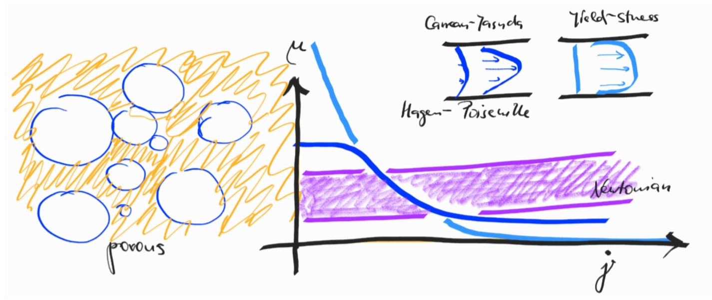

Towards functional characterization of soil pore size distribution using shear-thinning fluids: challenges and prospects

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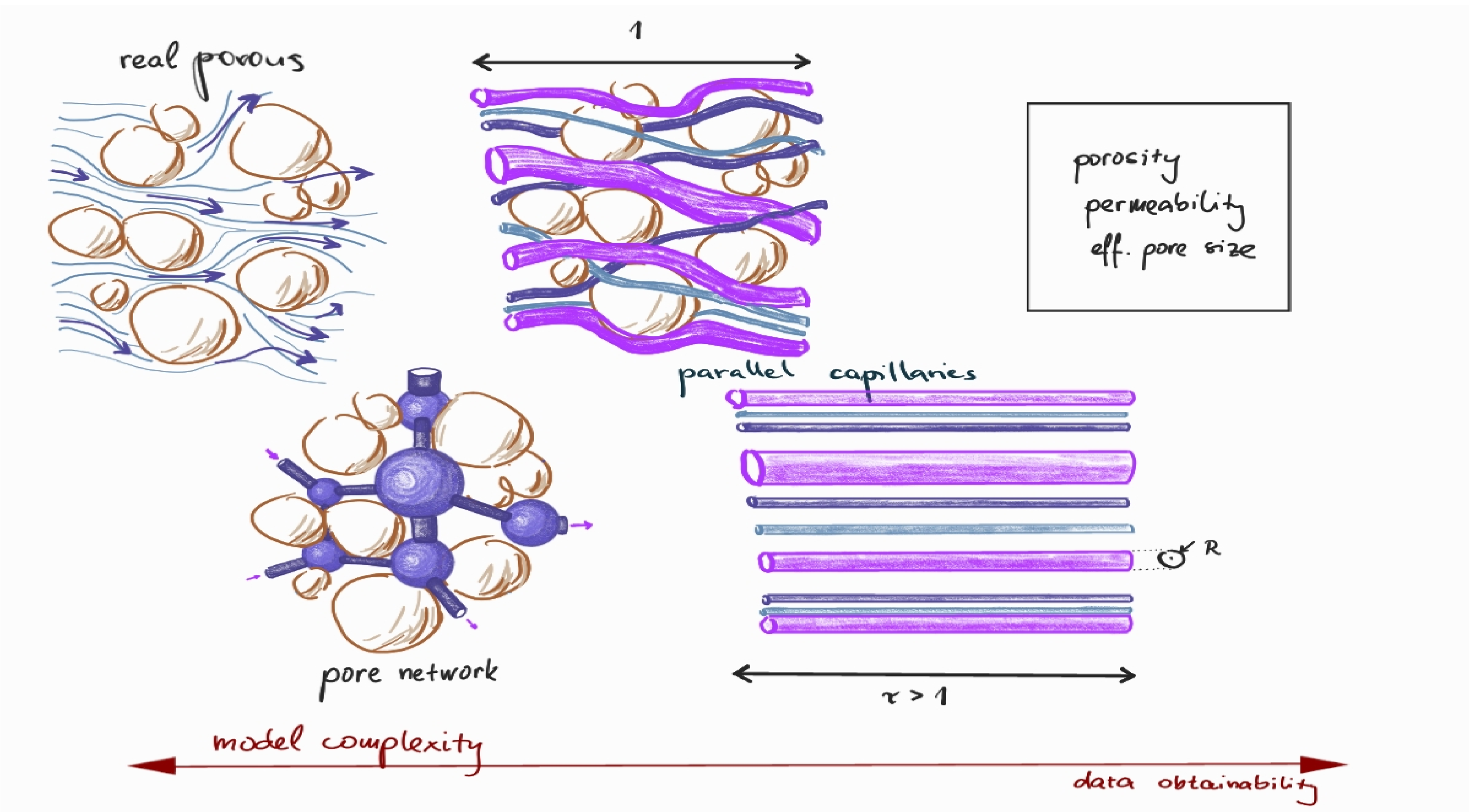
A: Hierarchy of models: idealised porous media

