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Planetary Interior Modeling Using Synthetic Gravity Simulator

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1. INTRODUCTION

In the frame of spherical harmonics expansion for gravitational data (see [1]), the **Stokes coefficients** [C_{nm}, S_{nm}] play a fundamental role. They are directly linked to the interior mass distribution of the planetary body and can be derived from that. In this work, we introduce an approach to evaluate these coefficients for an arbitrary **interior model** and to find the best internal parameter combination that produces the most similar gravitational response to the measured data. The data and evaluations are performed using **SHTOOLS** python routines developed by Wiecozrek et al. [6].

2. METHODOLOGY

Generating Stokes coefficients [C_{nm}, S_{nm}] for a chosen internal structure [1]:

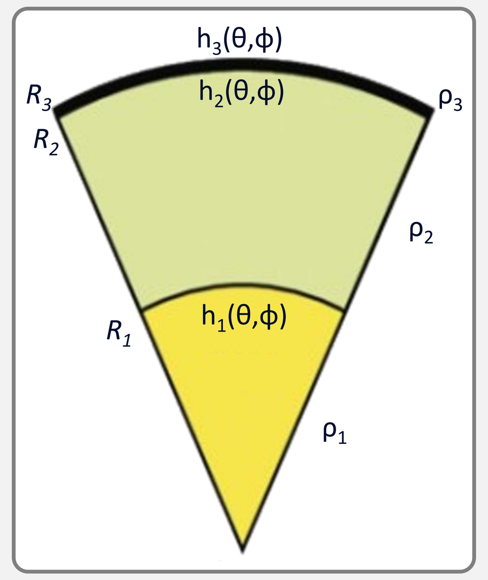
$$\begin{pmatrix} C_{lm}\\ S_{lm}\end{pmatrix} = \frac{1}{(2l+1)MR^l}\int_V \rho(\mathbf{r})\, r^{l+2}\, \bar{P}_{lm}(\cos\theta)\begin{pmatrix}\cos(m\phi)\\ \sin(m\phi)\end{pmatrix} dr\, d\Omega$$

For **each internal layer**, [C_{nm}, S_{nm}]$_i$ can be calculated as:

$$\begin{pmatrix} C_{lm}\\ S_{lm}\end{pmatrix}_i = \frac{\rho_i}{(2l+1)(l+3)}\frac{R^3}{M}\int_\theta\int_\phi\int_{R_{i-1}+h_{i-1}(\theta,\phi)}^{R_i+h_i(\theta,\phi)} \frac{r^{n+2}}{R^{n+3}}\bar{P}_{lm}(\cos\theta)\begin{pmatrix}\cos(m\phi)\\ \sin(m\phi)\end{pmatrix}\sin(\theta)\, dr\, d\theta\, d\phi$$

Obtaining the **global** coefficients as:

$$\begin{pmatrix} C_{lm}\\ S_{lm}\end{pmatrix} = \sum_{i=1}^{N}\begin{pmatrix} C_{lm}\\ S_{lm}\end{pmatrix}_i$$



To solve these, an **interior model** is needed:

- n = number of layers ($i \in [1,n]$);
- R_i = i-th layer thickness;
- ρ_i = i-th layer density (homogenous);
- $h_i(\theta,\phi)$ = i-th layer interface topography.

Interface topography $h_i(\theta,\phi)$:

- **Randomly** generated;
- Spheroid + scaled **polar flattening**;
- **Downwarding** the measured **Bouguer** anomaly to the chosen depth [2]. **NO isostasy model** compensation assumption (careful choice of the downward filter).

Constraints:

- External **shape** (R_n and $h_n(\theta,\phi)$) ;
- Total mass **M** (balancing ρ_i);
- Moment of Inertia **MoI** (core parameters [ρ_1, R_1]).

3. OBJECTIVES

Double purpose:

a) Testing an interior model for a studied body, **comparing** the synthetic gravitational response with the measured one [3].

Example: **Mercury**.

Having a reference dataset, an «inversion» procedure is performed, exploring the parameters combination that produces the **most similar** gravitational signal to the real measured one [4].

b) Simulating the gravitational signal for an «unknown» body (from a specific internal mass distribution) to evaluate the effects of the layers' features.

