A new Bayesian approach to the inverse modelling of modern sea level change

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Aim

- Estimates of mean sea level change in the 20th and 21st centuries are important for monitoring the effects of climate change
- In particular, there is increasing interest in attributing the relative contributions to observed sea level from surface mass change both globally and in specific regions
- Here we present a new method for obtaining such quantitative inferences from combinations of satellite gravity, satellite altimetry, and tide gauge data.
- Our Bayesian approach uses an (effectively) infinite dimensional model space, which allows for realistic priors and comprehensive treatment of uncertainties
- As part of this work, open source python libraries are being developed for sea level modelling and the solution of Bayesian problems within a function space setting

Mathematical background

Operator Formulation
A is an operator which determines how the Earth responds to a surface mass change:

$$A\zeta = (SL, \mathrm{u}, \phi, \boldsymbol{\omega}).$$

The operator P takes these response variables and maps them to some observable:

$$P(SL, \mathrm{u}, \phi, \boldsymbol{\omega}) = v$$

Composed together, $B\zeta = PA\zeta = v$

C is an operator which averages or sums the surface mass change over particular areas of interest. For a set of averaging functions, we have:

$$C\zeta = w$$

Bayesian Solution
Employing the above operators, we can form a pair of jointly distributed random variables:

$$v = Bu + z, \quad w = Cu.$$

Using the properties of Gaussian measures, it follows that

$$\begin{pmatrix} v \\ w \end{pmatrix} \sim \mathcal{N}_{F\oplus G}\left(\begin{pmatrix} B\overline{u} \\ C\overline{u} \end{pmatrix}, \begin{pmatrix} BQB^* + R & BQC^* \\ CQB^* & CQC^* \end{pmatrix}\right)$$

Our inference problem is posed such that we want to estimate w from a measurement of v. It can be shown that

$$w|v \sim \mathcal{N}_G(\overline{w}, S)\,,$$

with
$$\overline{w} = CQB^*(BQB^* + R)^{-1}(v - B\overline{u}),$$
$$S = C[Q - QB(BQB^* + R)^{-1}B^*Q]C^*.$$

Data Sources

Satellite gravity

$$P\zeta = \begin{pmatrix} \phi_{2-2} \\ \phi_{2-1} \\ \vdots \\ \phi_{LL} \end{pmatrix}$$