The pore space geometry can be represented in various levels of complexity, ranging from single valued material parameters (porosity, permeability, effective pore size,...) to the complete three-dimensional geometry.



Our ability to acquire model parameters decreases with increasing model complexity. On the other hand, the pore space geometry resolution allows to determine many properties of soils, including saturated and unsaturated hydraulic, transport and heat transport characteristics. We need to study models of favourable complexity, such that the data are obtainable by measurement.

B: Parallel straight capillaries with simple non-Newtonian fluids

Capillary bundle model is in general defined by functional pore-size distribution (fPSD), determined by the saturated hydraulic conductivity of pores), including the functional porosity and tortuosity. Here, the model is used to describe the flow of non-Newtonian (shear-thinning and/or yield-stress) fluids:

$$v(c,P') = \int_0^\infty q(c,P',\hat{R})\,\mathrm{d}W(\hat{R}) \approx \sum_{j=1}^{M} q(c,P',\hat{R}_j)\,w_j, \tag{1}$$

(With v total flux; c fluid rheology; P' total pressure gradient; q partial flux through pores of size R; $\mathrm{d}W$ or w_j functional PSD. It can be shown that the tortuosity can be "hidden" in the pore size by defining $\hat{R} = R/\tau$.)

Major feature: The distribution of total flux into pores of different size may differ with rheology and/or pressure gradient. That can be used to assess fPSD from observed total fluxes, using ANA, YSM or analogous methods...

Missing / idealised / unclear: Fluid pathways cannot change with rheology (in real medium or pore-network model, they can). Does the model represent the real medium? (When and how well?) How exactly is fPSD related to other notions of PSD? How is porosity?

C: Shear-thinning / yield-stress fluids

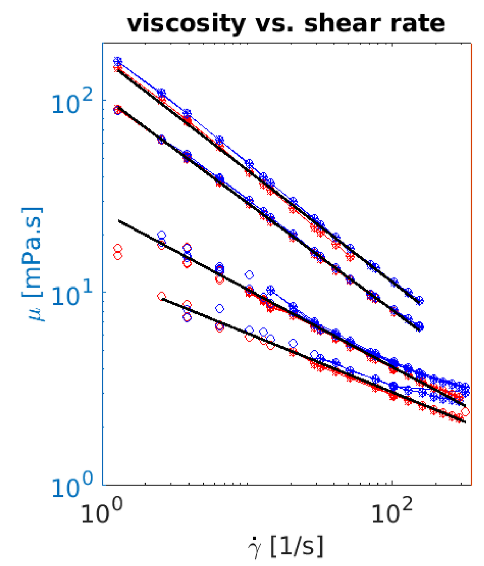
Simple example: power-law shear-thinning fluids.

