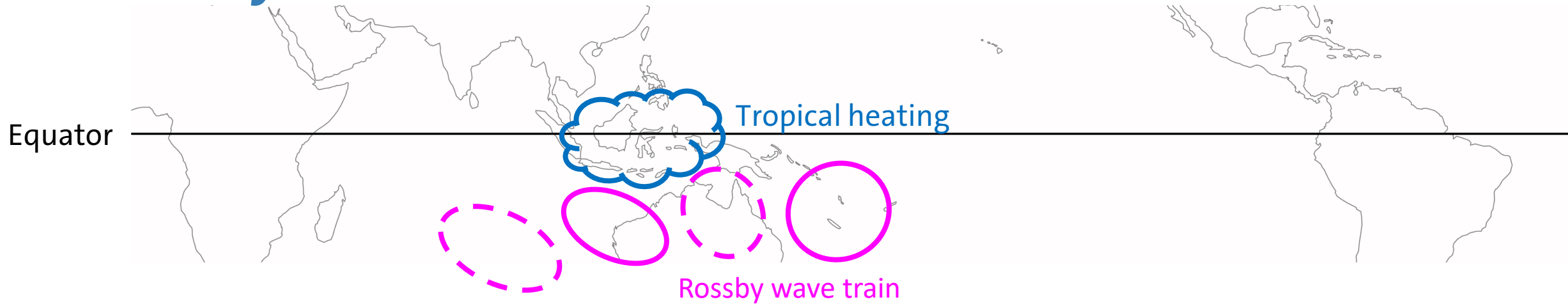


Adiabatic and diabatic energy tendencies of the equatorial Kelvin wave

Katharina M. Holube, Frank Lunkeit, Sergiy Vasylkevych, Nedjeljka Žagar

Atmospheric Dynamics and Predictability Group, Meteorological Institute

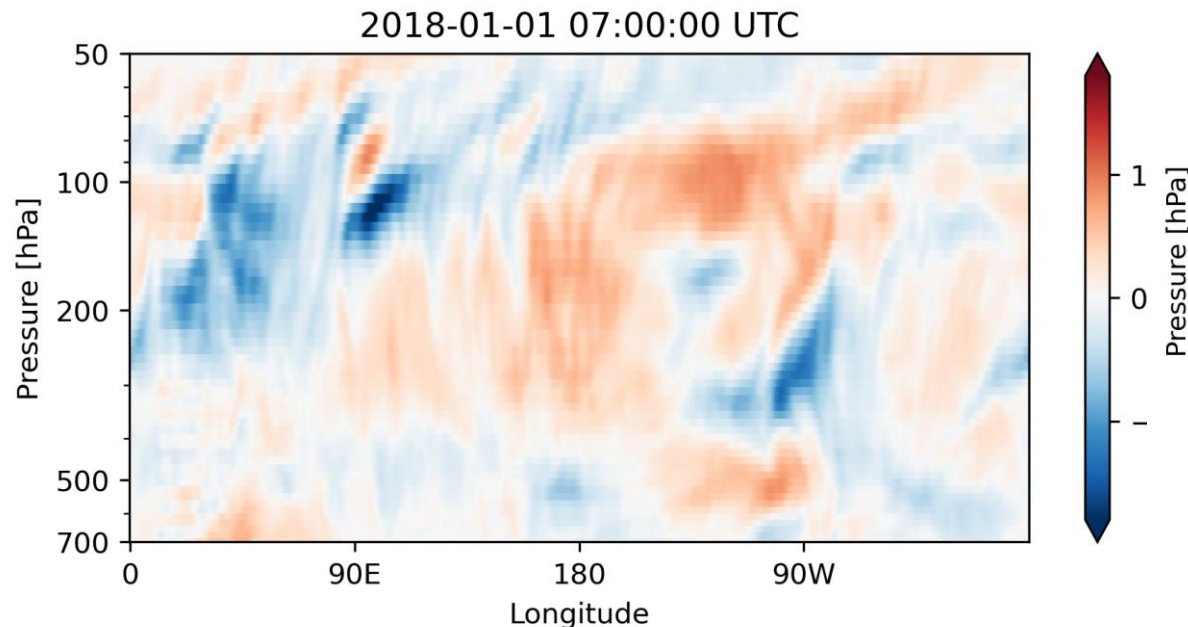
Dynamical and diabatic Kelvin wave sources



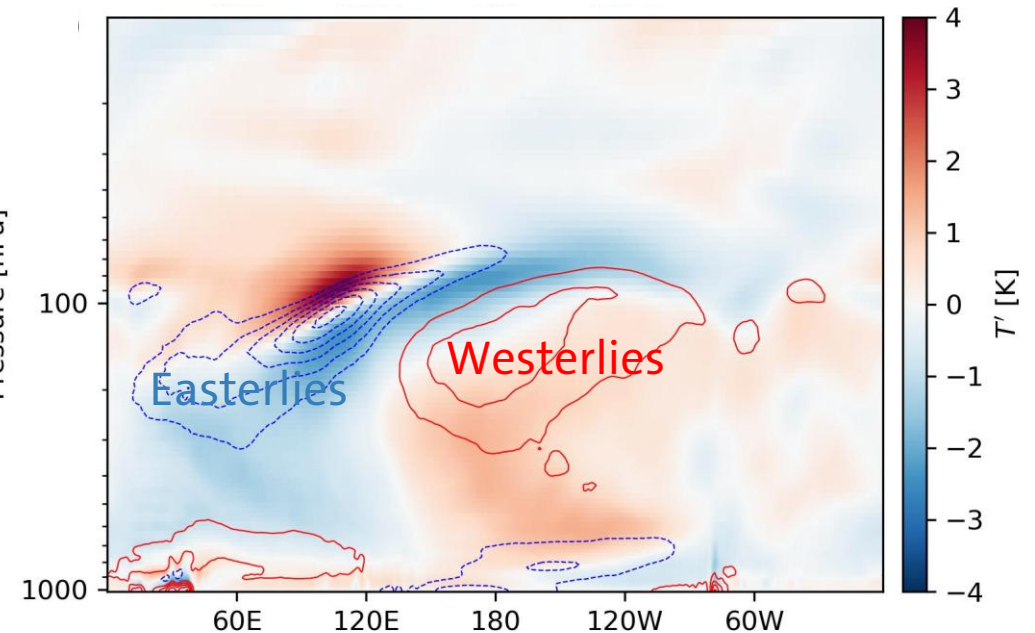
- Kelvin wave response to **latent heat release** (e.g. Salby and Garcia 1987)
- Momentum flux forcing associated with **extratropical Rossby waves** (e.g. Tulich and Kiladis 2021; Cheng et al. 2022; Holube et al. 2024)
- Dry dynamical processes are source in 3D normal mode energy budget of Kelvin waves (Tanaka 1985; Castanheira and Marques 2021)
- Are **dynamical** or **diabatic** Kelvin wave sources predominant in reanalysis data?
- We quantify all energy sources of Kelvin waves in the 3D normal mode energy budget in ERA5 forecasts, for which tendencies due to parametrizations are saved.

Identifying Kelvin waves using 3D normal-mode decomposition

Zonal velocity of Kelvin waves at the equator
from ERA5 forecasts



Time-average over January 2018



- Expansion of wind and geopotential from ERA5 into 3D normal-mode functions
- Part of Kelvin wave is apparently stationary (Žagar et al. 2022) (modes.cen.uni-hamburg.de)
 - **Westerlies**: “Gill’s response” to heating over warm pool
 - **Easterlies**: Forced by meridional advection of zonal momentum

Kelvin wave energy budget

- Kelvin wave energy I_K consists of kinetic energy and available potential energy (Tanaka 1985)
- Kelvin wave energy tendencies $\frac{dI_K}{dt}$ are computed from momentum and temperature tendencies

Saved in ERA5 forecasts

$$\frac{dI_K}{dt} = \text{Resolved nonlinear dynamics} + \text{Diabatic processes} + \text{Subgrid-scale parametrizations}$$

↓

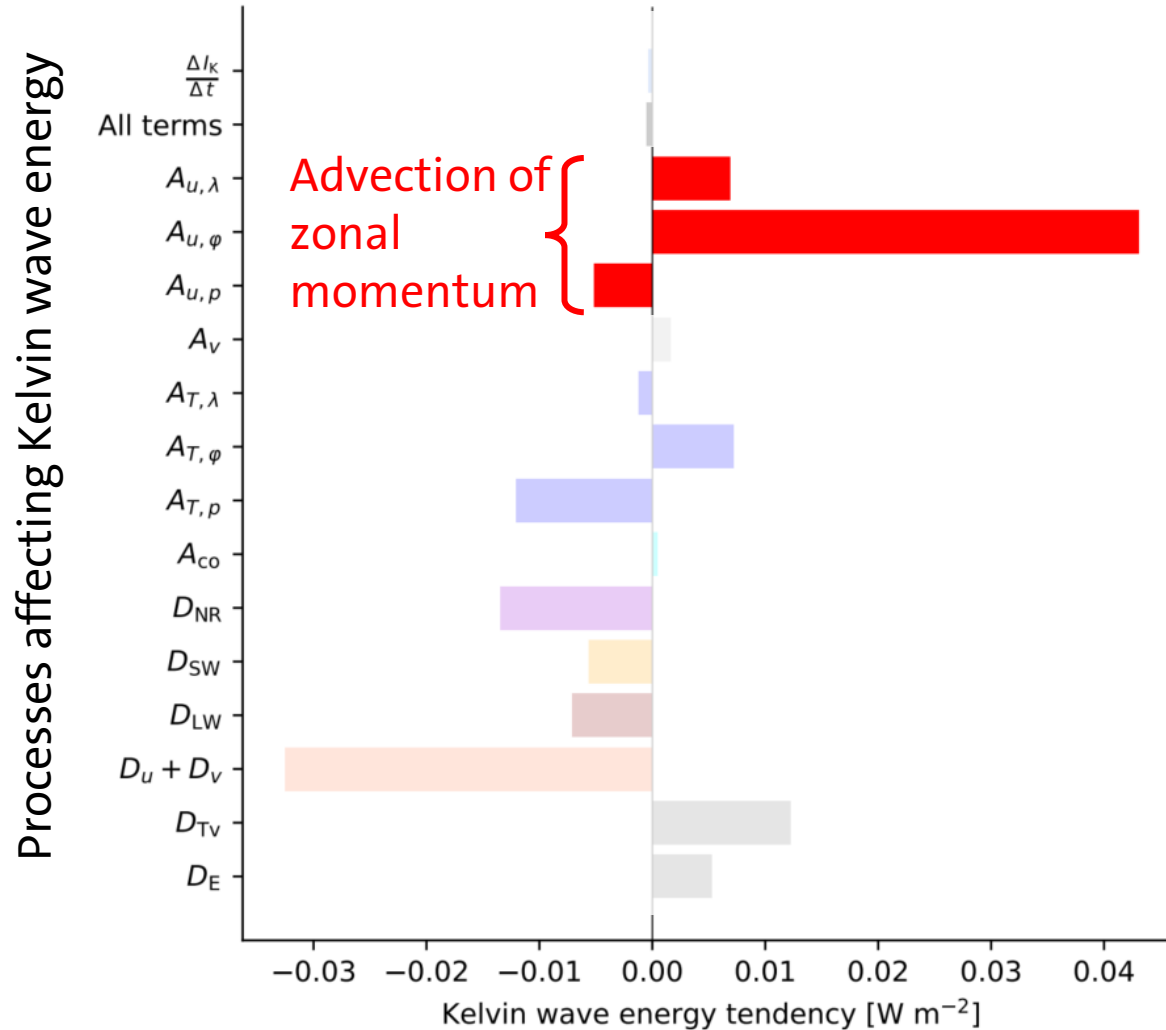
- Contain advection of momentum and temperature
- Computed from normal mode expansion, this allows decomposition into wave-mean flow and wave-wave interactions

