An analytical approach for modeling the initiation and early development of the Rayleigh-Taylor gravitational instability in subduction settings

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Appendix

A. Approximations

It is generally accepted that rocks behave mostly viscoelastically over short periods of time (<5x10^4 years), but over longer timescales they exhibit a ductile flow behaviour, thus undergoing major plastic deformation. Nevertheless, researchers mostly apply a more vague visco-elasto-plastic rheology in order to fully capture the geodynamic processes with a higher precision (Gerya and Yuen, 2007; Babeyko and Sobolev, 2008; Popov and Sobolev, 2008; Pusok et al., 2018; Gerya, 2019; Jacquey and Cacace, 2020; Duretz et al., 2021; Papadomarkakis and Athanassas, 2024). Moreover, for the present study both the flowing mantle and the subducting plate were considered to be viscoelastic rocks, a decision that was made due to the short time scale that the Rayleigh-Taylor instability will be studied. Additionally, rock type materials are compressible when subjected to elastic deformations, but almost incompressible when deformed in a viscous manner (Schubert et al., 2001). Hence a very common approximation that is utilised in geodynamic modeling is that rocks are completely incompressible. It is worth noting that the latter approximation was investigated numerically by some researchers, in order to essentially assess the influence of the compressibility of rocks on the growth rate of the Rayleigh-Taylor instability (Poliakov et al., 1993; Kaus and Becker, 2007) and on viscoelastic folding (Mancktelow, 2001). All these studies concluded that the effect of compressibility is negligible.

Furthermore, as it can be seen in Eq. 6 the rotational terms of the co-rotational time derivative of the stress tensor were completely neglected in the present study. This approximation does not limit the accuracy of the analytical model, since it was clearly shown by Mühlhaus and Regenauer (2005), as well as Kaus (2005) that the aforementioned rotational terms are important for unrealistically high stress values (Kaus and Becker, 2007). In addition, the general Navier-Stokes equation of motion (Eq. 8) was simplified to a Stokes equation of slow, viscous flow (Eq. 9). This approximation is very typically employed for the modelling of geodynamic processes (Gerya and Yuen, 2003; Gerya and Yuen, 2007; Kaus and Becker, 2007; Popov and Sobolev, 2008; Gerya, 2019; Jacquey and Cacace, 2020; Papadomarkakis and Athanassas, 2024), due to the highly viscous flows that are encountered. Specifically, the inertial forces (meaning the term $\rho\frac{dv_i}{dt} \approx 0$) can be neglected, due to their small impact when compared with viscous resistance and gravitational forces.

It should be noted that the x,y coordinates that were chosen for the computation of the growth rate of the Rayleigh-Taylor gravitational instability were picked on the following premise. Particularly, the x = 120 km coordinate was chosen after considering that the dip of the subducting plate at the shallow depth of 50 km is around 25° (Papazachos and Nolet, 1997; Tiberi et al., 2000; Pearce et al., 2012; Sodoudi et al., 2015; Halpaap et al., 2018), hence the length of the hypotenuse equals x = 50 km/sin(25) = 120 km. As for the y = 8 km coordinate, it was assumed that the initial vertical displacement of the interface caused by the small perturbation was around 0.1 the thickness of the subducting plate.

B. Determination of Constants

In this section the assumed boundary conditions are applied in order to analytically determine the two constants (C_1, C_2) from Eq. 16. Firstly, the lower (as well as the upper) boundary had an assigned fast erosion condition (see Fig. 2), consequently for the x,y coordinates of 120 km and -80 km, respectively, which correspond to the bottom of the subducting plate at the same horizontal position as point A, the following expression can be written for the vertical velocity of the initial perturbation:

$$\frac{\partial v_y(120,-80)}{\partial y} = 0 \quad (21),$$

where, $v_y(120, -80)$: represents the vertical velocity of the perturbation at the x,y coordinates 120 km and -80 km respectively. Moreover, for the right (as well as the left) boundary an external free slip condition was assumed (see Fig. 2), as a result for the x,y coordinates of (120+λ/2) km and 8 km, respectively, which correspond to the right side boundary at the same vertical position as point A, the following expression can be produced:

$$\frac{\partial v_y(120+\lambda/2,8)}{\partial y}\Delta L + v_y(120 + \lambda/2, 8) = 0 \quad (22),$$

where, $v_y(120 + \lambda/2, 8)$: illustrates the vertical velocity of the perturbation at the x,y coordinates (120+λ/2) km and 8 km respectively, and ΔL: is considered to be the distance from the actual boundary (in km), in the present study it was assumed to be equal to 10λ.

