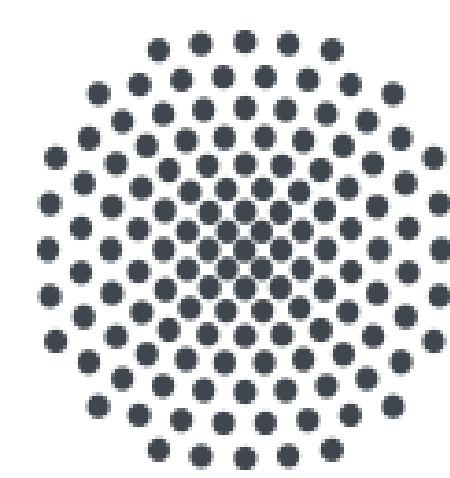


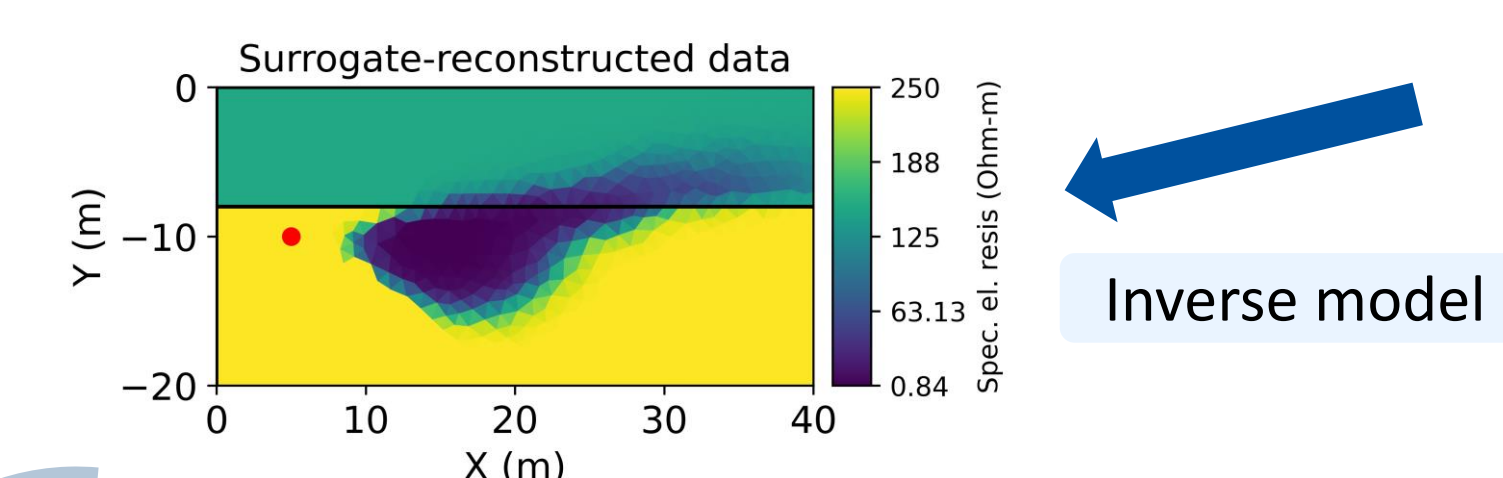


Surrogate-assisted Bayesian inference with ERT data for contaminant transport modeling in the subsurface

Maria Fernanda Morales Oreamuno¹, Nino Menzel², Florian Wagner², Sergey Oladyshkin¹, Wolfgang Nowak¹¹ Institute for Modelling Hydraulic and Environmental Systems - Department of Stochastic Simulation and Safety Research for Hydrosystems, University of Stuttgart, Stuttgart, Germany² Geophysical Imaging and Monitoring (GIM), RWTH Aachen University, North Rhine-Westphalia, Germany

Electrical Resistivity Tomography (ERT) Data

- ✓ Non-invasive geophysical tool
- ✓ Can provide insights into subsurface properties and fluid movement → sensitive to T and P changes

