Third-Order Law for MHD Turbulence Varying the Dissipation Mechanisms

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Introduction

In this study, we investigate the energy cascade in Alfvénic solar wind turbulence under non-ideal MHD conditions, where viscosity (ν) and resistivity (μ) differ and act at separate scales [1]. Unlike the ideal case (ν = μ), recent observations suggest that viscous effects dominate at larger scales than magnetic dissipation. Adopting a phenomenological model with ν ≠ μ , we study how this imbalance impacts the energy transfer, focusing on the third-order Yaglom law reformulated using Elsässer variables. This relation directly measures the cascade rate and reveals asymmetries in kinetic and magnetic dissipation. Through theoretical analysis and ongoing simulations, our goal is to quantify these effects and improve the interpretation of in-situ measurements in the solar wind and magnetosheath.

Analitical Results

The starting points are the incompressible MHD equations

$$\begin{cases} \dfrac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v} = -\vec{\nabla}P + \vec{B}\cdot\vec{\nabla}\vec{B} + \nu\nabla^2\vec{v} \\ \dfrac{\partial \vec{B}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{B} = \vec{B}\cdot\vec{\nabla}\vec{v} + \mu\nabla^2\vec{B} \end{cases}$$

We introduce a new variable z, which represents a linear combinations of the velocity field and the magnetic field, normalized by the mass density.

$$\vec{z}^{\pm} = \vec{v} \pm \frac{\vec{B}}{\sqrt{4\pi\rho}}$$

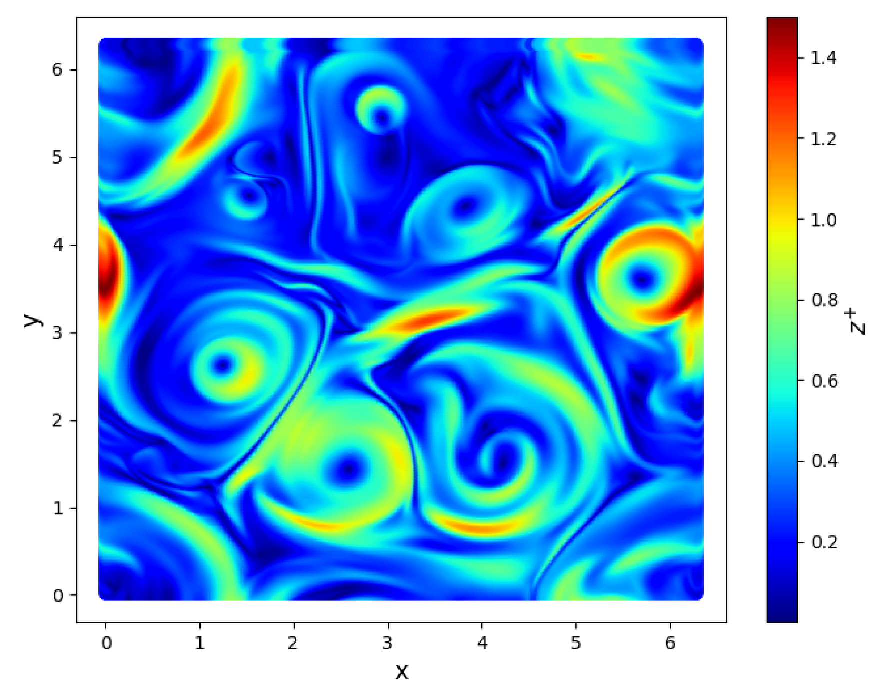
These quantities represent Alfvénic fluctuations propagating along the background magnetic field in opposite directions.
MHD equations can be immediately written in terms of these variables as [2].

$$\frac{\partial}{\partial t}\vec{z}^{\pm} + (\vec{z}^{\mp}\cdot\vec{\nabla})\vec{z}^{\pm} = -\vec{\nabla}P + \frac{\nu+\mu}{2}\nabla^2\vec{z}^{\pm} + \frac{\nu-\mu}{2}\nabla^2\vec{z}^{\mp}$$

