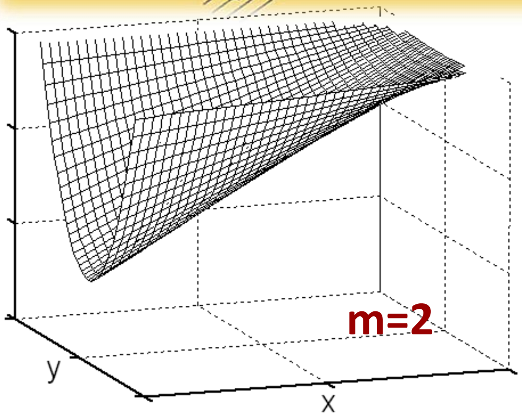
Sognefjoren fjord (Norway)

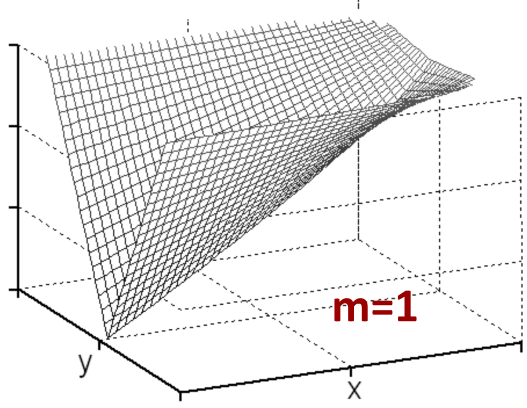


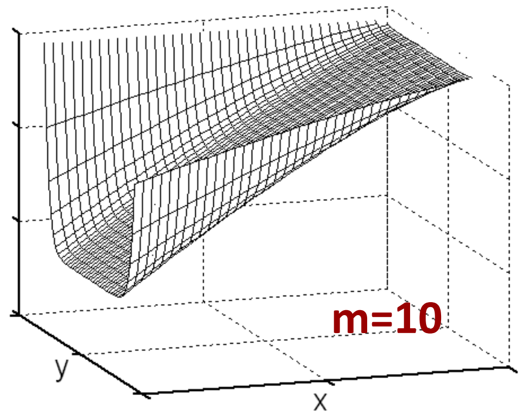
Abnormal wave amplification
In the coastal zone

$$h(x) \sim x^{\frac{4m}{3m+2}}$$

$$z \sim |y|^m$$

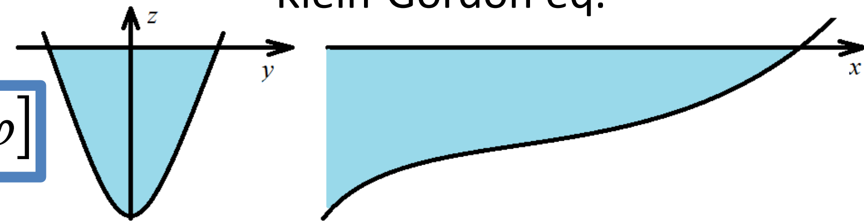






Other bottom geometries

Initial linearized shallow water eqs. are reduced to the constant-coefficient Klein-Gordon eq.