↘

- Latent heat release
- Shortwave radiation
- Longwave radiation

Kelvin wave energy budget

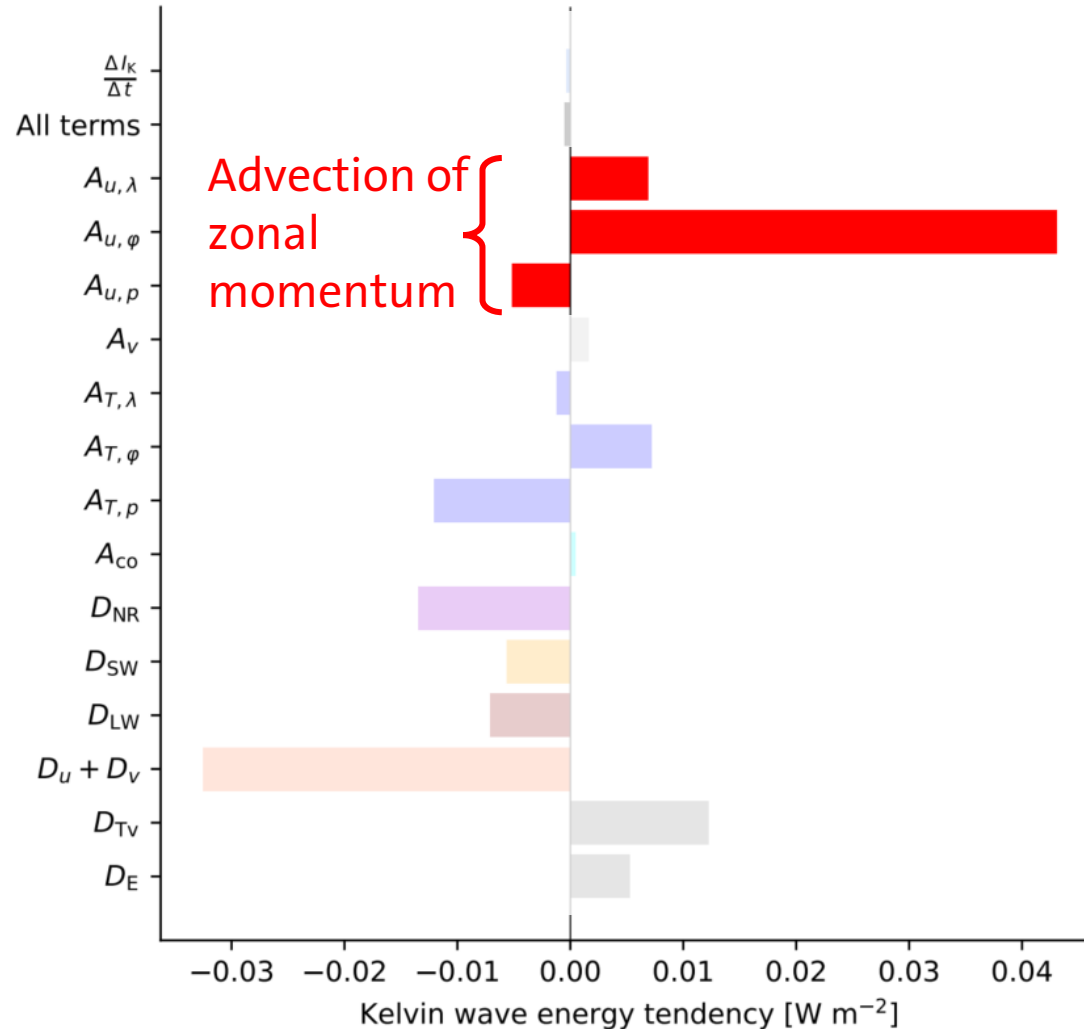
Time-average over December 2017 – November 2018



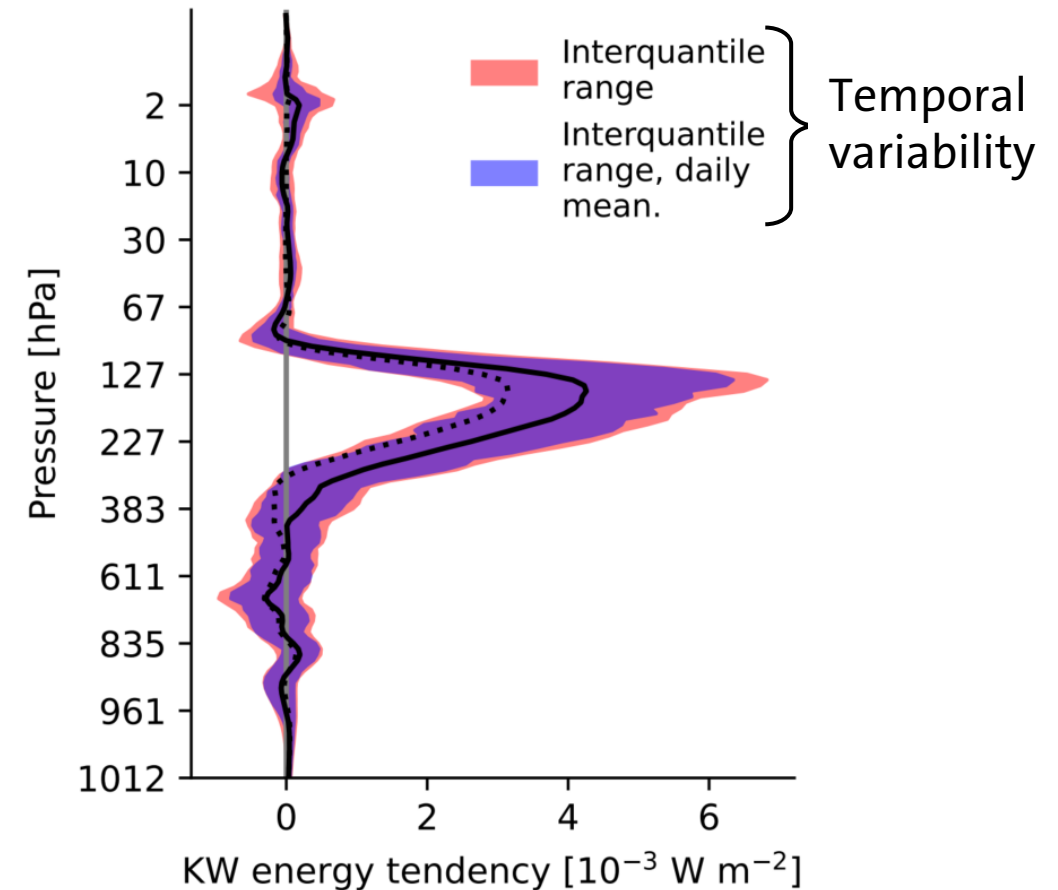
Kelvin wave energy budget

Time-average over December 2017 – November 2018

Processes affecting Kelvin wave energy

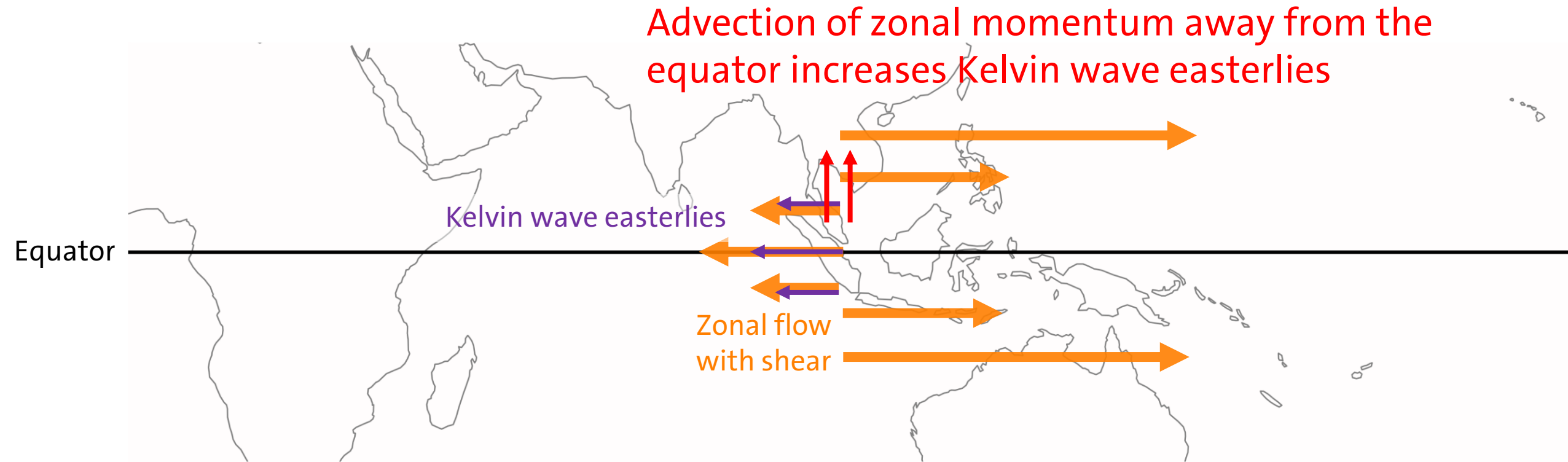


Takes place in the upper troposphere



Vertical profile is consistent with aquaplanet simulations by Tulich and Kiladis (2021)

