

Vienna, Austria (27 April – 2 May 2025)



Session ST4.3:

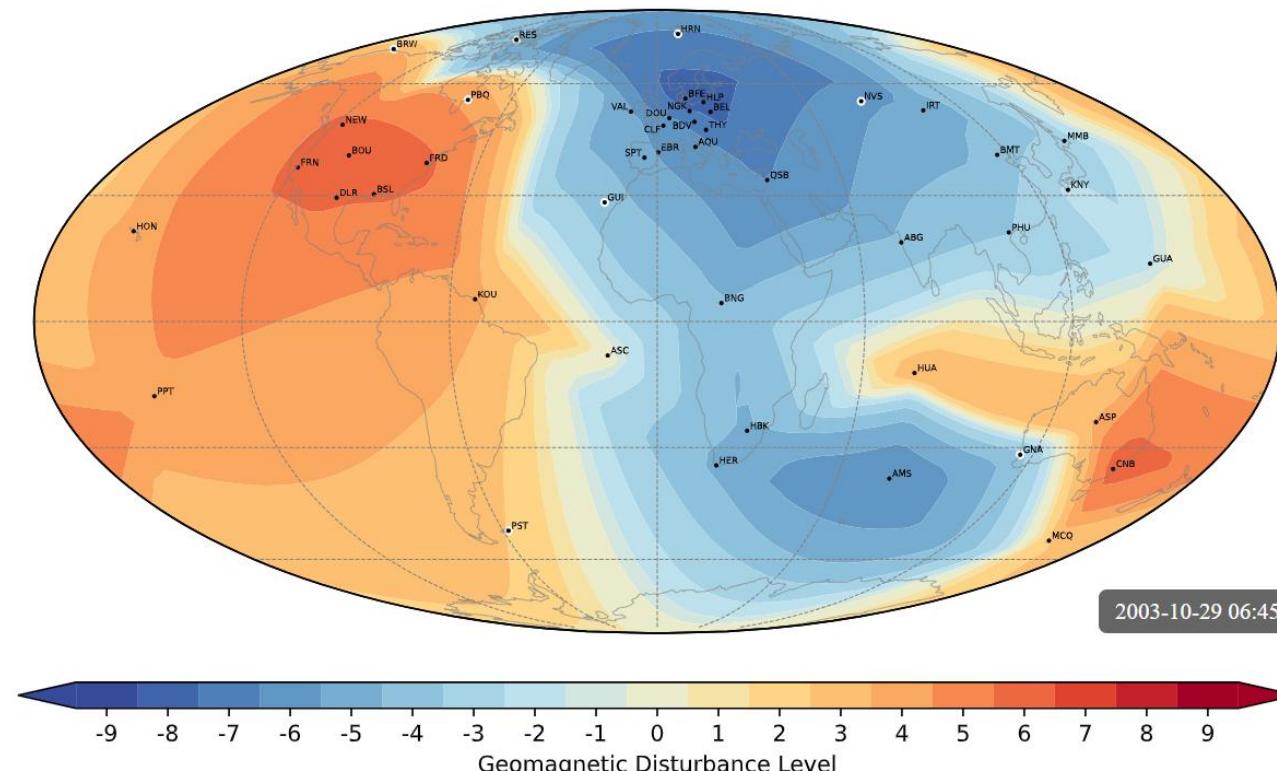
Nowcasting, forecasting, operational monitoring and post-event analysis of the space weather and space climate in the Sun-Earth system.

ST4.3 Wed, 30 Apr | 14:00-18:00,
Room 0.94/95

This presentation at 14:05 GMT

WORLD MAP OF GEOMAGNETIC DISTURBANCES FOR MID AND LOW LATITUDES (LDI-GMAP)

Antonio Guerrero (aguerrero@uah.es), Physics and Mathematics Dpt., University of Alcalá, SPAIN



Selected day: 29/10/2003 (Data available only for 2003)



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- a) The need
- b) Geom. Indices
- c) Prev. Work

2. The LDi Procedure

- a) The method
- b) Retrospective
- c) Real-time

3. The GMAP

- a) Stations
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- e) Scale and values

4. Validation

- a) Regions of interest
- b) Procedures
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5. Results

- a) Web features
- b) 2003 examples

6. Conclusions

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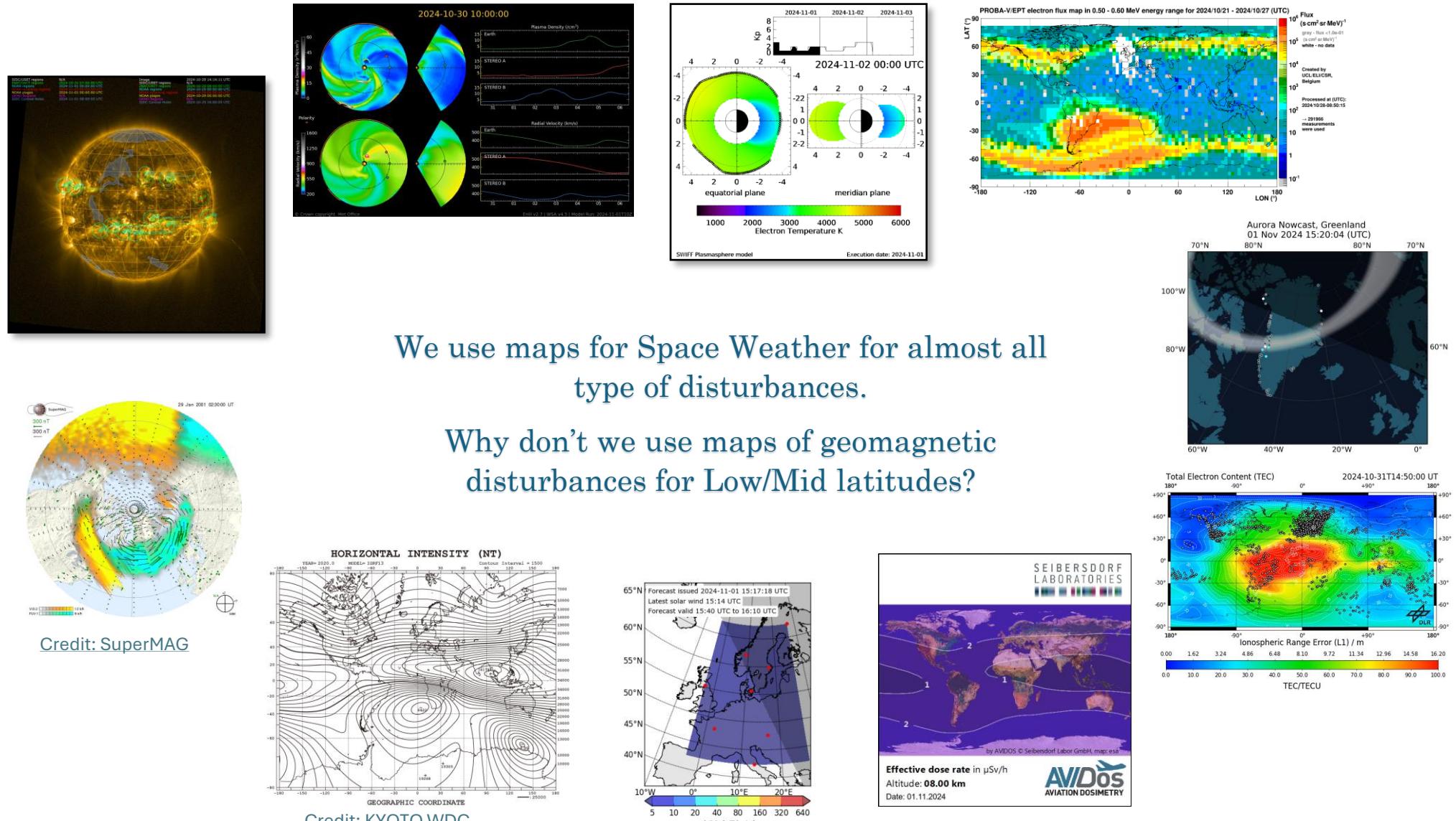
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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

1. Introduction

a) The need



We use maps for Space Weather for almost all type of disturbances.

Why don't we use maps of geomagnetic disturbances for Low/Mid latitudes?

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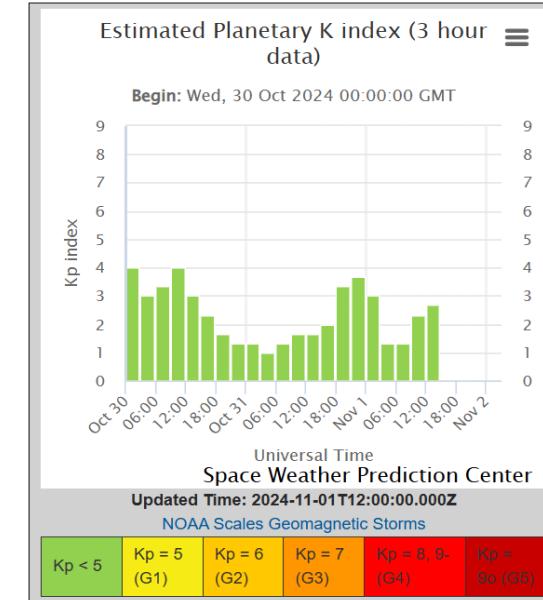
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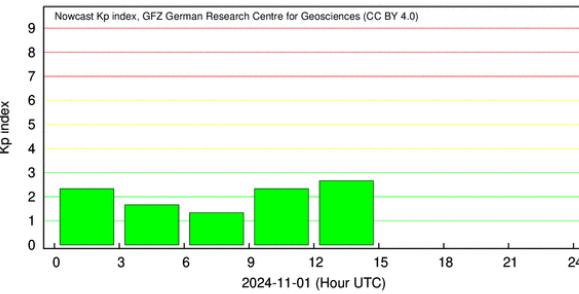
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1. Introduction

