A phase-field description of crevasse growth: comparison of Elastic, Maxwell, and Burgers models for ice

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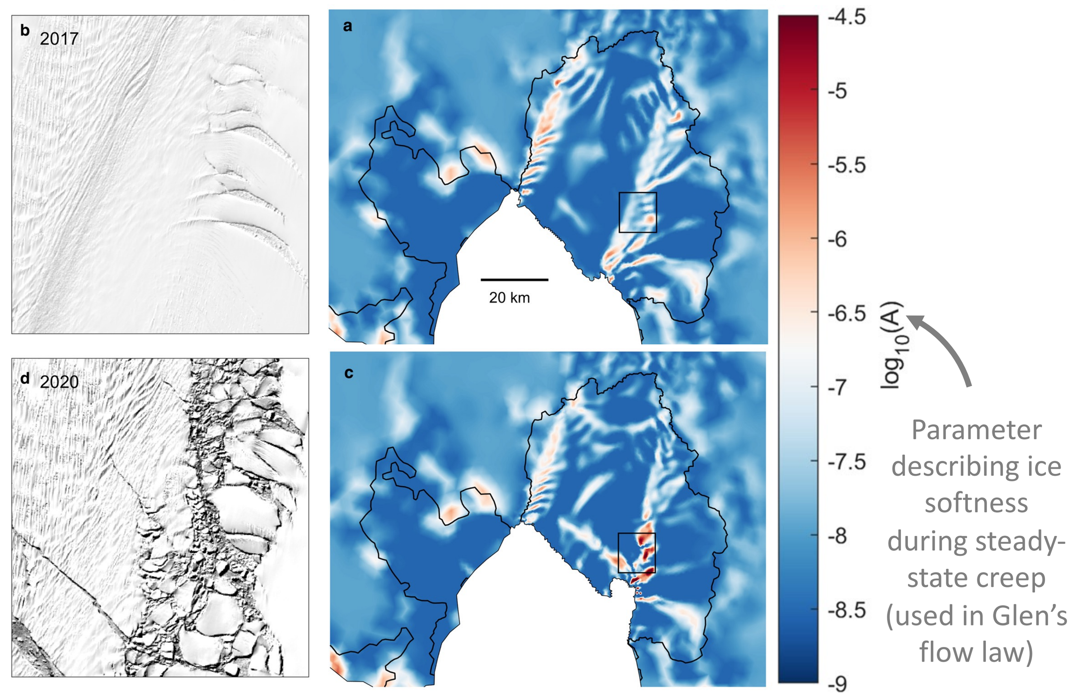
Motivating work

Predictions of the rate and extent of ice lost from Antarctica is the one of the largest sources of uncertainty in estimating global sea-level rise, largely due to a poor understanding of the mechanisms governing iceberg calving. Ice-shelf fracture models typically employ a linear elastic model for ice. However, ice exhibits both elastic and viscous behavior in response to a load. This is evidenced by the observation that ice flow speeds and therefore ice viscosity are influenced by the evolution of damage (Fig. 1; Sun and Gudmundsson, 2023).