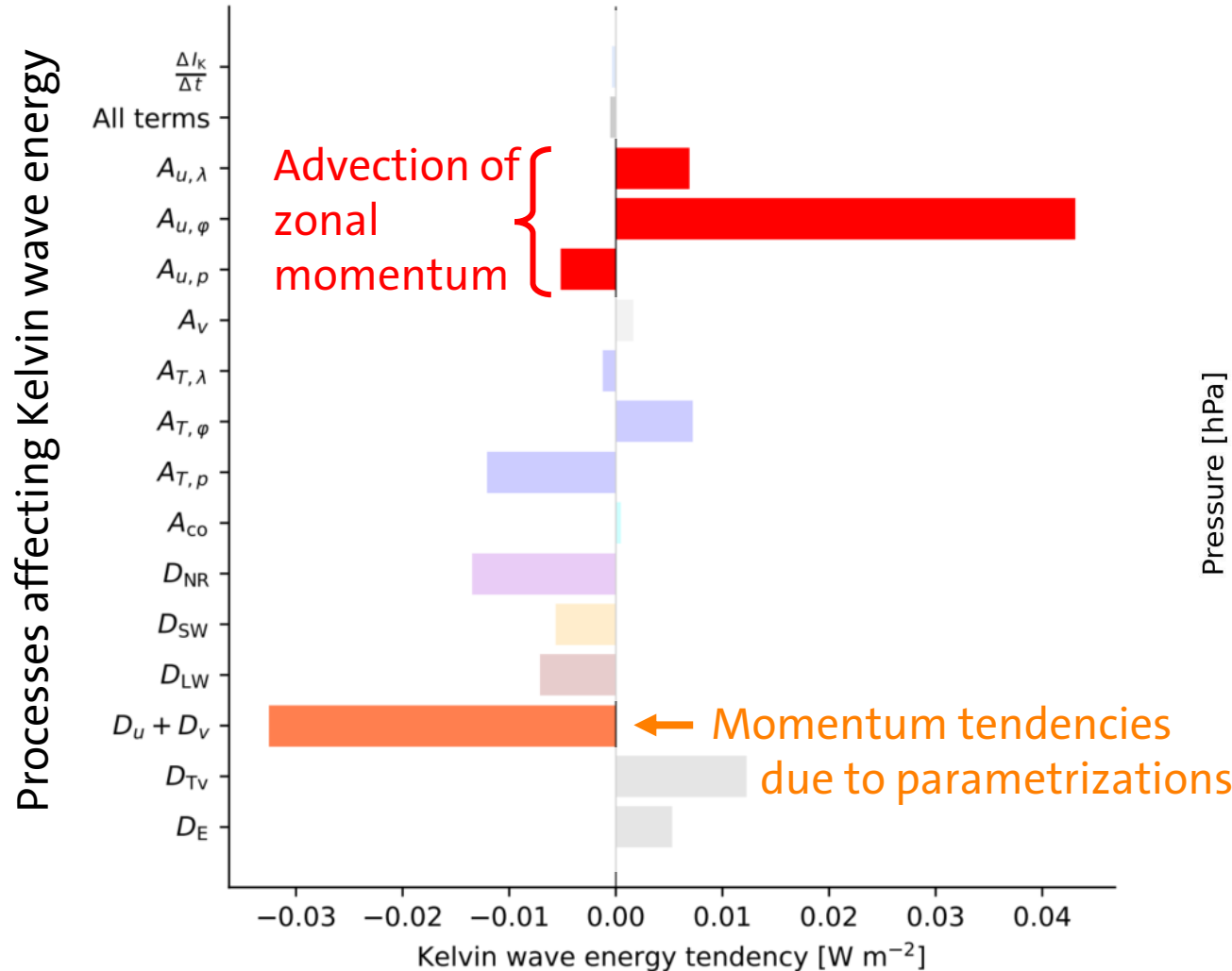
Meridional momentum advection over Indian Ocean is source of Kelvin wave energy



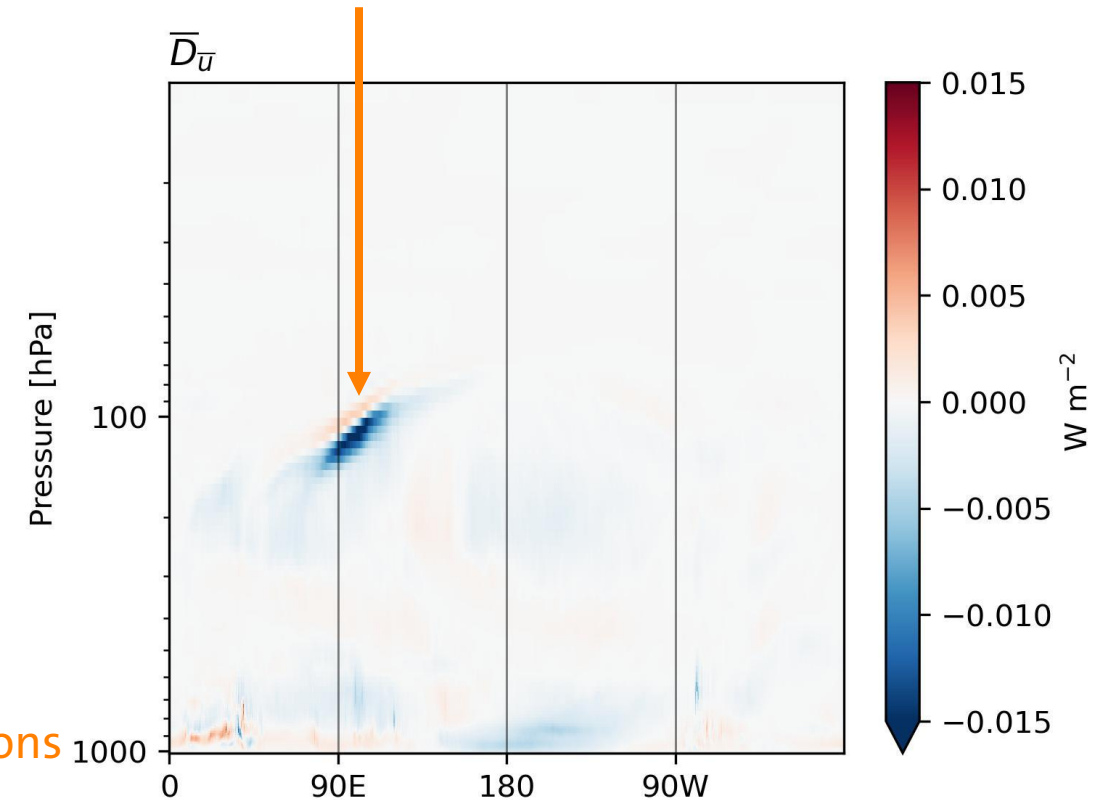
Sketch of upper-tropospheric horizontal flow

Momentum tendencies result in Kelvin wave energy sink

Time-average over December 2017 – November 2018



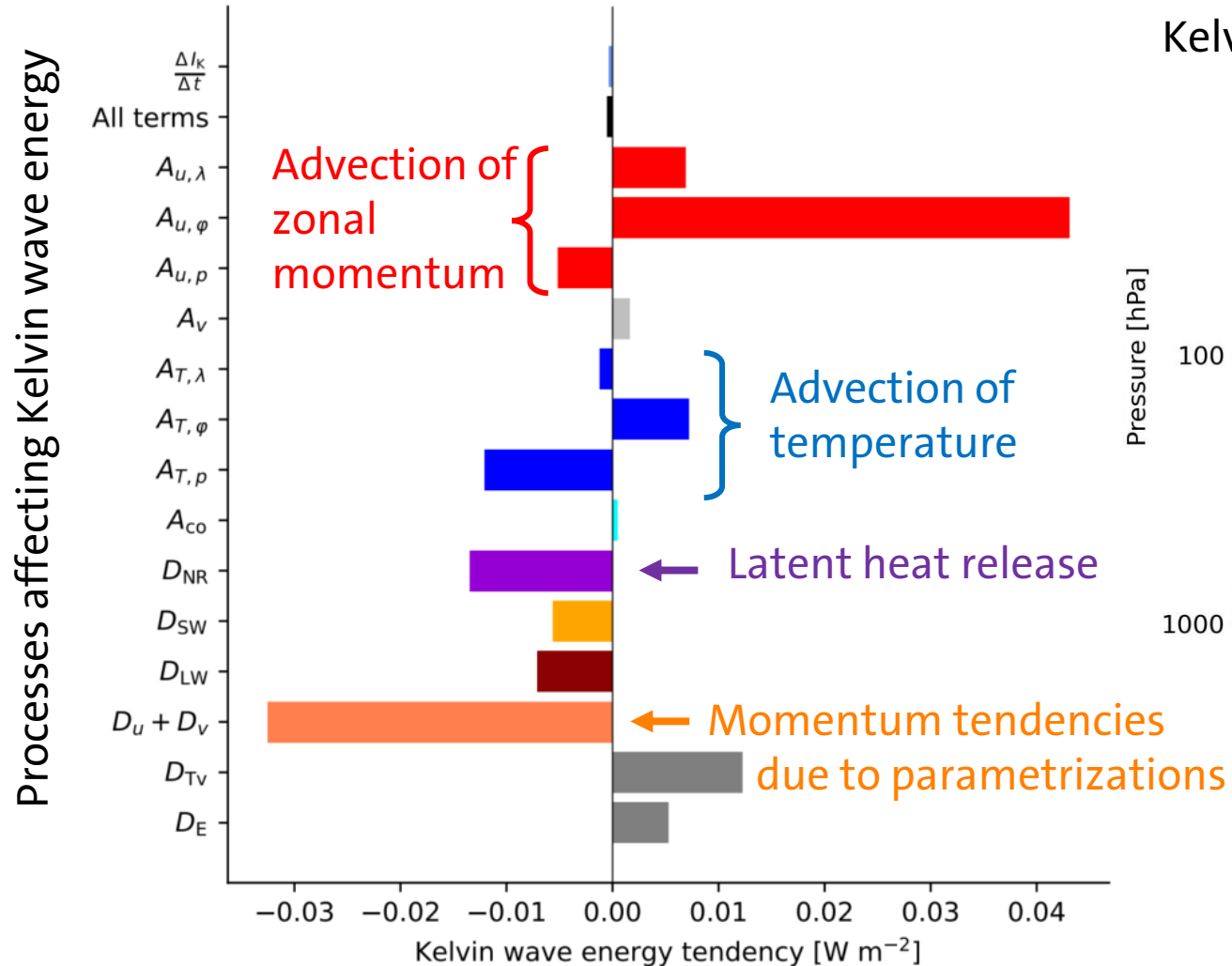
Energy sink in regions of strong shear (vertical diffusion)