b) Geomagnetic indices

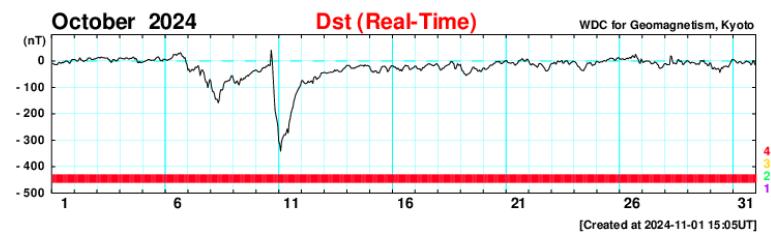
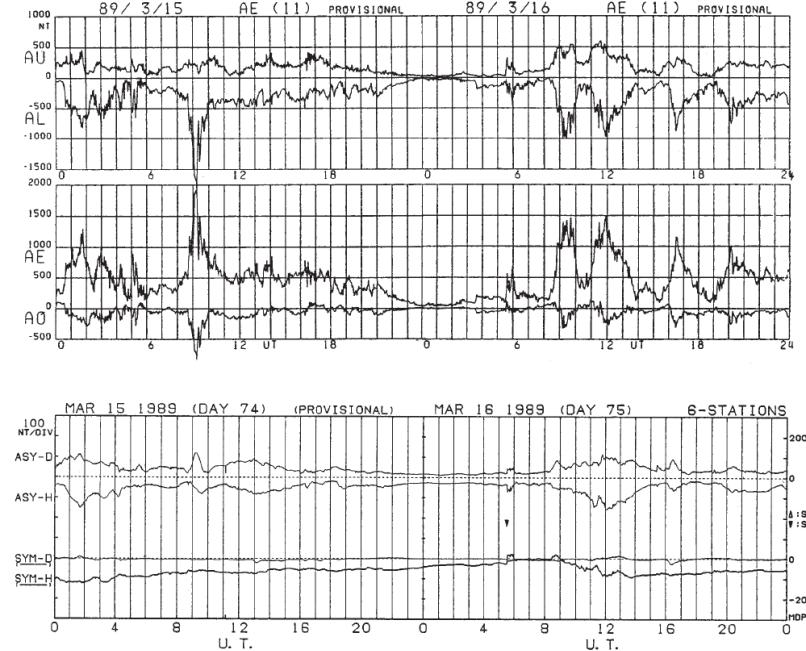
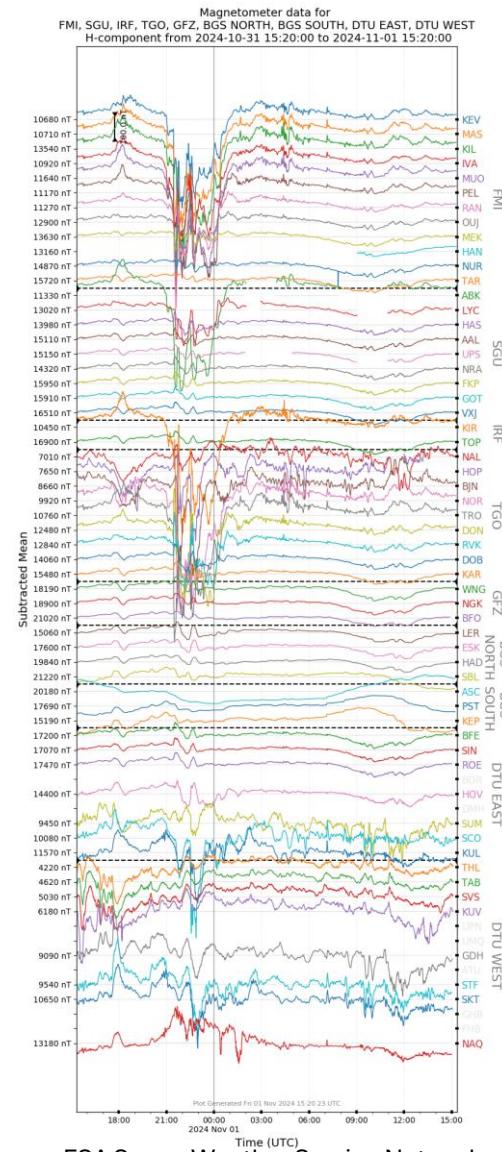


SWPC NOAA
<https://www.swpc.noaa.gov/>



2024-10-26 2024-10-27 2024-10-28 2024-10-29 2024-10-30 2024-10-31

ESA Space Weather Service Network
<https://swe.ssa.esa.int/>



Kyoto WDC: <https://wdc.kugi.kyoto-u.ac.jp/dstdir/>

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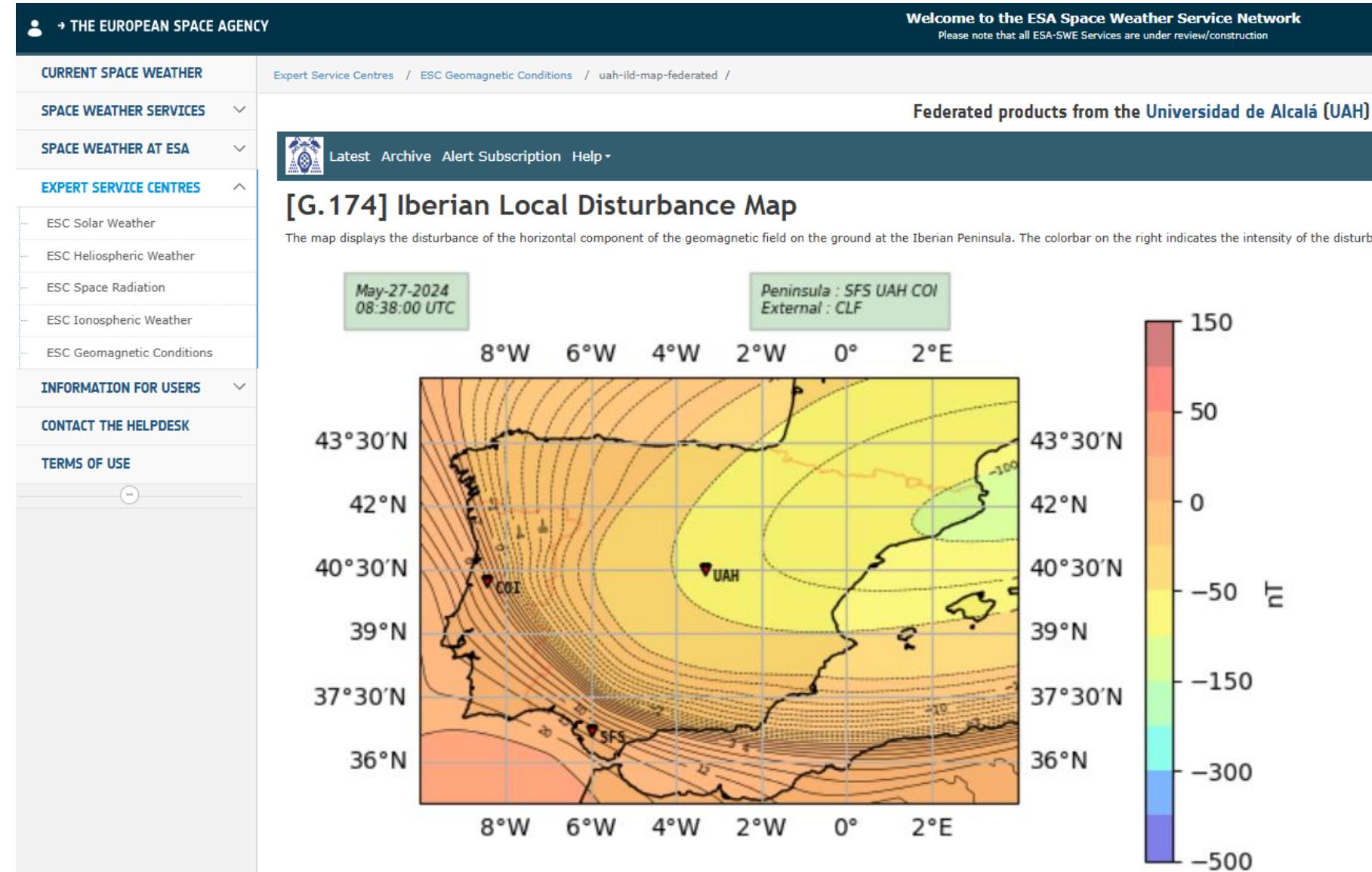


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1. Introduction

c) Prev. Work



2. The LDi Procedure

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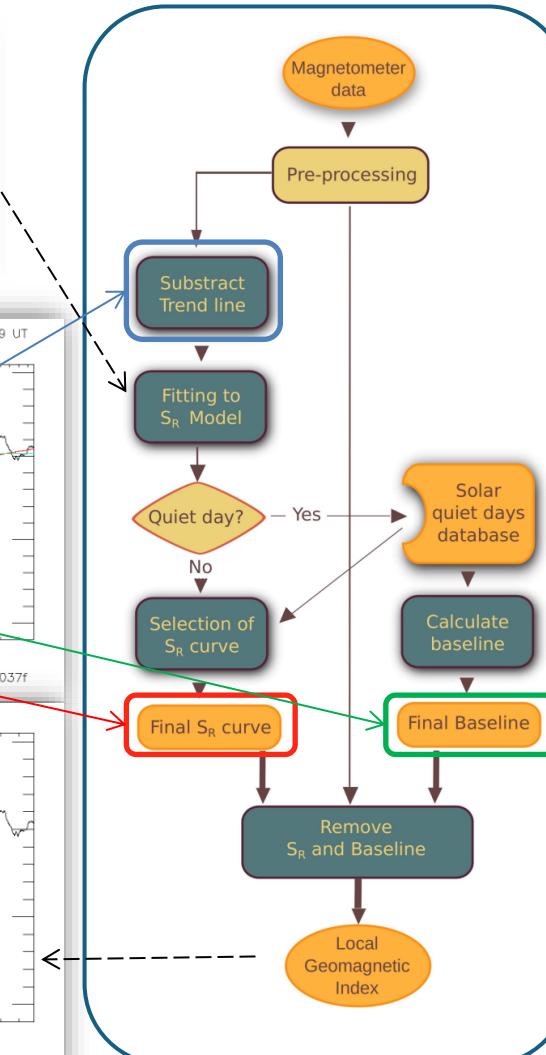
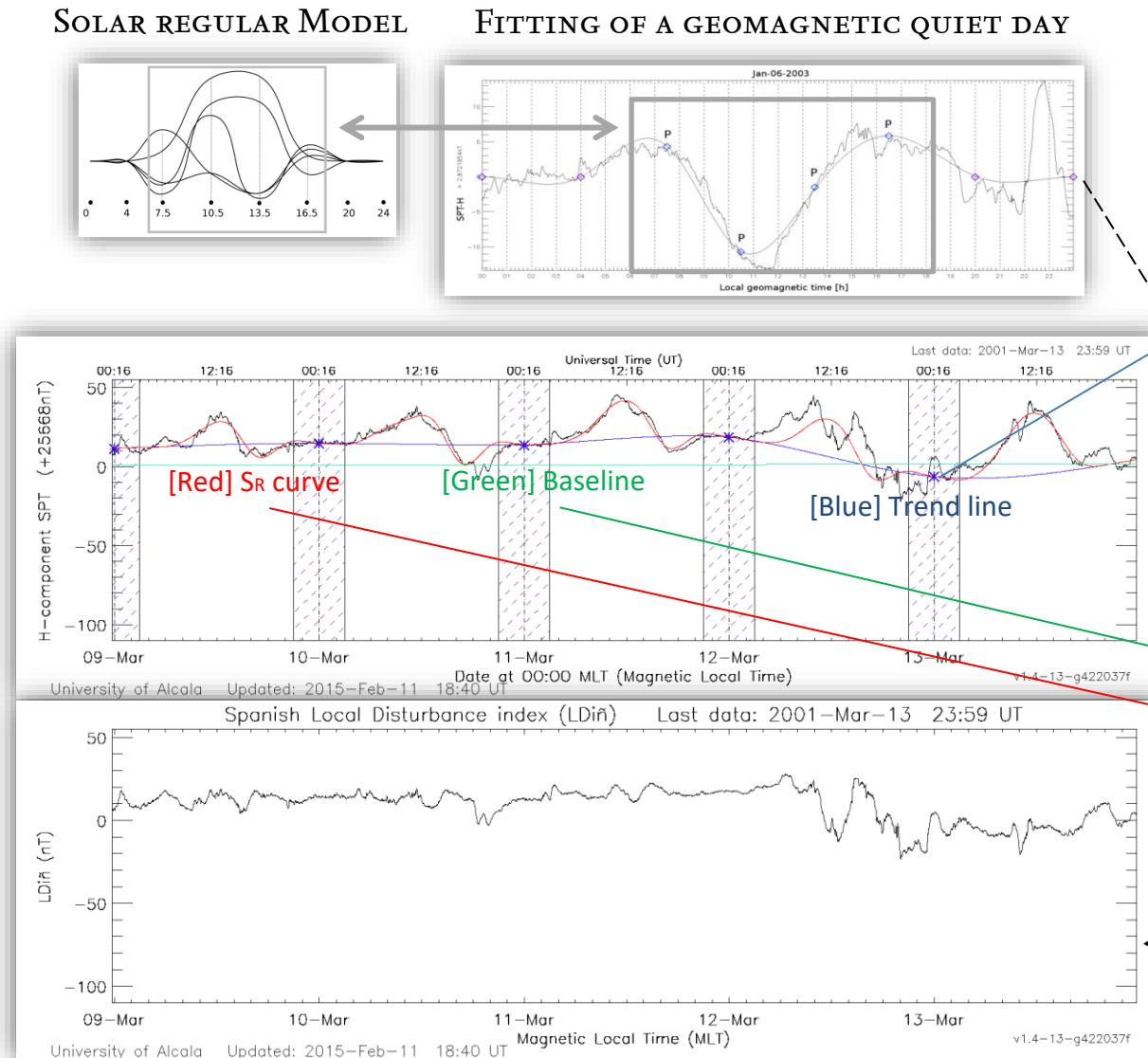
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2. The LDi Procedure