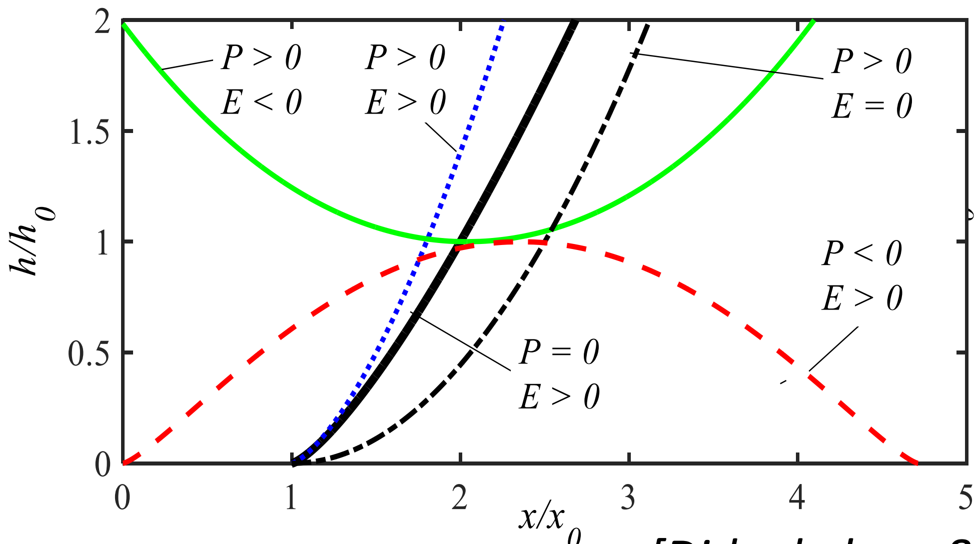


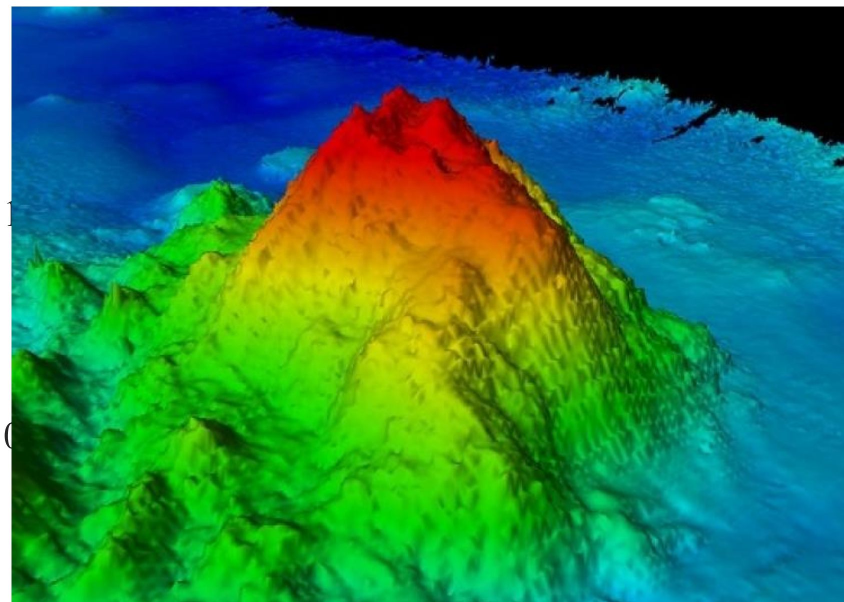
$$\frac{\partial^2\Phi}{\partial t^2}-\frac{\partial^2\Phi}{\partial\tau^2}+P\Phi=0$$

$$\Phi(t,\tau)=A_0\sin\left[\omega t-K\tau+\varphi\right]$$

$$\omega^2=K^2+P$$

Rectangular channel





x/x0

[Didenkulova & Pelinovsky 2016]

