Supplement to EGU25-4125

THE TEMPORAL MEAN OF TRANSIENT ROSSBY WAVE PACKETS

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MOTIVATION

- Extreme weather events are commonly associated with (recurrent) Rossby wave packets (Fragkoulidis et al., 2018; Röthlisberger et al. 2019).
- We lack detailed understanding of the drivers of spatially compounding extreme events (Zscheischler et al., 2020).
- The phase speed of synoptic-scale waves influences the duration and location of heatwaves (Wicker et al., 2024).

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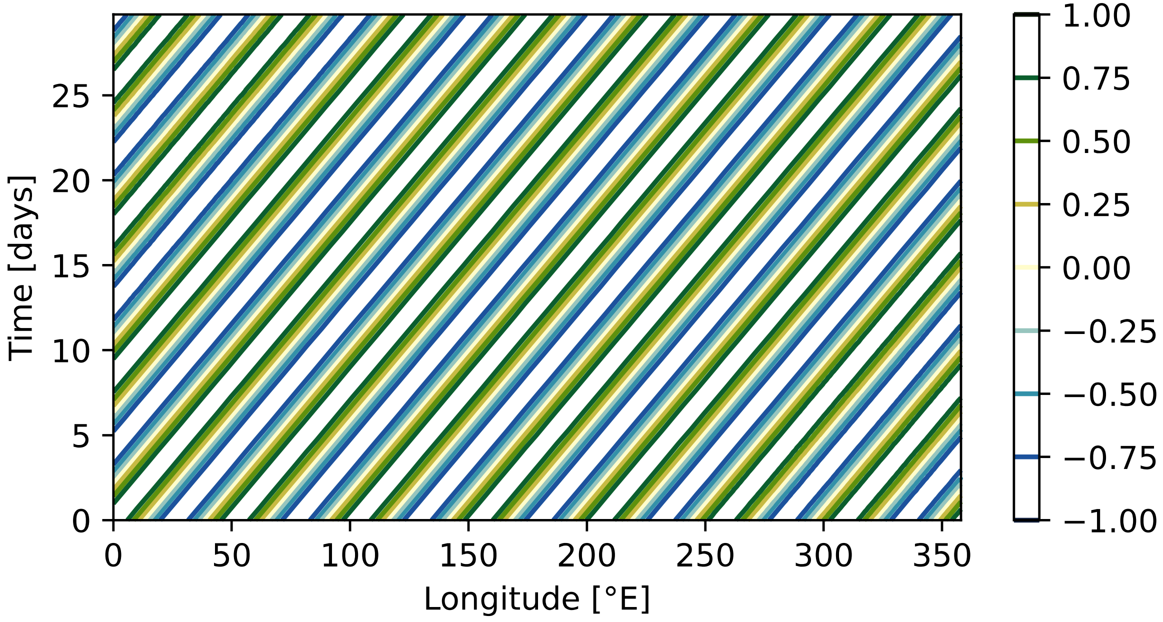
Investigate the temporal mean of a transient Rossby wave packet in two simple models.

A KINEMATIC MODEL WITH CONSTANT GROUP VELOCITY

Hovmöller diagram of a carrier wave with $k_0 = 7$ and c_p=5 m/s.



