Flow-dependent large-scale blending for limited-area ensemble assimilation

Saori Nakashita[1], Takeshi Enomoto[1]

1. Disaster Prevention Research Institute, Kyoto University, Japan

nakashita@dpac.dpri.kyoto-u.ac.jp

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Background & Objectives

- A limited-area model (LAM) sometimes degrades the large-scale structures compared to a global model (GM)[1].
- To mitigate the large-scale errors, **large-scale blending (LSB)** has been used in several operational NWP centers[2].
- While flow-dependent background error plays an important role in convective-scale DA, traditional LSB methods often ignore the flow-dependent error of GM large-scales.

This study proposes a novel LSB method within an **ensemble variational (EnVar)** framework to clarify **the impact of flow dependency on LSB**.

Variational large-scale blending methods

3DVar cost function with augmented GM information[1]

$$J(\boldsymbol{x}) = \frac{1}{2}\begin{pmatrix} \boldsymbol{x}^b - \boldsymbol{x} \\ \boldsymbol{y} - H(\boldsymbol{x}) \\ H_1(\boldsymbol{x}^B) - H_2(\boldsymbol{x}) \end{pmatrix}^{\mathrm{T}} \mathbf{W}^{-1} \begin{pmatrix} \boldsymbol{x}^b - \boldsymbol{x} \\ \boldsymbol{y} - H(\boldsymbol{x}) \\ H_1(\boldsymbol{x}^B) - H_2(\boldsymbol{x}) \end{pmatrix}$$

$\boldsymbol{x}^b, \boldsymbol{x}^B$: backgrounds of LAM and GM
$\boldsymbol{y}$: observations
H : observation operator
H_1 : GM ↦ truncated LAM
H_2 : LAM ↦ truncated LAM

$$\mathbf{W} = \begin{bmatrix} \langle \boldsymbol{\varepsilon}^b (\boldsymbol{\varepsilon}^b)^{\mathrm{T}} \rangle & \cancel{\langle \boldsymbol{\varepsilon}^b (\boldsymbol{\varepsilon}^o)^{\mathrm{T}} \rangle} & \cancel{\langle \boldsymbol{\varepsilon}^b (\boldsymbol{\varepsilon}^v)^{\mathrm{T}} \rangle} \\ \cancel{\langle \boldsymbol{\varepsilon}^o (\boldsymbol{\varepsilon}^b)^{\mathrm{T}} \rangle} & \langle \boldsymbol{\varepsilon}^o (\boldsymbol{\varepsilon}^o)^{\mathrm{T}} \rangle & \cancel{\langle \boldsymbol{\varepsilon}^o (\boldsymbol{\varepsilon}^v)^{\mathrm{T}} \rangle} \\ \cancel{\langle \boldsymbol{\varepsilon}^v (\boldsymbol{\varepsilon}^b)^{\mathrm{T}} \rangle} & \cancel{\langle \boldsymbol{\varepsilon}^v (\boldsymbol{\varepsilon}^o)^{\mathrm{T}} \rangle} & \langle \boldsymbol{\varepsilon}^v (\boldsymbol{\varepsilon}^v)^{\mathrm{T}} \rangle \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{B} & 0 & 0 \\ 0 & \mathbf{R} & 0 \\ 0 & 0 & \mathbf{V} \end{bmatrix}$$

$\boldsymbol{\varepsilon}^b = \boldsymbol{x}^b - \boldsymbol{x}^t$ background error
$\boldsymbol{\varepsilon}^o = \boldsymbol{y} - H(\boldsymbol{x}^t)$ observation error
$\boldsymbol{\varepsilon}^v = H_1(\boldsymbol{x}^B) - H_2(\boldsymbol{x}^t)$ large-scale GM error

ignore based on uncorrelated assumptions

ignore based on error statistics[1]

Nested 3DVar[1, 3]: $\mathbf{B}\,(\mathbf{L}^b)$ and $\mathbf{V}\,(\mathbf{L}^v)$ are empirically determined.

$$J(\chi) = \frac{1}{2}\|\chi\|^2 + \frac{1}{2}\|\mathbf{d}^o - \mathbf{H}\mathbf{L}^b\chi\|^2_{\mathbf{R}^{-1}} + \frac{1}{2}\|(\mathbf{L}^v)^{-1}(\mathbf{d}^v - \mathbf{H}_2\mathbf{L}^b\chi)\|^2$$