Cylindrical wave propagation

$$\frac{\partial^2 \eta}{\partial t^2} - \nabla[c^2(x,y)\nabla\eta] = 0$$

$$c = \sqrt{gh(x,y)}$$

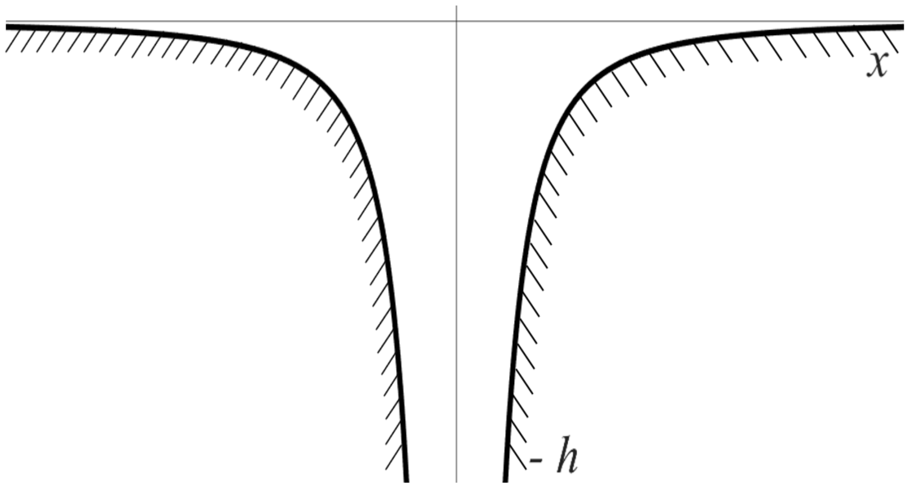
speed of tsunami propagation

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho c^2(\rho)\frac{\partial \eta}{\partial \rho}\right] = 0$$

ρ is radial coordinate

equation with constant coefficients:

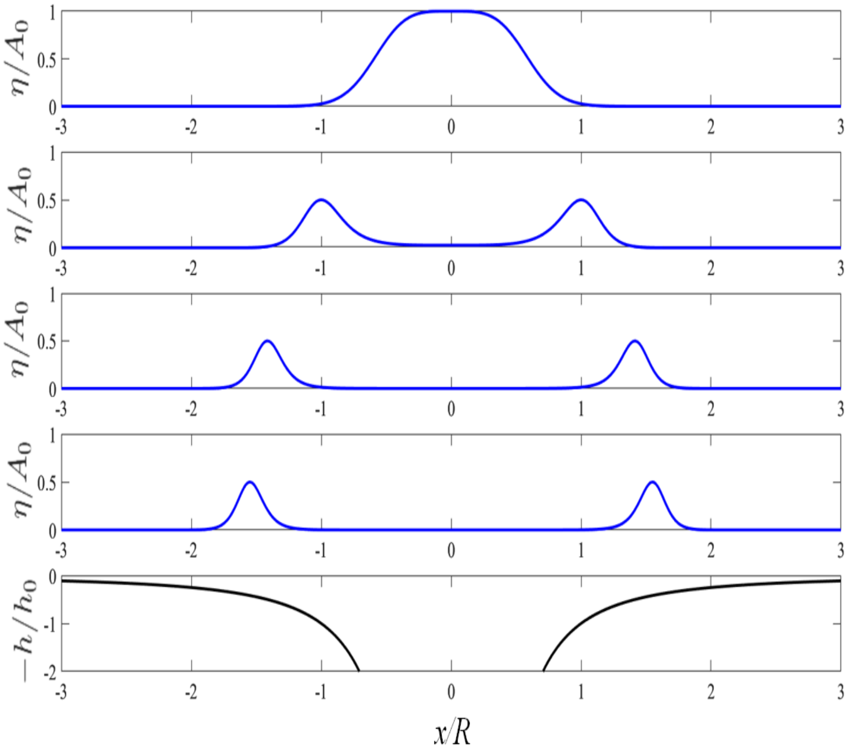
"blue hole"