a) The method



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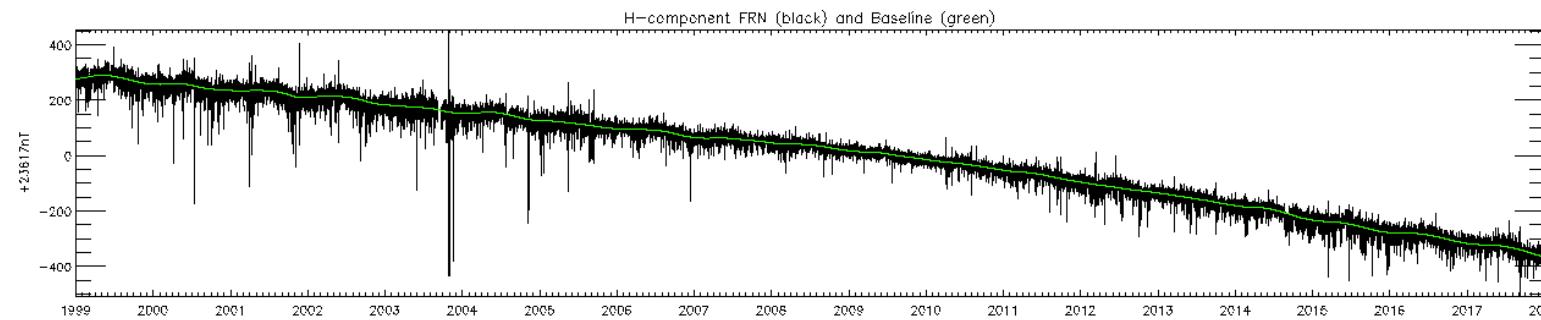
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2. The LDi Procedure

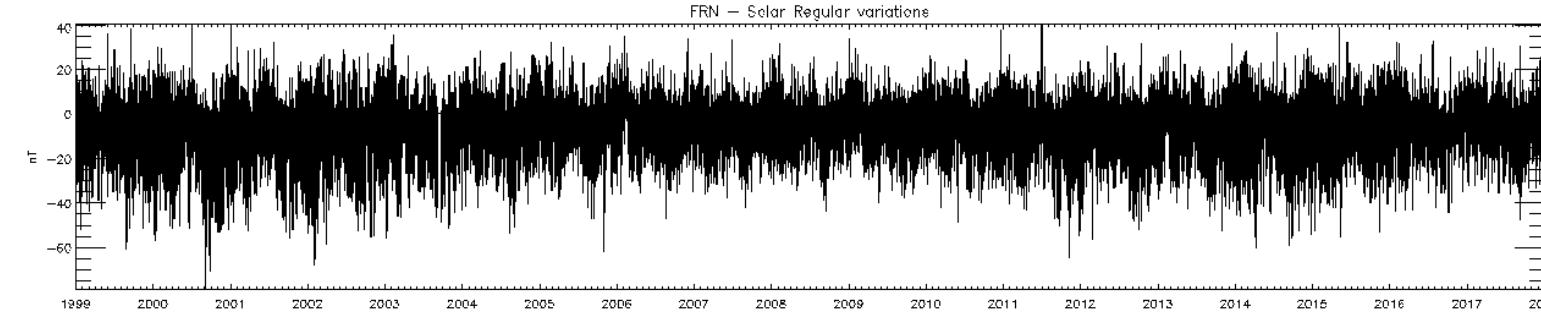
b) Retrospective

LDi-FRN

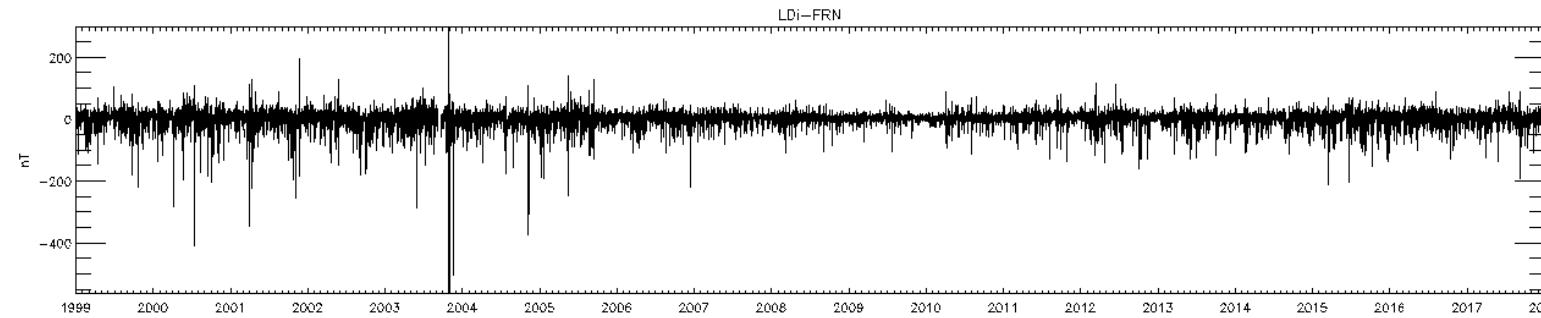


OBSERVATORY DATA

BASELINE



SOLAR REGULAR
VARIATIONS



LOCAL GEOMAGNETIC
INDEX FOR FRN

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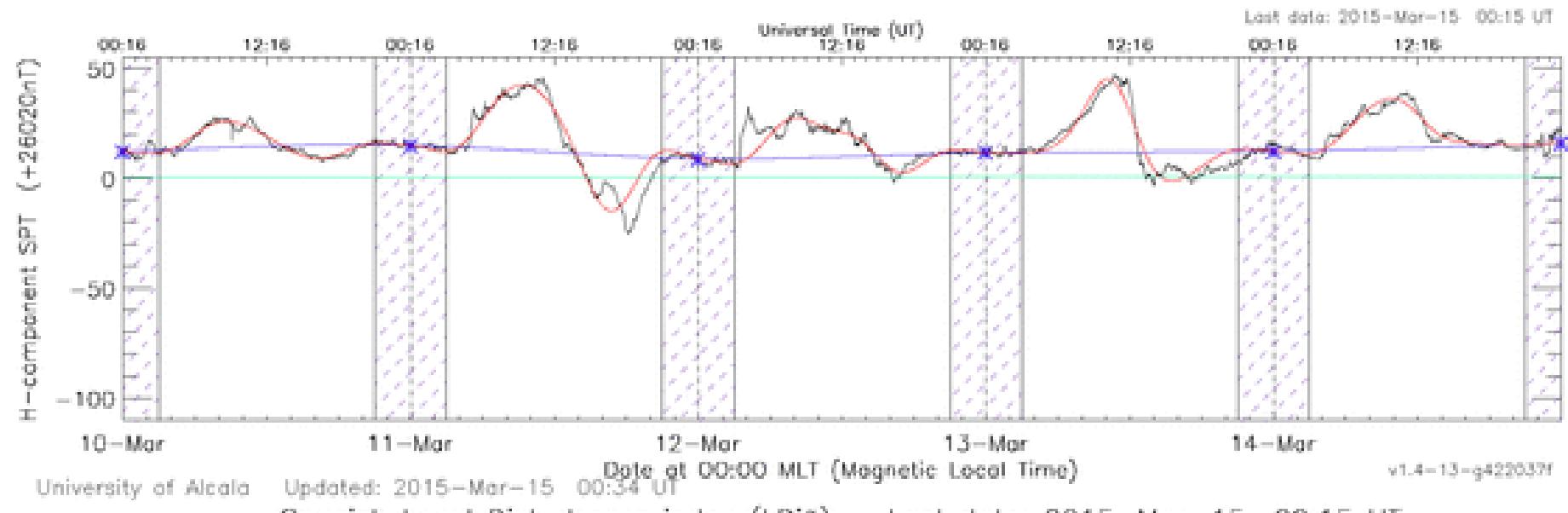
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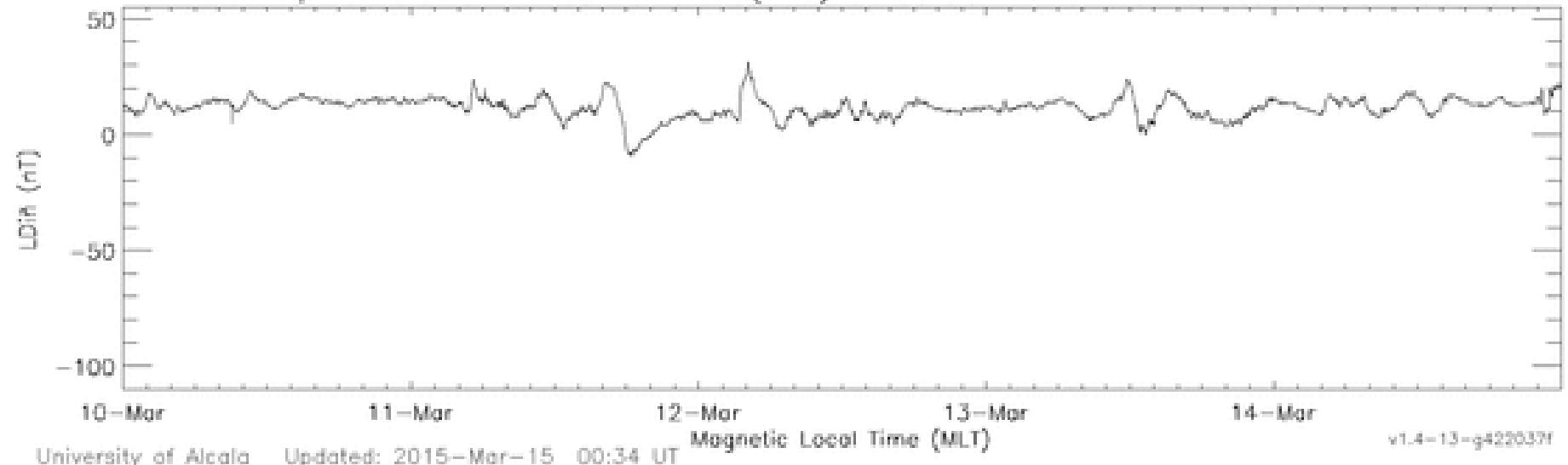
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c) Real-time