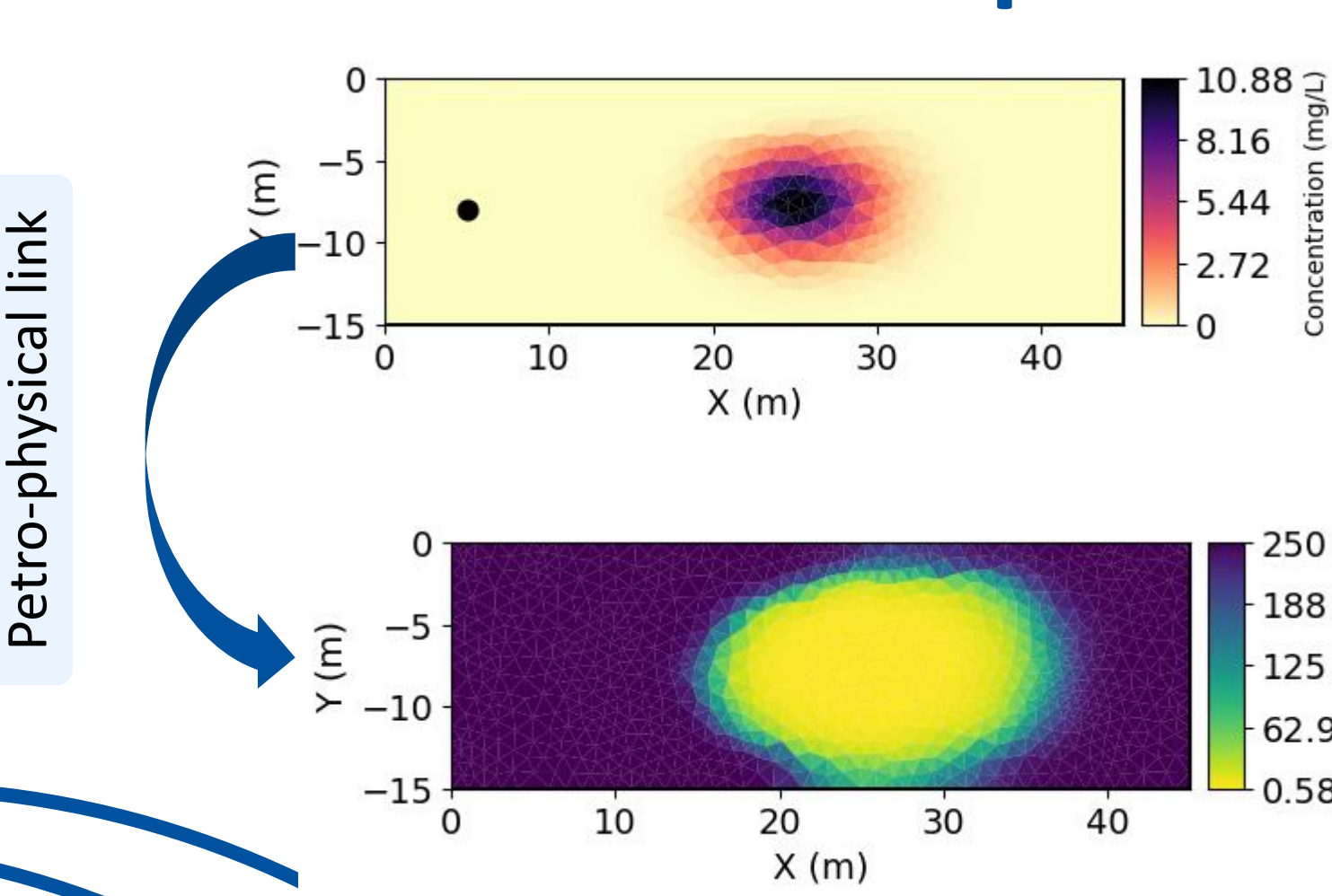


Inverse model

We use ERT measurements to do inference

- High dimensional measurement data

Contaminant Transport Model



Petro-physical link

2D non-reactive transport

- Homogeneous test case
- We consider 4 uncertain parameters (ω)

- K
- Φ
- α_L
- Contaminant strength (s_i)

We map from concentrations to **electrical resistivity data** through a petro-physical link

Objective

Use ERT measurements to reduce predictive uncertainty for contaminant transport simulations

Challenges

- Uncertainties in subsurface and contaminant-related parameters
- Computationally-expensive simulations
- ERT data is high dimensional, which is a challenge for likelihood estimations and surrogate training

Surrogate Model

Surrogate models $\mathcal{S}$ are used to approximate a forward model's outputs (y), given variable inputs (ω), at a fraction of the time.