$\mathbf{B} = \mathbf{L}^b(\mathbf{L}^b)^{\mathrm{T}}$, $\boldsymbol{x} = \boldsymbol{x}^b + \mathbf{L}^b\chi$ — Control variable transformation for efficient minimization[3]

$\mathbf{d}^o = \boldsymbol{y} - H(\boldsymbol{x}^b)$ — Innovation

$\mathbf{V} = \mathbf{L}^v(\mathbf{L}^v)^{\mathrm{T}}$, $\mathbf{d}^v = H_1(\boldsymbol{x}^B) - H_2(\boldsymbol{x}^b)$ — Large-scale innovation

NEW Nested EnVar: $\mathbf{P}^b\,(\mathbf{X}^b)$ and $\mathbf{P}^v\,(\mathbf{Z}^v)$ are dynamically estimated.

$$J(\boldsymbol{w}) = \frac{1}{2}\|\boldsymbol{w}\|^2 + \frac{1}{2}\|\mathbf{d}^o - \mathbf{Y}^b\boldsymbol{w}\|^2_{\mathbf{R}^{-1}} + \frac{1}{2}\|\underbrace{(\mathbf{Z}^v)^{\dagger}}_{\text{Pseudoinverse}}(\mathbf{d}^v - \mathbf{Z}^b\boldsymbol{w})\|^2$$

$$\mathbf{X}^b = \frac{1}{\sqrt{K-1}}[\boldsymbol{x}^b_k - \overline{\boldsymbol{x}^b}]_{k=1,\cdots,K} \quad \mathbf{P}^b = \mathbf{X}^b(\mathbf{X}^b)^{\mathrm{T}} \quad \boldsymbol{x} = \overline{\boldsymbol{x}^b} + \mathbf{X}^b\boldsymbol{w}$$

$$\mathbf{Y}^b = \frac{1}{\sqrt{K-1}}[H(\boldsymbol{x}^b_k) - \overline{H(\boldsymbol{x}^b)}]_{k=1,\cdots,K}$$

K: Ensemble size

$$\mathbf{Z}^v = \frac{1}{\sqrt{K-1}}[H_1(\boldsymbol{x}^B_k) - \overline{H_1(\boldsymbol{x}^B)}]_{k=1,\cdots,K} \quad \mathbf{P}^v = \mathbf{Z}^v(\mathbf{Z}^v)^{\mathrm{T}} \quad \mathbf{Z}^b = \frac{1}{\sqrt{K-1}}[H_2(\boldsymbol{x}^b_k) - \overline{H_2(\boldsymbol{x}^b)}]_{k=1,\cdots,K}$$

Ensemble update[4] $\mathbf{X}^a = \mathbf{X}^b[\nabla^2_{\boldsymbol{w}}J]^{-\frac{1}{2}}$ $\nabla^2_{\boldsymbol{w}}J = \mathbf{I} + (\mathbf{Y}^b)^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y}^b + [(\mathbf{Z}^v)^{\dagger}\mathbf{Z}^b]^{\mathrm{T}}(\mathbf{Z}^v)^{\dagger}\mathbf{Z}^b$