PROCEDURE



LDI_n



3. The GMAP

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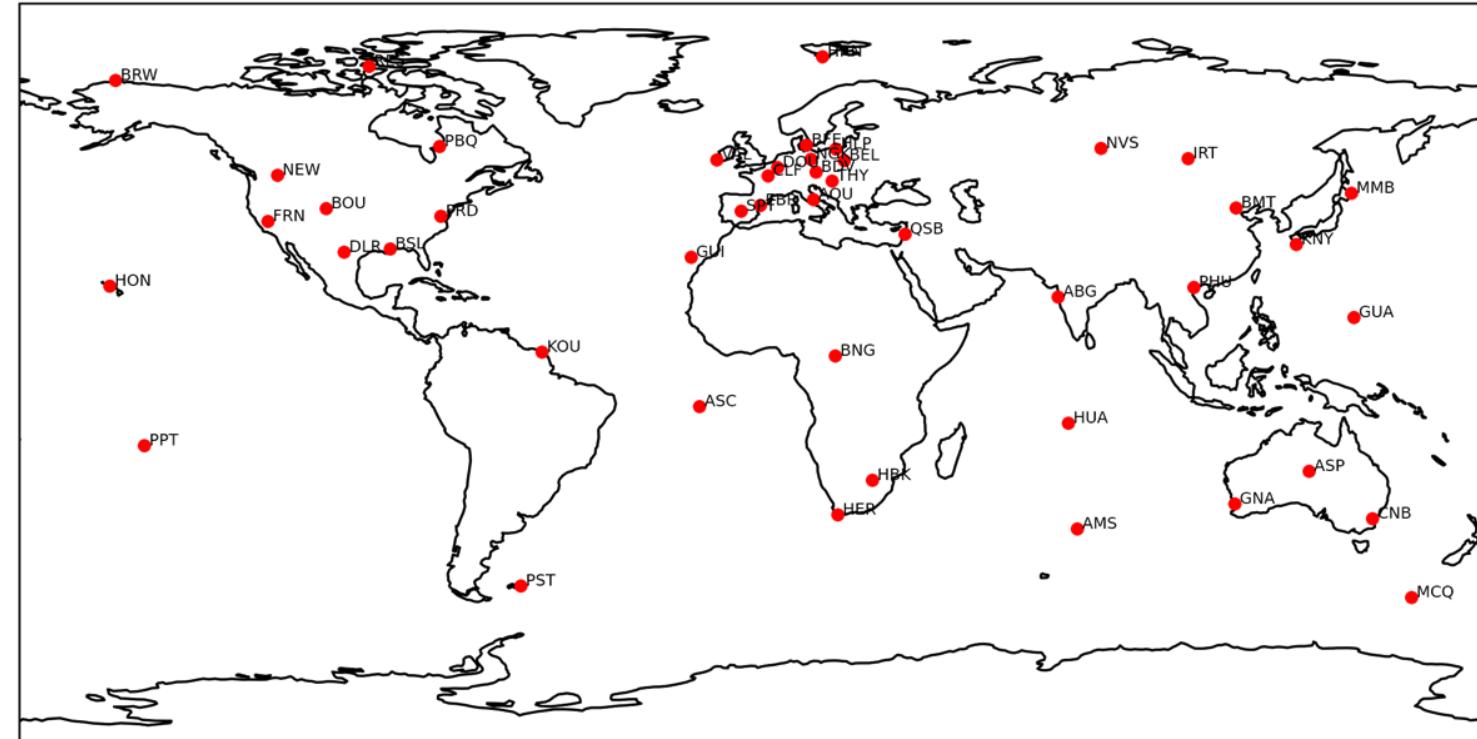
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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

3. The GMAP

a) Observatories



A total of 46 stations have been selected from INTERMAGNET as those with reliable data and extended through out all longitudes for geomag. latitudes up to 82 degrees.

IAGA CODE	DLR	MMB	GUI	BMT	QSB	HON	KNY	KOU	ABG	PHU	GUA	BNG	HUA	ASC	PPT	HBK	ASP	HER	PST	CNB	GNA	AMS	MCQ
LON	259	144	343	116	35	202	130	307	72	105	144	18	284	345	210	27	133	19	302	149	116	77	158
LAT	29	43	28	40	33	21	31	5	18	21	13	4	-12	-7	-17	-25	-23	-34	-51	-35	-31	-37	-54
GLON	327	211	60	186	113	269	200	19	146	177	215	91	356	56	285	94	208	83	11	226	188	144	244
GLAT	38	35	33	30	30	21	21	14	10	10	5	4	-1	-2	-15	-27	-32	-33	-41	-42	-42	-46	-60

IAGA CODE	RES	HRN	BRW	PBQ	BFE	VAL	NEW	HLP	DOU	NGK	BEL	CLF	BDV	BOU	FRD	NVS	THY	EBR	FRN	AQU	SPT	IRT	BSL
LON	265	15	203	282	11	349	242	18	4	12	20	2	14	254	282	83	17	0	240	13	355	104	270
LAT	74	77	71	55	55	51	48	54	50	52	51	48	49	40	38	54	46	40	37	42	39	52	30
GLON	302	126	246	351	98	74	304	104	88	97	105	85	97	320	353	159	100	81	305	94	75	176	339
GLAT	82	73	69	65	55	55	54	53	51	51	50	49	48	48	48	45	45	43	43	42	42	41	40

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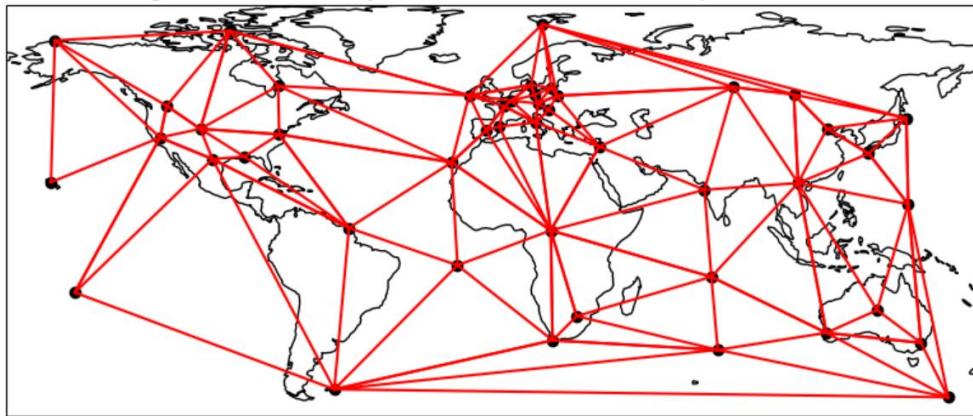
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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

3. The GMAP

b) Grid and coordinates

First, we need to triangulate the positions where the observatories are located.



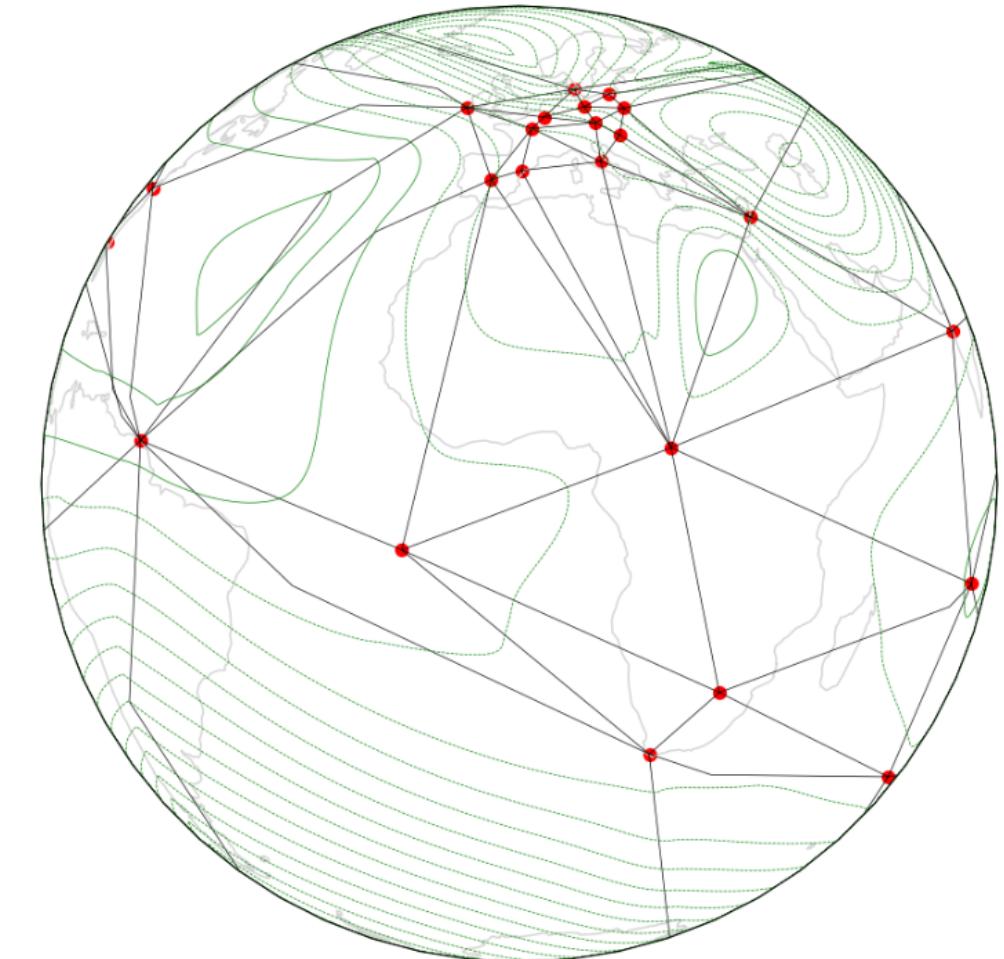
A planar triangulation like the one above does not work well, for several reasons, i.e. distances are not real.

