Fractional Fick's Law and Anomalous Transport of Energetic Particles at Interplanetary Shocks

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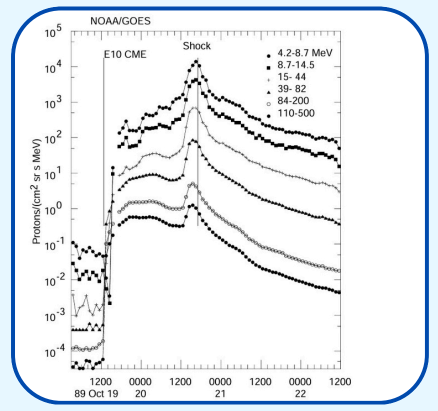
1 Introduction

Data analysis and numerical simulations of shock accelerated particles indicate that the downstream region exhibits a decreasing number density profile of energetic particles. However, in stationary conditions, the total particle flux should be uniform, according to the continuity equation; this requires a flat density profile on the dwonstream side, as prescribed by Diffusive Shock Acceleration theory (DSA), which expresses the total flux as the contribution of advection by the plasma flow and diffusion processes. To solve this discrepancy, we propose to change the expression for the contribution to the total flux due to diffusion, namely the standard flux-gradient relation given by the Fick's law $J = -D\frac{\partial n}{\partial x}$. In the present work, this new formulation is based on non-local phenomena, which are plausible in collissionless plasmas. The "non-local" diffusive flux is numerically calculated in several cases.



Intensity-time profiles at different energies for an event detected on the 19th October 1989 by one of the Geostationary Satellites launched by NOAA .

2 Fractional Fick's Law

As described by Calvo et al. (2007), we replace the ordinary Fick's Law with the non-local expression

[1] $$J(x) = \frac{D_s}{2\cos[(1+\beta)\pi/2]}\frac{\beta}{\Gamma(1-\beta)}\int_0^\infty \frac{n(x+\xi)-n(x-\xi)}{\xi^{1+\beta}}\,d\xi \qquad 0<\beta<1$$

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0 \quad \Rightarrow \quad \frac{\partial n(x,t)}{\partial t} - D_s R_x^{1+\beta} n(x,t) = 0$$

$R_x^{1+\beta}$ is the Riesz fractional derivative!

We obtain the diffusion equation describing an anomalous transport regime in which

$\langle \Delta x^2 \rangle \propto t^\gamma \quad \gamma > 1$ → Superdiffusion!

This new formulation, instead of the first-order spatial derivative of the number density, involves the so-called Fractional Derivatives, which extend the concept of " derivative" to non-integer derivation orders!

3 Numerical implementation

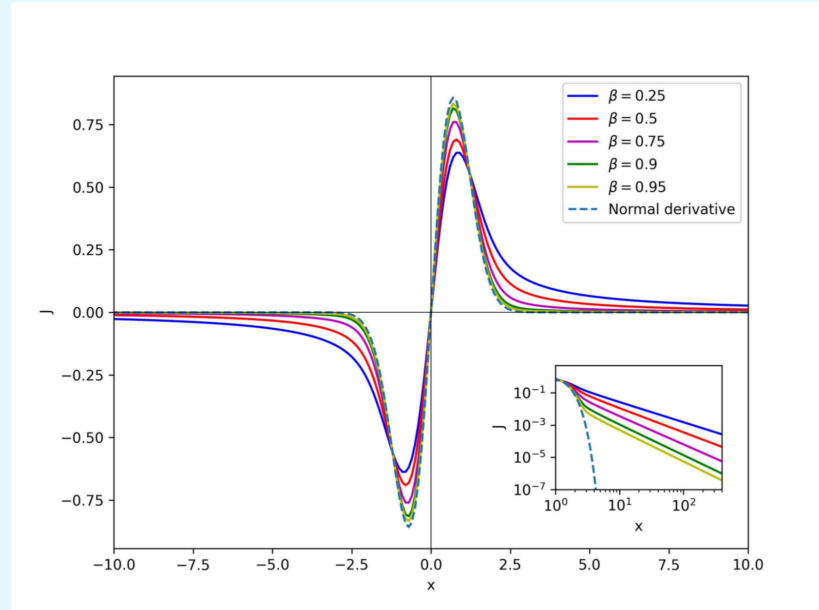
- Finite set of values of $n(x)$ → Truncated integral $$I(x) = \int_0^{\xi_N} \frac{n(x+\xi)-n(x-\xi)}{\xi^{1+\beta}}d\xi$$
- Singularity in correspondence of $\xi = 0$ $\quad I = I_1 + I_2 \quad$ with $$I_1 = \int_0^{\Delta} \frac{n(x+\xi)-n(x-\xi)}{\xi^{1+\beta}}\,d\xi$$

If Δ is small $$\frac{n(x+\xi)-n(x-\xi)}{\xi} \approx \frac{n(x+\Delta)-n(x-\Delta)}{\Delta} \approx 2n'(x) \qquad I_1 = \frac{1}{1-\beta}\frac{n(x+\Delta)-n(x-\Delta)}{\Delta^\beta}$$

- Trapezoidal method for I_2 → $$I_2 = \int_\Delta^{\xi_N} \frac{n(x+\xi)-n(x-\xi)}{\xi^{1+\beta}}\,d\xi = \int_\Delta^{\xi_N} f(\xi)\,d\xi \qquad I_2 \approx \sum_{j=1}^{N}\left[\frac{f(\xi_{j-1})+f(\xi_j)}{2}\right]\Delta_j$$
- We assume $D_s = 1$ for the anomalous diffusion coefficient. $\qquad \Delta_j = \xi_j - \xi_{j-1}$