$$y \equiv \hat{y} = \mathcal{S}(\omega, \theta)$$

Noisy GPR¹ approach

$$\hat{y} = \mathcal{S}(\omega, c) \approx N(\mu^*(\cdot), K(\cdot, \cdot))$$

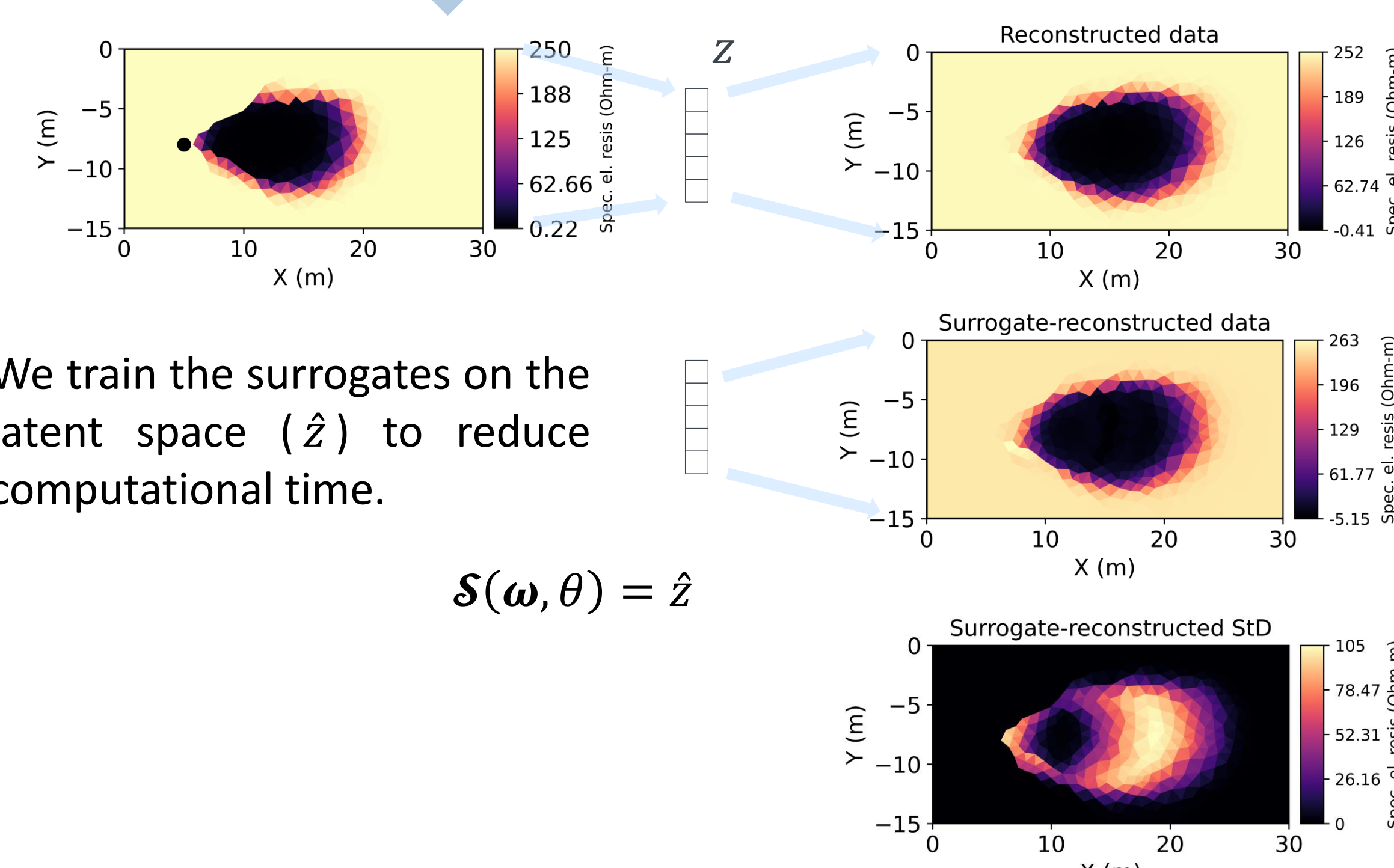
PCE² approach with Bayesian solver

$$\hat{y} = \mathcal{S}(\omega, c) \approx \sum_{i=0}^P c_i \psi_i(\omega)$$

We build surrogates to estimate electrical resistivity data for different time steps of interest

Dimension Reduction Approaches

We represent our quantity of interest in a low-dimensional latent space (z).