We need to work on the surface of a sphere.
On the right we have the triangulation of the position of the stations in a spherical surface, created using the cited code.

Delaunay Triangulation on a Sphere

See TRIPACK [Renka, 1996a] and STRIPACK [Renka 19997a]

We use a Python interface to the FORTRAN code (See [Python1] and [Python2])



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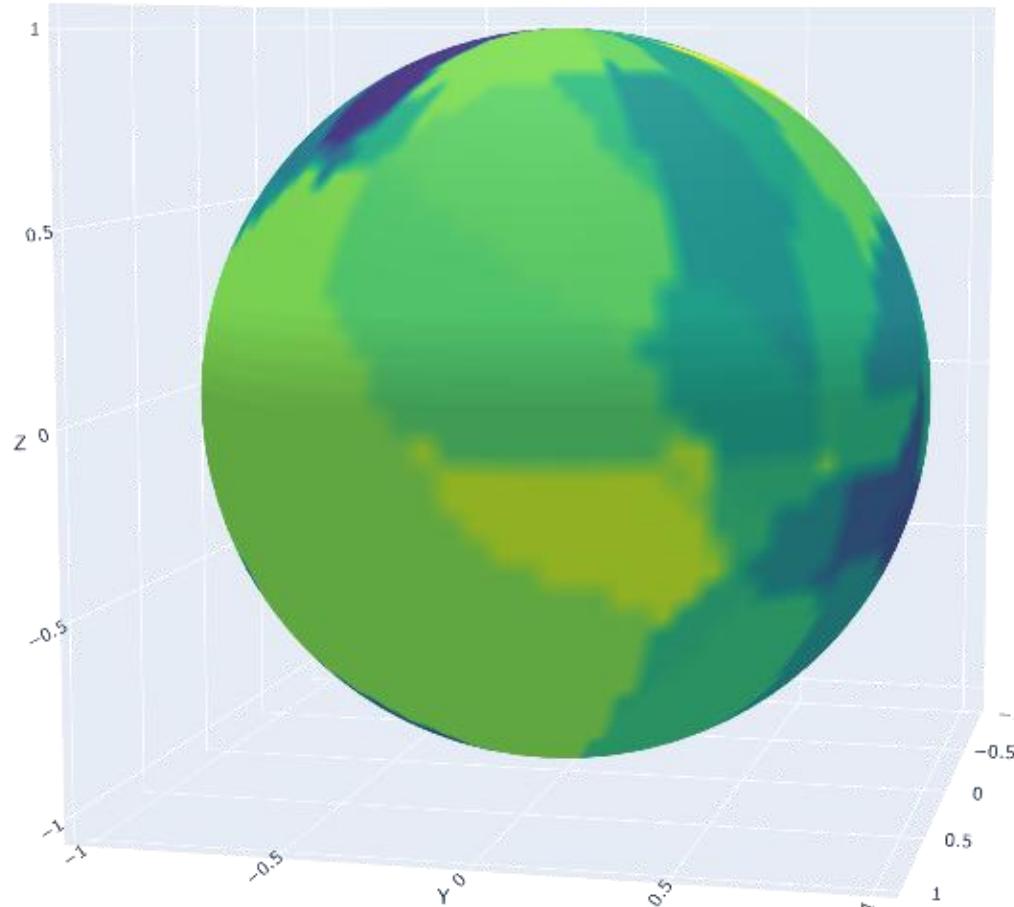
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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

3. The GMAP

c) Interpolation

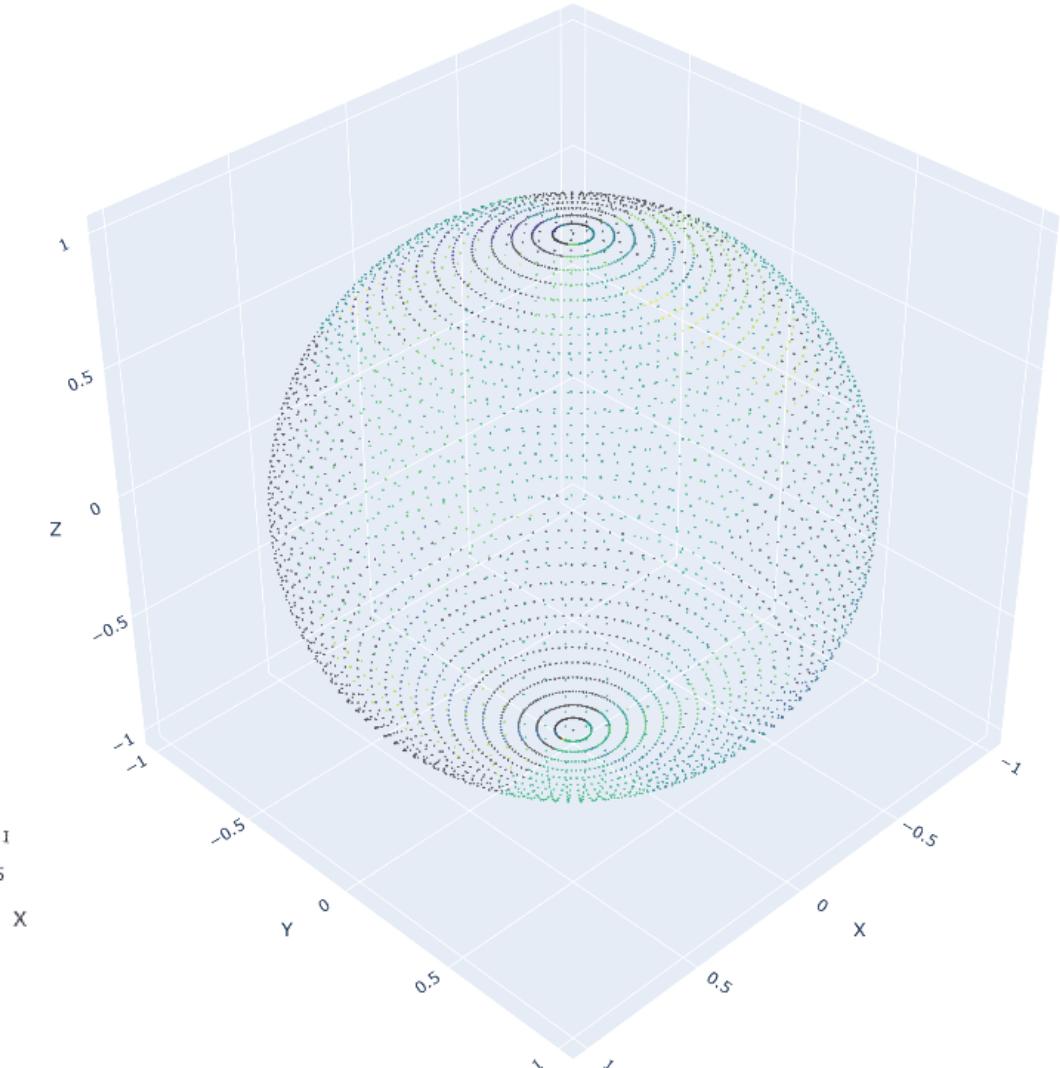
Second, we need a spherical grid (on the right) and an interpolation method for points on the surface of a sphere (spherical distances).



Interpolation on the Surface of a Sphere

See SRFPACK [Renka, 1996b] and SSRFPACK [Renka 19997b]

We use a Python interface to the FORTRAN code (See [Python1] and [Python2])



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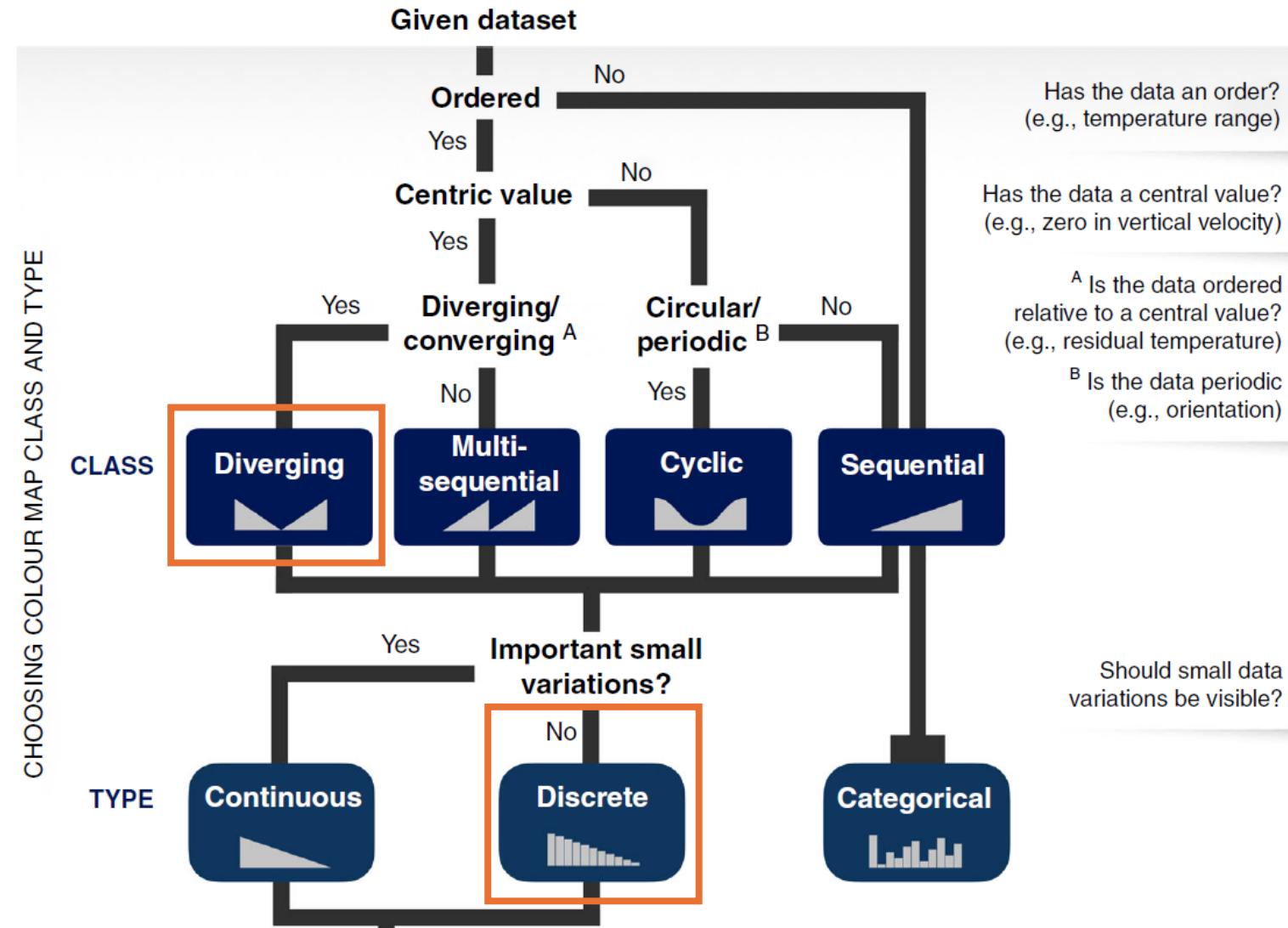
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3. The GMAP

d) Colour palette



From Cramer et al., 2020: “The misuse of colour in science communication”