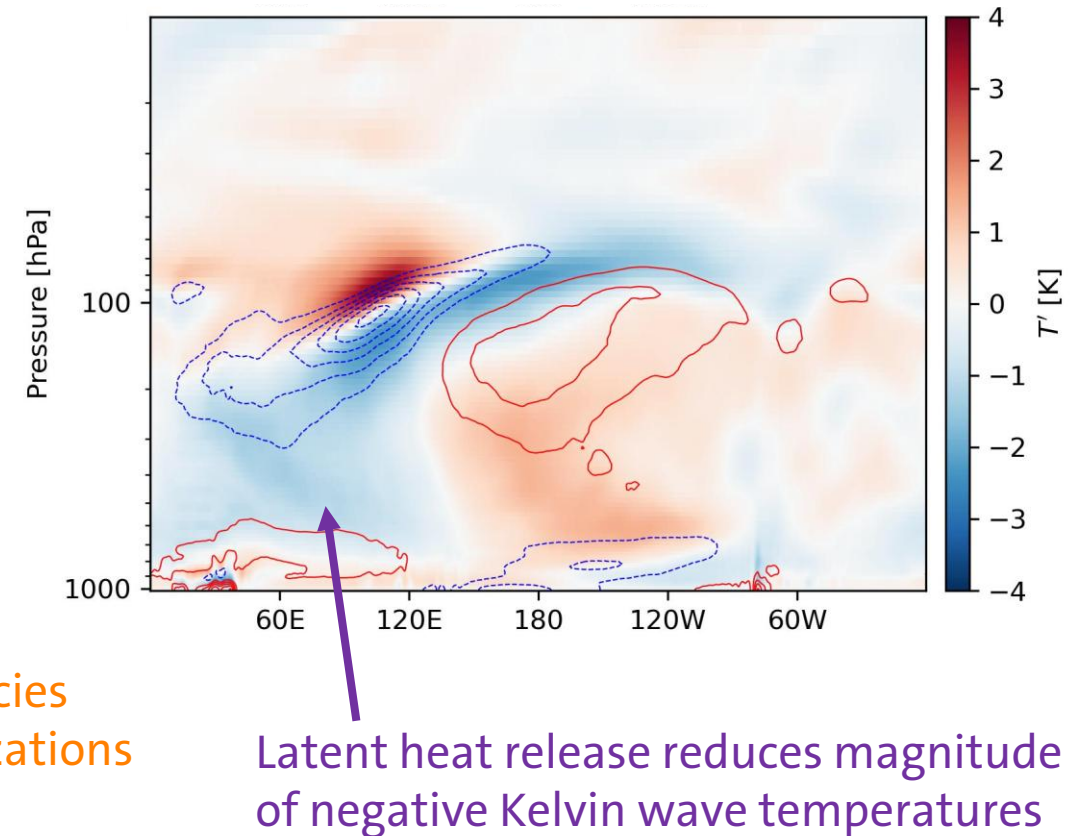
Kelvin wave energy tendency resulting from averaged momentum tendencies due to parametrizations in January 2018.

Latent heat release results in Kelvin wave energy sink

Time-average over December 2017 – November 2018

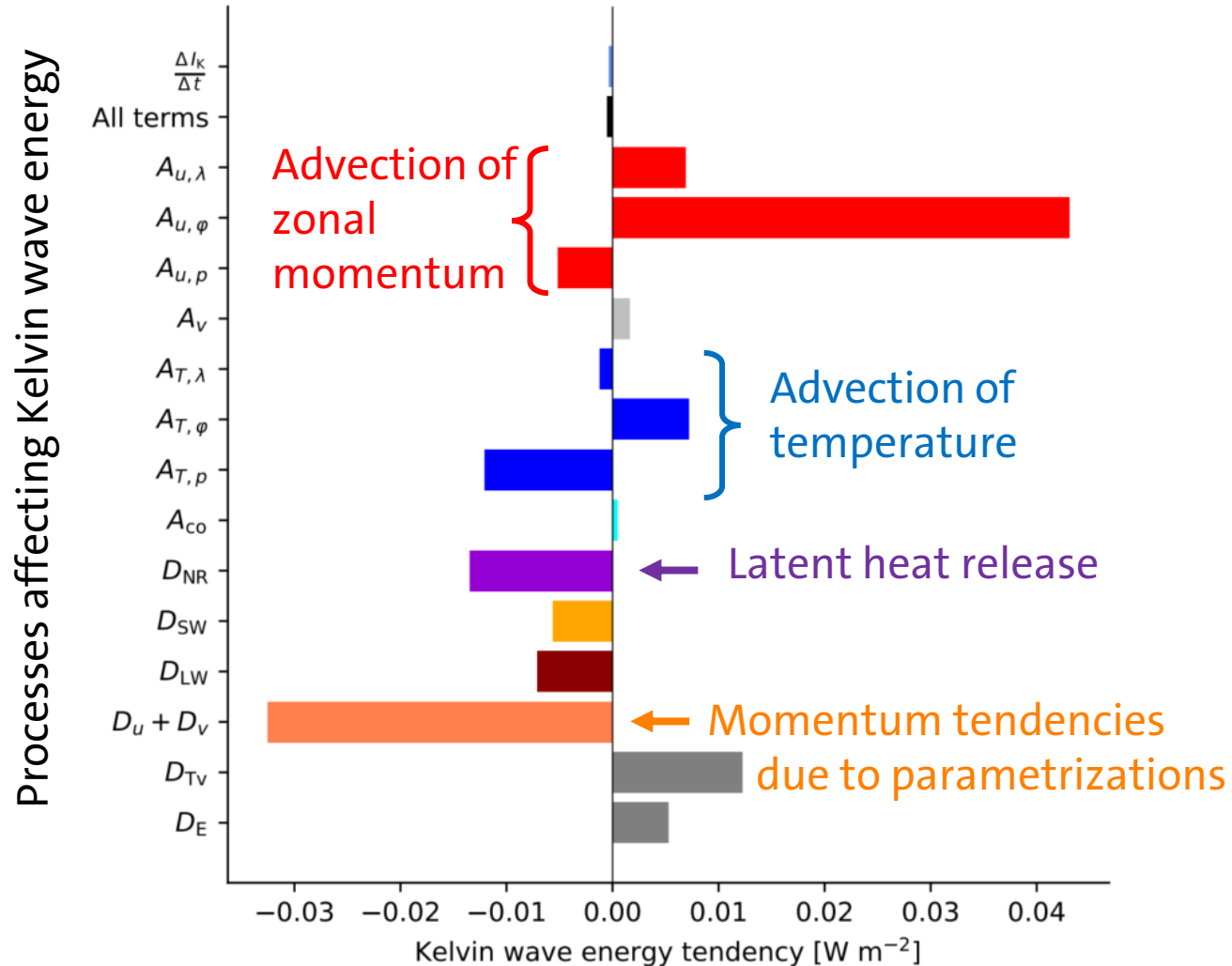


Kelvin wave: Time-average over January 2018

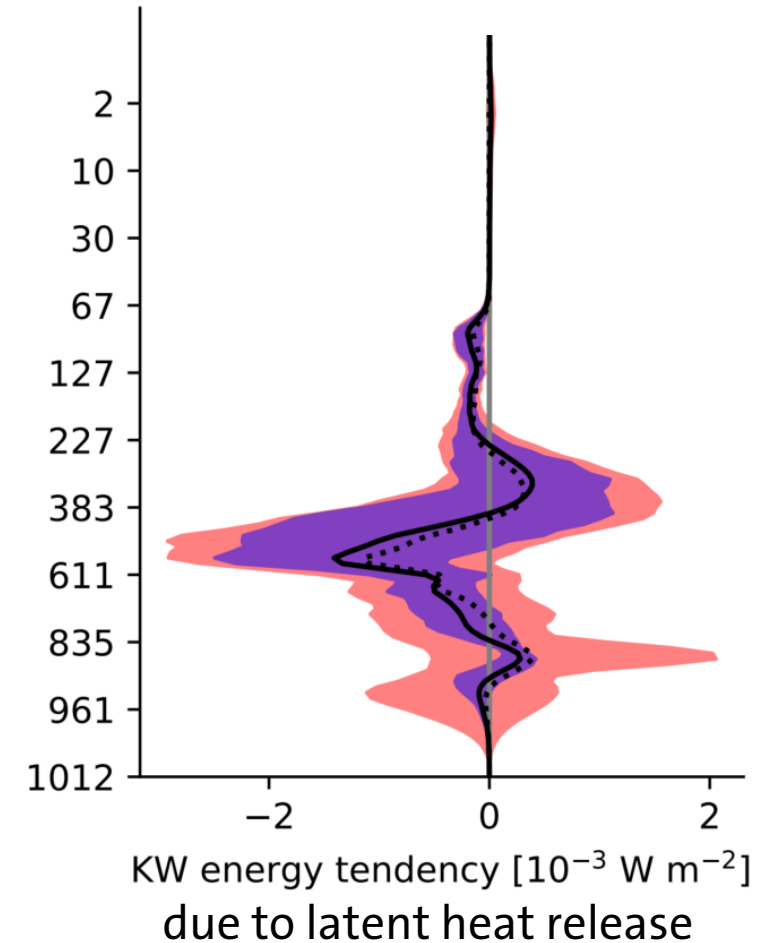


Latent heat release results in Kelvin wave energy sink

Time-average over December 2017 – November 2018



Energy sink in mid-troposphere



Consistent with aquaplanet simulations by Tulich and Kiladis (2021)

Conclusions

- In ERA5 forecasts, the main source of Kelvin wave energy arises from meridional advection of zonal momentum. This strengthens Kelvin wave easterlies in the upper troposphere.
- In the time average, Kelvin wave energy sinks result from latent heat release and other parametrized processes.