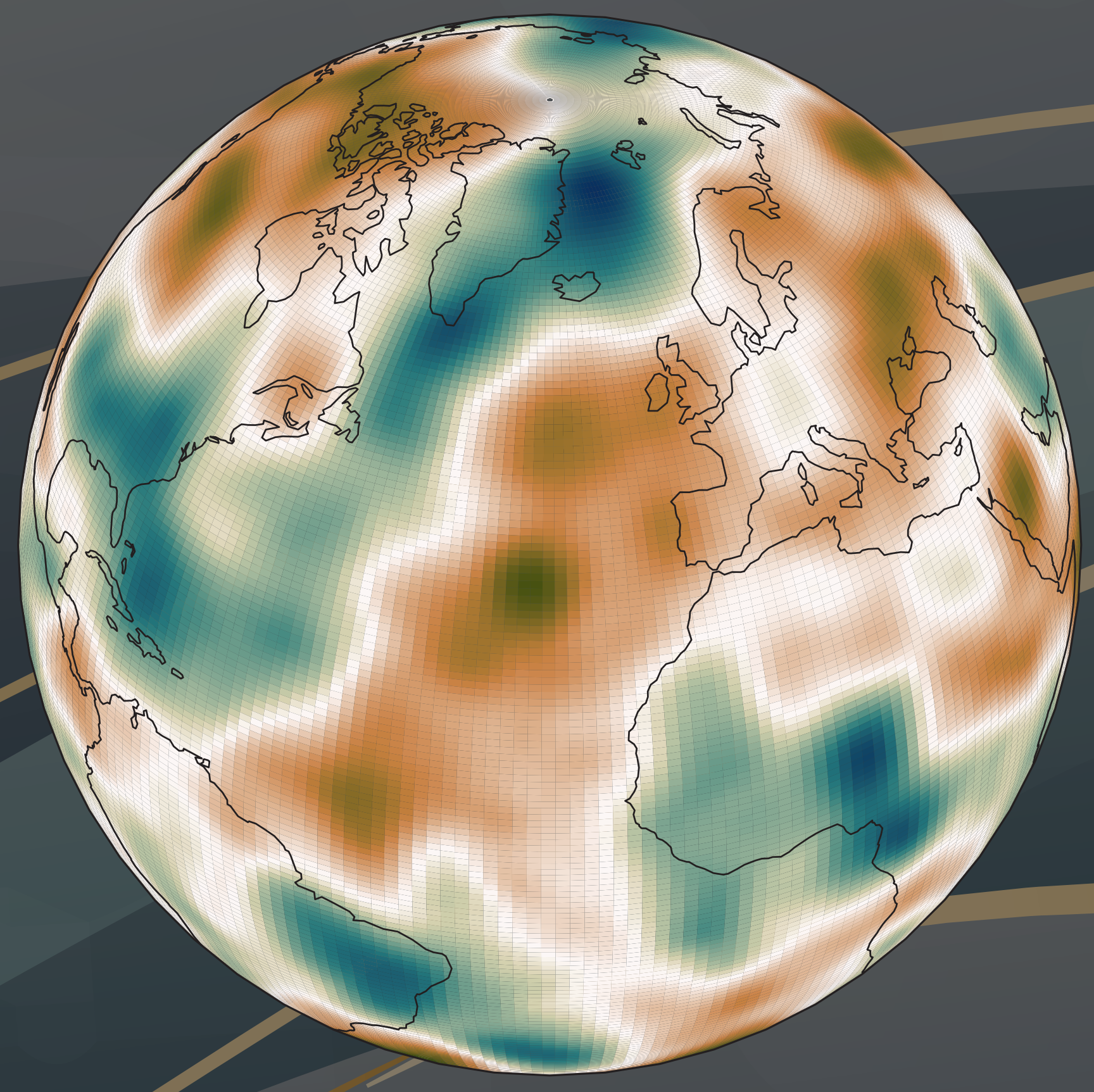
Tide gauge data

$$P\zeta = \begin{pmatrix} SL_0 \\ SL_1 \\ \vdots \\ SL_n \end{pmatrix}$$