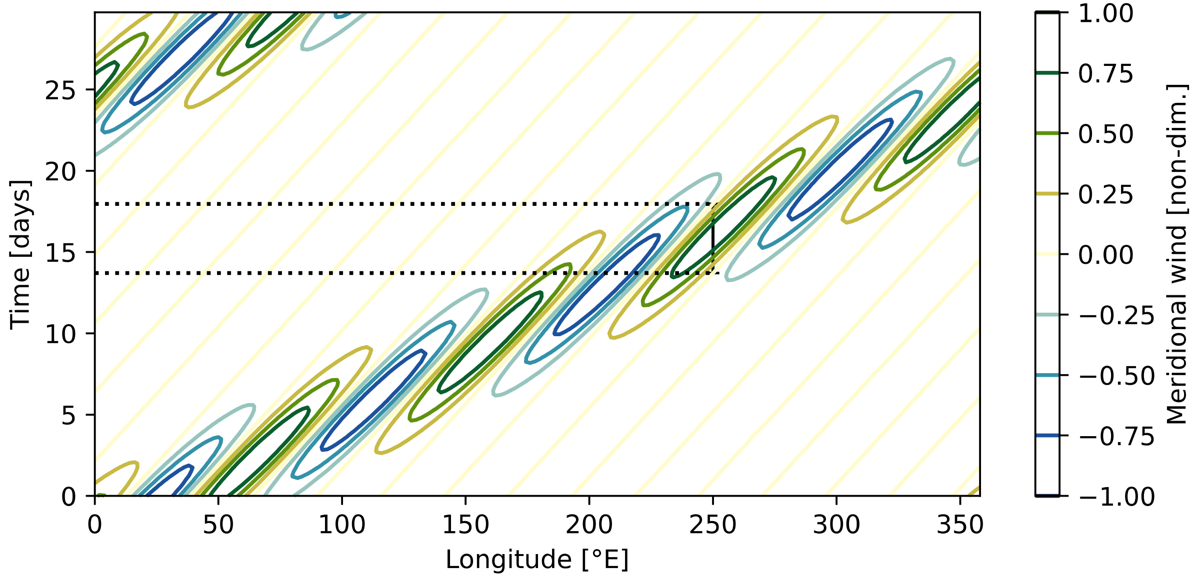
Abbreviation: $c_p^* = \frac{c_p}{a \cos \phi}$

Let us represent the upper-tropospheric flow $\Psi(\lambda, t)$ as the product of a sinusoidal carrier $e^{ik_0(\lambda - c_p^* t)}$ wave with wavenumber k_0 and phase speed c_p and an envelope function Ψ_0.

- When averaged over a long enough time frame, the temporal mean of the carrier wave becomes zero.

A KINEMATIC MODEL WITH CONSTANT GROUP VELOCITY

Idealized wave packet with a Gaussian envelope and $c_g = 11$ m/s



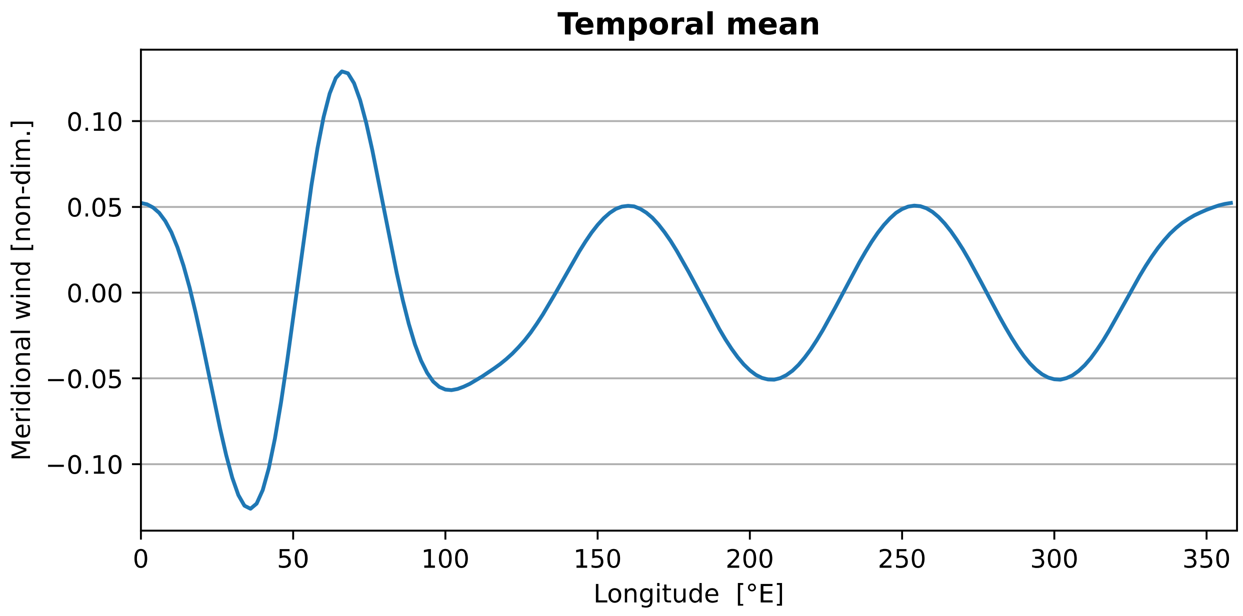
Abbreviation: $c_g^* = \frac{c_g}{a \cos \phi}$