Other results (not shown):

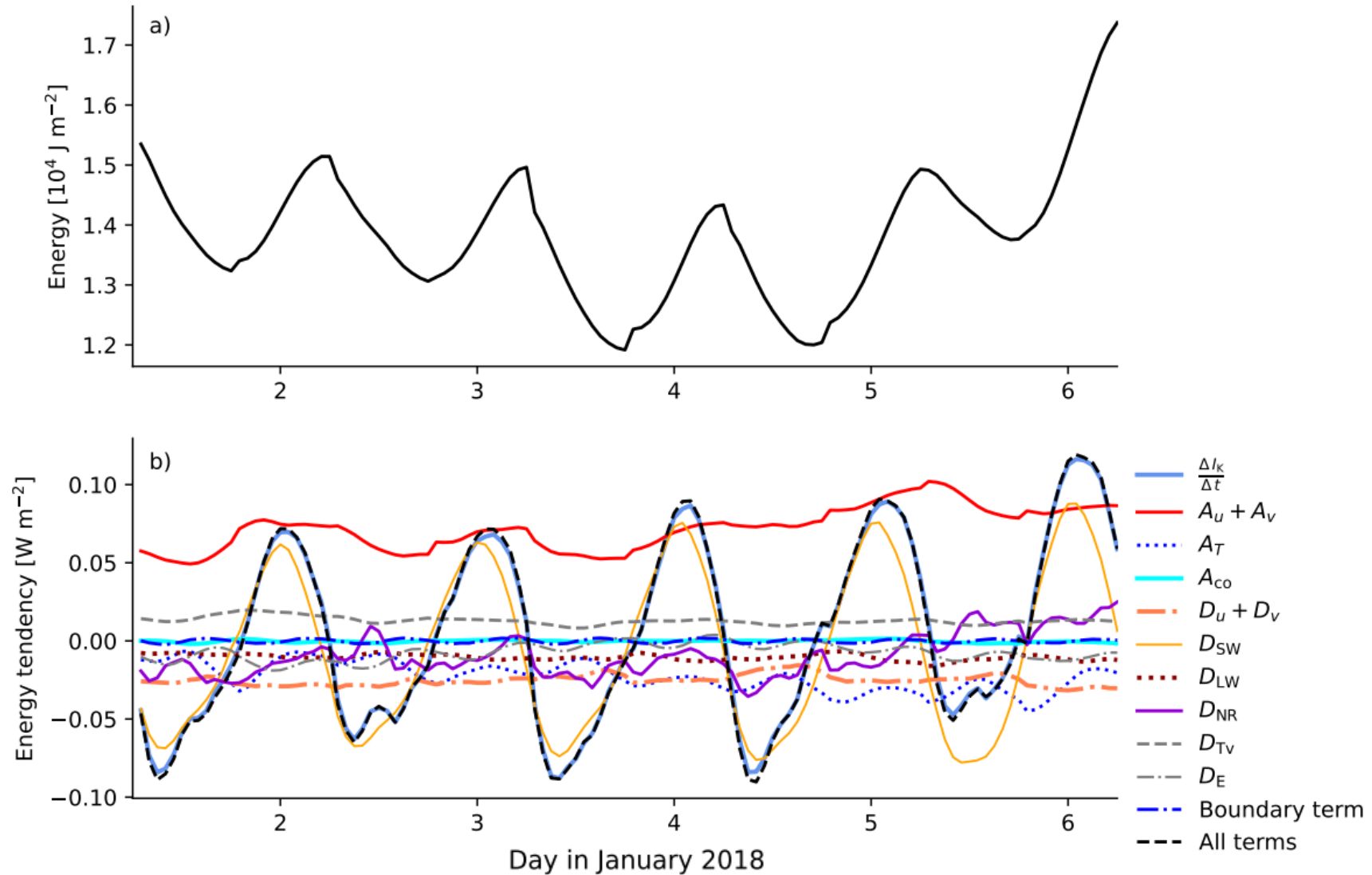
- Latent heat release governs variability of Kelvin wave energy on a time scale of several days.
- Shortwave radiation induces a diurnal cycle in Kelvin wave energy.
- Kelvin wave energy source due to meridional advection is located in winter hemisphere.

Further studies: Decomposition of dynamical Kelvin wave energy sources into wave-wave interactions and wave-mean flow interactions

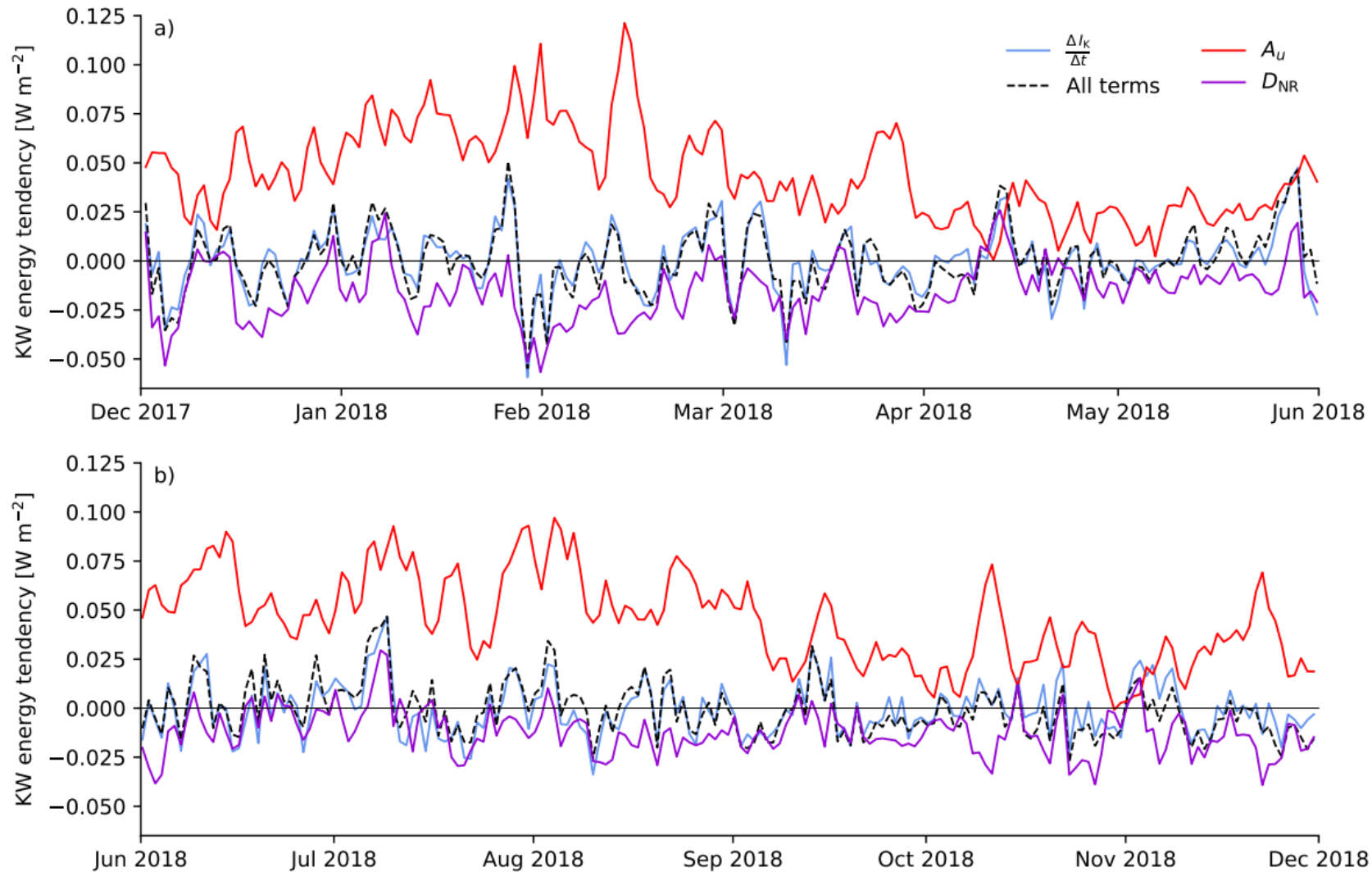
References

- Castanheira and Marques 2021: The equatorial wave skeleton of the Madden-Julian Oscillation, QJRM.
- Cheng et al. 2022: Two Extratropical Pathways to Forcing Tropical Convective Disturbances, J. Climate.
- Hersbach et al. 2020: The ERA5 global reanalysis, QJRM.
- Holube et al. 2024: Resonant Excitation of Kelvin Waves by Interactions of Subtropical Rossby Waves and the Zonal Mean Flow, JAS.
- Salby and Garcia 1987: Transient Response to Localized Episodic Heating in the Tropics. Part I: Excitation and Short-Time Near-Field Behavior, JAS.
- Tanaka 1985: Global Energetics Analysis by Expansion into Three-Dimensional Normal Mode Functions during the FGGE Winter, Journal of the Meteorological Society of Japan.
- Tulich and Kiladis 2021: On the Regionality of Moist Kelvin Waves and the MJO: The Critical Role of the Background Zonal Flow, JAMES.
- Žagar et al. 2022: Three-dimensional structure of the equatorial Kelvin wave: Vertical structure functions, equivalent depths, and frequency and wavenumber spectra, J. Climate.