4 Results

- **Gaussian Profile** $\quad n(x) = e^{-x^2}$



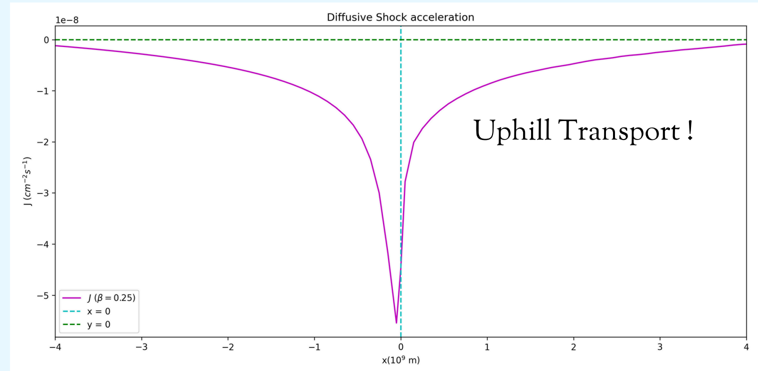
Discretization of the *x* axis:

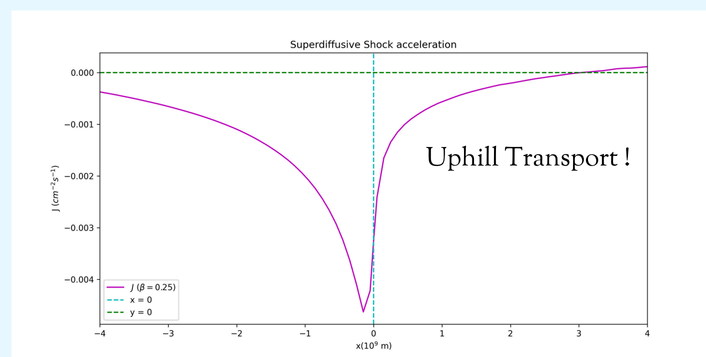
$x_j = j\Delta, \quad j = -N_x, ..., N_x$

$\Delta = 0.1 \qquad N_x = 10000$

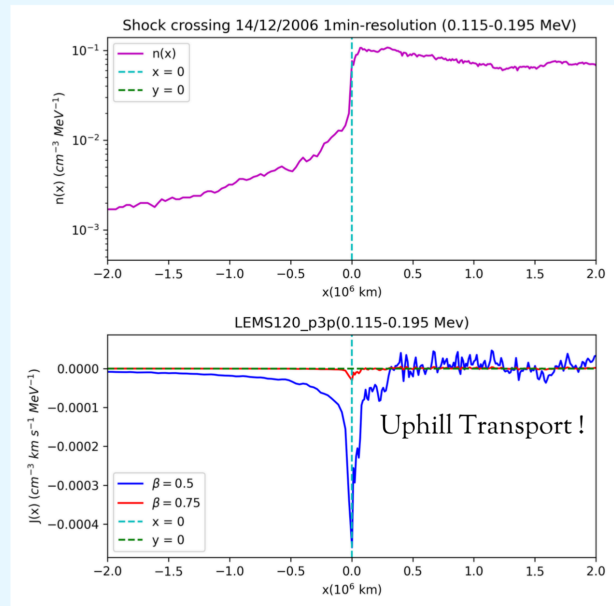
- Number of upstream and downstream points in which we evaluate the flux: $N_1 = 4999$
- Number of points in the integration domain: $N_2 = 5000$

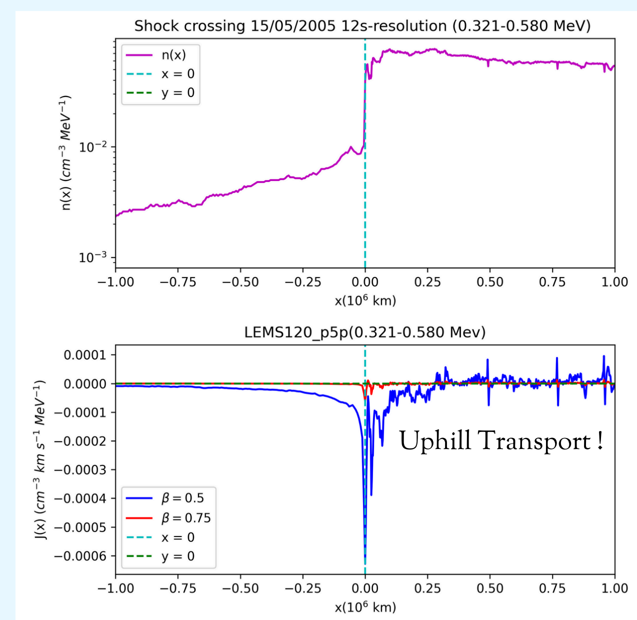
- **Density profile for a shock crossing simulated by *Prete et al.*** [2]





- **Interplanetary shocks observed by ACE and Wind** *(work in progress)*





5 Conclusions

- The numerical evaluation of the fractional flux highlights the possibility of uphill transport, that is, transport in the same direction as the density gradient, which is an interesting and counterintuitive result!
- An uphill negative downstream flux can balance the positive advective flux due to plasma flow, thus giving a conserved total flux.

References:

[1] Calvo I., et al., 2007, *Fractional Generalization of Fick's Law: A Mi croscopic Approach*, Physical Review Letters, Vol. 99.

[2] Prete G., et al., 2019, *Influence of the transport regime on the energetic particle density profiles upstream and downstream of interplanetary shocks*, Advances in Space Research, Vol. 63, pag. 2659 2671.