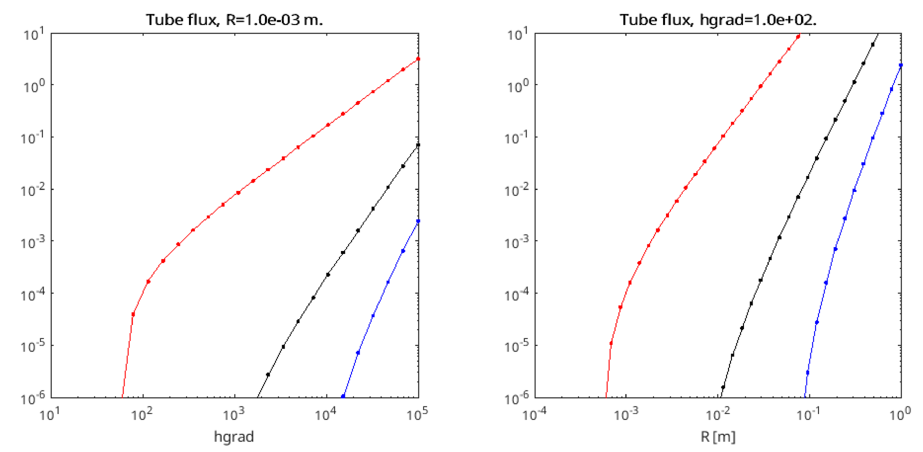
Aqueous solutions of xanthan gum (here, of concentrations 0.3, 0.4, 0.6 and 0.8 g/l) have been used for the hydraulic conductivity experiments. Their viscosity (measured on low-viscosity rotational viscometer at shear-rates $\dot\gamma$ of approx. 1–$250\,\mathrm{s}^{-1}$) follow the power-law:

$$\mu = \mu_0 \dot\gamma^{n-1} \qquad \text{with} \qquad 0 < n < 1,\ \mu_0 > 0.$$

The corresponding partial flux (through simple cylindrical pores) is then:

$$q(n,P',R) \sim R^{2+\nu}\,P'^{1+\nu} \qquad \text{with} \qquad \nu = \tfrac{1}{n} - 1 > 0.$$





Simple yield-stress fluids: Herschel–Bulkley fluids.

With larger concentrations of the polymer, yield-stress behaviour is observed. Herchel–Bulkley model reads (simplified):

$$\begin{aligned} \tau &= \tau_0 + \mu_0 \dot\gamma^n, \\ \dot\gamma &= 0 \text{ for } \tau < \tau_0, \end{aligned} \qquad \text{with} \qquad 0 < n < 1,\ \mu_0, \tau_0 > 0.$$

The corresponding partial flux is then:

$$q(\ldots,P',R) = \begin{cases} \sim R^{2+\nu}\,P'^{1+\nu} & \text{for } RP' \gg 2\tau_0, \\ 0 & \text{for } RP' \le 2\tau_0. \end{cases}$$

D: Functional PSD approximation inversion problem

Various methods have been introduced for approximating the fPSD based on measured saturated flows.

For illustration, the ANA method, see [1, 2]. has been originally developed for using a set of power-law fluids: With each measured power-law fluid with index n (with $\nu = 1/n - 1$), we observe from each experiment

$$v = C_{\mu_0,n} P'^{1+\nu} \int_0^\infty \hat{R}^{2+\nu}\,\mathrm{d}W(\hat{R}) \approx C_{\mu_0,n} P'^{1+\nu} \sum_{j=1}^{M} \hat{R}_j^{2+\nu} w_j. \tag{2}$$

Using a number of distinct observations, we can obtain a number of R_j, w_j values by solving a discrete inverse problem.

By contrast, the yield-stress method (YSM) requires only one Herschel–Bulkley fluid, but a whole range of pressure gradients, see [3, 4].