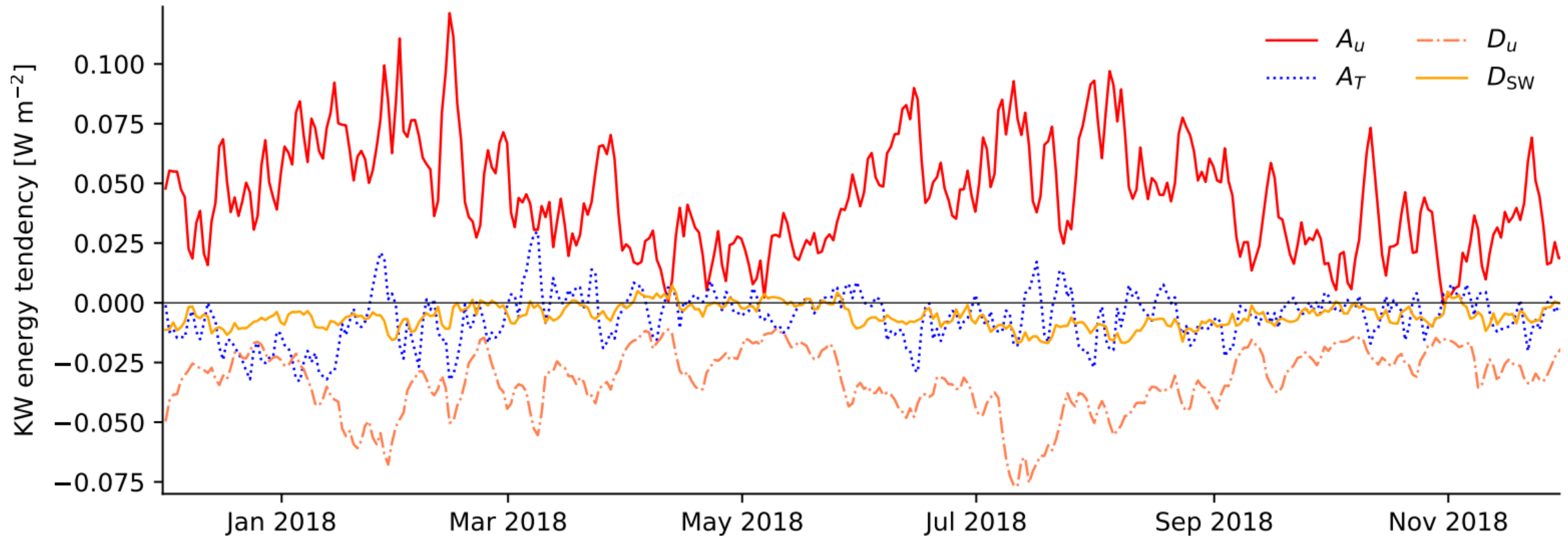
Short timeseries of Kelvin wave energy and energy tendencies



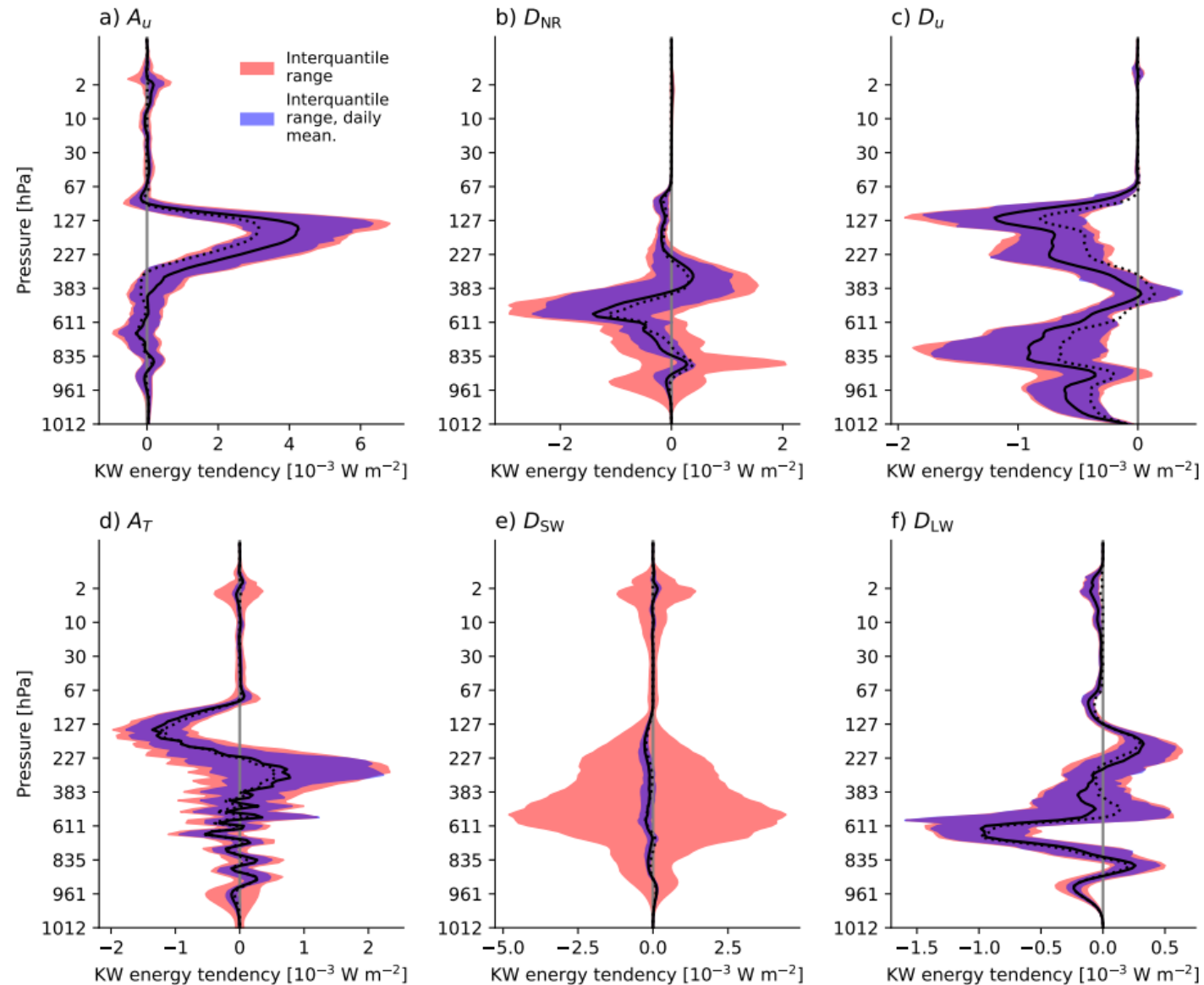
Kelvin wave energy tendencies for one year



Selected Kelvin wave energy tendencies



Vertical localization of Kelvin wave energy sources



Vertical profiles of energy tendencies in aquaplanet simulations

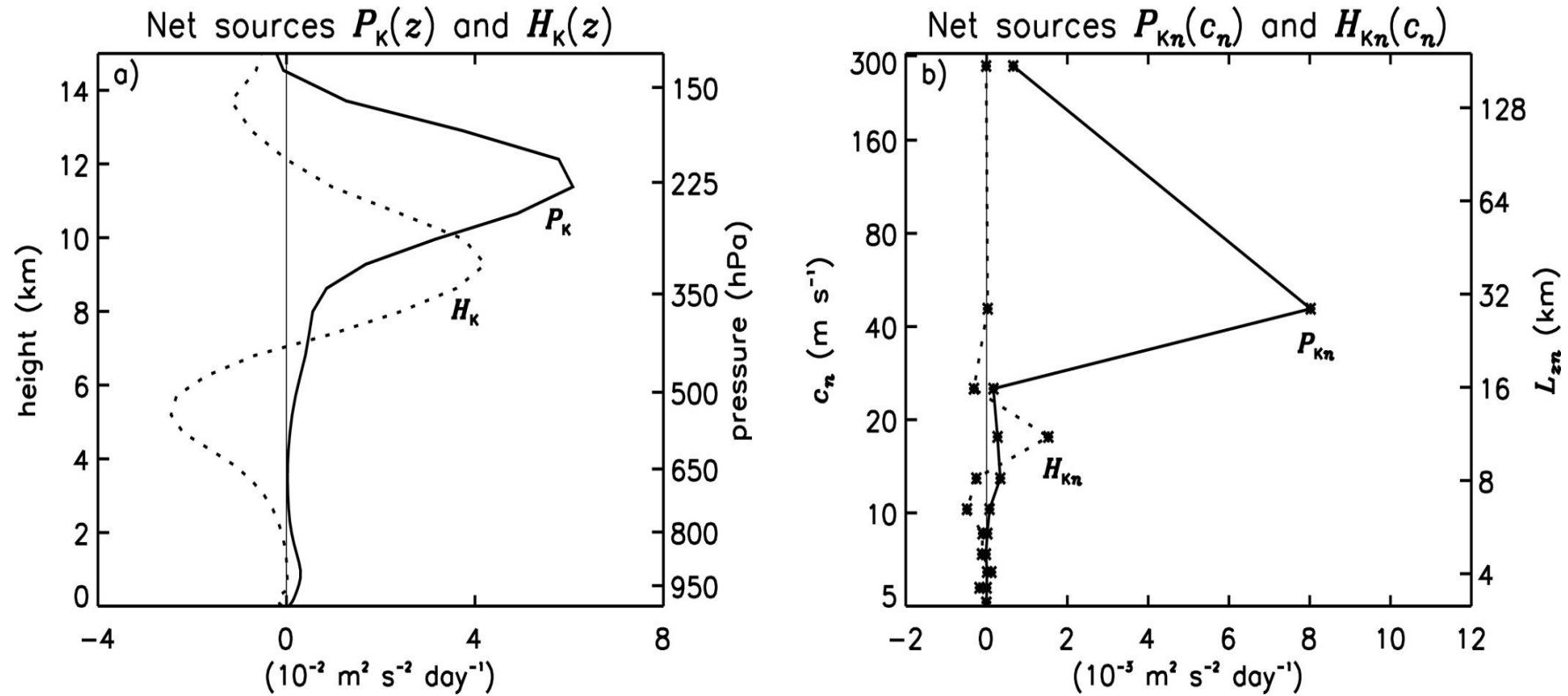
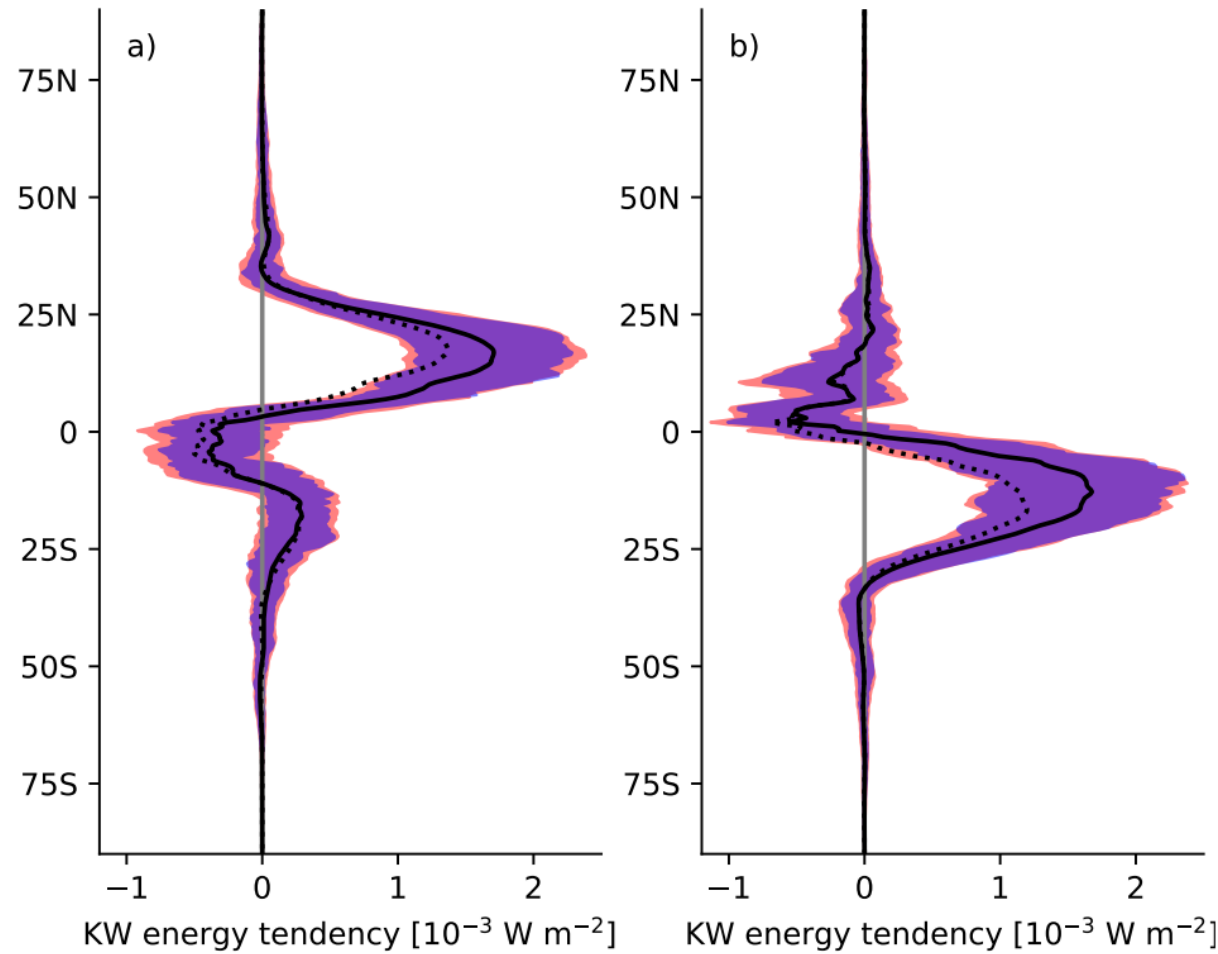