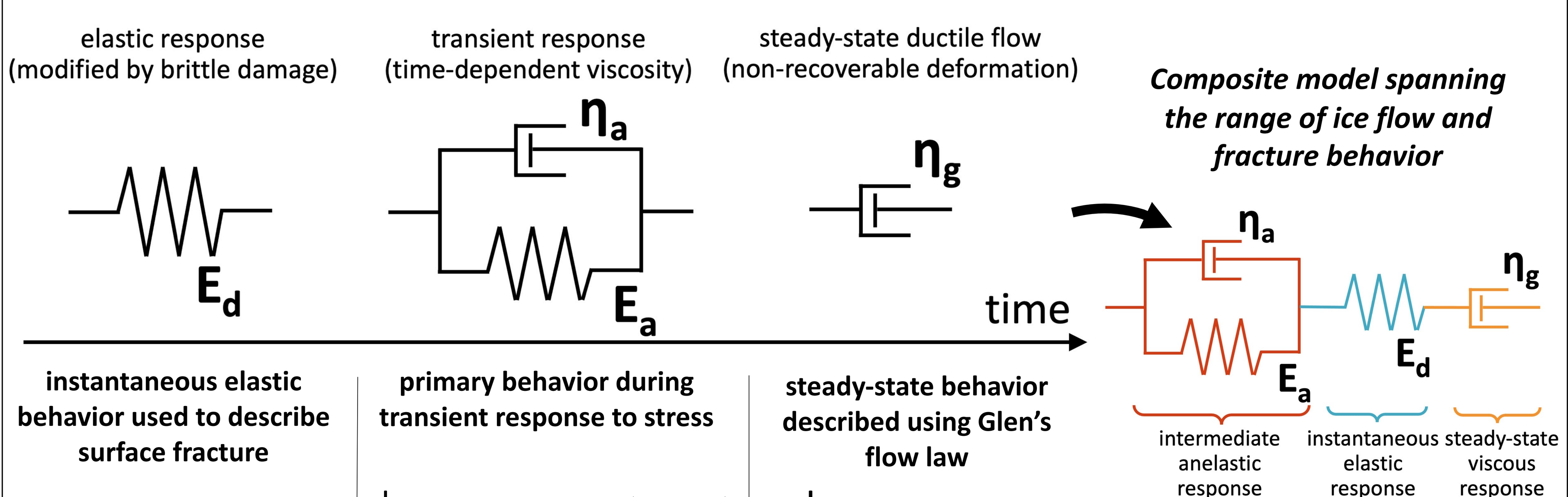


Left: Response of ice flow to damage within Pine Island Ice-shelf (Sun and Gudmundsson, 2023).

To examine the relationship between ice flow and fracture, we use a phase-field description of ice fracture that is coupled to a transient and steady-state model of ice flow. Ice viscosity is transient (changing in time) over short strains, and so inclusion of a model for the transience in ice can fill the gap in describing brittle to ductile behavior in ice-shelves.

