A Scalable Approach for Non-Gaussian Bayesian Emissions Inference

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The problem

- Non-Gaussian methods needed for inference of non-CO_2 emissions
- Model hierarchy allows for "uncertainties in uncertainties" to be incorporated, leading to more complete estimates
- Posterior sampling methods (MCMC) are flexible, but computationally expensive and so *dimension reduction required*
- They rely on "batch" data usage, which is problematic with growing volumes of remotely sensed data

The inversion

- Surface fluxes are inferred from mole fraction observations
- Large volumes of remotely sensed data not compatible with inferential frameworks designed for in-situ measurements
- New methods required

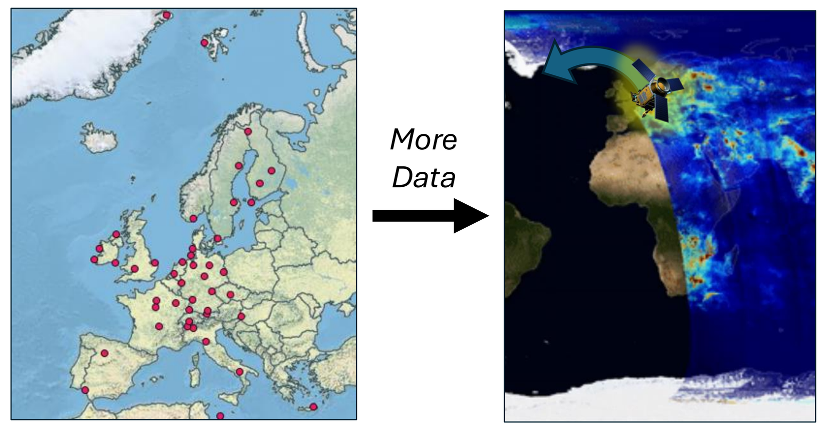


Fig.1. *Left: ICOS Atmosphere network[2], providing high frequency in-situ measurements of greenhouse gas concentrations. Right: TROMOPI[3] satellite observations, with a full set of global observations completed roughly once daily.*

Why non-Gaussian?

Gaussian emissions prior:

- Non-zero probability of negative surface fluxes
- Light tailed, may not account for extreme pollution events

Lognormal emissions prior:

- Non-negative and heavy tailed

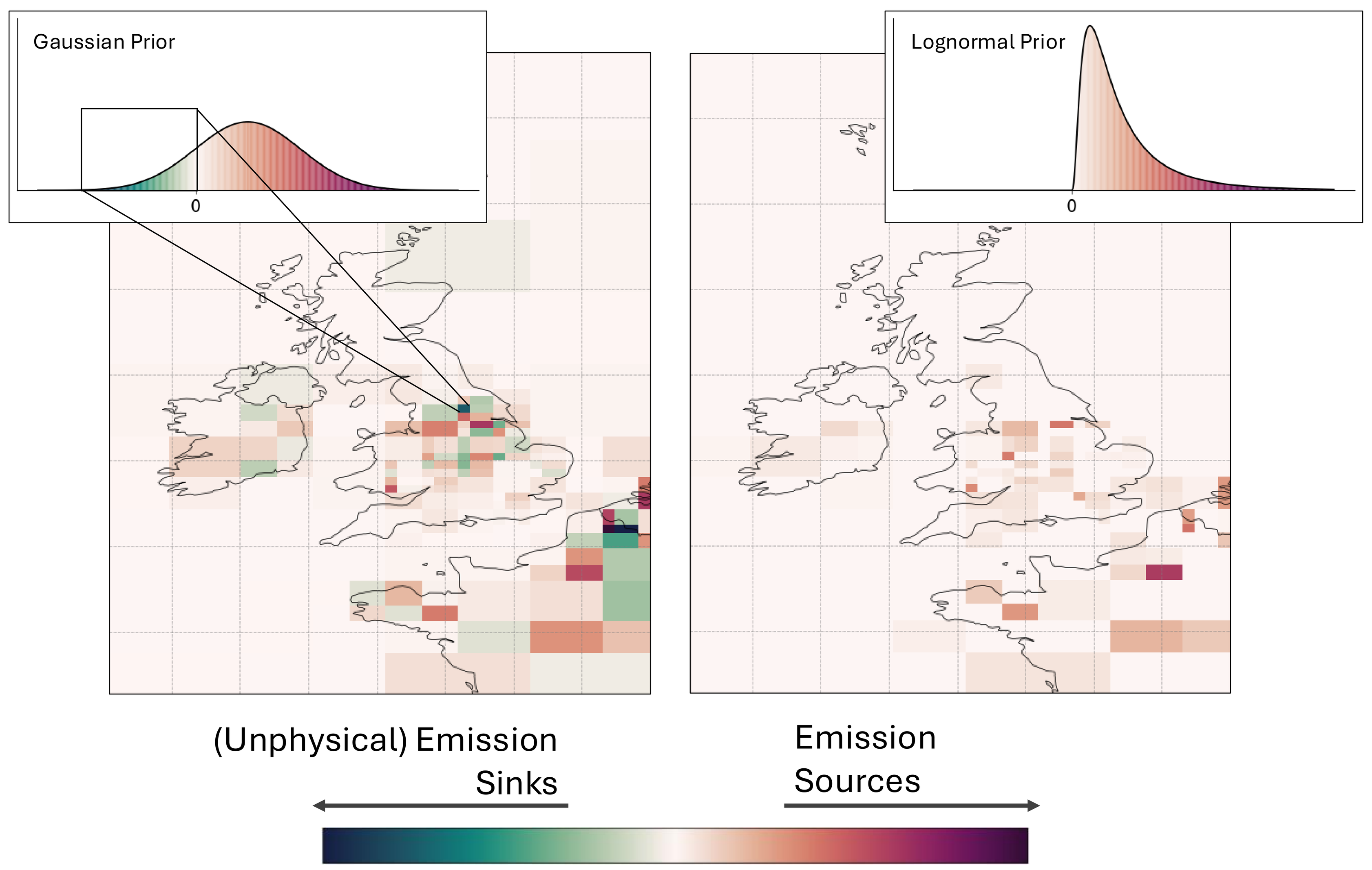