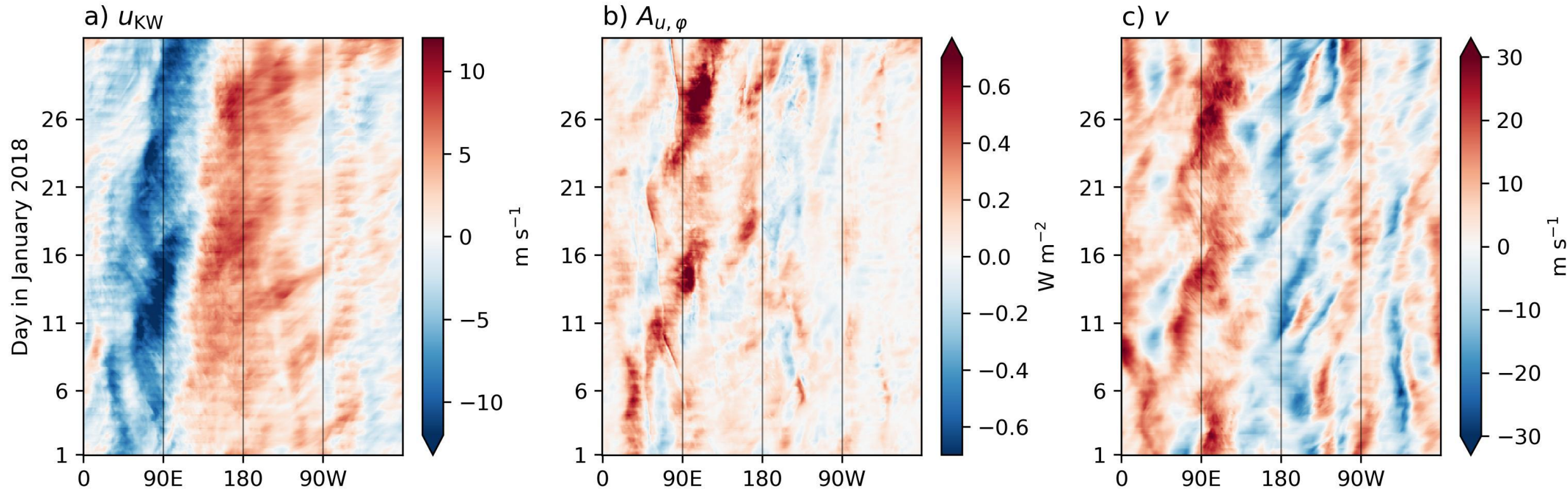
Fig. 7 in Tulich and Kiladis (2021).

Latitudinal localization of Kelvin wave energy source due to advection of zonal momentum



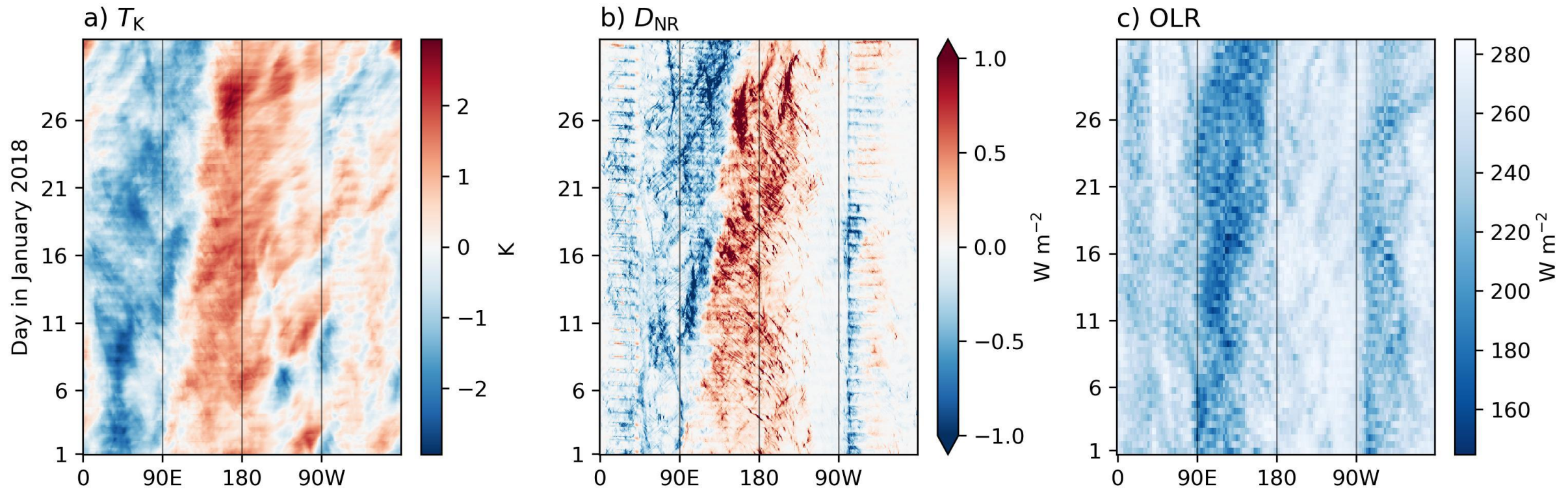
a) January, b) July

Longitude-time variability



- a) Kelvin wave zonal velocity averaged over 15N to 15S,
- b) Kelvin wave energy tendency due to meridional advection of zonal momentum,
- c) Meridional velocity averaged over 10N to 20N.