Model description

Rheological descriptions of ice viscoelasticity

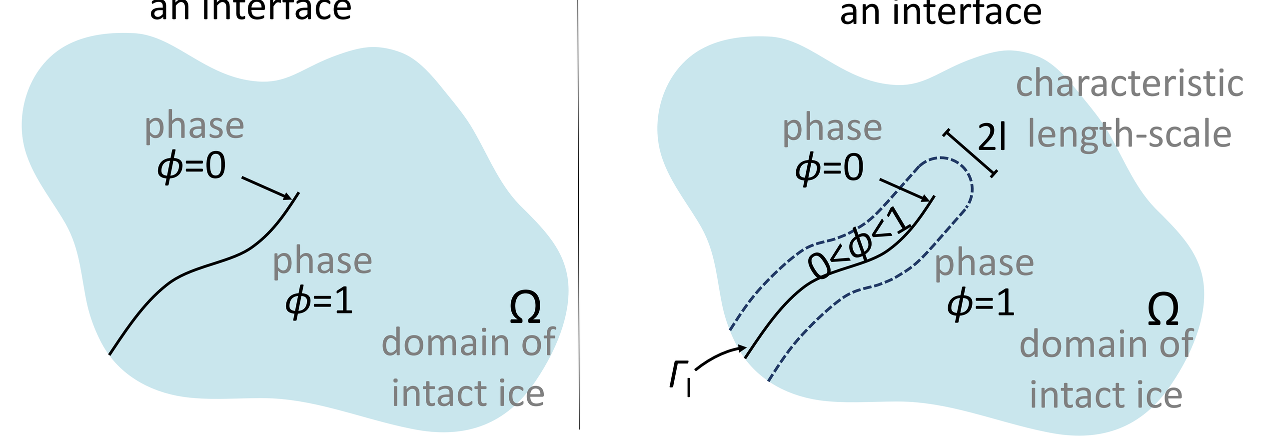


instantaneous elastic behavior used to describe surface fracture	primary behavior during transient response to stress	steady-state behavior described using Glen's flow law
surface energy $\Gamma_l = \int G_C \gamma \, dV$	$\frac{d\varepsilon}{dt} = A(T,\rho)\sinh\left(\frac{\sigma_e}{\sigma_{ref}(T)}\right)$	$\frac{d\varepsilon}{dt} \approx A(T)\,\sigma(t)^3$
critical energy release rate $G_C = K^2_{IC}/E'$ (mode-I fracture toughness; Young's modulus)	$\approx A(T,\rho)\,\sigma_e/\sigma_{ref}(T)$	$A(T = 0^oC) = 2.4\times10^{-24}\ Pa^{-3}s^{-1}$
	approximately linear stress dependence	*non-linear stress dependence*

Phase-field model of ice fracture

Discrete representation of a crack: a sharp change in structure along an interface

Phase-field representation of a crack: a continuous change in structure along an interface



$$\begin{cases} \phi = 0 & \text{fully intact material} \\ 0 < \phi < 1 & \text{damaged material} \\ \phi = 1 & \text{fully fractured material} \end{cases}$$

mass conservation

$$0 = -\nabla D\sigma$$

stress reduction

$$\sigma_d = (1-D)\sigma$$

damage function

$$D(\phi) = 1-(1-\phi)^2$$

energy budget

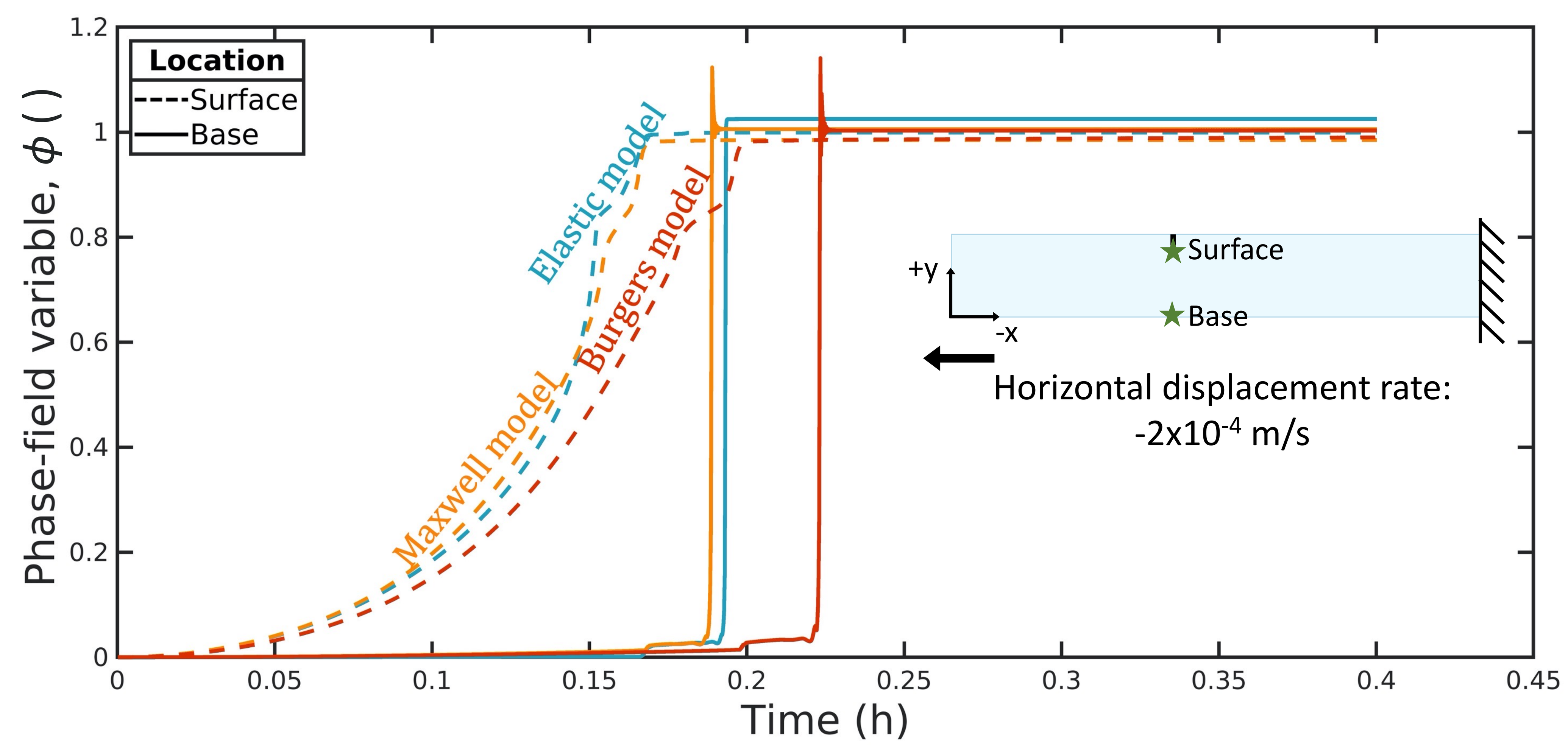
$$E = \Gamma_l + \text{stored energy} + \text{lost energy}$$