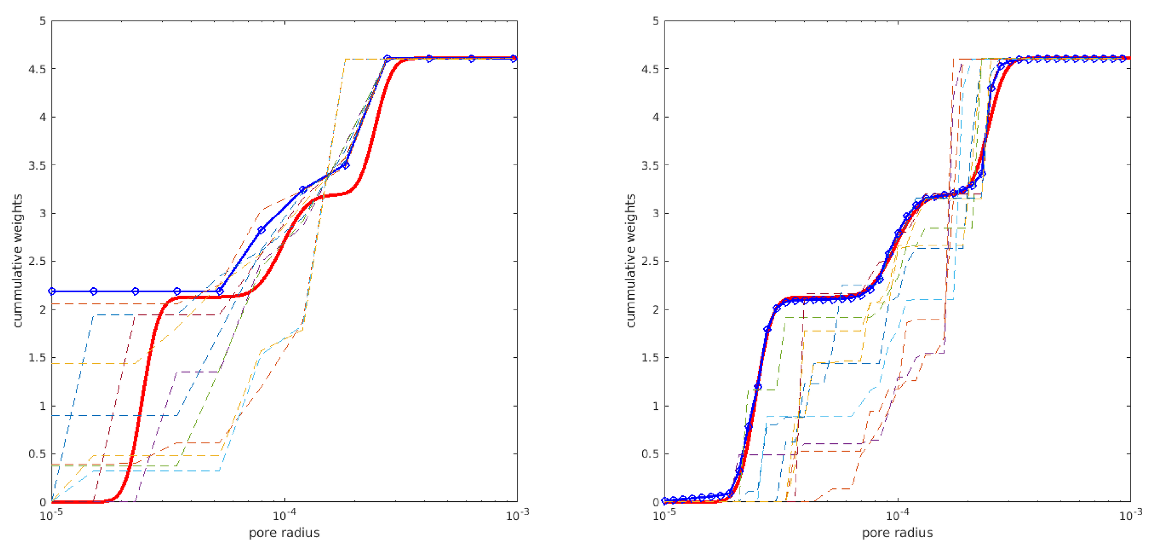
I1: Sensitivity of the inverse problem to measurement errors

(Not surprisingly) the inverse problem can sometimes be very sensitive to observation errors.

For illustration, here we compute the inversion of artificial data with as low as 1 % noise!

How to best analyse this issue? So far, we address this by numerical experiments.

In particular, how to choose the set of experiments for optimal robustness?



I2: Polymer entrapment

In comparison to other techniques, non-Newtonian porosimetry is quite non-invasive. Yet, the effect of polymer entrapment may pose constraints to the ANA method, since:

- A sequence of distinct fluids needs to pass through the porous sample.
- Fluid replacement is particularly demanding at low hydraulic gradients.
- Gradual clogging of the pores implies that a slightly different porous medium is tested with each fluid.

E: Radial flows

Motivated by the prospect of borehole testing, we study the possible generalisation of the method to isotropic radial flows.

For given total discharge Q, the specific flux decreases with the radial coordinate r as $v = Q/(2\pi r)$ and we can observe the pressure drop ΔP between two radial positions $0 < r_1 < r_2$:

$$\Delta P(c,Q) = \int_{r_1}^{r_2} P'\left(c, \tfrac{Q}{2\pi r}\right)\,\mathrm{d}r$$

Importantly, the local pressure gradient $P'(c,v)$ is given by (1) implicitly. In general, two nested non-linear inverse problems need to be solved.

The problem remains easy for power-law fluids, where (using (2) and $1/(1+\nu) = n$)

$$\Delta P(c,Q) = \left(\frac{Q}{2\pi}\right)^n C_{\mu_0,n}^{-n} \left(\sum_{j=1}^{M} \hat{R}_j^{2+\nu}\,w_j\right)^{-n} \frac{r_2^{1-n} - r_1^{1-n}}{1-n}.$$

For other fluids, however, the distribution of total flux into individual pore sizes differ along the radial coordinate. Can YSM be generalised similarly?

Perspective

Experiments and numerical experiments are in progress... So far, the main issues are:

- Lack of analysis. Relation of various problem's levels (continuous, discrete, numerical).
- Missing sensitivity analysis. Which settings of the experiments are safe?
- Polymer entrapment. In which setting is it (not) a limiting factor?

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