Retrieving climate proxies from an Invertible Glacier Evolution Model (IGEM)

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Background & Motivation

Glacier footprints provide valuable potential for paleo-climate reconstructions through inversions problems. However, existing inversion methods remain constrained by computational cost, rigid assumptions, and **lack of gradient information**. We introduce IGEM, an invertible glacier model based on the SIA and PDD surface mass balance. Featuring a tensor-based design, **IGEM efficiently computes gradients** with respect to climate inputs like temperature and precipitation, enabling accurate and flexible climate reconstructions from glacier geometries.

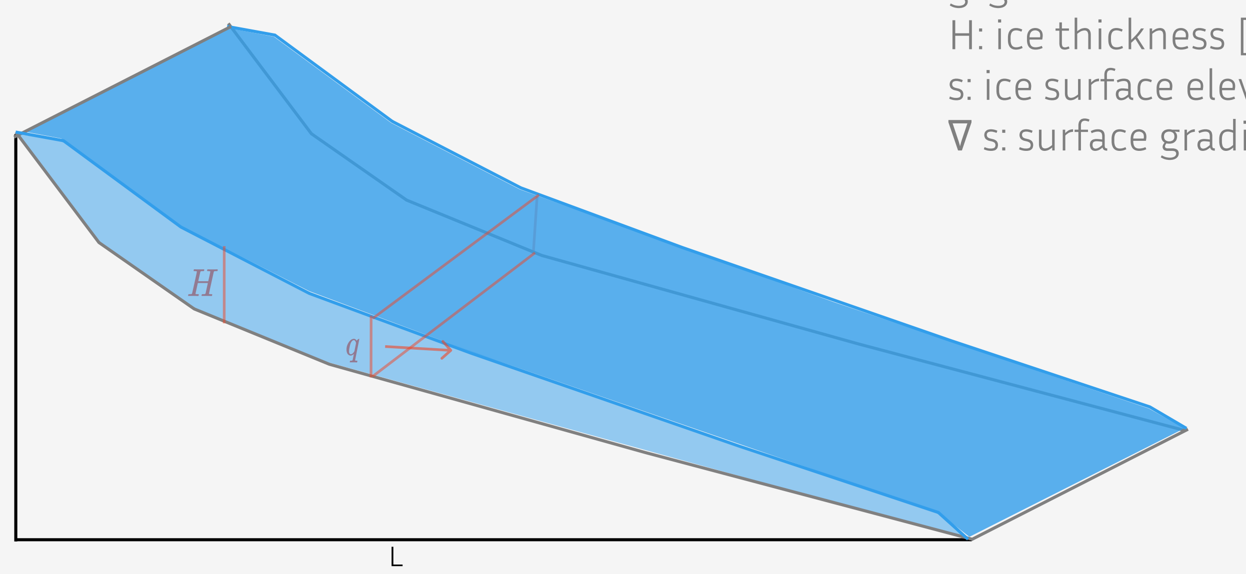
Forward Modelling

$$\frac{\partial H}{\partial t} + \nabla \cdot \mathbf{q} = smb$$

$$\mathbf{q} = f_d(\rho g)^n H^{n+2} |\nabla s|^{n-1} \nabla s$$

Shallow Ice Approximation (SIA) assumptions : $\epsilon = \dfrac{H}{L} << 1$

Mass conservation
H: ice thickness [m]
t: time [a]
smb: surface mass balance [m/a]
q: horizontal ice flux [m^2/a]
A: flow rate factor [Pa^{-n} s^{-1}]
n: Glen's flow law exponent (= 3)
ρ: ice density [kg/m^3]
g: gravitational acceleration [m/s^2]
H: ice thickness [m]
s: ice surface elevation [m]
∇ s: surface gradient (slope)