Choose an envelope $\Psi_0 = \Psi_0(\lambda - c_g^* t)$.

- Zonal translation with constant group velocity c_g ; shape of the envelope is time-invariant.
- When averaged over a long enough time frame, the temporal mean of the envelope is zonally symmetric and equals the zonal mean of a snapshot.

A KINEMATIC MODEL WITH CONSTANT GROUP VELOCITY

Temporal average of the idealized wave packet over 30 days



- The temporal mean of the product of carrier wave and envelope is zonally asymmetric. The example shows peak power at zonal wavenumber 4.
- Wave envelope takes 27.1 days to propagate around the globe.
- The phase of the carrier wave undergoes 3.2 oscillations in those 27.1 days.

SPECTRAL ANALYSIS OF THE IDEALIZED WAVE PACKET

For the idealized example with a Gaussian envelope, we can compute the frequency-wavenumber spectrum analytically.

➢ *The Fourier transform of a product of two functions is the convolution of their Fourier transforms.*

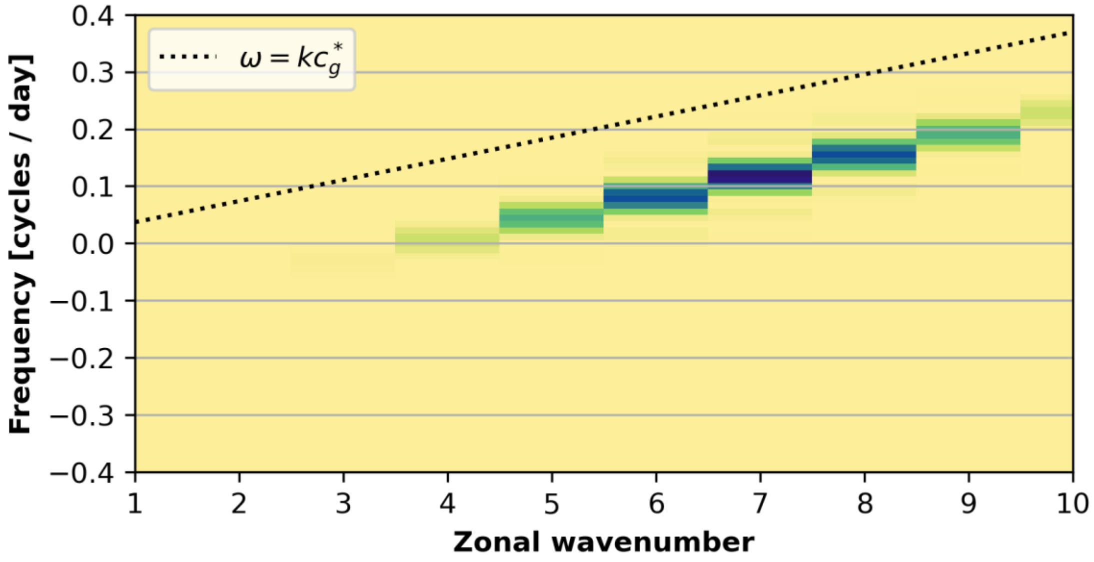
Step-by-step calculation of Fourier coefficients of the Gaussian envelope $\Psi_0 = \Psi_0(\eta)$ for wavenumber k and angular frequency ω.