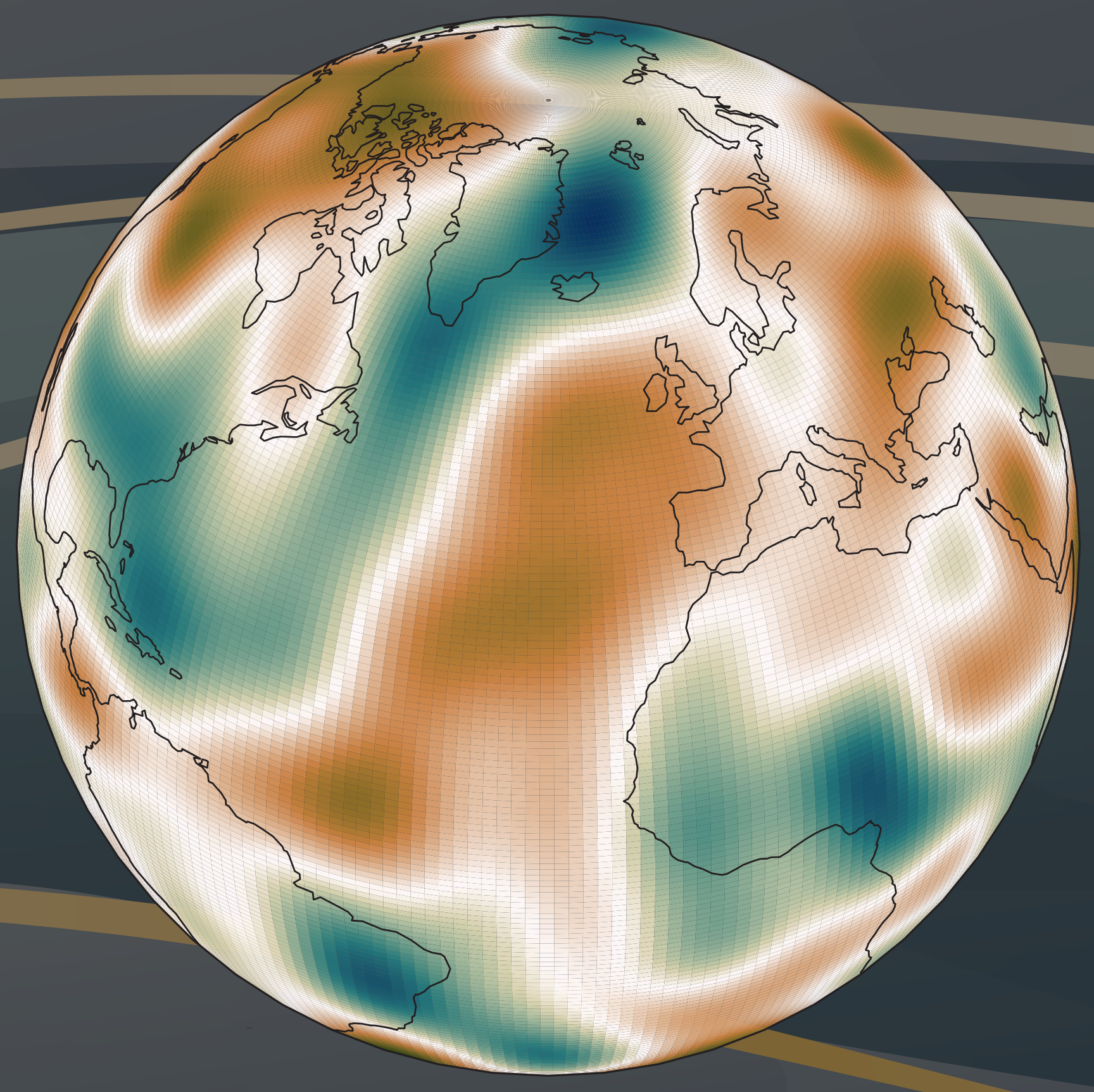
Satellite altimetery

$$P\zeta = \begin{pmatrix} SSH_{00} \\ SSH_{01} \\ \vdots \\ SSH_{nm} \end{pmatrix}$$