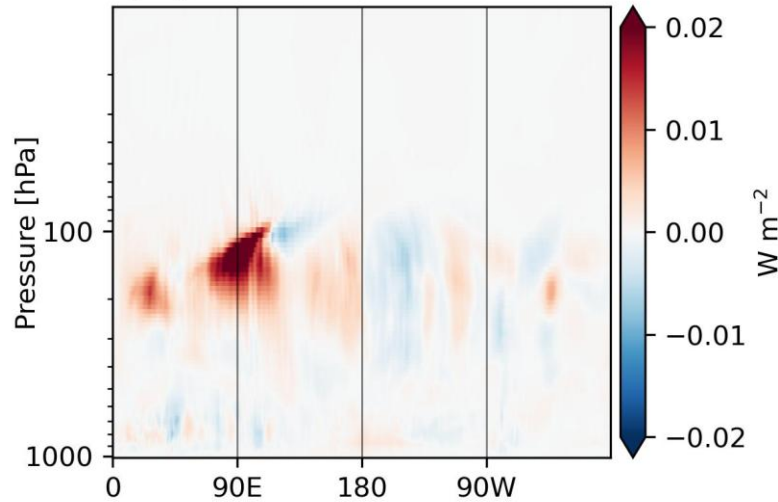
Longitude-time variability



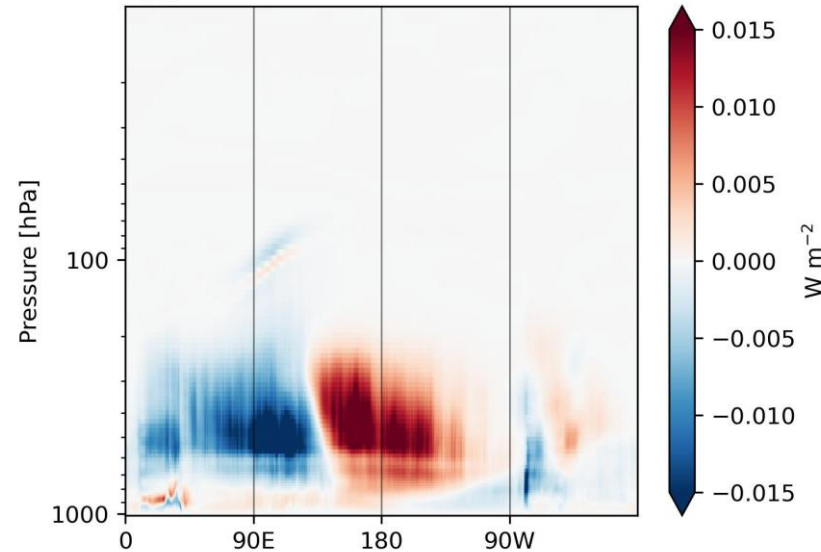
- a) Kelvin wave temperature averaged over 15N to 15S,
- b) Kelvin wave energy tendency due to non-radiative diabatic heating (latent heat release),
- c) Outgoing longwave radiation averaged over 15N to 15S.

Energy tendencies involving apparently stationary Kelvin wave

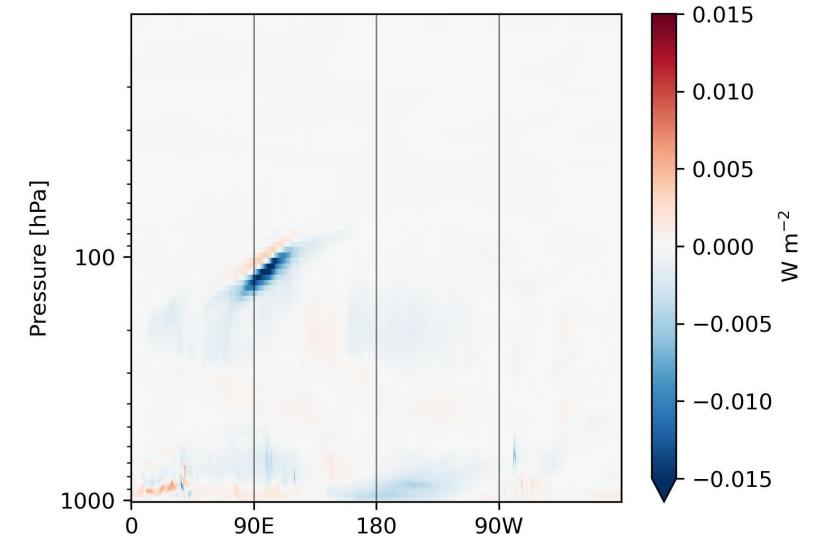
Advection of zonal momentum



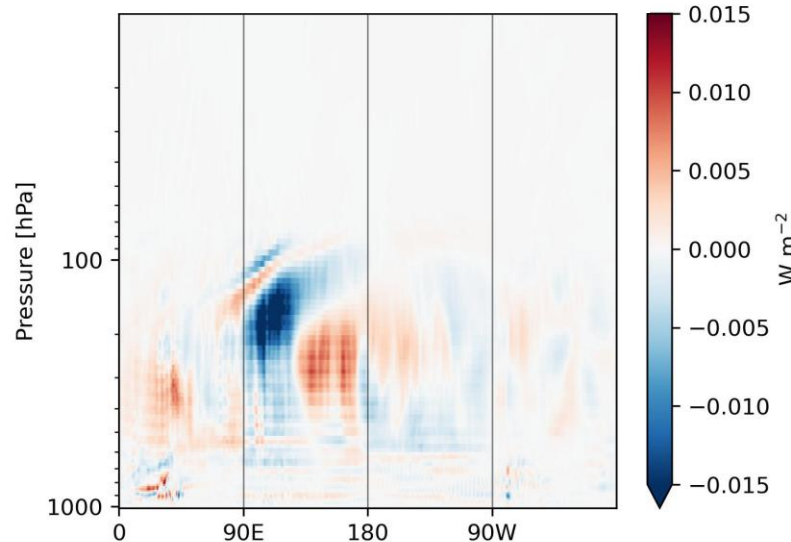
Latent heat release



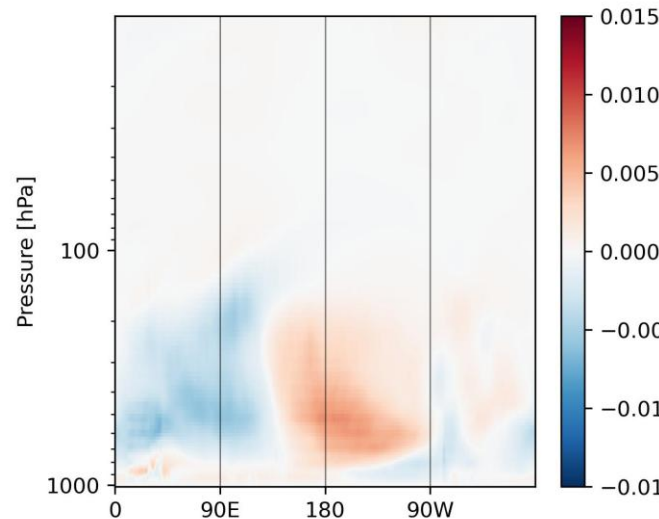
Momentum tendencies due to parametrizations



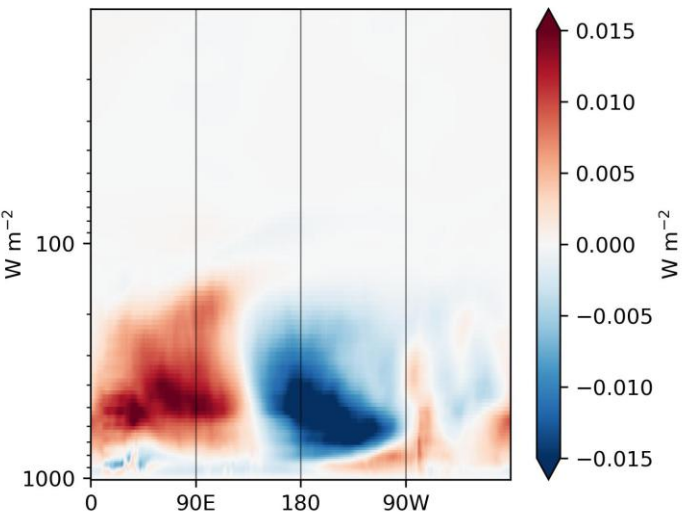
Advection of temperature



Short-wave radiation



Long-wave radiation



Kelvin wave energy budget

$$\frac{dI_K}{dt} = A_u + A_v + A_T + A_{co} + D_u + D_v + D_{SW} + D_{LW} + D_{NR} + D_p$$

Dynamical sources of KW energy due to

total advection of u

$$A_u = A_{u,\lambda} + A_{u,\varphi} + A_{u,\omega} = \int_V N_u u_K dV$$

zonal advection of u

$$A_{u,\lambda} = - \int_V \frac{u}{a \cos \varphi} \frac{\partial u}{\partial \lambda} u_K dV$$

meridional advection of u

$$A_{u,\varphi} = - \int_V \frac{v}{a \cos \varphi} \frac{\partial(u \cos \varphi)}{\partial \varphi} u_K dV$$

vertical advection of u

$$A_{u,p} = - \int_V \omega \frac{\partial u}{\partial p} u_K dV$$

advection of v

$$A_v = \int_V N_v v_K dV$$

total advection of T

$$A_T = A_{T,\lambda} + A_{T,\varphi} + A_{T,\omega} = \int_V \frac{R}{\gamma_0} N_T T_K dV$$

zonal advection of T

$$A_{T,\lambda} = - \int_V \frac{R}{\gamma_0} \frac{u}{a \cos \varphi} \frac{\partial T}{\partial \lambda} T_K dV$$

meridional advection of T

$$A_{T,\varphi} = - \int_V \frac{R}{\gamma_0} \frac{v}{a} \frac{\partial T}{\partial \varphi} T_K dV$$

vertical advection of T'

$$A_{T,p} = - \int_V \frac{R}{\gamma_0} \omega \frac{\partial T}{\partial p} T_K dV$$

conversion term

$$A_{co} = \int_V \frac{\omega R T'}{\gamma_0 c_p} T_K dV$$

Kelvin wave energy budget

$$\frac{dI_K}{dt} = A_u + A_v + A_T + A_{co} + D_u + D_v + D_{SW} + D_{LW} + D_{NR} + D_p$$

Sources of KW energy due to parametrizations for

zonal momentum	$D_u = \int_V S_u u_K dV$
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meridional momentum	$D_v = \int_V S_v v_K dV$
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short-wave radiation	$D_{SW} = \int_V \frac{R}{\gamma_0} Q_{SW} T_K dV$
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long-wave radiation	$D_{LW} = \int_V \frac{R}{\gamma_0} Q_{LW} T_K dV$
---------------------	--

non-radiative processes ^a	$D_{NR} = \int_V \frac{R}{\gamma_0} (S_T - Q_{LW} - Q_{SW}) T_K dV$
--------------------------------------	---

Differences between Eq. (1) and ERA5 result in KW energy sources $D_p = D_{Tv} + D_E$ involving

gradients of difference Φ_v between Φ and geopotential in ERA5 (Appendix C)	$D_{Tv} = - \int_V \left(\frac{1}{a \cos \varphi} \frac{\partial \Phi_v}{\partial \lambda} u_K + \frac{1}{a} \frac{\partial \Phi_v}{\partial \varphi} v_K \right) dV$
--	--

geopotential gradients below the surface	D_E (Eq. D1)
--	----------------