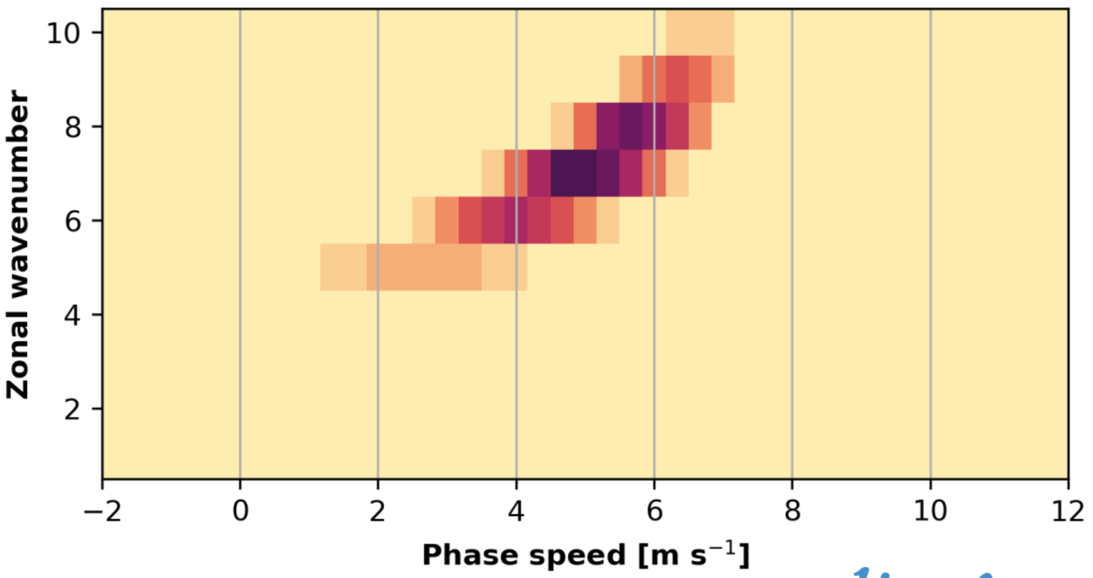
- $\eta = \left[\lambda - \frac{c_g}{a\cos\phi} t - (\lambda_0 - \pi)\right] \mathrm{mod}\ 2\pi$
- $\hat{A}_{k,\omega} = A_0 \frac{\sqrt{2\Delta t}}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-i\omega n\Delta t} \int_0^{2\pi} e^{\frac{-(\eta-\pi)^2}{2\sigma^2}} e^{-ik\lambda} d\lambda$
- $\hat{A}_{k,\omega} = A_0 \frac{\sqrt{2\Delta t}}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-in\Delta t\left(\omega + k\frac{c_g}{a\cos\phi}\right)} \int_{-\pi}^{\pi} e^{\frac{-\eta^2}{2\sigma^2}} e^{-ik\eta} d\eta$
- $\hat{A}_{k,\omega} = A_0 \frac{\sqrt{2\Delta t}}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-in\Delta t\left(\omega + k\frac{c_g}{a\cos\phi}\right)} \sqrt{2\pi}\sigma e^{\frac{-k^2\sigma^2}{2}} \left[\mathrm{erf}\left(\frac{\pi}{\sqrt{2}\sigma} + \frac{ik\sigma}{\sqrt{2}}\right) + \mathrm{erf}\left(\frac{\pi}{\sqrt{2}\sigma} - \frac{ik\sigma}{\sqrt{2}}\right)\right]$

There is no closed form for the error functions in the last expression. But, to good approximation, the wavenumber spectrum of Gaussian envelope is again a Gaussian. The frequency-wavenumber spectrum describes a straight line through the origin. A convolution with the Fourier transfrom of the carrier wave, *a delta function* $\delta\left(k - k_0, \omega - c_p^* k_0\right)$, centers the spectrum around $\left(k_0, c_p^* k_0\right)$.