Climate Input/ SMB Forcing

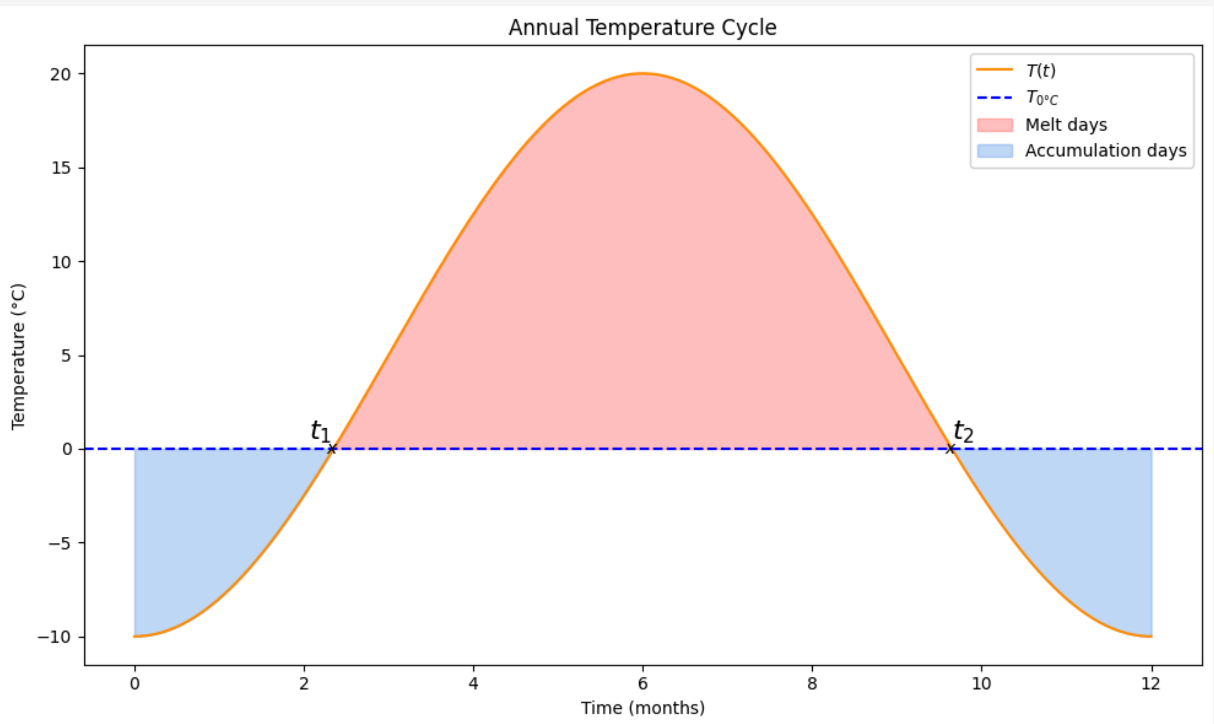
smb= accumulation- ablation

$$T(t) = T_{ma} - (T_{mj} - T_{ma})\cos\left(\frac{2\pi t}{A}\right)$$

acc= Precip * Neg_temp_rat

abl = melt_f * $\int_{T>0} T(t)\, dt$

Neg_temp_rat: days with negative temperature /total no. of days in a year
T_{ma}: mean temperature in one year
T_{mj}: mean temperature on the hottest month (June)
Percip: Precipitation [m/year]