Example: **Ganymede**.

It allows to investigate the influence of the **hydrosphere** over the lithosphere, studying different internal layer parameter configurations (see [5]).

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4a. GRID SEARCH FOR REAL DATA COMPARISON

Calculating different interior models, **randomly** changing the interior parameters (R_i, ρ_i, n_{half})

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Two tested configurations: 3 (core, mantle, crust) and 4 internal layers (differentiated core)

($>10^6$ tested models each)

Grid ranges	
ρ_i [kg/m³]:	ρ_{orig} +/- 200
R_i [km]:	R_{orig} +/- 200
n_{half} :	3:100

Evaluate by n normalized **statistical metrics** the "distance" between synthetic results and real data:

- Δ,σ
- Mean Absolute Error (MAE)
- Root Mean Square Error (RMSE)
- Pearson Correlation Coefficient (R^2)
- Structure Similarity Index (SSIM)
- Peak Signal-to-Noise Ratio (PSNR)
- Normalized Cross-Correlation (NCC)

Combining them into a single final metric:

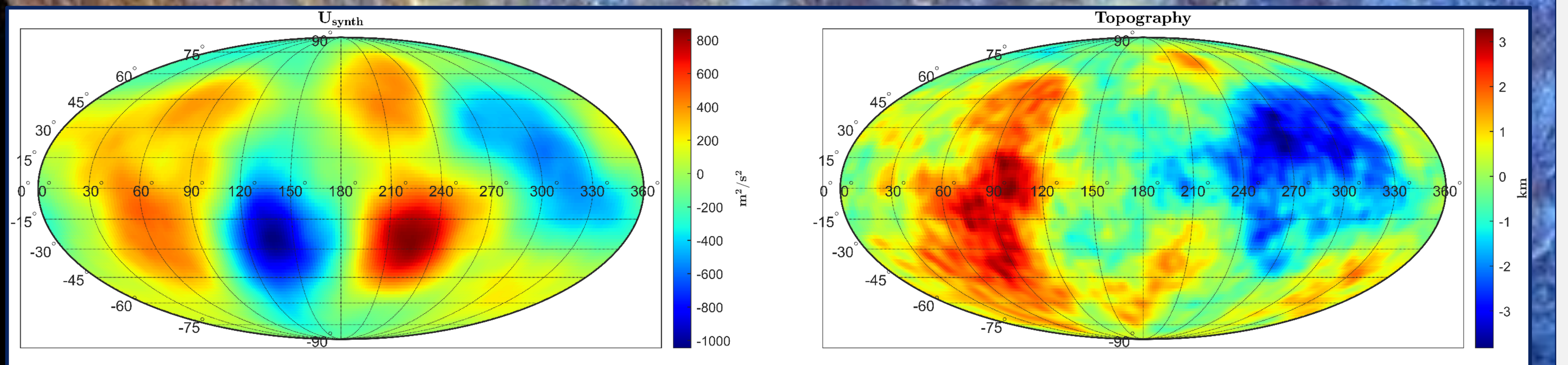
$$final\_metric = \sqrt{\frac{\sum_i^n \left(metric_U^2 + metric_{FreeAir}^2 + metric_{Bouguer}^2\right)}{3n}}$$