damage evolution

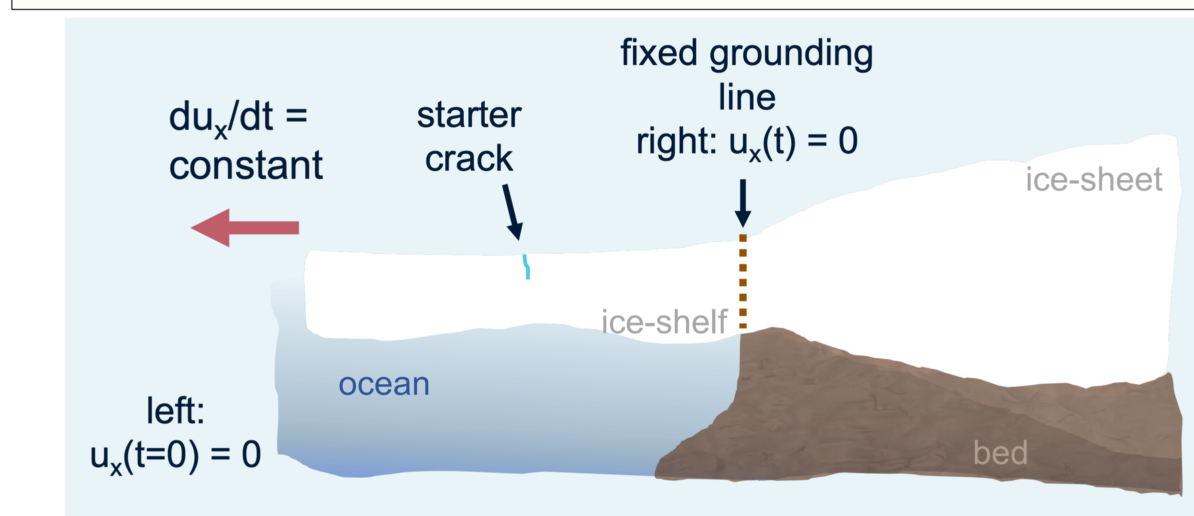
$$\tau\frac{\partial\phi}{\partial t} = \frac{\partial D}{\partial t} H_D - \phi - l^2\nabla^2\phi$$

(τ: characteristic time; H_D: driving force)