When Eqs. 10 and 16 are inserted into Eqs. 21 and 22, a simple linear system of equations is derived that has the following general form:

$$BC_1 + DC_2 = E \quad (23), \text{ and}$$

$$FC_1 + HC_2 = J \quad (24),$$

where, B, D, E, F, H, and J: are all constants that can easily be determined by applying the aforementioned x,y coordinates in Eqs. 21 and 22. Consequently, when this simple system of equations is solved the desired C_1, C_2 constants of Eq. 16 are predicted, and as a result Eqs. 10 and 16 can be written in their full form. It should be noted that depending on the λ/h ratio that is utilised the values of the two coefficients will vary, hence when the λ/h ratio value is altered the two constants have to be computed again, using the methodology written above.

C. List of Symbols

In the following table below all the symbols that were utilised throughout the present work are explained, along with their respective units.

Table 3 The symbols that were used throughout the present study are thoroughly showcased.

Symbol	Meaning	Units
i,j	Symbolic coordinate indices (x,y) which vary in vertical and horizontal directions respectively	-
$\frac{d\varepsilon'_{ij(viscous)}}{dt}$	Viscous deviatoric strain rate	s^{-1}
$\frac{d\varepsilon'_{ij}}{dt}$	Total deviatoric strain rate	s^{-1}
$\frac{D\varepsilon'_{ij(elastic)}}{Dt}$	Rotational elastic deviatoric strain rate	s^{-1}
n	Viscosity	Pa s
σ'_{ij}	Deviatoric stress tensor	MPa
σ_{ij}	Normal stress tensor	MPa
P	Pressure	MPa
δ_{ij}	Kronecker’s delta	-
ρ	Density	kg/m^3
g	Gravitational acceleration	m/s^2
y	Depth from the surface	km
G	Elastic shear modulus	GPa
$\frac{D\sigma'_{ij}}{Dt}$	Objective co-rotational time derivative of the stress tensor	MPa/s

$\frac{d\sigma'_{ij}}{dt}$	Stress rate tensor	MPa/s
k	Index indicating summation	-
ω_{ij}	Rotation rate tensor	s^{-1}
v_x	Horizontal velocity component	m/s
v_y	Vertical velocity components	m/s
x_i	Spatial coordinate	m
x_j	Spatial coordinate	m
g_i	The i-th component of the gravity vector	m/s^2
g_x	Horizontal component of the gravitational acceleration	m/s^2
g_y	Vertical component of the gravitational acceleration	m/s^2
$\frac{dv_i}{dt}$	Substantive time derivative of the i-th component of the velocity vector	m/s^2
$v_y(x, y)$	Vertical velocity of the perturbation (as a function of x,y coordinates inside the plate)	m/s
$P(x, y)$	Pressure of the perturbation (as a function of x,y coordinates inside the plate)	MPa
x	Horizontal coordinate inside the subducting plate	km
y	Vertical coordinate inside the subducting plate	km
k	Wavenumber of the instability	m^{-1}
λ	Wavelength of the instability	m
$v_x(x, y)$	Horizontal velocity of the subduction plate and the perturbation (as a function of x,y coordinates inside the plate)	m/s
v_{sub}	Subduction rate of the plate	m/s

C_1	Constant from Eq. 16	m/s
C_2	Constant from Eq. 16	m/s
$n(t)$	Time-dependent viscosity	Pa s
$n(0)$	Constant initial viscosity	Pa s
$\sigma_V(0)$	Initial vertical stress that is exerted in the subducting plate	GPa
$\frac{d\varepsilon(0)}{dt}$	Initial strain rate that is exerted in the subducting plate	s^{-1}
q	Growth rate of the Rayleigh-Taylor gravitational instability	m/s^2
$\frac{qk^2}{g}$	Dimensionless growth rate of the Rayleigh-Taylor gravitational instability	-
$\frac{\lambda}{h}$	Ratio of the wavelength of the instability to the thickness of the subducting plate	-
$\frac{\Delta qk^2}{g}$	Dimensionless growth rate difference of the Rayleigh-Taylor gravitational instability between two distinct time moments that were utilised, particularly the 10,000th and the 50,000th year	-
ΔL	Assumed distance from the actual side boundaries of the model	km
B	Constant from Eq. 23	m^{-1}
D	Constant from Eq. 23	m^{-1}
E	Constant from Eq. 23	s^{-1}
F	Constant from Eq. 24	-
H	Constant from Eq. 24	-
J	Constant from Eq. 24	m/s

References (Appendices A, B, C)

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