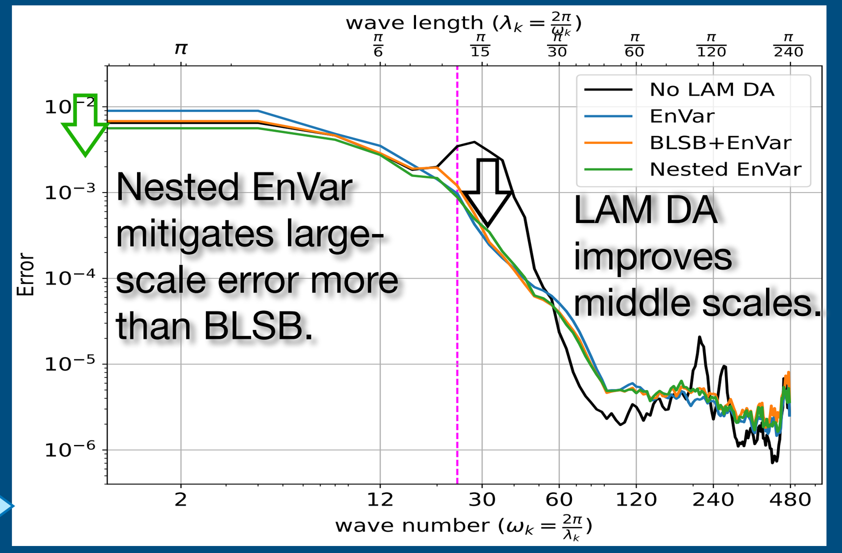
[1] Guided and Fischer 2008: *QJRMS*, 134, 723–735.
[2] Milan et al. 2023: *QJRMS*, 149, 2067–2090.
[3] Dahlgren and Gustafsson 2012: *Tellus A*, 64, 15836.
[4] Zupanski 2005: *MWR*, 133, 1710–1726.
[5] Lorenz 2005: *JAS*, 62, 1574–1587.
[6] Kretschmer et al. 2015: *Tellus A*, 67, 26495.
[7] Davies 1976: *QJRMS*, 102, 405–418.
[8] Denis et al. 2002: *MWR*, 130, 1812–1829.
[9] Sakov and Oke 2008: *Tellus A*, 60, 361–371.
[10] Farchi and Bocquet 2019: *FAMS*, 5(3).
[11] Whitaker and Hamill 2012: *MWR*, 140, 3078–3089.

Nested EnVar has the potential to enhance the effectiveness of high-resolution LAM DA for spatially localized observations.

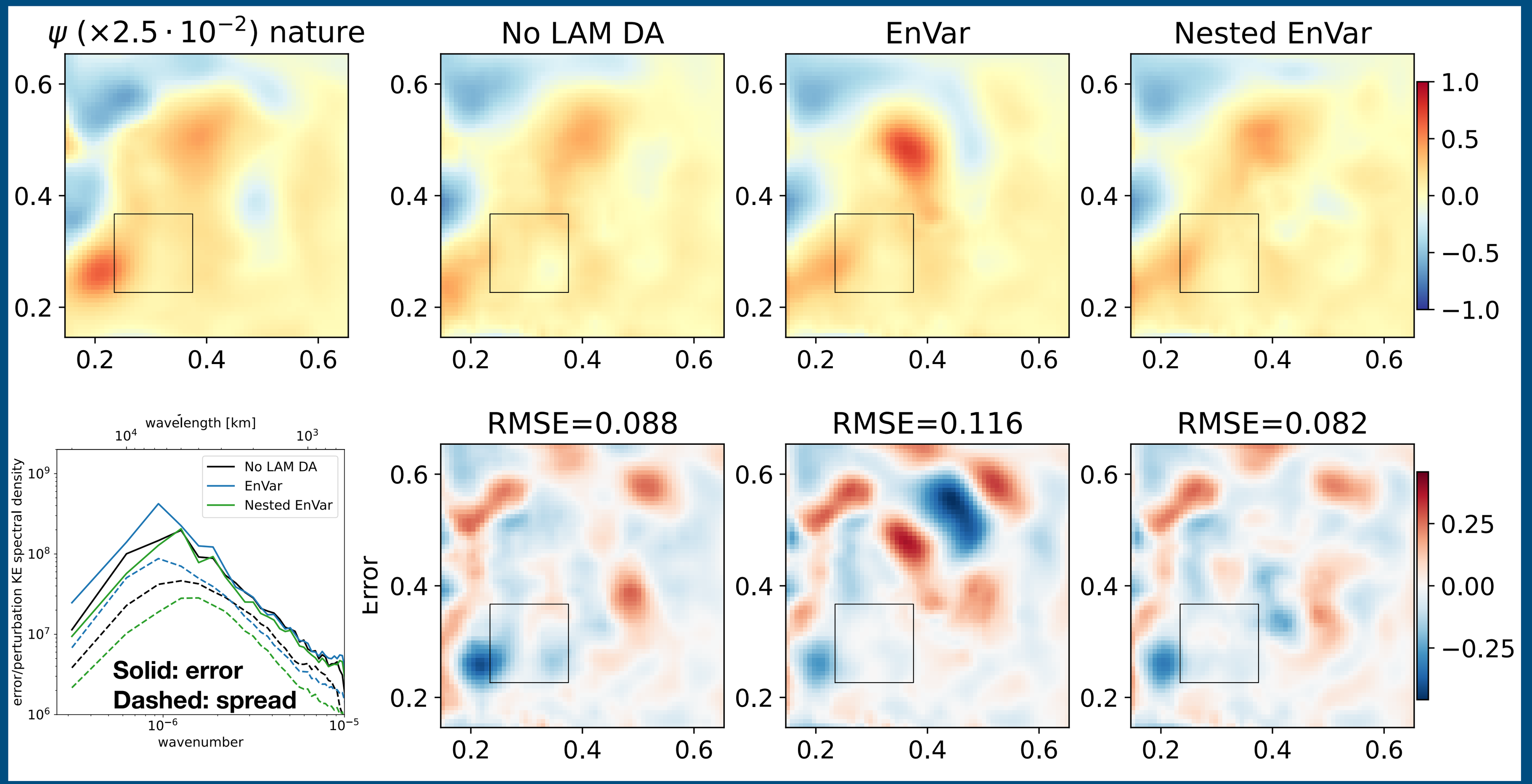
Time averaged analysis RMSE of nested Lorenz OSSE
Degrade or improve relative to No LAM DA (0.334)