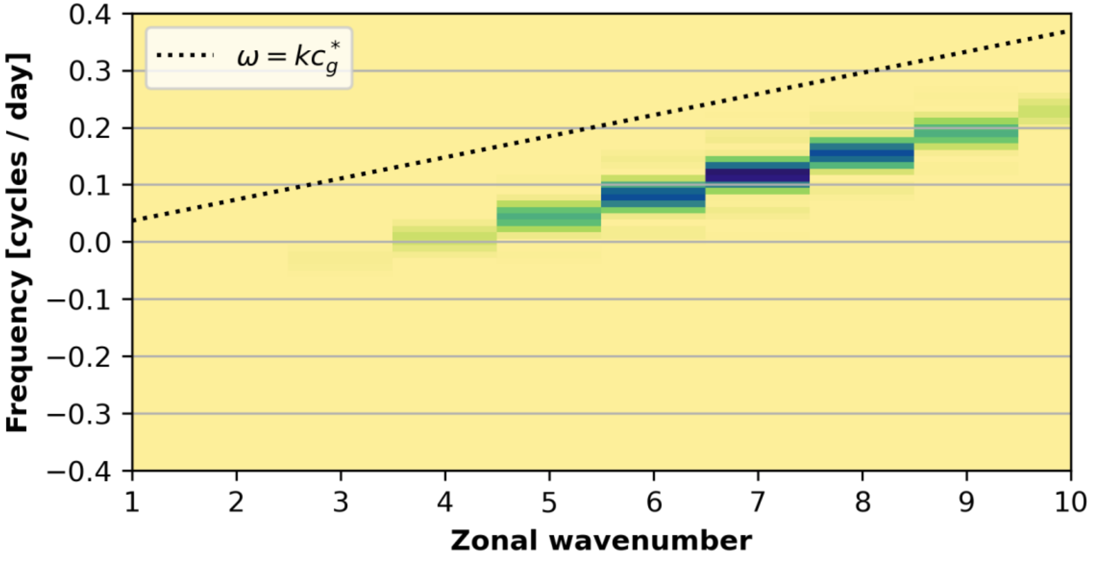
SPECTRAL ANALYSIS OF THE IDEALIZED WAVE PACKET

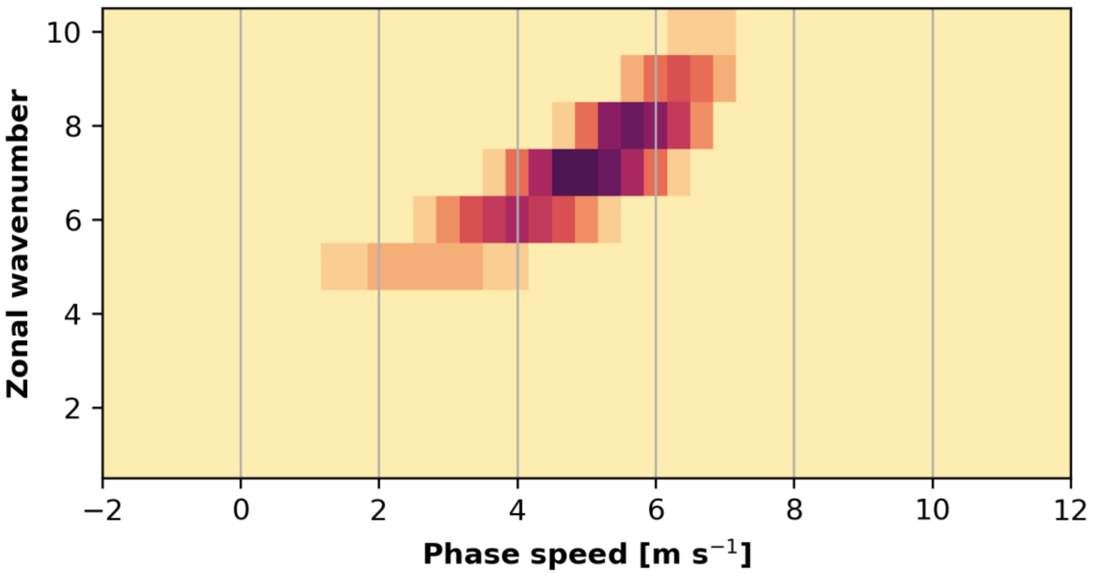
Alternatively, we can perform the Fourier transformation numerically. Following Randel & Held (1991), we can also convert the wavenumber-frequency spectrum into a phase speed-wavenumber spectrum.