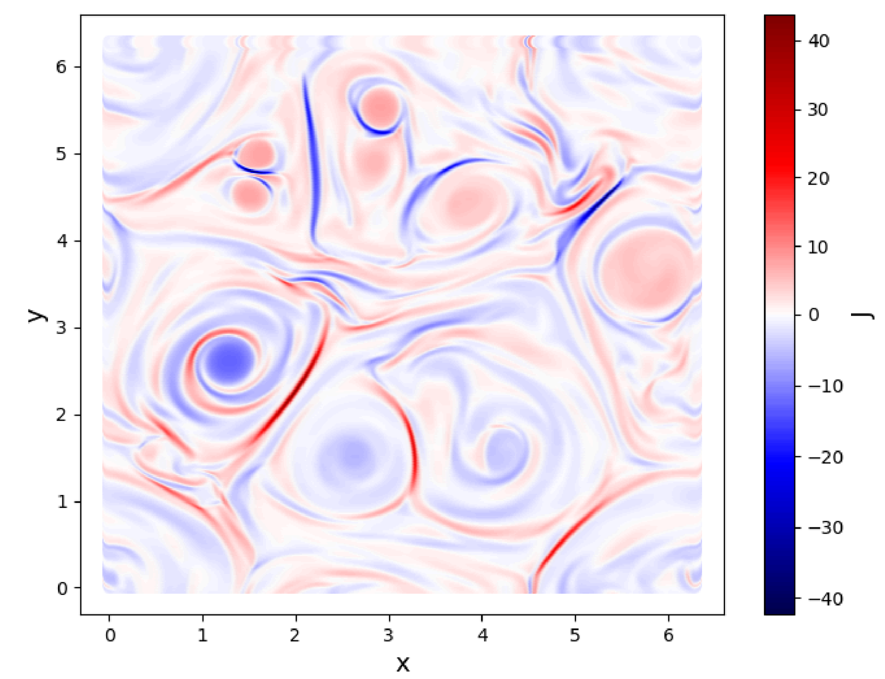
Preliminary Results

Data shown corresponds:

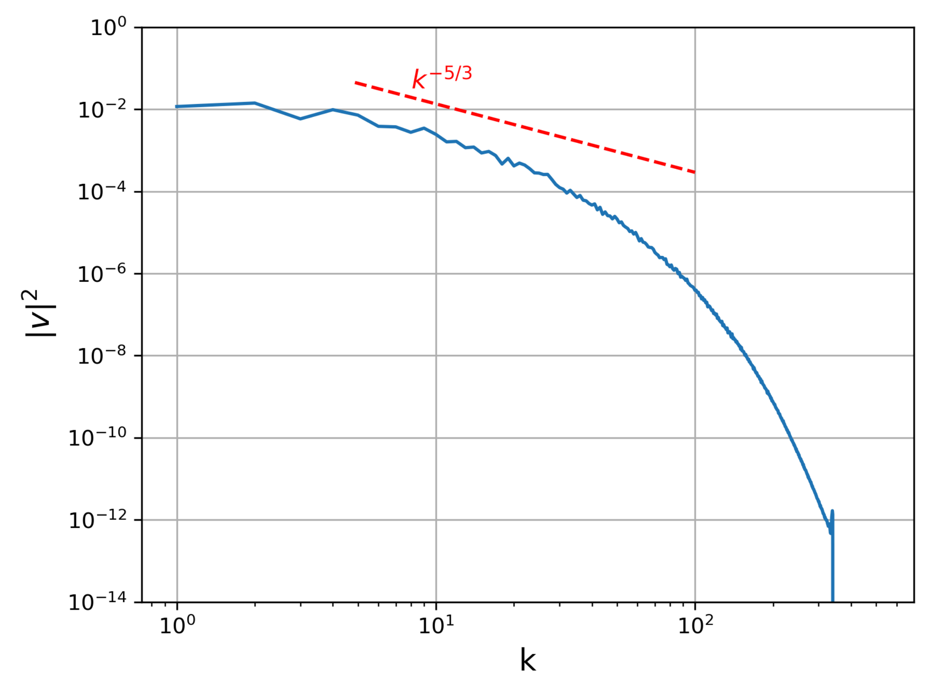
- Grid size: Nx=Ny=1024
- Time snapshot: t=152
- Time step: Δt= 5×10^{-4}
- Domain size: Lx=Ly=2π
- ν = μ = 6×10^{-4}