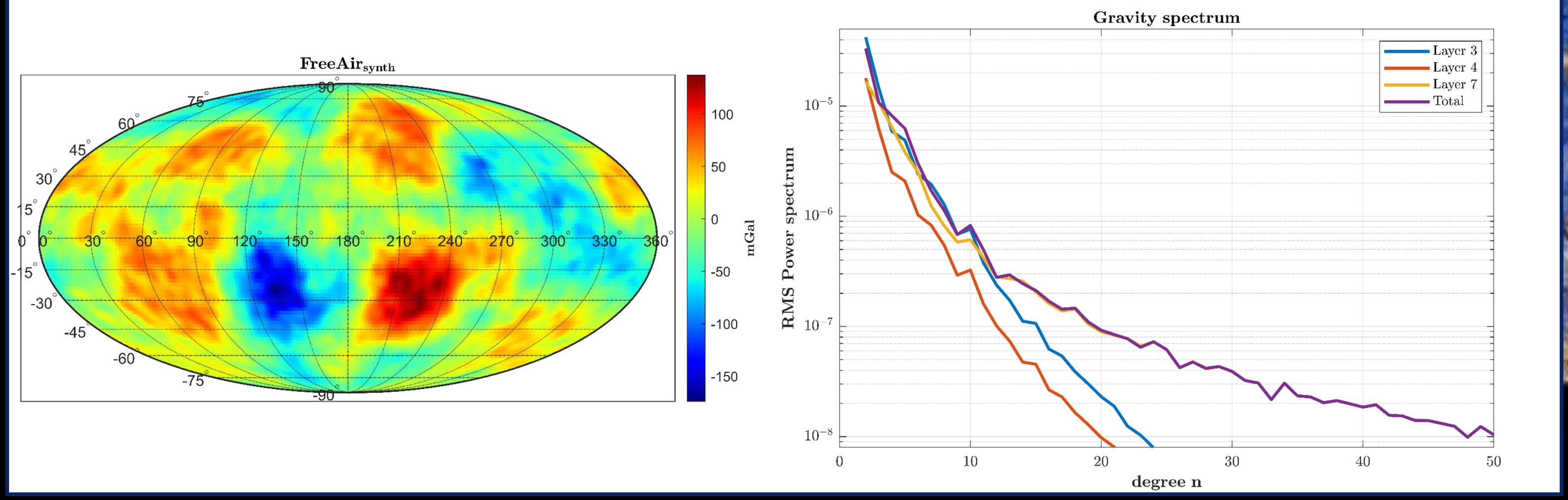
Final ranking to find the best (**top n%**) models and analyze their parameter distribution (es: Gaussian)

4b. "Synthetic" Ganymede

Interior model:

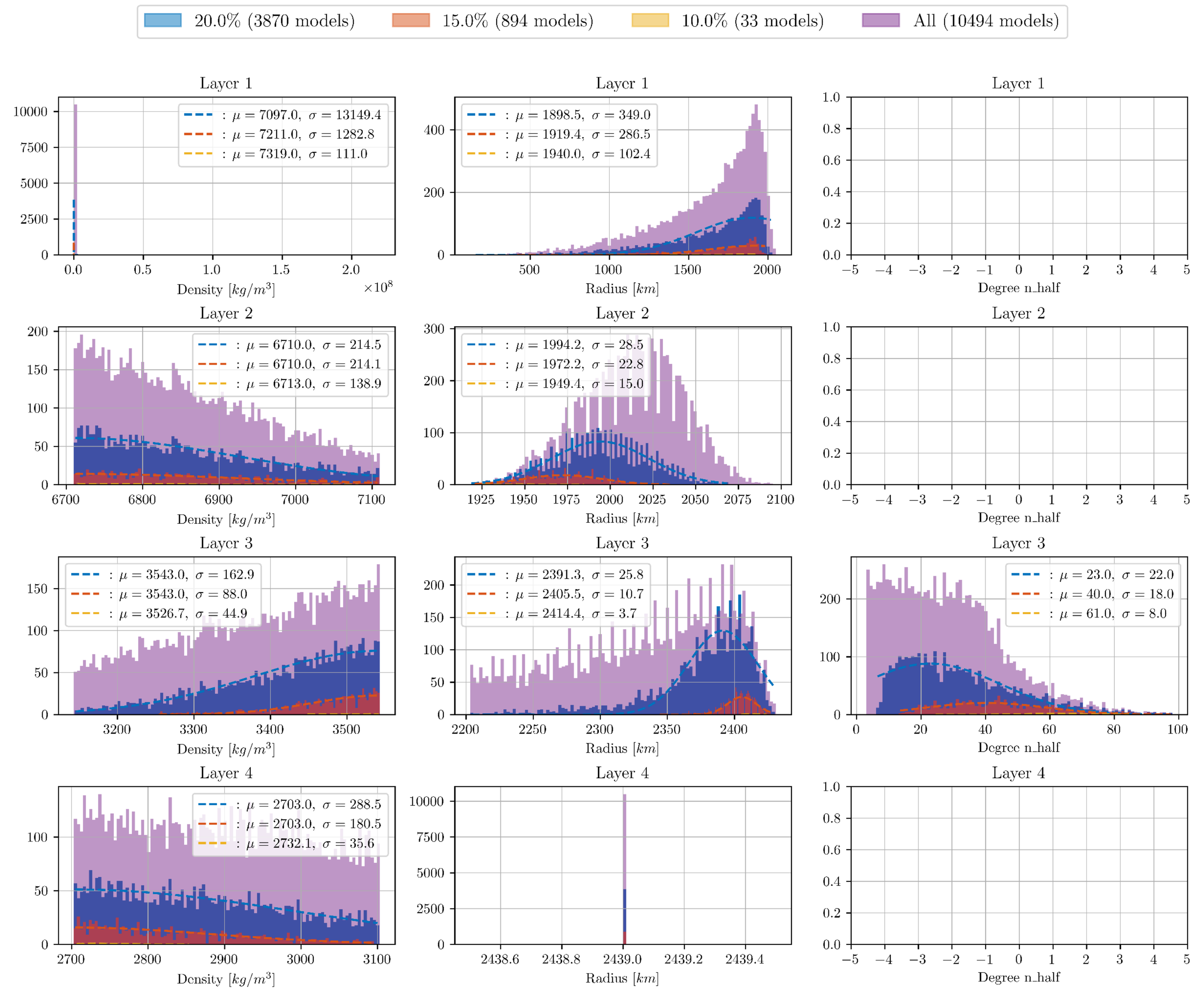
	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5	Layer 6	Layer 7
ρ [kg/m³]:	8000	3400	3100	1320	1235	1100	920
R [km]:	570	1820	1870	2000	2280	2460	2631,2
Interface:	sphere	flat	random	sphere	sphere	sphere	random