SPECTRAL ANALYSIS OF THE IDEALIZED WAVE PACKET

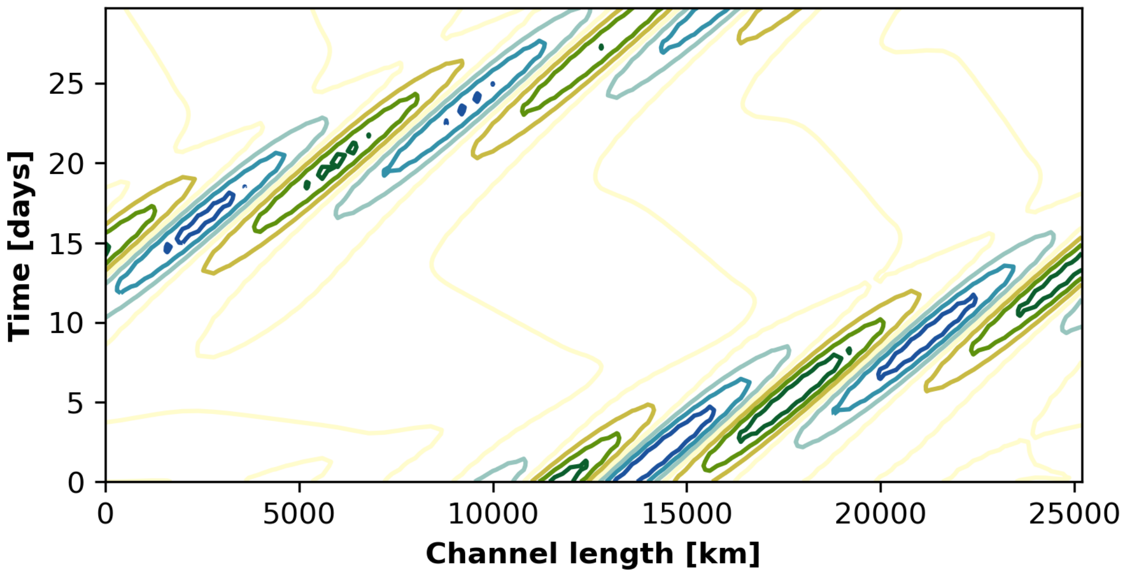




- Peak power at $\left(k_0, k_0 c_p^*\right)$ and $\left(c_p, k_0\right)$ respectively.
- The frequency spectrum follows a straight line parallel to $\omega = kc_g^*$.
- The Hayashi spectrum follows a curved line limits at c_g for $k \rightarrow \infty$ and $-\infty$ for $k \rightarrow 0$.
- Crossing zero phase speed and zero frequency at $\mathrm{k} = \frac{c_g - c_p}{c_g} k_0$.
- Width of the wavenumber spectrum is inversely proportional to the zonal width of the wave packet.

BAROTROPIC QUASI-GEOSTROPHIC CHANNEL MODEL

Downstream development of a barotropic Rossby wave packet with an initially Gaussian envelope in a channel of width $L = 3500$ km centered around $\phi = 51$ °N with a uniform background flow of 8.8 m/s.