observation (points)	3DVar	EnVar	BLSB+ 3DVar	BLSB+ EnVar	Nested 3DVar	Nested EnVar
uniform (7)	0.203	0.014	0.048	**-0.040**	-0.007	-0.024
dense, left (30)	2.95	0.328	-0.003	-0.039	-0.011	**-0.047**
dense, center (30)	0.99	0.037	-0.007	-0.049	-0.021	**-0.061**
dense, right (30)	3.00	0.209	-0.006	**-0.043**	-0.021	-0.027
dense, moving (30)	0.318	-0.019	-0.005	-0.051	-0.023	**-0.076**

Time averaged analysis error spectra with dense moving observations



Snapshot of stream function in the LAM domain at the final cycle of nested QG OSSE



NP1.1 EGU25-16805 X3.41 Attendance: 28 Apr, 10:45–12:30

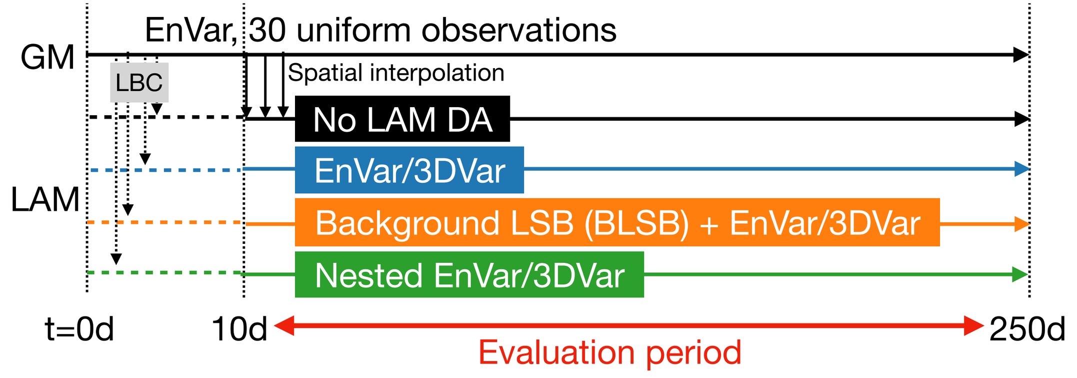
OSSE 1: nested Lorenz system[5, 6]

Nature: Lorenz III $(2\pi,\ \Delta x_N = 2\pi/960)$

GM: Lorenz II $(2\pi,\ \Delta x_G = 4\Delta x_N)$

LAM: Lorenz III $(\pi/2,\ \Delta x_L = \Delta x_N)$

- LBC: Davies[7] relaxation with 10-grid sponge regions
- Lorenz II & III are modified to contain multiple wavelengths.