4a. RESULTS

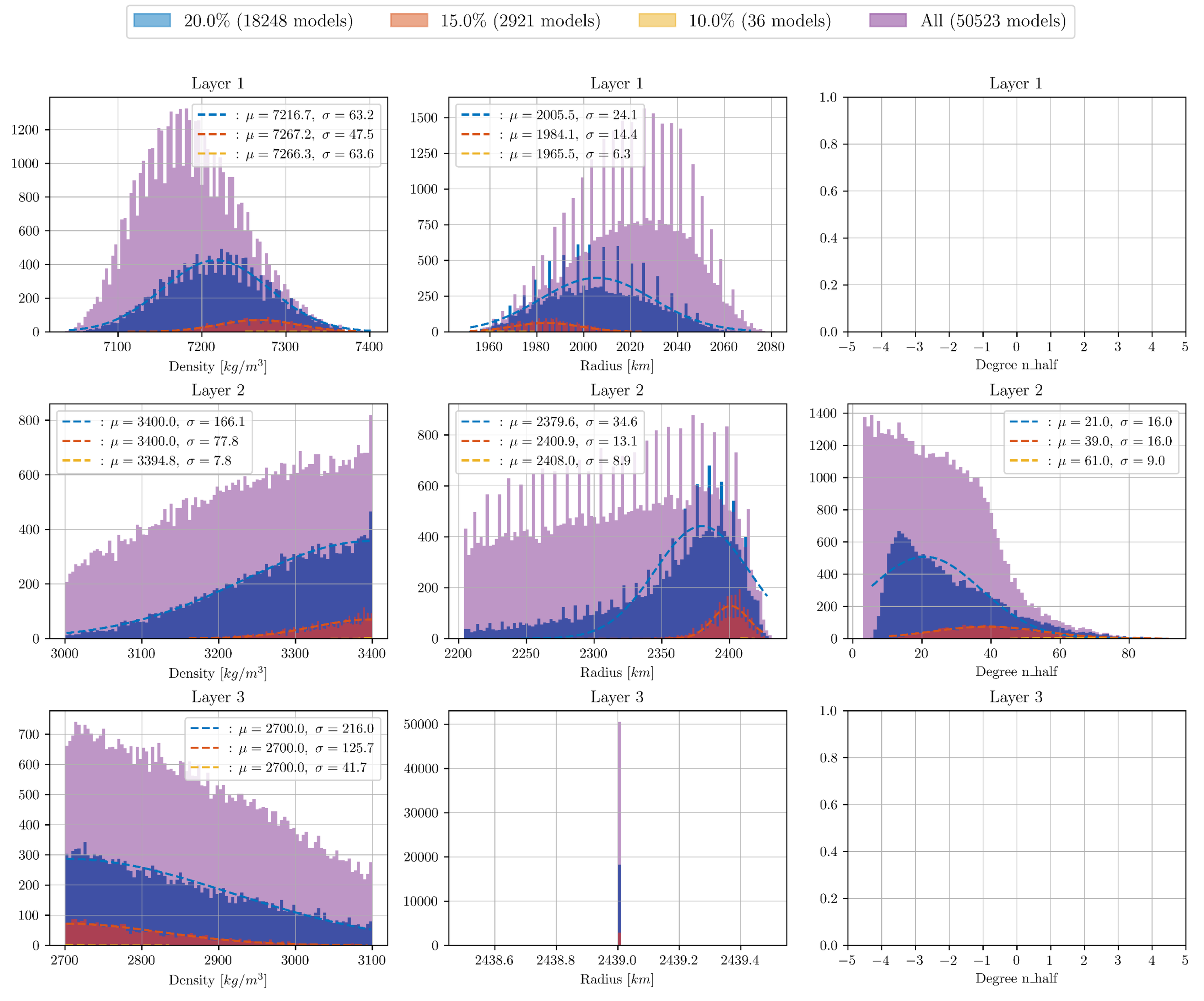
4 layers



Parameters distribution:

	Radius R [km]	Density ρ [kg/m³]	Filter degree $n_{1/2}$
Inner Core:	1919.4 +/- 286.5	7211.0+/- 1282.8	-
Outer Core:	1972.2 +/- 22.8	6710.0 +/- 214.1	-
Mantle:	2405.5 +/- 10.7	3543.0 +/- 88.0	40 +/- 18
Crust:	-	2700.3 +/- 180.5	-