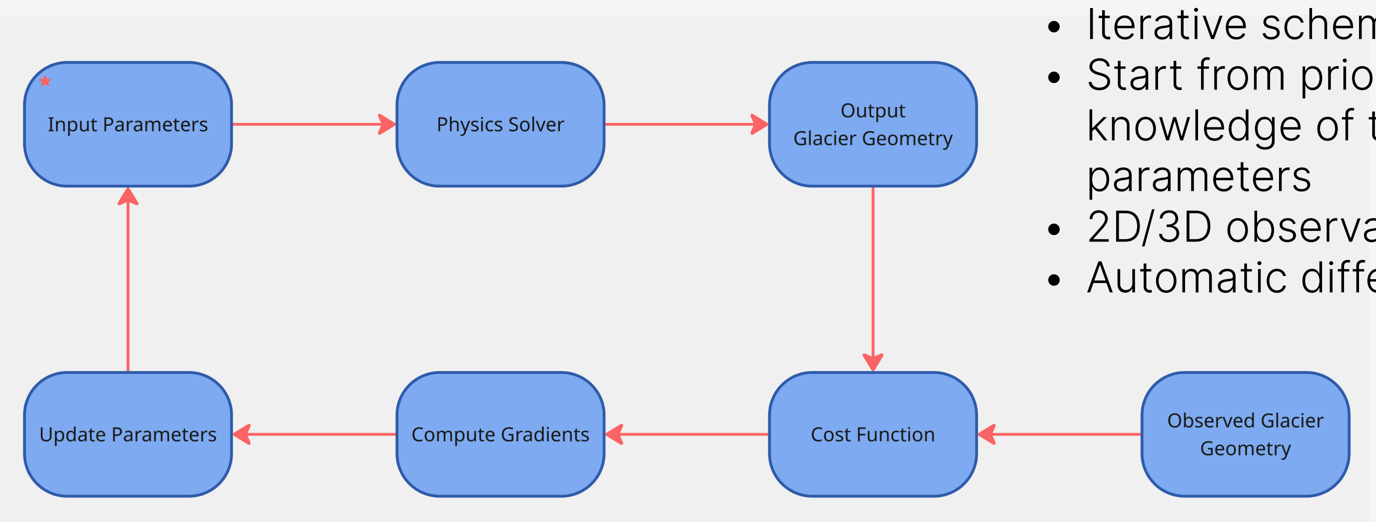


$$t_1 = \frac{A}{2\pi} \arccos\left(\frac{T_{ma}}{T_{mj} - T_{ma}}\right)$$

$$t_2 = A - t_1$$

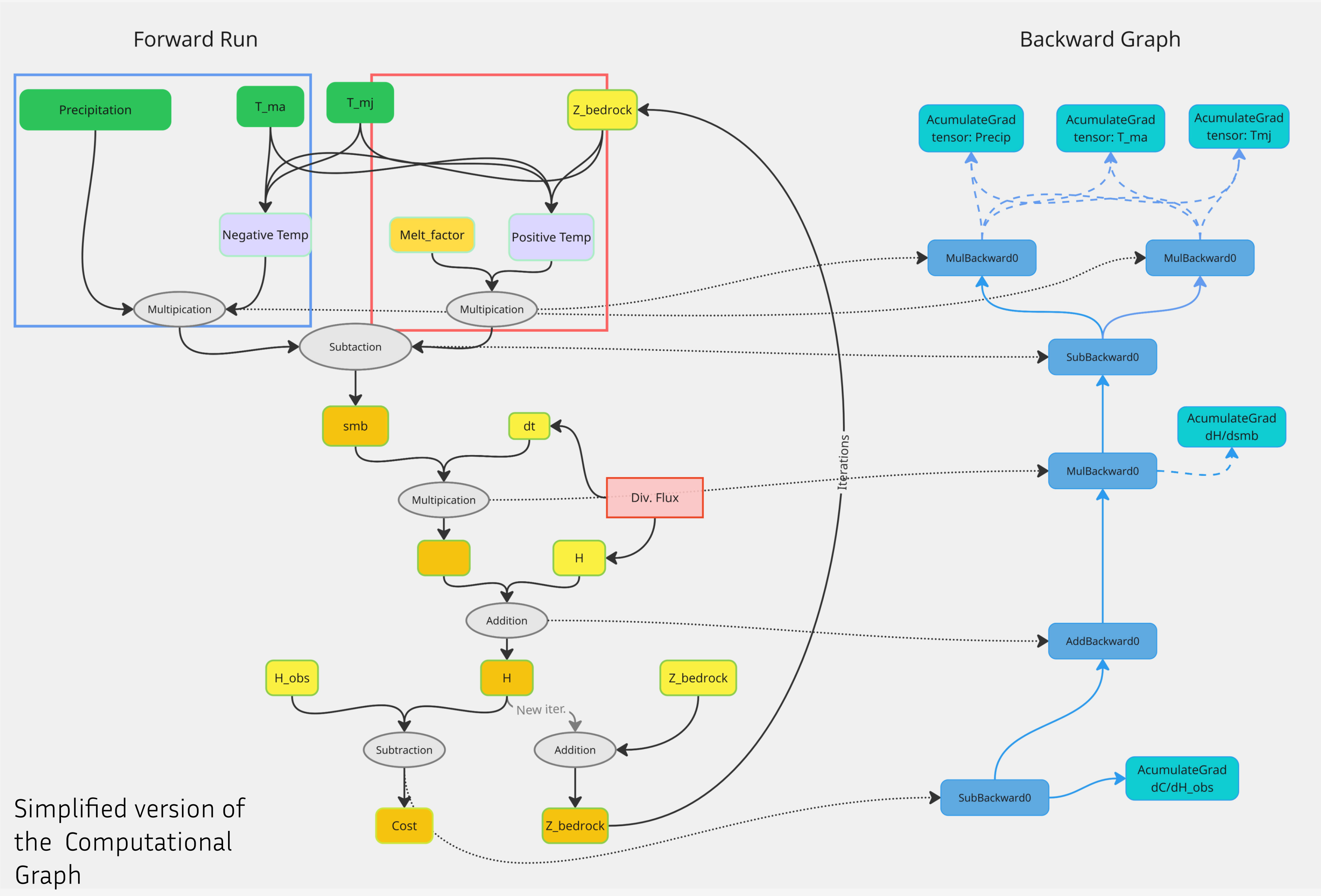
Inversion scheme

Overview



- Iterative scheme
- Start from prior knowledge of the parameters