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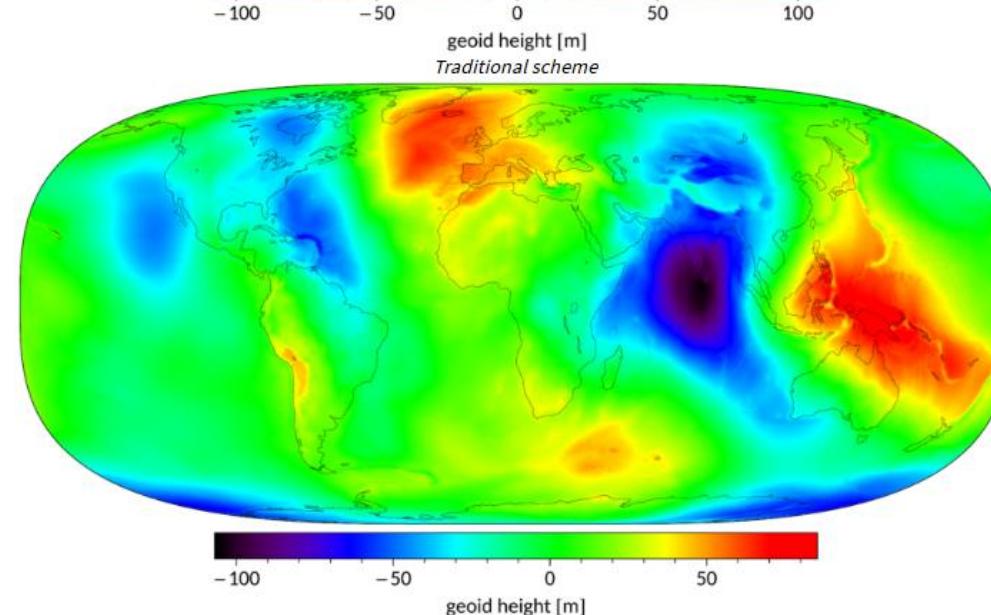
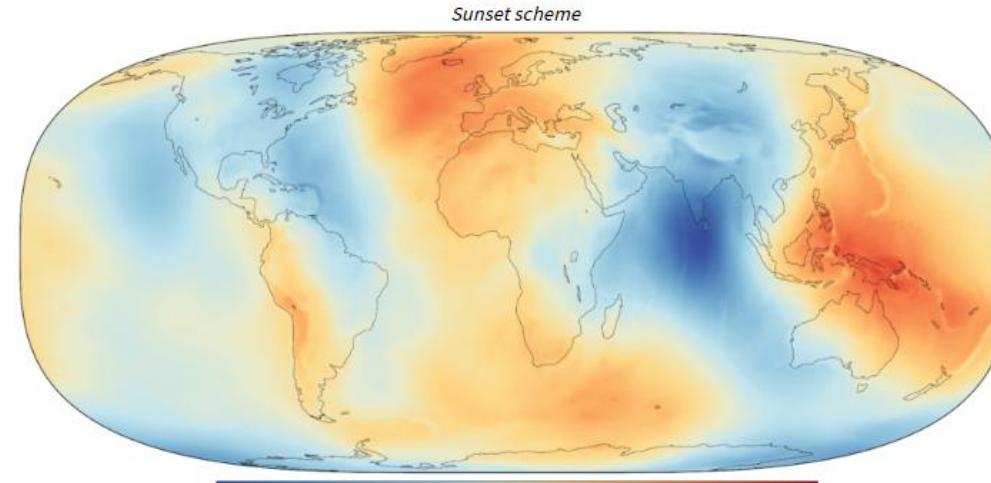
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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

3. The GMAP

d) Colour palette

Good and bad uses of colour schemes

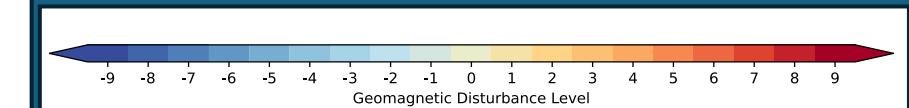


From Paul Tol's Notes web:

<https://personal.sron.nl/~pault/>

See also Fabio Crameri colour palettes at:
<https://www.fabiocrameri.ch/colourmaps/>

We are using a discrete adapted version of Paul Tol colour palette called "Sunset" shown at the top.



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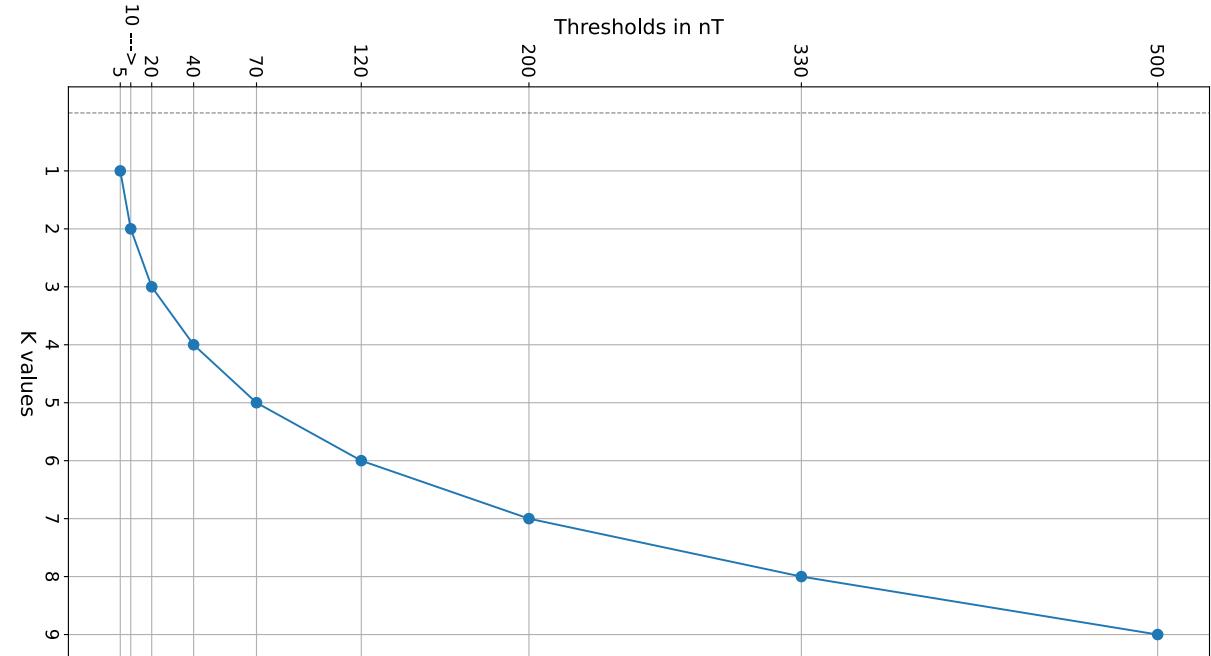
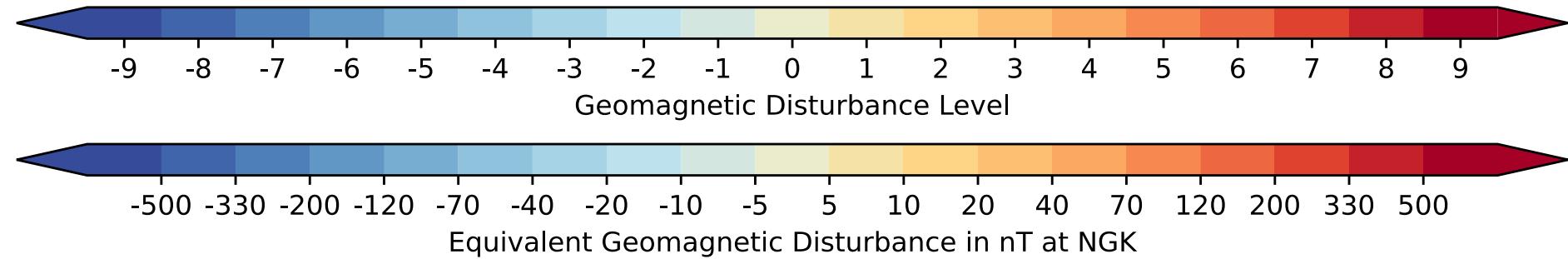
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3. The GMAP

e) Scale and Values

It uses a similar scale to that of the K index [Bartels, 1939]



We use the same scale as the K index

The K scale, developed by Bartels in 1939, is a quasi-logarithmic scale that relates the amplitude of geomagnetic disturbances to specific ranges.

You might wonder: *why not just use the raw disturbance values?*

Because the disturbance amplitude varies with latitude. (See next slide.)