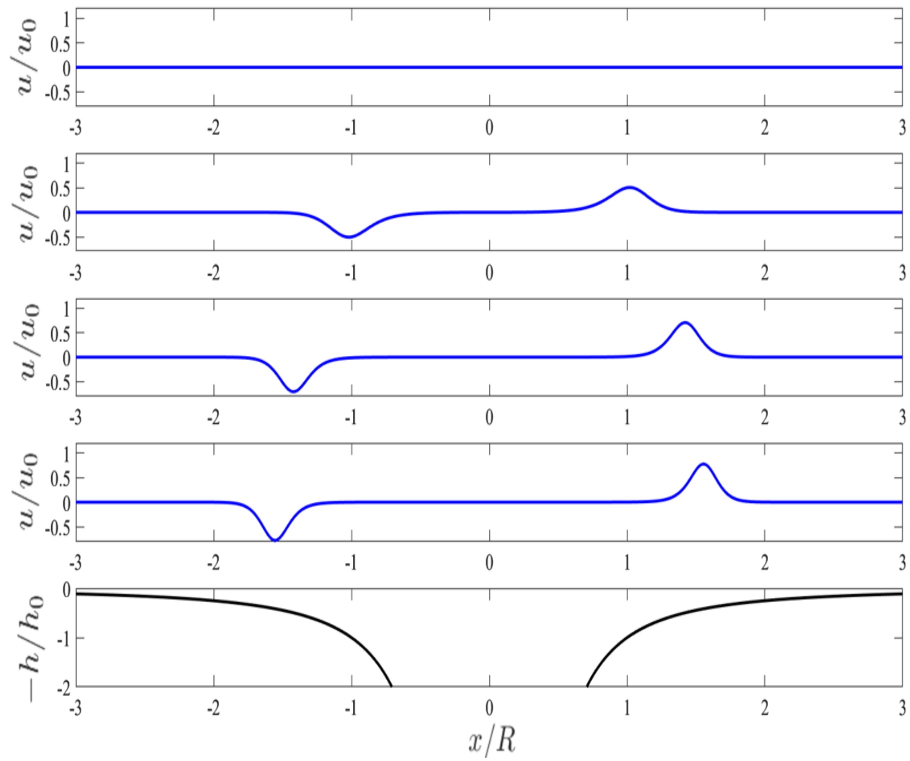
Wave propagation above blue hole

Initial conditions in the centre of the basin:

$$\eta(\rho, 0) = A_0 sech^2(\sigma\tau(\rho)), \qquad u(\rho, 0) = 0$$



Evolution of the solitary pulse along the non-reflective bottom geometry



Evolution of the flow velocity along the non-reflective bottom geometry

Reduction to the Klein-Gordon equation

$$\eta = A(\rho)G[t,\tau(\rho)]$$

$A(\rho)$, $G[t,\tau(\rho)]$, $\tau(\rho)$ are three unknown functions

$$\frac{\partial^2\eta}{\partial t^2} - \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho c^2(\rho)\frac{\partial\eta}{\partial\rho}\right] = 0$$

Initial linearized shallow water eqs. can also be reduced to the Klein-Gordon eq.

$$A\left[\frac{\partial^2 G}{\partial t^2} - c^2\left(\frac{d\tau}{d\rho}\right)^2\frac{\partial^2 G}{\partial\tau^2}\right] - \left[\frac{c^2}{\rho}\frac{d\tau}{d\rho}\frac{d(A\rho)}{d\rho} + \frac{d}{d\rho}\left(c^2 A\frac{d\tau}{d\rho}\right)\right]\frac{\partial G}{\partial\tau} - \frac{1}{\rho}\frac{d}{d\rho}\left[\rho c^2\frac{dA}{d\rho}\right]G = 0$$

$$\frac{\partial^2 G}{\partial t^2} - \frac{\partial^2 G}{\partial\tau^2} = 0$$

$$c^2\left(\frac{d\tau}{d\rho}\right)^2 = 1$$

$$\frac{c^2}{\rho}\frac{d\tau}{d\rho}\frac{d(A\rho)}{d\rho} + \frac{d}{d\rho}\left(c^2 A\frac{d\tau}{d\rho}\right) = 0$$

$$\frac{d}{d\rho}\left[\rho c^2\frac{dA}{d\rho}\right] = 0$$

$$A = \frac{-1}{\left(\frac{3D}{2}\rho^2 + 3E\right)^{1/3}}$$

$$c^2 = gh(\rho) = \frac{B}{D\rho^2}\left(\frac{3D}{2}\rho^2 + 3E\right)^{\frac{4}{3}}$$