The phase speed of a barotropic Rossby wave is given by

$$\omega/_{k^*} = U - \frac{\beta}{k^{*2} + \frac{\pi^2}{L^2}}$$

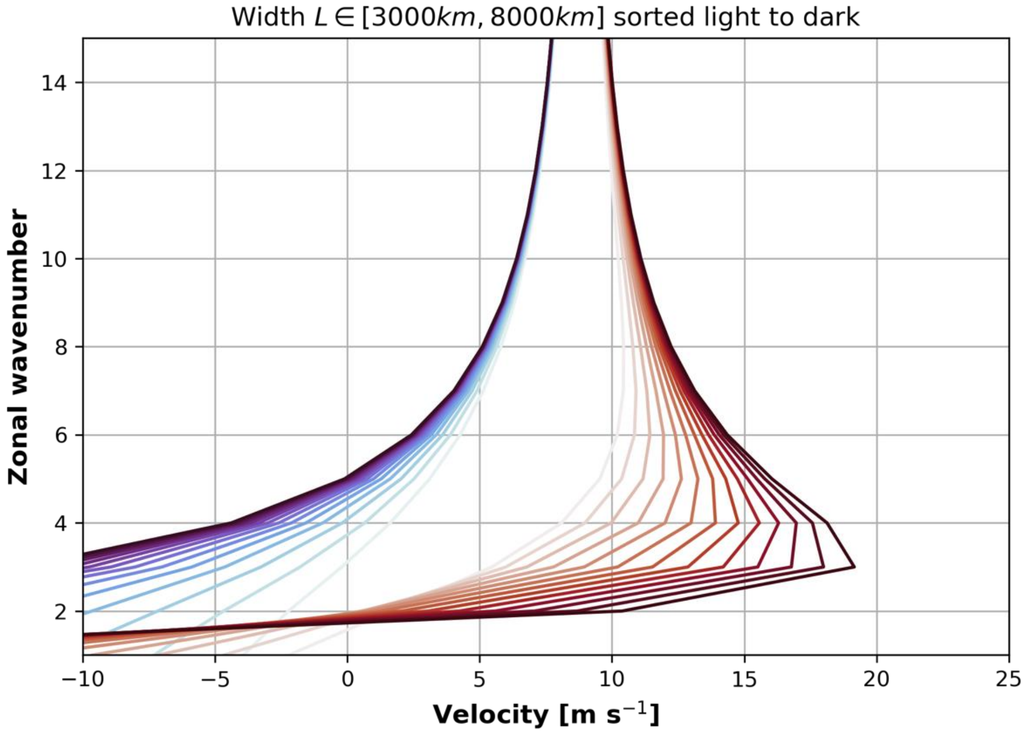
while the group velocity is

$$\partial\omega/_{\partial k^*} = U - \frac{\beta}{k^{*2} + \frac{\pi^2}{L^2}} + \frac{\beta 2k^{*2}}{\left[k^{*2} + \frac{\pi^2}{L^2}\right]^2}$$

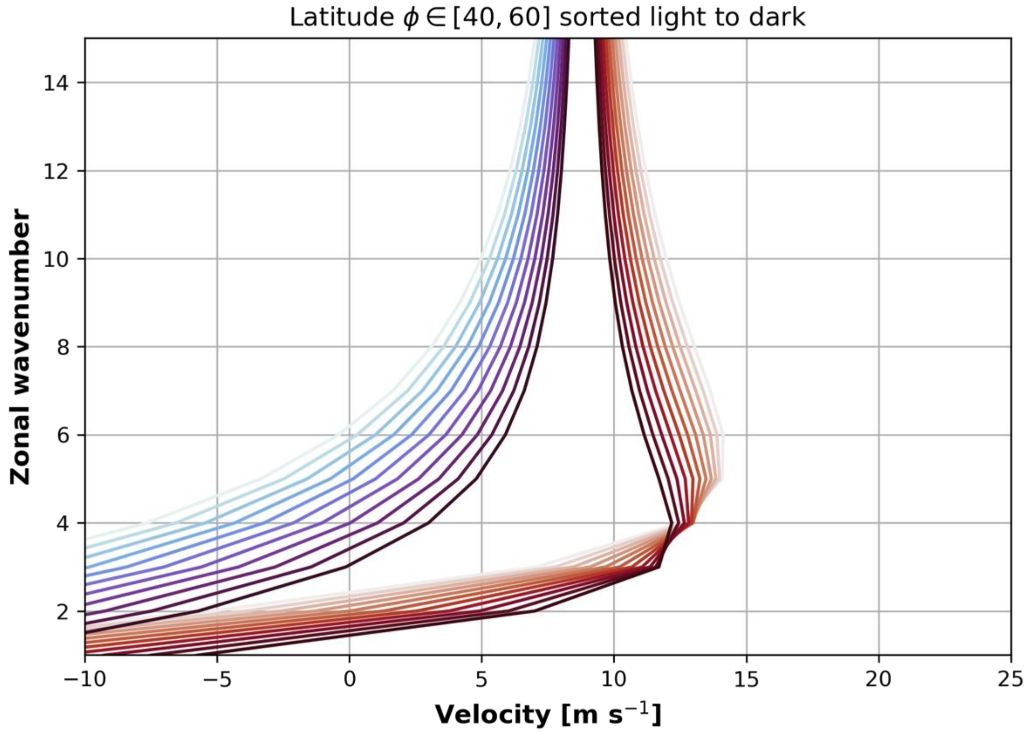
Abbreviation: $k^* = \frac{k}{a \cos \phi}$