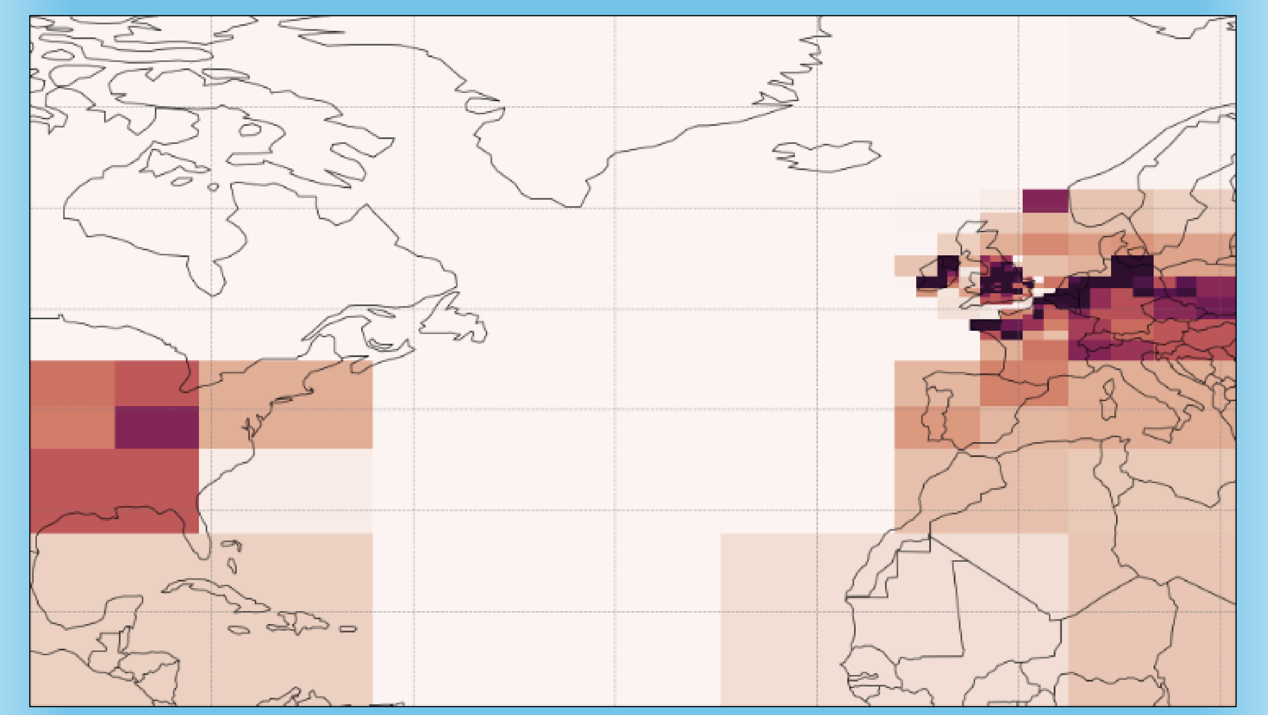
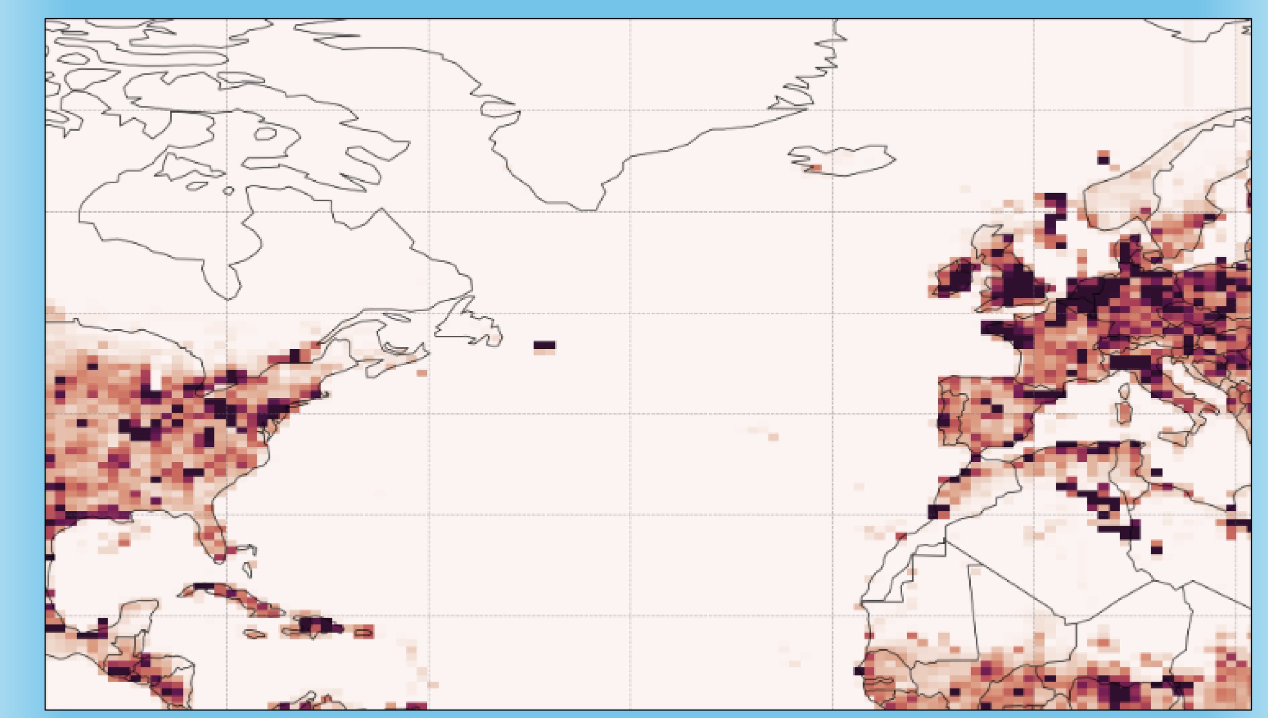