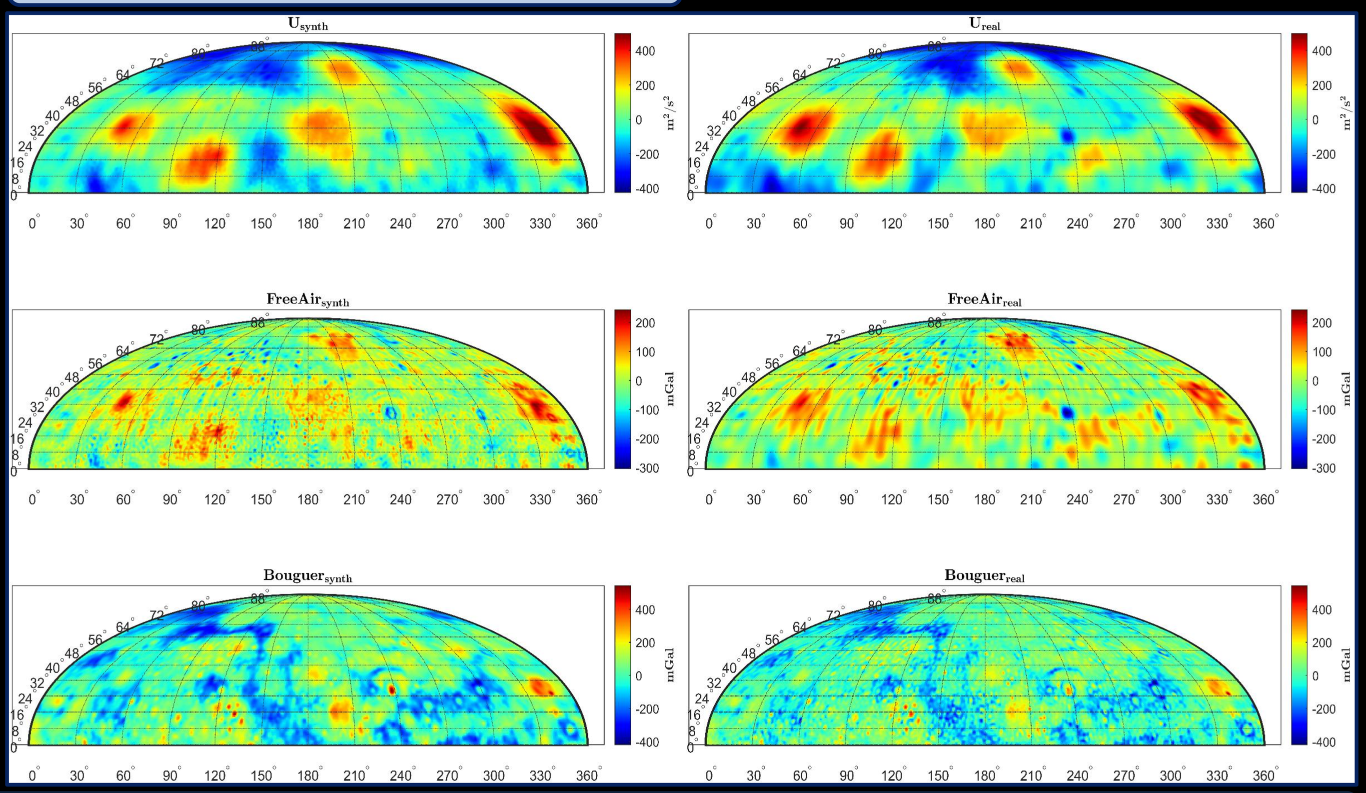
3 layers



Parameters distribution:

	Radius R [km]	Density ρ [kg/m³]	Filter degree $n_{1/2}$
Core:	1984.1 +/- 14.4	7267.2 +/- 47.5	-
Mantle:	2400.9 +/- 13.1	3400.0 +/- 77.8	39 +/- 16
Crust:	-	2700.0 +/- 125.7	-

Best model (3 layers):



5a. DISCUSSION and CONCLUSIONS

4 layers:

- Inner Core and Outer Core are **not distinguishable**: densities and radii tend to each other;
- the mantle and crust characterisation is slightly better determined within the explored domain thanks to the **M and MoI constraints** and the DoF due to the additional layer.

3 layers:

- **Better** overall characterization of the **core** (as a whole);
- **Densities' smaller uncertainties**.

Conclusions:

- Results **compatible** with existing literature (see [4]);
- **Crust thickness**: 34.5 +/- 10.7 [km];
- **Core** parameters: 7267.2 +/- 47.5 [kg/m³], 1984.1 +/- 14.4 [km];
- Mantle and crust densities suffer from lateral variation anisotropy.

REFERENCES

- [1] M. A. Wieczorek, 'Gravity and Topography of the Terrestrial Planets', in Treatise on Geophysics, Elsevier, 2015.
- [2] M. A. Wieczorek and R. J. Phillips, 'Potential anomalies on a sphere: Applications to the thickness of the lunar crust', Journal of Geophysical Research: Planets, 1998.
- [3] A. Genova et al., Regional variations of Mercury's crustal density and porosity from MESSENGER gravity data, Icarus, 2023.
- [4] S. Buoninfante, M. Milano, B. Negri et al. 'Gravity evidence for a heterogeneous crust of Mercury'. Sci Rep 13, 2023.
- [5] D. M. Fabrizio *et al.*, 'Observability of Ganymede's gravity anomalies related to surface features by the 3GM experiment onboard ESA's JUpiter ICy moons Explorer (JUICE) mission', *Icarus*, 2021.
- [6] Mark A. Wieczorek and Matthias Meschede (2018). SHTools — Tools for working with spherical harmonics, Geochemistry, Geophysics, Geosystems, 19, 2574-2592, doi:10.1029/2018GC007529.