- EnVar: 80 member without localization
- H: linear, $\mathbf{R} = \mathbf{I}$
- $H_{1,2}$: truncation at k=12 with DCT[8]
- 5% multiplicative inflation for LAM EnVar except for Nested EnVar (25%)

Background LSB[2]

$$\boldsymbol{x}^{\mathrm{bld}} = \boldsymbol{x}^b + H_1'(\boldsymbol{x}^B) - H_2'(\boldsymbol{x}^b)$$

H_1', H_2': low-pass filtering for GM and LAM

OSSE 2: nested QG system

Non-dimensional 1.5-layer reduced-gravity QG model with double-gyre wind forcing[9]

$$\partial_t q = -\partial_x\psi - \epsilon J(\psi, \nabla^2\psi) + A\nabla^6\psi + W_E$$
$$q = \nabla^2\psi - F\psi$$

$F = 1600,\ \epsilon = 10^{-5},\ A = 2\times10^{-12},\ W_E = 2\pi\sin(2\pi y)$

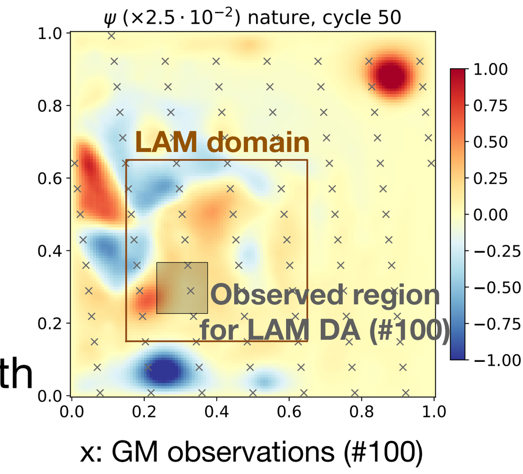
Domain: $0 \le x, y \le 1$

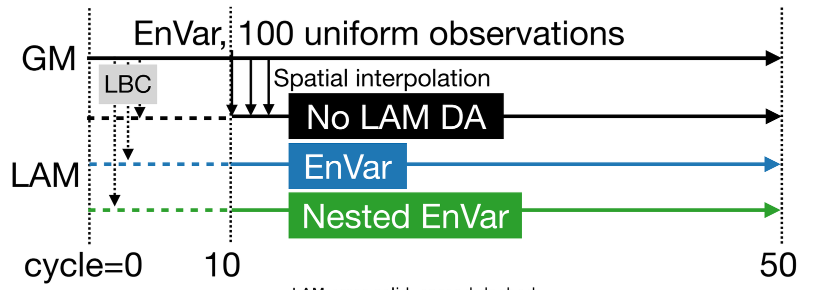
Nature: $\Delta x_N = \Delta y_N = \frac{1}{128}$

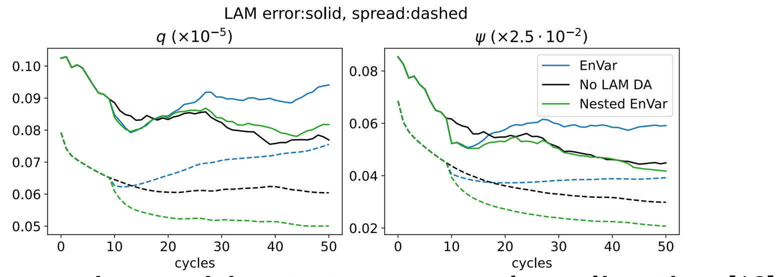
GM: $\Delta x_G = \Delta y_G = 2\Delta x_N$

LAM: $\Delta x_L = \Delta y_L = \Delta x_N$, $0.15 \le x, y \le 0.65$

LBC: Davies relaxation with exponential smoothing







- 60 member with state-space localization[10]
- H: observed ψ, linear, $\mathbf{R} = 4.0\,\mathbf{I}$
- $H_{1,2}$: truncation at k=4
- No inflation for LAM EnVar
- RTPS[11] with 50% for Nested EnVar

Summary & Discussion

- Dynamical blending of Nested EnVar improves analysis across scales by reducing large-scale errors while retaining DA effects on middle scales when dealing with dense and unevenly distributed observations.
- Simultaneous assimilation of large-scale and observational information by variational LSB is more beneficial than background LSB with severe large-scale degradation.
- Scale-dependent covariance inflation should be introduced for more effective performance of Nested EnVar to mitigate the underestimation of the ensemble spread.

Reference

Nakashita, S. and Enomoto, T. 2025: *Tellus A*, 77(1), 1–19. DOI:10.16993/tellusa.4089

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