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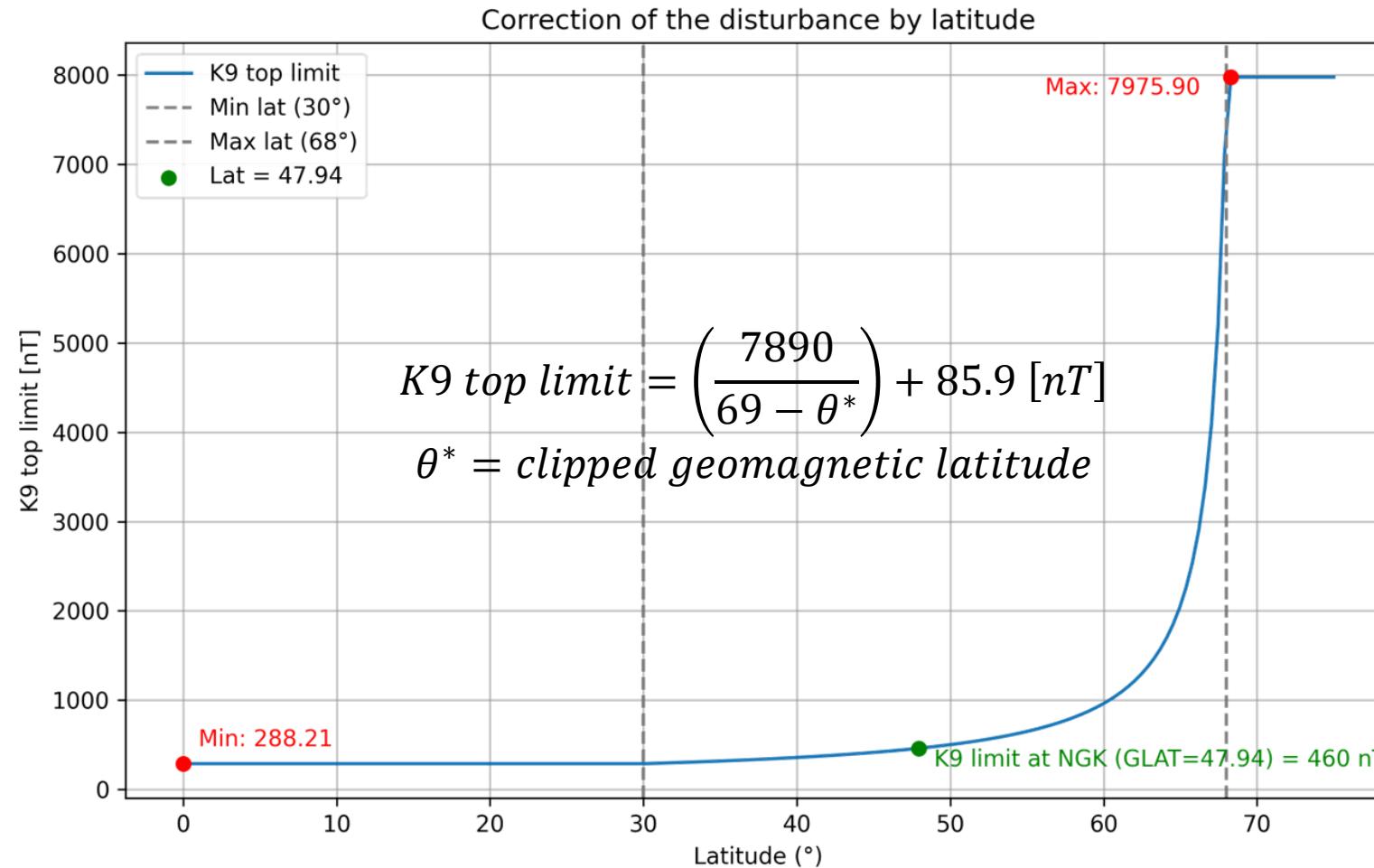
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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

3. The GMAP

e) Scale and Values

For different observatories, the top limit of the K9 value should be obtained empirically using long time series. Regi et al., 2019 obtained an almost perfect fitting of the K9 limit to the latitude of the observatory. We are using this information to be able to adapt the K-scale to any observatory.



Regi et al., 2019 obtained the relation on the left.

The latitudes are clipped at the lower level to 30° and at the higher level to 68° as recommended by Bartels.

4. Validation

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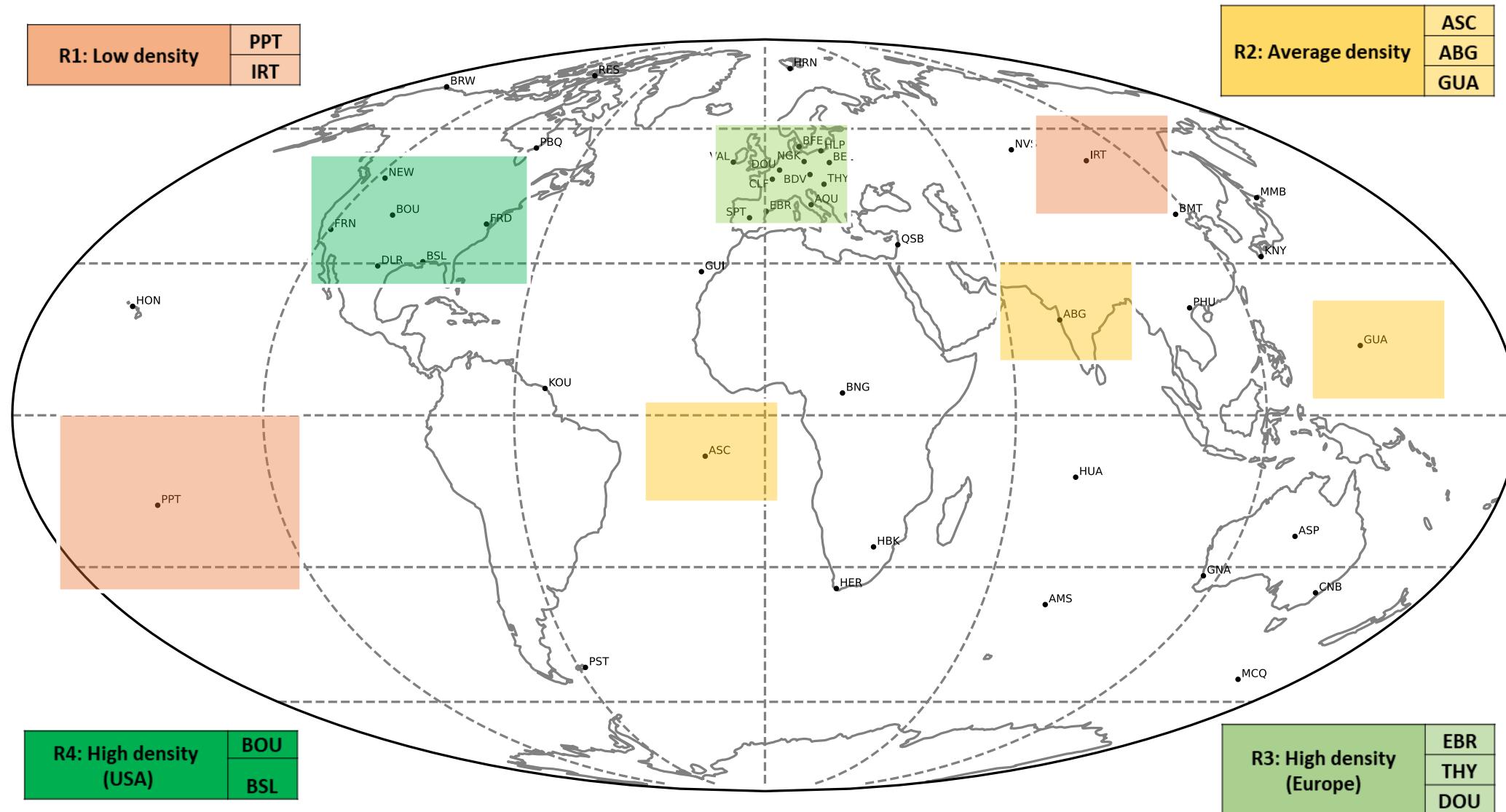
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4. Validation

a) Regions of interest



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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

4. Validation

b) Procedure

We use the method **Leave One Out (LOO)** for validation:

For each selected station (10) and each type of period (6) we compare the minute value at the station (\hat{y}_i) with the interpolated value at that location when we remove that station from the map creation procedure.

We use four type of measures:

The Root Mean Square Error:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}$$

The Mean Absolute Error:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

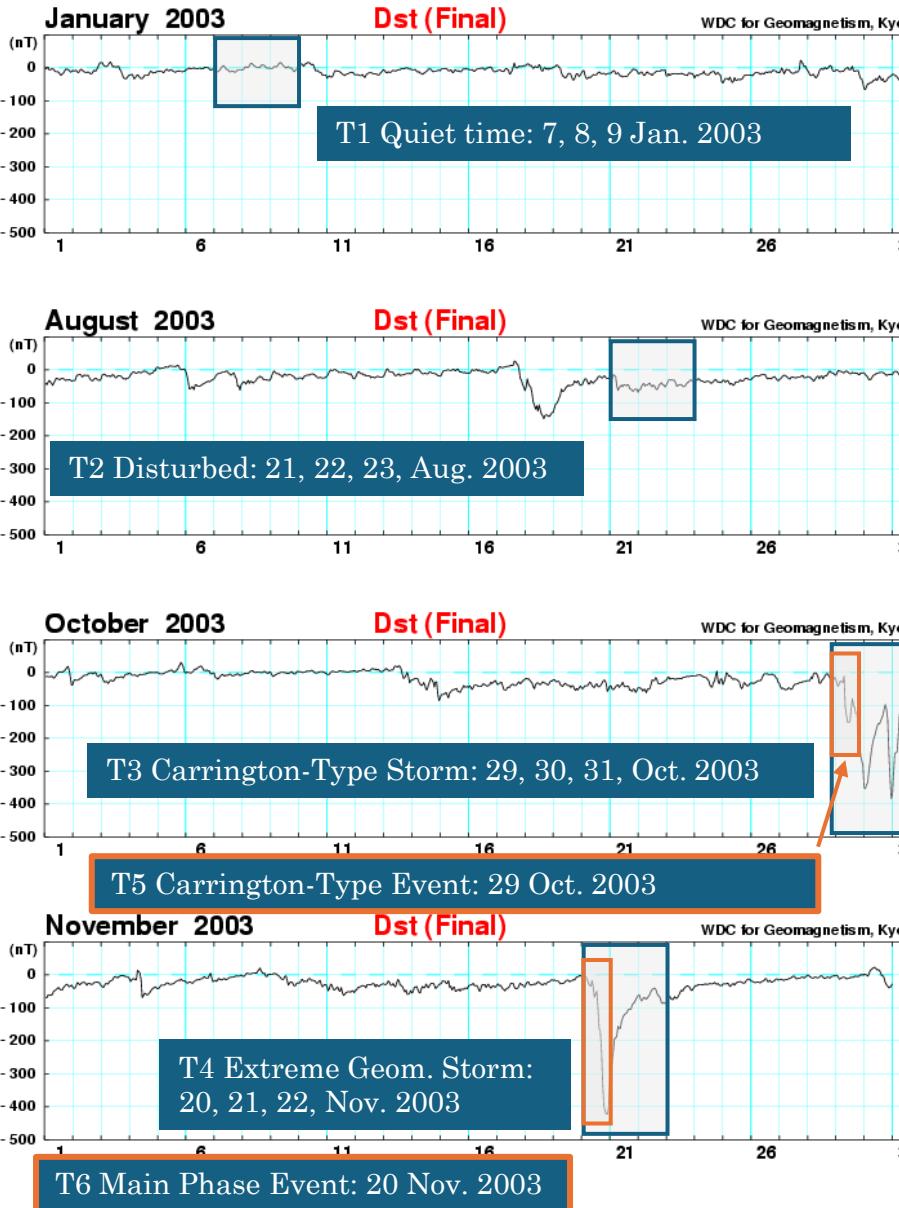
The Normalized RMSE:

$$\overline{RMSE} = RMSE / D_{max}$$

The Normalized MAE:

$$\overline{MAE} = MAE / D_{max}$$

The Normalized measures are used for better comparison between events of low and high activity. Where D_{max} is the maximum value of the disturbance at the observatory during the event period.