Evolution of a crevasse using different ice behaviors



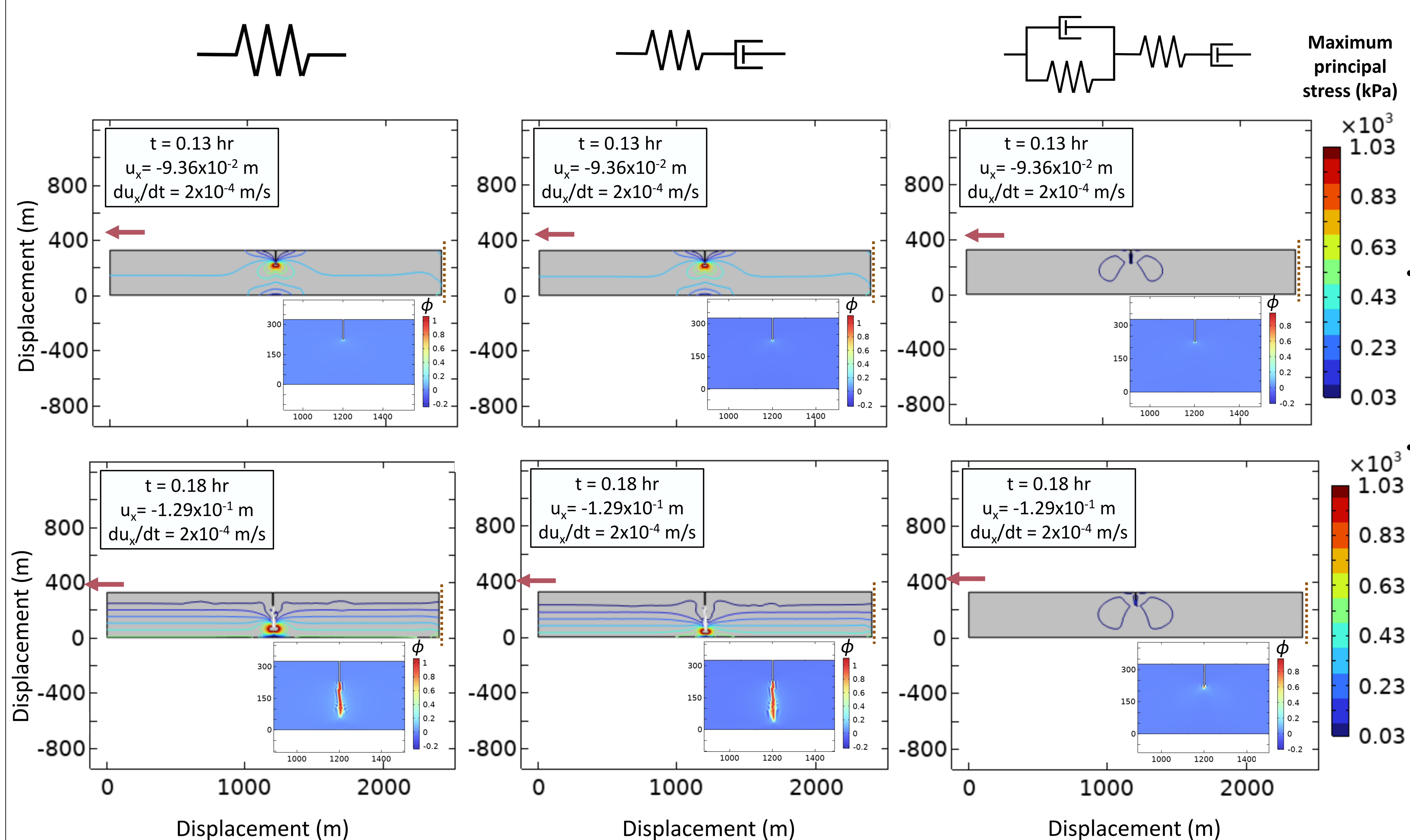
Boundary Conditions



Remarks

- The ice is initially isotropic and homogeneous with a constant density of 918 kg/m^3.
- The shelf is at a constant temperature of -20^{o}C.
- The domain is defined over a refined mesh, with a finer mesh spacing around the anticipated path of the crevasse.

Stress in ice during crevasse propagation across brittle to ductile regimes



Key Takeaways

*There is a trade off between **how fast** an ice-shelf is being strained and **how much** displacement is needed for a crevasse to propagate.*

- Transience (as represented by a Kelvin-Voight element) primarily reduces the rate of crack propagation by relaxing stress around a damaged region.
- Models of crack growth that exclude the transient viscosity in ice will **overpredict** the rate at which a crevasse will grow, which can lead to an **underprediction** of the buttressing potential of fast-flowing ice-shelves. Including a Kelvin-Voight model for ice can fill the gap in descriptions of ice fracture and flow.

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