- 2D/3D observations
- Automatic differentiation

Computation Graph/ Automatic Differentiation



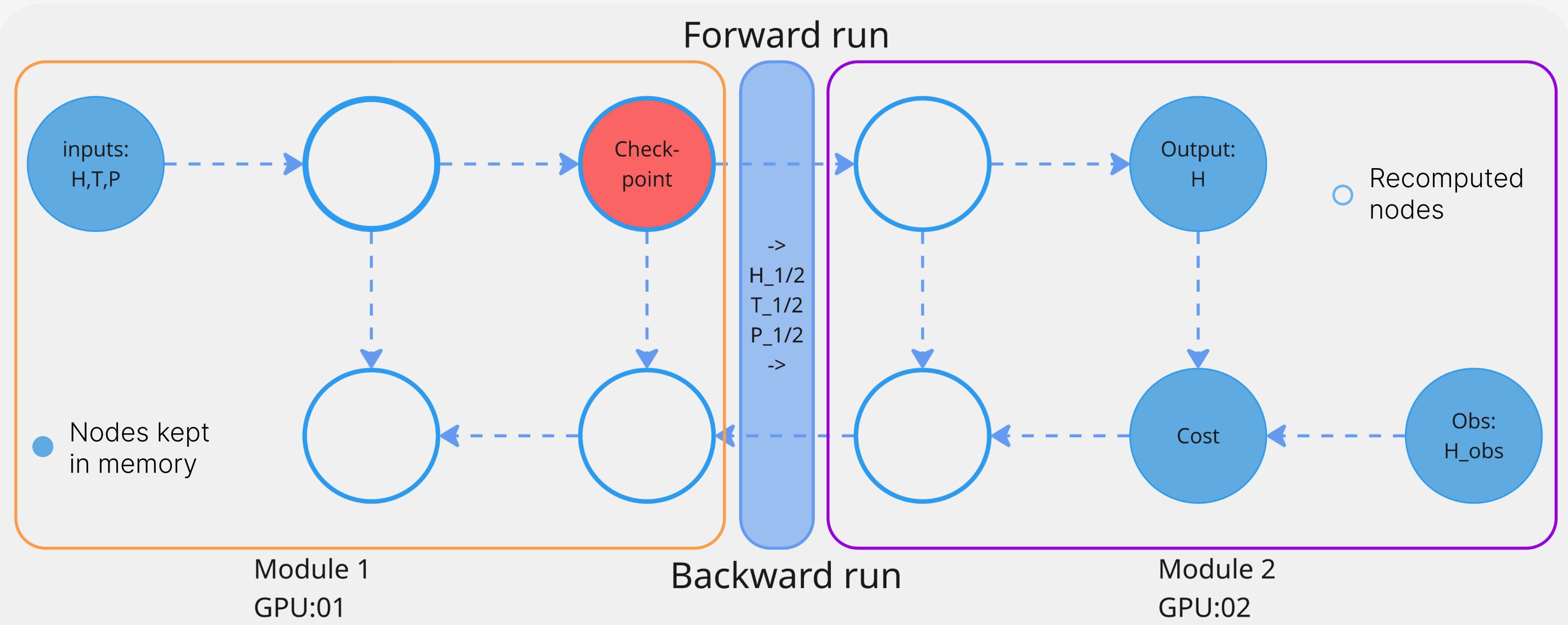
Challenges vs Solutions

Main Challenges :

1. Memory intensive
2. Exploding/vanishing gradients

Solutions :