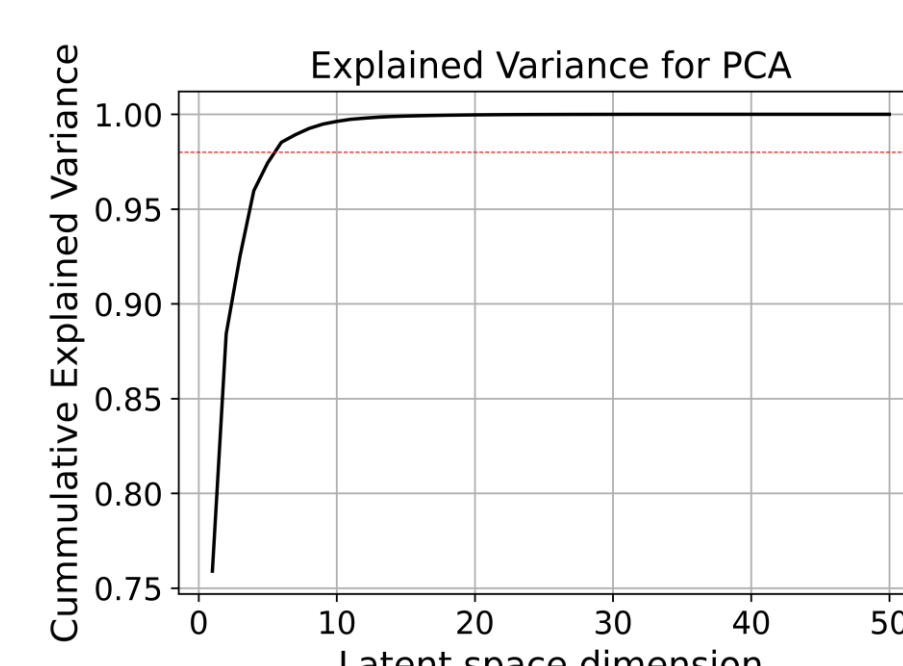


We train the surrogates on the latent space ($\hat{z}$) to reduce computational time.

$$\mathcal{S}(\omega, \theta) = \hat{z}$$

We consider PCA:

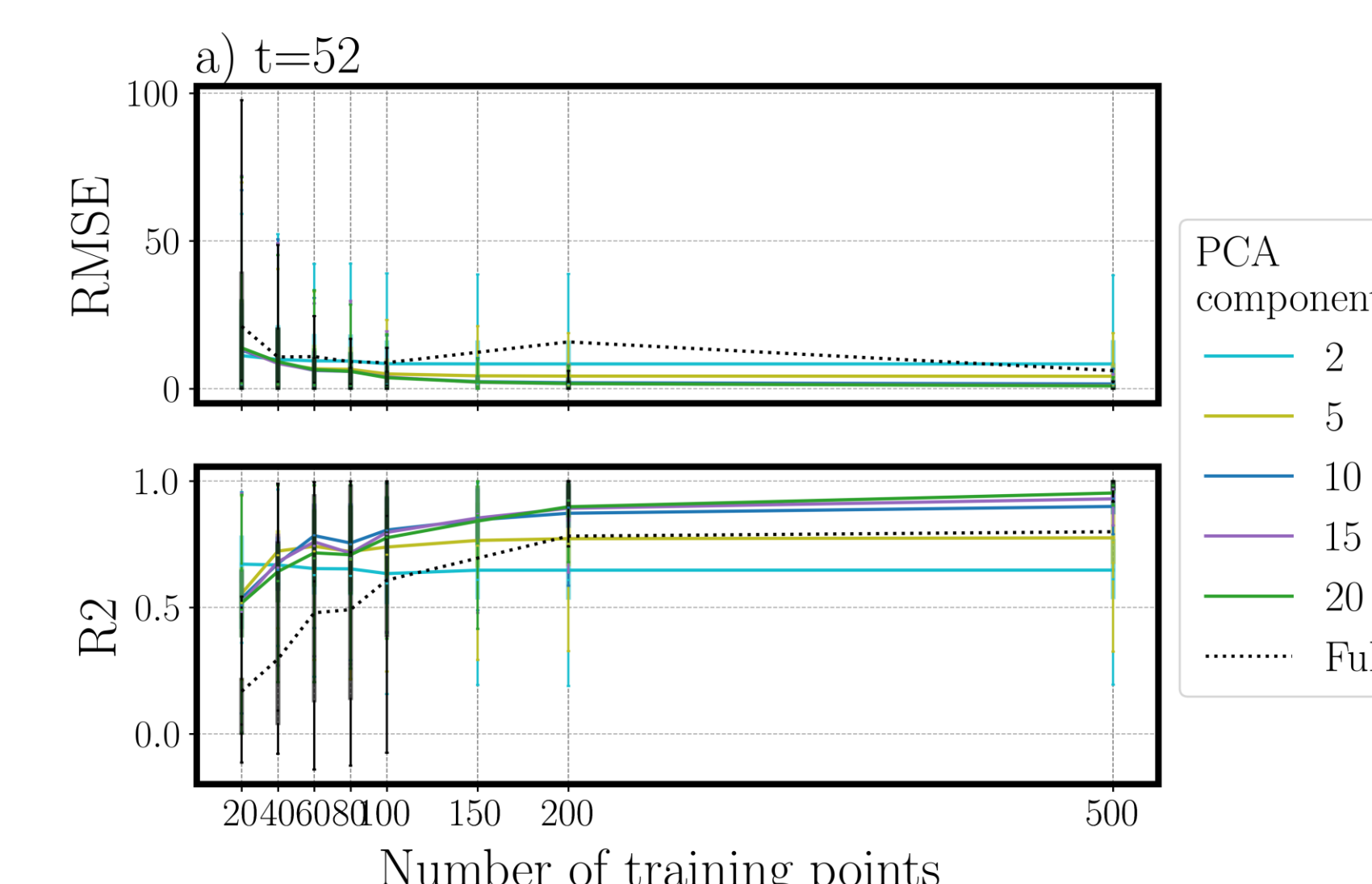
- ✓ Linear transformation
- Global encoder: Smoothens data