$$\tau = \frac{1}{\sqrt{DB}}\left(\frac{3D}{2}\rho^2 + 3E\right)^{1/3}$$

Travelling wave solutions with variable amplitude

$$A = \frac{-1}{\left(\frac{3D}{2}\rho^2 + \cancel{3E}\right)^{1/3}}$$

$$c^2 = gh(\rho) = \frac{B}{D\rho^2}\left(\frac{3D}{2}\rho^2 + \cancel{3E}\right)^{\frac{4}{3}}$$

$$\tau = \frac{1}{\sqrt{DB}}\left(\frac{3D}{2}\rho^2 + \cancel{3E}\right)^{1/3}$$

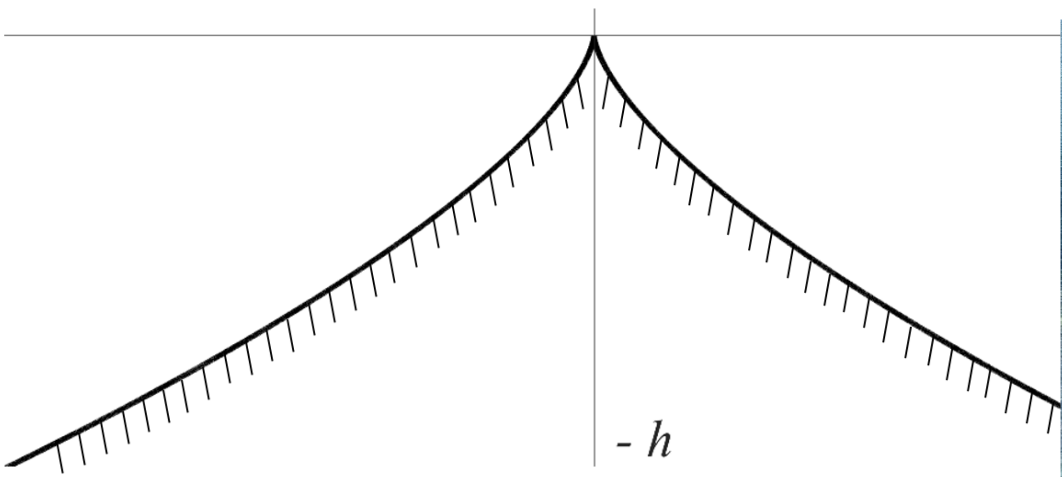
Consider $E = 0$

$$h(\rho) = \frac{c_1^2}{g}\left(\frac{\rho}{L}\right)^{\frac{2}{3}}$$

$$A = A_1\left(\frac{L}{\rho}\right)^{\frac{2}{3}}$$

$$\tau = \tau_1\left(\frac{\rho}{L}\right)^{\frac{2}{3}}$$

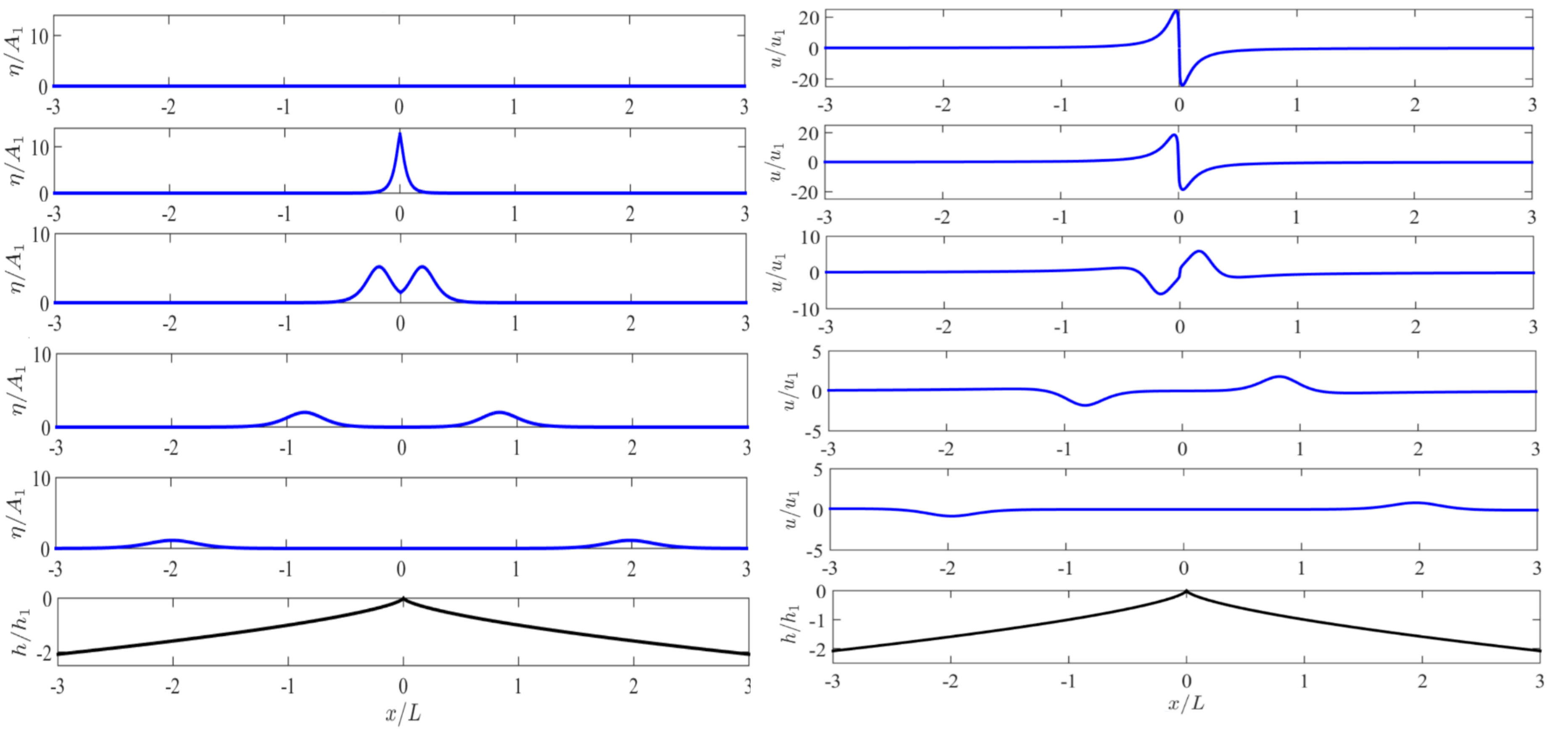
Bottom geometry $h(\rho)$ resembles an underwater volcano:



Wave propagation above underwater volcano

$$\eta = \frac{A(\rho)}{2} sech^2\left(\sigma\big(t-\tau(\rho)\big)\right) - \frac{A(\rho)}{2} sech^2\left(\sigma\big(t+\tau(\rho)\big)\right)$$

$$u = -\frac{g}{2\sigma}\frac{dA}{d\rho}\left[tanh\left(\sigma\big(t-\tau(\rho)\big)\right) - tanh\left(\sigma\big(t+\tau(\rho)\big)\right)\right] + \frac{gA}{2}\frac{d\tau}{d\rho}\left[sech^2\left(\sigma\big(t-\tau(\rho)\big)\right) + sech^2\left(\sigma\big(t+\tau(\rho)\big)\right)\right]$$



Other non-reflected bottom geometries

$$A = \frac{-1}{\left(\frac{3D}{2}\rho^2 + 3E\right)^{1/3}}$$

$$c^2 = gh(\rho) = \frac{B}{D\rho^2}\left(\frac{3D}{2}\rho^2 + 3E\right)^{\frac{4}{3}}$$