Bayesian Inversion

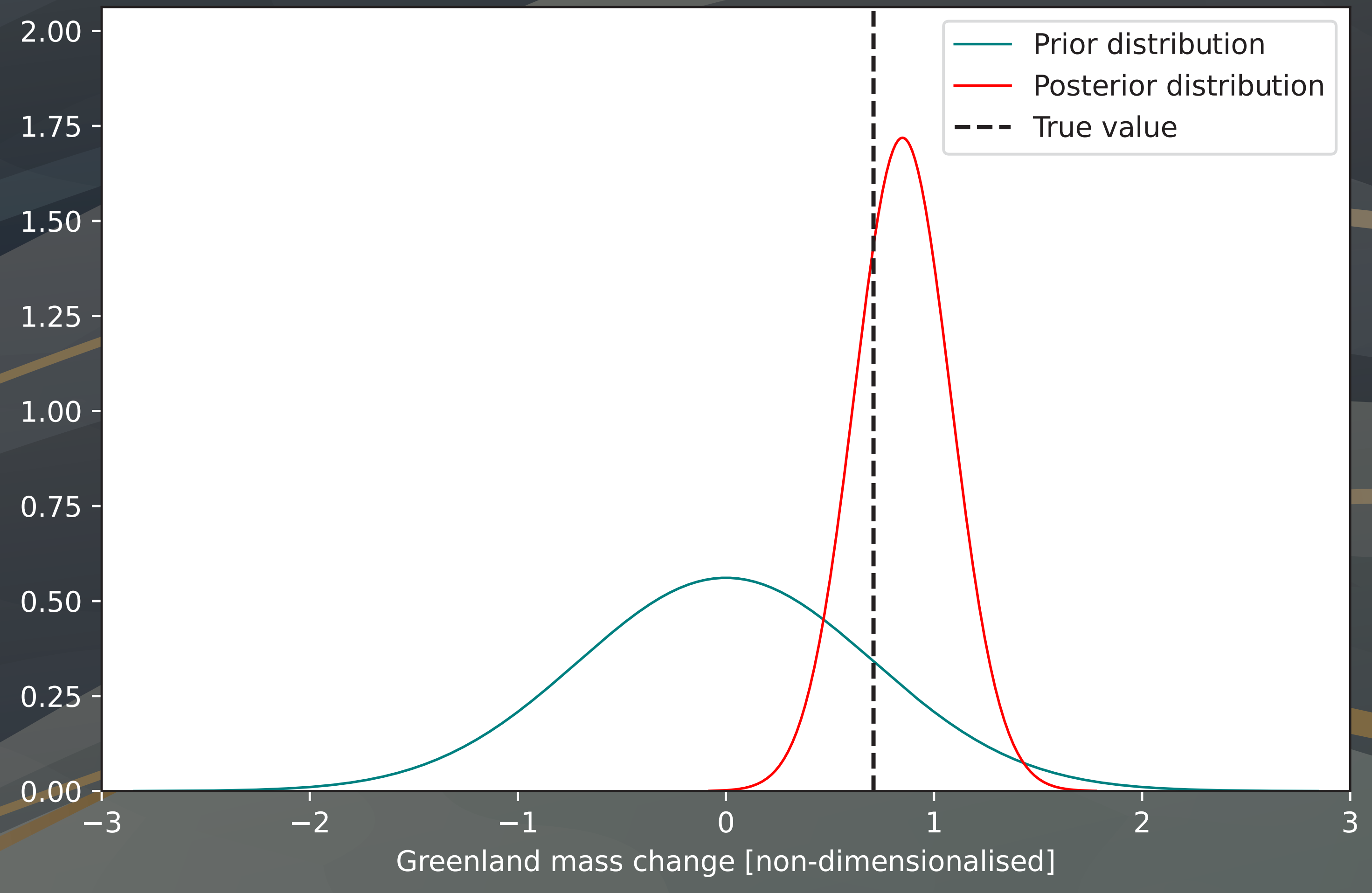


Surface mass change - True model



Surface mass change - Posterior mean

Bayesian Inference

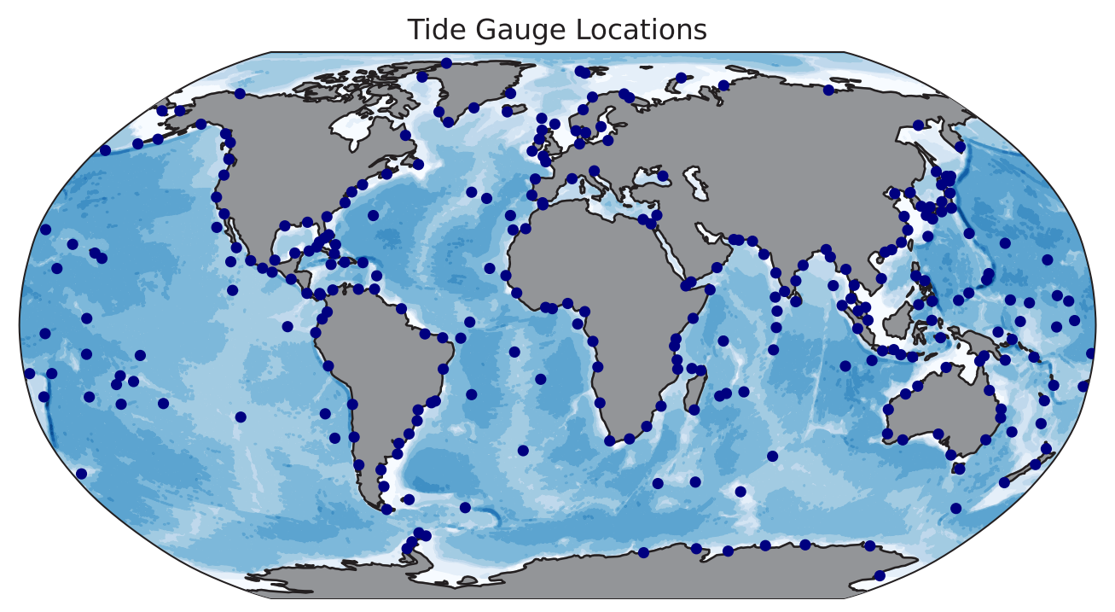


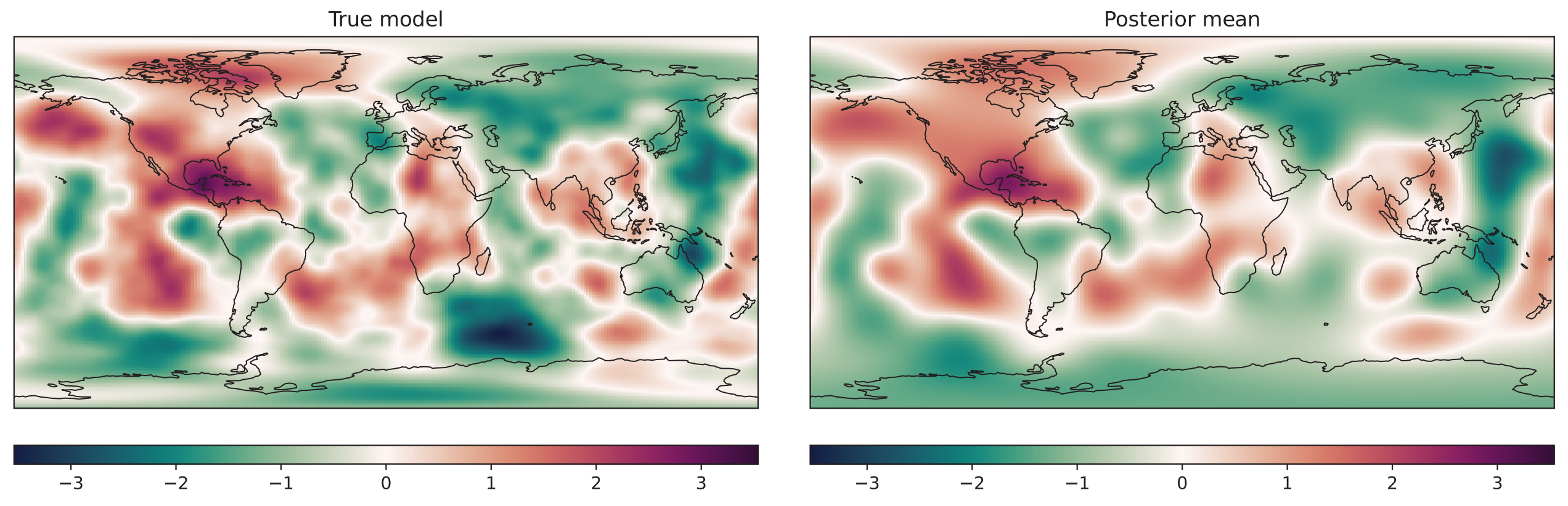
Inversion/inference using synthetic satellite gravity data.

Truncation degrees:
Model space L = 128
Data space L = 20

Using different observations

The example shown to the left demonstrates our method using satellite gravity data. We can easily adjust to other data sources, however, by changing the operator P. Here we show an inversion using tide gauge data from 290 different locations.





Model space spherical harmonic truncation degree: L = 128

To extend this, we can also combine multiple data sources into one inversion, by giving P a block matrix form.

Choosing the prior

A very high dimensional model space facilitates the use of realistic priors. Here we show inversion results for three different priors - two rotationally invariant and one geographically informed. The sensitivity of the result to the choice of prior depends on the number of data points being observed; a higher dimension data space results in lower prior sensitivity.