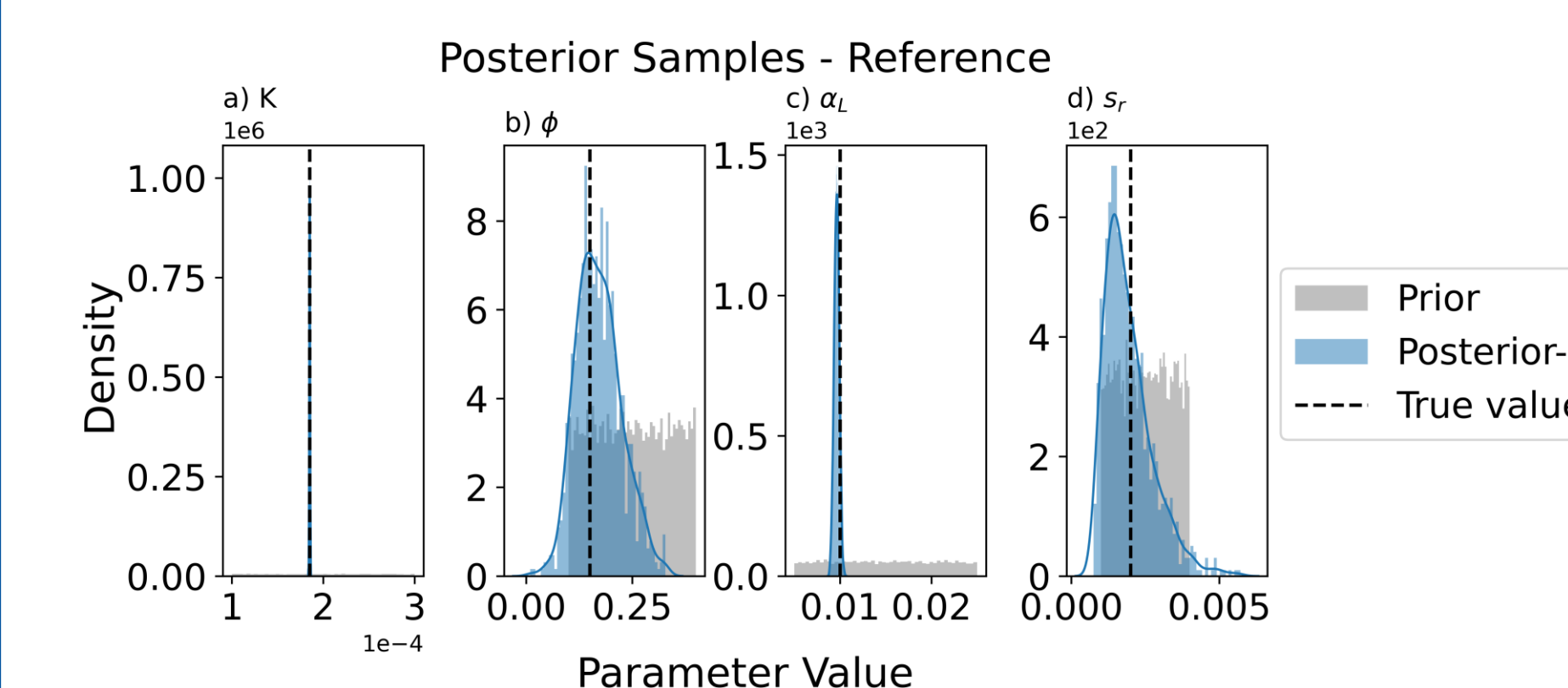
Results

Predictive accuracy

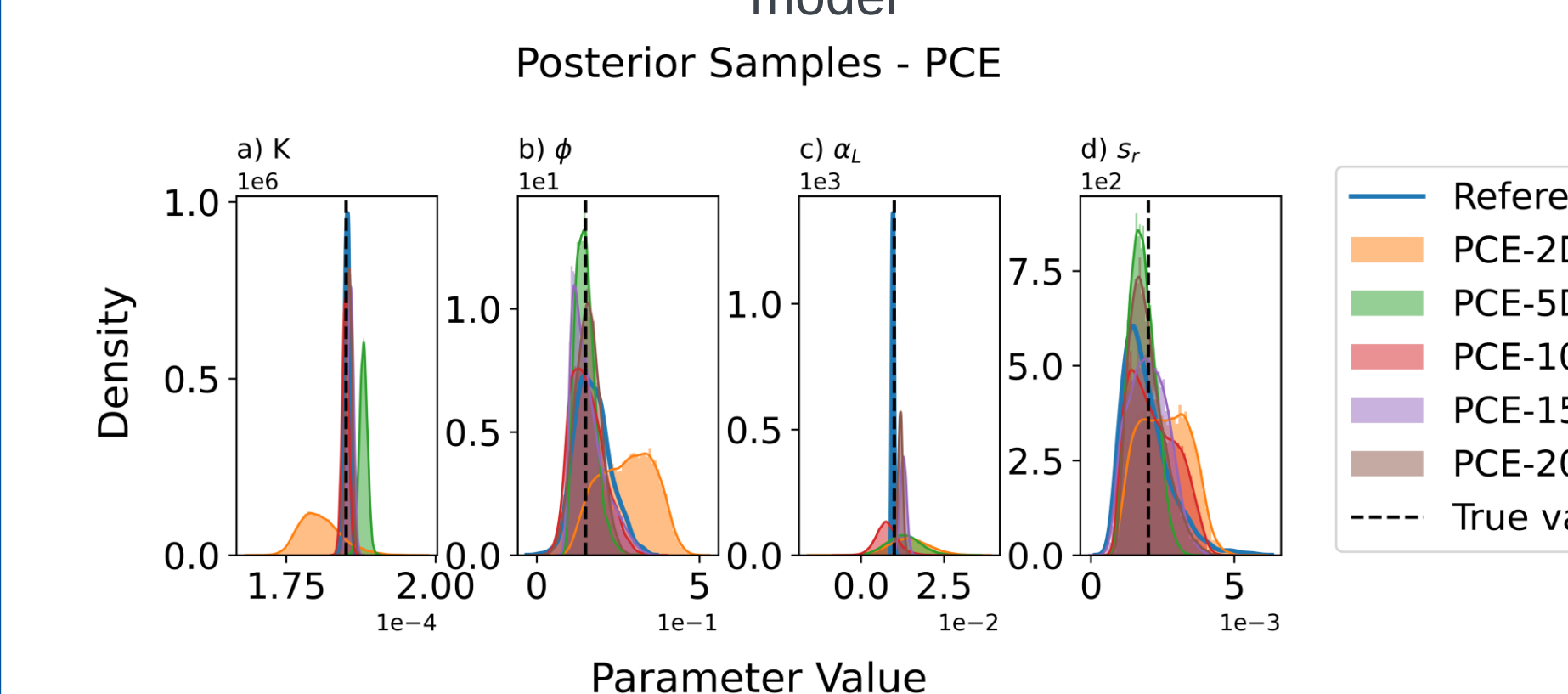
- ✓ PCA-based surrogates show similar or higher predictive accuracy as per-cell surrogate approach (full)
- ✓ PCA-based surrogates exhibited slightly higher predictive uncertainty, but overall good generalization.