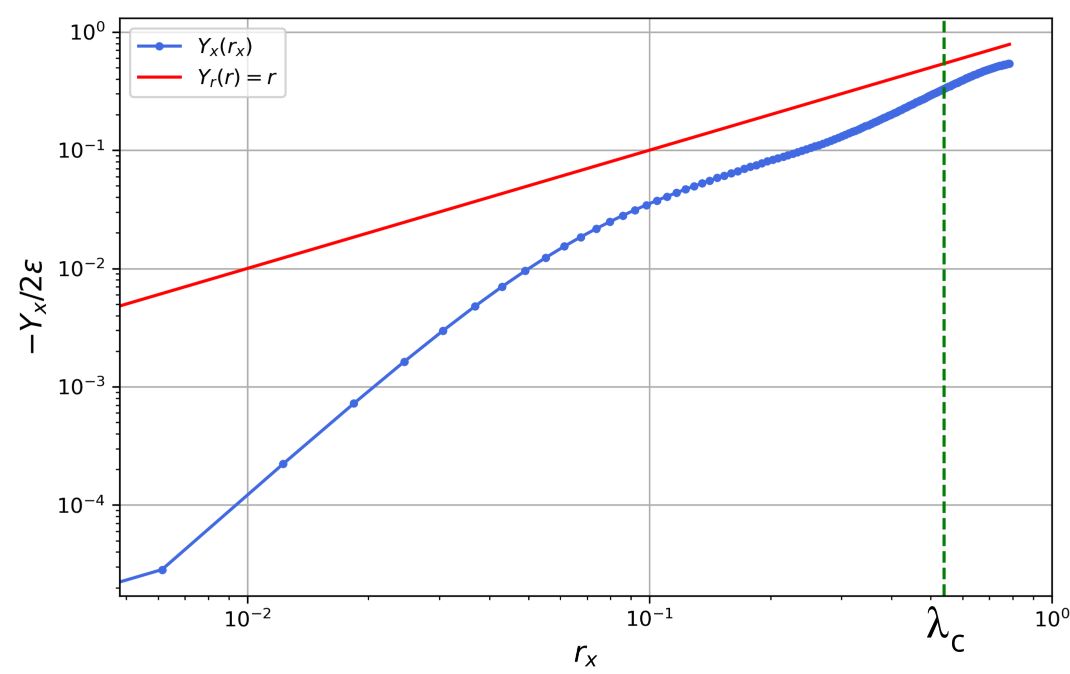
This distribution illustrates the variation of Z across the domain, offering insights into the interaction between velocity and magnetic fields in MHD turbulence.



The plot shows the spatial distribution of the Jz component of current density, highlighting the flow along the z-axis and its role in electromagnetic interactions in MHD turbulence.



Velocity energy spectrum $|v(k)|^2$ vs. wavenumber k, showing Kolmogorov $k^{-5/3}$ scaling typical of inertial-range turbulence.



The plot shows the radial component of the Yaglom flux, normalized by −2ε, compared to the theoretical prediction r.

Conclusions

- The radial Yaglom flux confirms the presence of a well-defined inertial range, with a constant transfer of energy from large to small scales, indicating a stable cascade in agreement with turbulence theory.
- The velocity spectrum follows a law similar to $k^{-5/3}$, in line with Kolmogorov's theory, suggesting a self-similar behavior at small scales.
- The calculation of the third-order Yaglom law in non-ideal conditions will provide a direct measure of the energy transfer rate and clarify the role of viscosity and resistivity in the energy cascade.

References

1. Sorriso-Valvo, L.,et al., 2007. Observation of inertial energy cascade in interplanetary space plasma. Physical review letters, 99(11), 115001.
2. Carbone V.,et al., 2009, On the turbulent energy cascade in anisotropic magnetohydrodynamic turbulence. Europhysics Letters, 88(2), 25001.