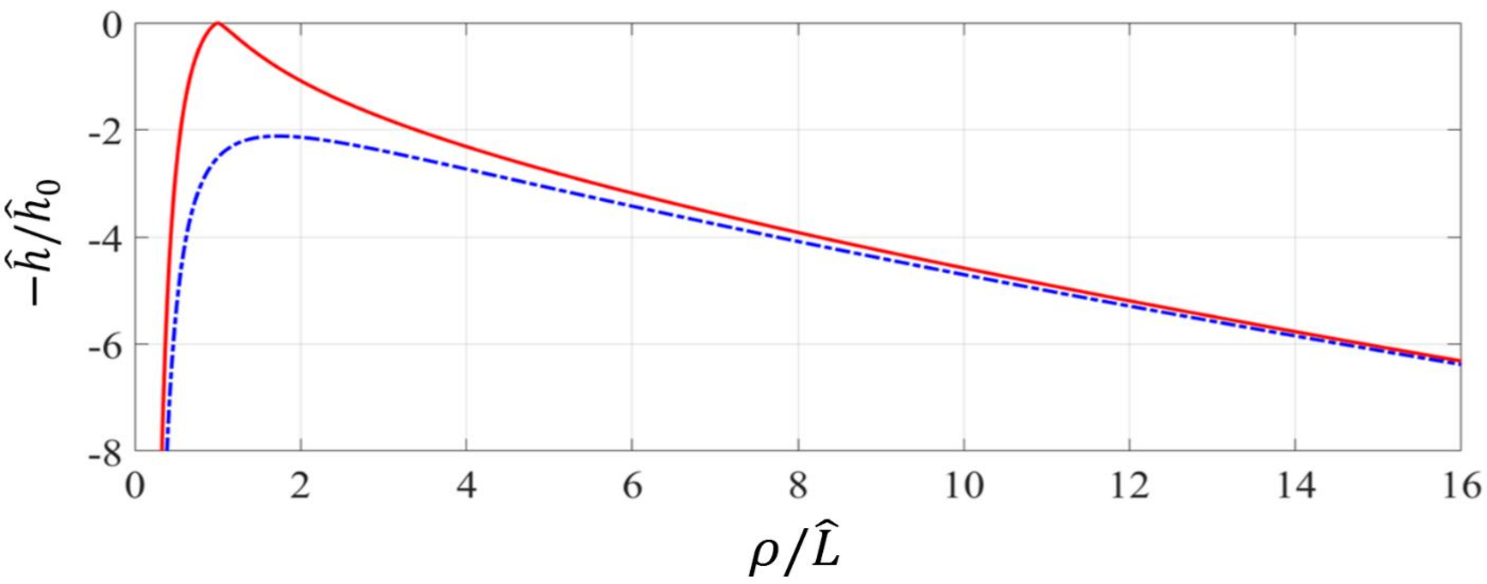
$$\tau = \frac{1}{\sqrt{DB}}\left(\frac{3D}{2}\rho^2 + 3E\right)^{1/3}$$

$E \neq 0$

$$c^2(\rho) = gh(\rho) = \hat{c}_0^2\left(\frac{\hat{L}}{\rho}\right)^2\left(1 \pm \left(\frac{\rho}{\hat{L}}\right)^2\right)^{\frac{4}{3}}$$

$$A = \frac{\hat{A}_0}{\left(1 \pm \left(\frac{\rho}{\hat{L}}\right)^2\right)^{1/3}}$$

$$\tau = \hat{\tau}_0\left(1 \pm \left(\frac{\rho}{\hat{L}}\right)^2\right)^{1/3}$$



Conclusion

- Several new non-reflective bottom geometries allowing for cylindrical wave propagation are found
- Geometries represent a blue hole and an underwater volcano
- The results can be used for model benchmarking

Didenkulova et al. Non-reflected cylindrical propagation of tsunami waves in the ocean of changing depth. Submitted to Ocean Engineering (2025)

Thank you for your attention!