Validation criteria for PCA-PCE surrogate models for different latent space size ($\hat{z}$)



SBI-obtained posterior samples using the reference model



SBI-obtained posterior samples using PCA-PCE surrogates for training

Bayesian Inference

- SBI using 10,000 model runs successfully recovered posterior distributions close to the synthetic truth.
 - Computationally intensive due to large number of model runs needed
- PCA-trained surrogates enabled a **faster** SBI training
- Using $\geq 98\%$ retained variance yielded posterior convergence comparable to full-dimensional reference models.
- ERT data provides insights into field-scale parameters (e.g., permeability, porosity, dispersivity)

Outlook

Implement non-linear dimension reduction techniques to reduce surrogate-related uncertainty.

Implement methodology to optimal experimental design simulations for continuous monitoring.

Sources

^[1] Williams, C. K., & Rasmussen. Gaussian processes for machine learning, Volume 2. MIT press Cambridge, MA, 2006.

^[2] Oladyshkin, S. and Nowak, W. Data-driven uncertainty quantification using the arbitrary polynomial chaos expansion. Reliability Engineering and System Safety, 106:179–190, 2012. ISSN 0951-8320.

^[3] Greenberg, D., Nonnenmacher, M., & Macke, J. (2019, May). Automatic posterior transformation for likelihood-free inference. In International Conference on Machine Learning (pp. 2404–2414). PMLR.

^[4] Tejero-Cantero et al., (2020). sbi: A toolkit for simulation-based inference. Journal of Open Source Software, 5(52), 2505, <https://doi.org/10.21105/joss.02505>

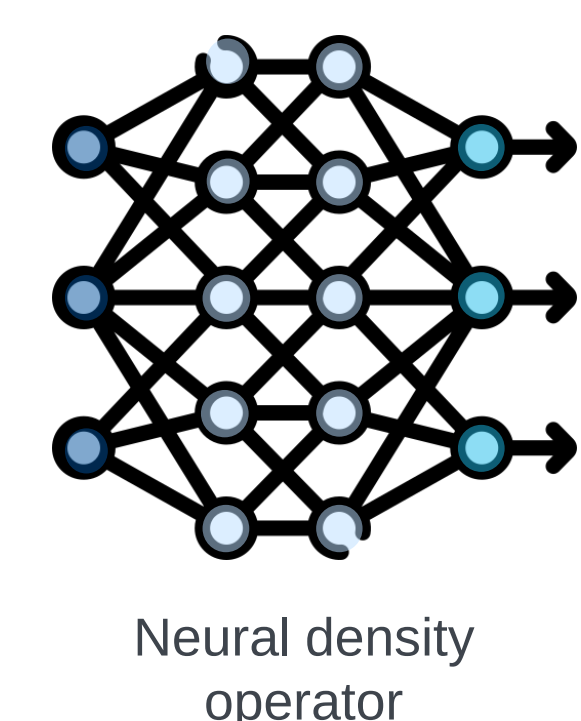
Bayesian Inference

To quantify uncertainty in contaminant transport predictions by updating prior parameter beliefs using ERT data (y_o)

$$p(\omega|y_o) = \frac{p(y_o|\omega)p(\omega)}{p(y_o)}$$

Likelihood free inference methods

- ✓ Avoids intractable likelihood computations in high dimensions
- ✓ Our approach: Simulation-based inference³
- ✓ Surrogate-assisted approach: to generate training samples



Research questions:

- How much does **ERT** data help reduce uncertainty in contaminant transport predictions?
- What are the impacts of dimensionality reduction on the accuracy of **surrogate-assisted Bayesian inference**?