Fig.2. *Inferred mean emissions field for a single month, demonstrating the impact of a Gaussian emission prior distribution. Negative surface fluxes are unphysical for many greenhouse gases and ozone depleting substances.*

This enables efficient, high-resolution, non-Gaussian emissions inference



250 Emissions Regions



10,000 Emissions Regions

Mixed Gaussian-Lognormal Sequential Approach

- Fletcher et al.[4] derived equations for a mixed Gaussian-lognormal Kalman filter (MXKF)
- By propagating the median state in log-space, the form of the standard Gaussian Kalman filter is maintained
- This provides a pseudo-analytical sequential framework for non-Gaussian inference
- As a proof of concept, a lognormal prior distribution was used as the forecast uncertainty (no data filtering or smoothing)

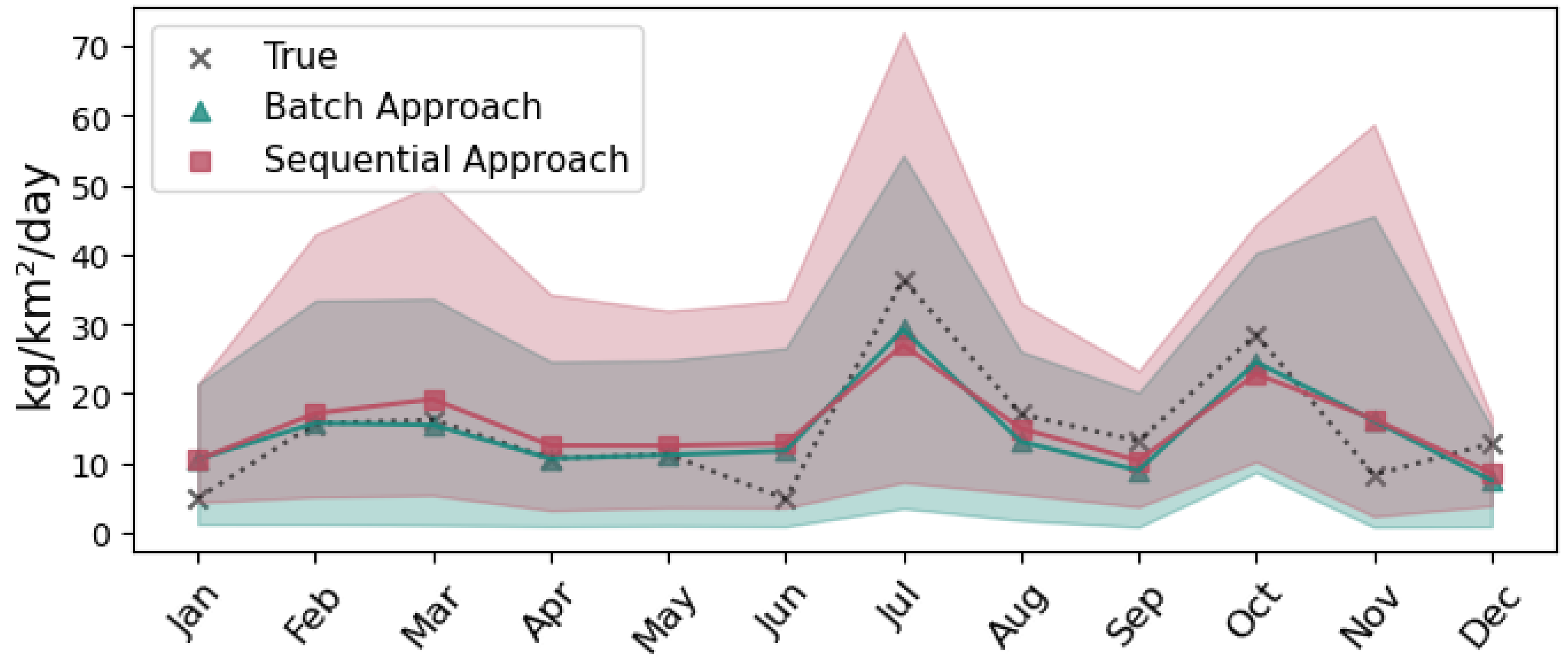


Fig.3. *To enable validation, pseudo data is used to simulate mole fraction CH_4 observations for 4 in-situ UK sites. Shown here are the inferred monthly mean emission fluxes for a region in Lancashire. A conventional stochastic batch approach (NUTs MCMC) is shown, alongside a sequential approach based on the MXKF derived by Fletcher et al.*

Advantages of Kalman filtering

- Kalman filters are highly efficient recursive estimators
- Data assimilated sequentially, well suited to handle large volumes
- Temporal correlation implicitly included in the forecast step
- Does not rely on posterior sampling and so can be scaled more effectively

Regions	Observations	Batch MCMC time	Sequential solver time
9	7,960	16 mins	< 1 s
50	8,230	19 mins	2 s
50	43,865	~1.5 hrs	36 s
250	43,865	~11.5 hrs	46 s
500	43,865	~36.5 hrs	47 s
1,000	43,865	~88 hrs	56 s
10,000	43,865	N/A	41 mins

Fig.4. *This table demonstrates the significant time savings associated with the sequential lognormal solver. This approach avoids any posterior sampling, and instead directly calculates the posterior mean and standard deviation.*

Next Steps

- Implement full mixed Gaussian-lognormal Kalman Filter with observational data
- Introduce spatiotemporal correlations
- Explore hierarchical methods to estimate uncertainties and correlations

Get in touch!
If you have any suggestions or would like a chat, please drop me a line - ***s.pearson@bristol.ac.uk***

[1]Ganesan et al. (2014) Characterization of uncertainties in atmospheric trace gas inversions using hierarchical Bayesian methods. Doi: 10.5194/acp-14-3855-2014
[2]ICOS. (n.d.). *Atmosphere stations*. Integrated Carbon Observation System. Available from: https://www.icos-cp.eu/observations/atmosphere/stations
[3]Aeronomie.be. (2021). *Three years TROPOMI measurements*. [online] Available from: https://www.aeronomie.be/en/news/2021/three-years-tropomi-measurements
[4]Fletcher et al. (2023) Lognormal and Mixed Gaussian–Lognormal Kalman Filters. *Monthly Weather Review*. Doi: 10.1175/MWR-D-22-0072.1