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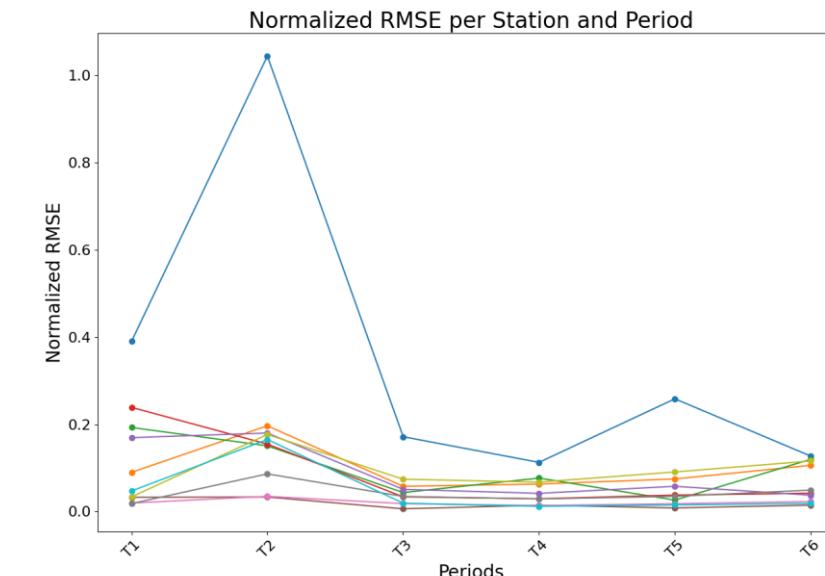
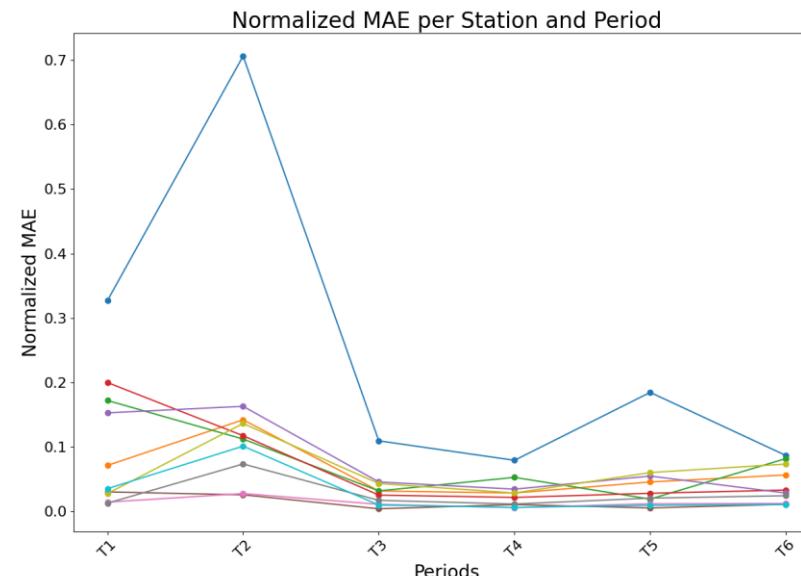
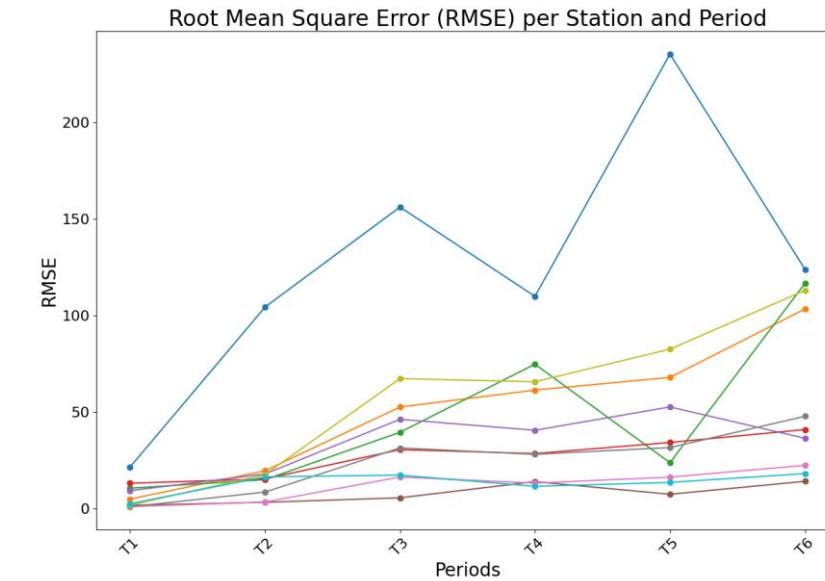
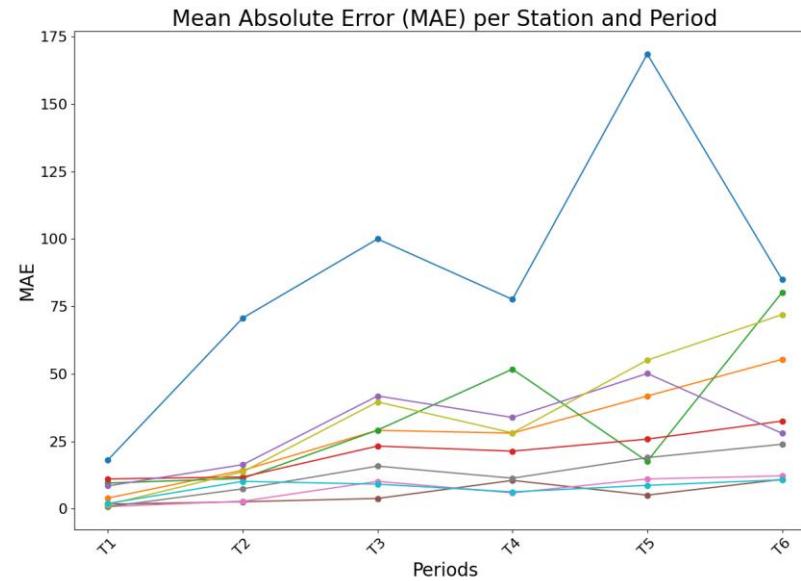
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4. Validation
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4. Validation

c) Results



5. The Website

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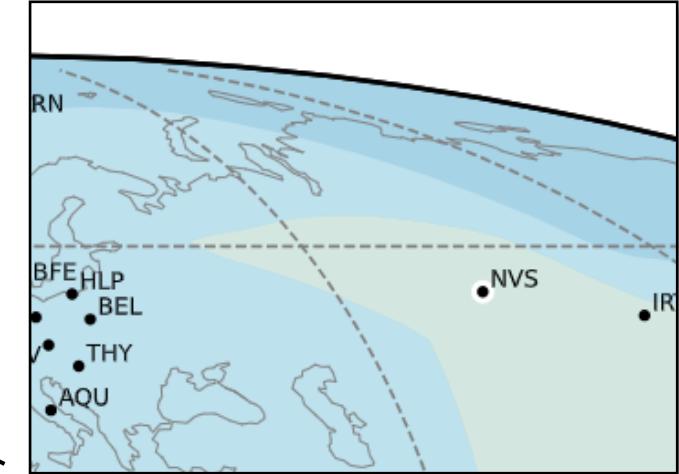
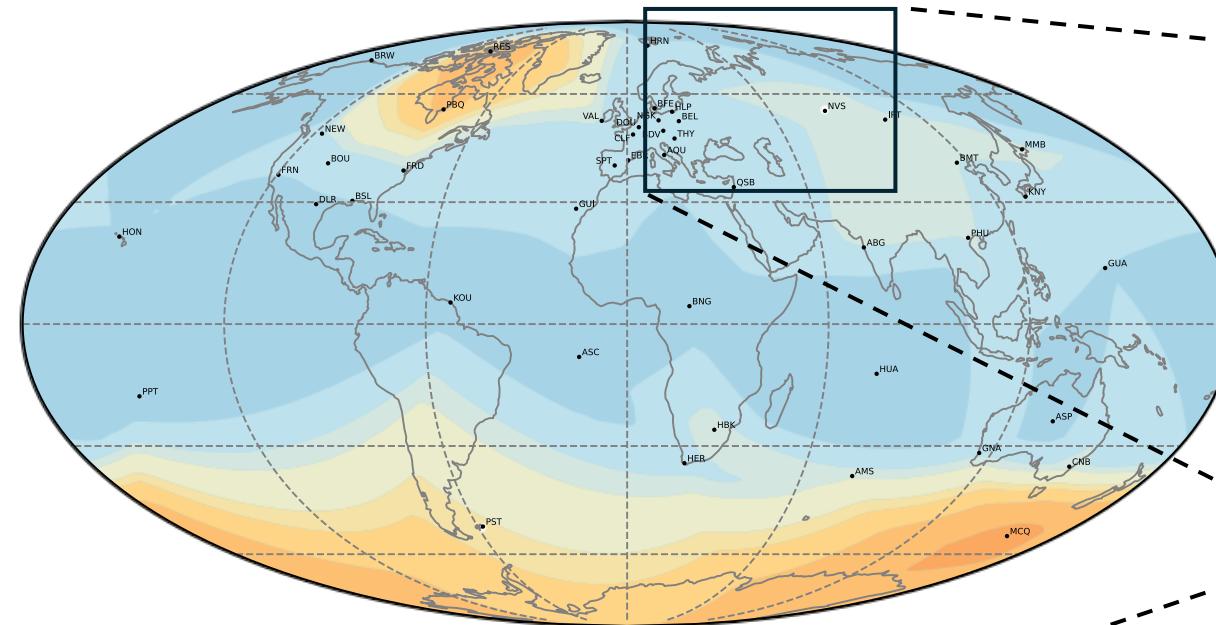
References

Acknowledgements

WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

5. Results

a) Web features



How we indicate that data from one observatory is missing and was not used to create the frame.

The frames load in the background while you can scroll to other positions.

Loaded frames are indicated in the slider.



Selected day: 29/10/2003 (Data available only fo

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5. Results

a) 2003 examples



Strong asymmetries in longitude (Click to go to Map online)

Geomagnetic Bubble? through the US (Click to go to Map online)



The Carrington-type disturbances of the Halloween event
(Click to go to Map online)
[See Cid et al., 2015; Saiz et al., 2021]

Recovery of geomagnetic storms
(Click to go to Map online)

6. Conclusions

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WORLD MAP OF GEOMAGNETIC ACTIVITY FOR MID AND LOW LATITUDES (LDI-GMAP)

6. Conclusions

Presenting GMAP

A New Approach to Monitoring Geomagnetic Disturbances

GMAP is a global map designed to visualize geomagnetic disturbances in a clear, intuitive, and accessible way. Inspired by the well-known K-indices but enhanced with modern advantages, GMAP offers:

- Regional information for the entire globe in a single glance.
- One-minute time resolution for detailed temporal tracking.
- Real-time execution capability.
- Differentiation between positive and negative disturbances.

Explore GMAP data for the year 2003 here: <https://doi.org/10.21950/UKR6XJ>

We are actively seeking collaborators to help us expand and enhance GMAP.

Join us in advancing real-time space weather monitoring!



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