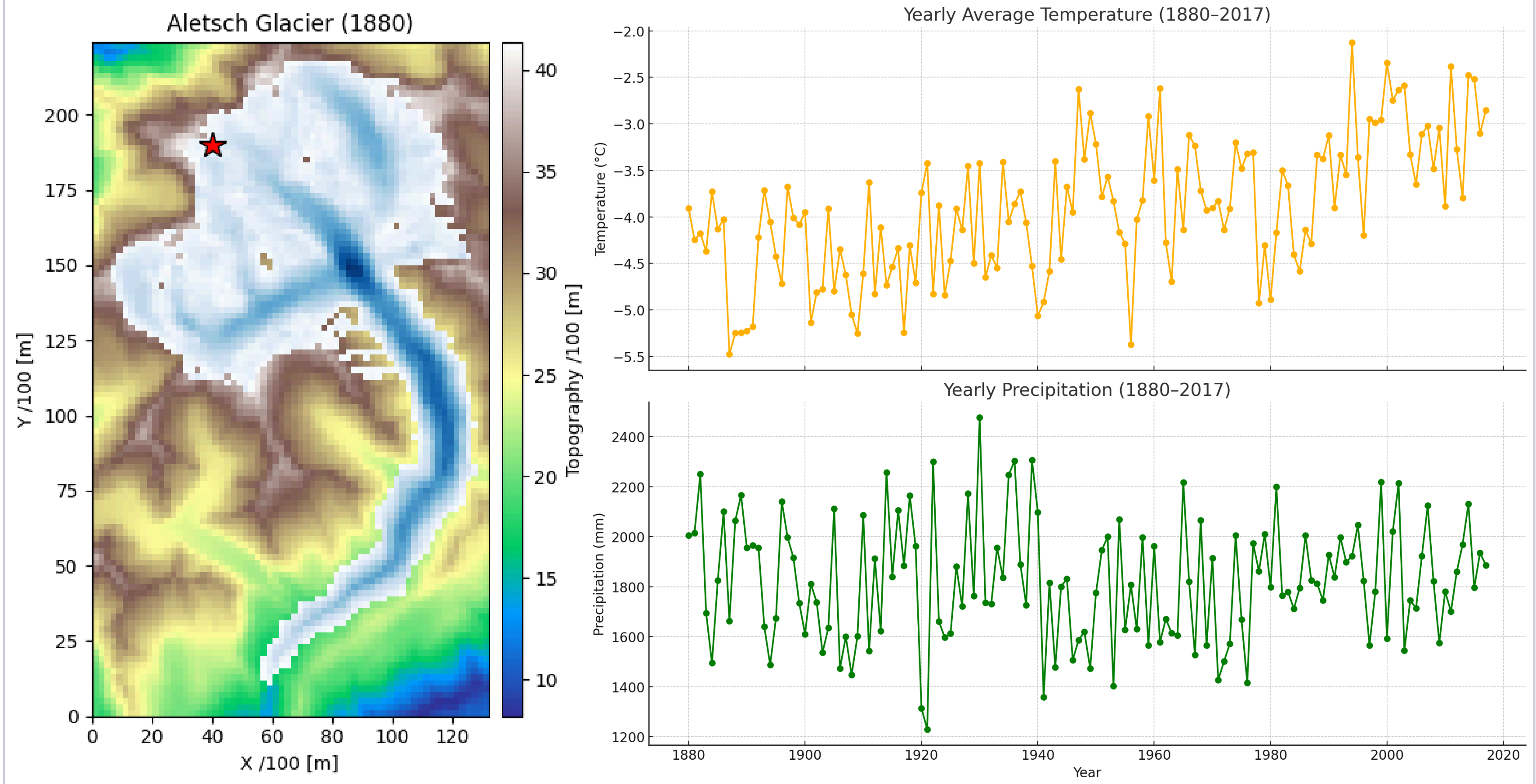
1 a) Check-pointing the forward run
1 b) Using multiple GPUs
2 a) Breaking and manipulating gradients.
2 b) Gradient hooks



Application

Set up

The Little Ice Age (1300–1850) brought marked glacial advance in the Alps, with the **Aletsch Glacier** reaching its maximum extent around 1850. Since 1880, its retreat has been reconstructed using historical records. A weather station near **Jungfraujoch** provides key data on temperature and precipitation.



Results