BAROTROPIC QUASI-GEOSTROPHIC CHANNEL MODEL

Phase speed & group velocity for a narrow channel in *light* blue & red.
Phase speed & group velocity for a wide channel in **dark** blue & red.



Phase speed & group velocity for at low latitudes in *light* blue & red.
Phase speed & group velocity for at high latitudes in **dark** blue & red.



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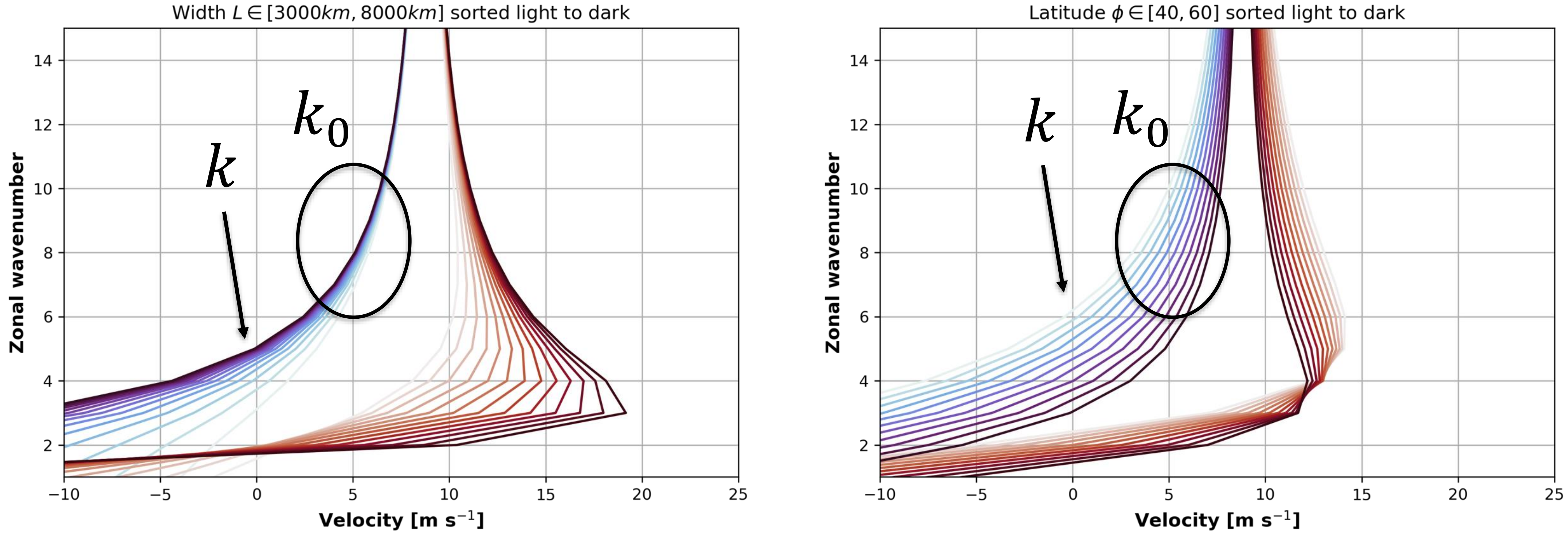

For an increasing channel width L or a reducing central latitude ϕ, the phase speed c_p reduces and the group velocity c_g increases. Hence, the dominant wavenumber k of the temporal mean approaches the wavenumber k_0 of the carrier wave and the power of the power of the temporal mean increases.

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