Stochastic Modeling in GRACE-FO Gravity Field Estimation Using Two Types of Satellite-to-Satellite Tracking Observations

F. Öhlinger , T. Mayer-Gürr

Institute of Geodesy, Graz University of Technology

TU Graz

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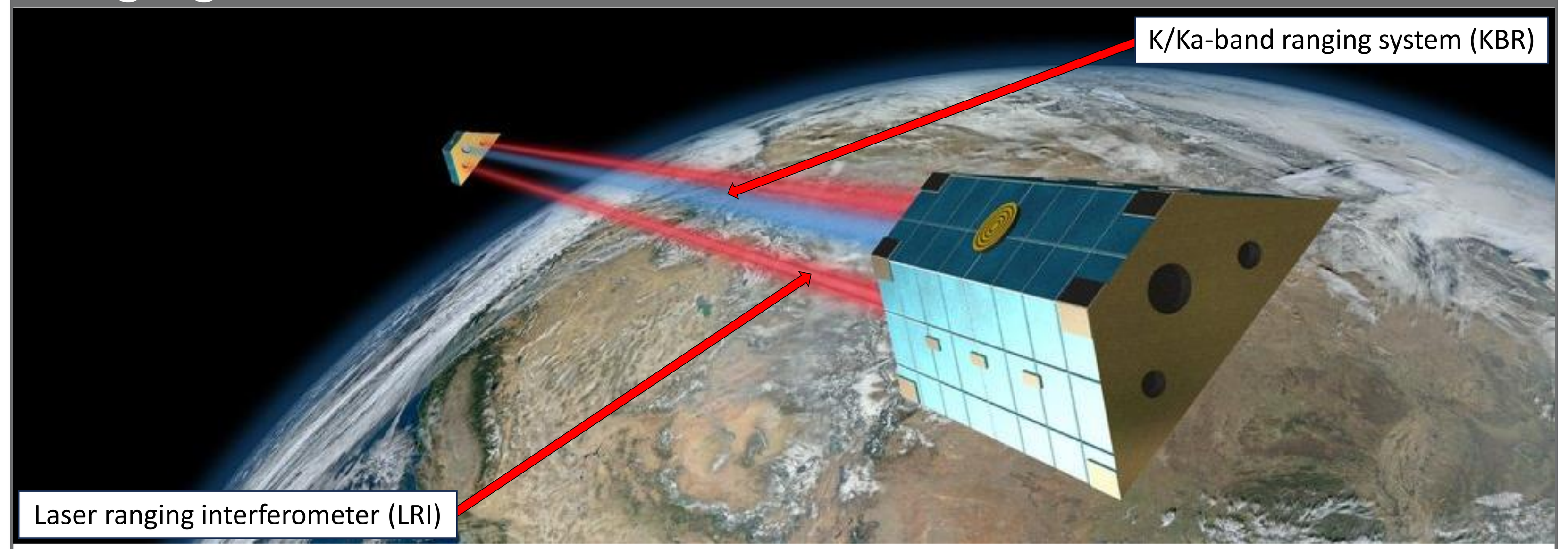
Ranging instruments



Source: NASA „Blue Marble" and Schütze (AEI, MPG)

- GRACE-FO carries two independent inter-satellite ranging systems: the microwave K/Ka-band ranging instrument (KBR) and the laser ranging interferometer (LRI).
- The inter-satellite distance variations can be measured with micrometer-level and nanometer-level accuracy for the KBR and the LRI, respectively [1].
- LRI will also serve as the ranging instrument for future gravity field missions such as GRACE-C and NGGM.
- The goal of this study is to exploit the full potential of the high LRI accuracy through improved stochastic modeling of the observations.

Stochastic modeling of observation data

- Stochastic modeling is required to properly weight the observations and consequently compute an optimal least-squares adjustment solution and obtain realistic formal errors for the monthly gravity field.
- The observations comprise KBR at 5 s, LRI at 2 s and kinematic orbit (POD) data at 60 s sampling.
- Since the functional model relating the observations to the estimated parameters is non-linear, forward-modeled observations are reduced, and correction terms are determined.

$$\Delta l = \begin{bmatrix} l_{kbr} \\ l_{lri} \\ l_{pod,C} \\ l_{pod,D} \end{bmatrix} - \begin{bmatrix} S_1 f(x_0,y_0) \\ S_2 f(x_0,y_0) \\ S_3 f(x_0,y_0) \\ S_3 f(x_0,y_0) \end{bmatrix}, \qquad \Sigma_{\Delta l} = \begin{bmatrix} \Sigma_{kbr} & 0 & 0 & 0 \\ 0 & \Sigma_{lri} & 0 & 0 \\ 0 & 0 & \Sigma_{pod,C} & 0 \\ 0 & 0 & 0 & \Sigma_{pod,D} \end{bmatrix} + \begin{bmatrix} S_1\Sigma_{cmn}S_1^T & S_1\Sigma_{cmn}S_2^T & 0 & 0 \\ S_2\Sigma_{cmn}S_1^T & S_2\Sigma_{cmn}S_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- The forward-modeled observations comprise a priori values for the monthly gravity field, as well as values for all other forces acting on the satellite. These include tides, variations in atmosphere and ocean, and non-conservative forces, which are accounted for by the accelerometer and the star camera measurements.
- Therefore, the reduced observations are contaminated not only by the actual observations noise, but also by noise from the accelerometer and star camera, and the uncertainties of the employed background models.
- This common component (CMN) in the reduced observations correlates the different observation types and fully populates the covariance matrix [2].
- For a proper stochastic modeling, these correlations must be considered.

Frequency-wise variance component estimation (VCE)

- Several assumptions are made concerning the observations:
  - Noise of SST and POD is wide sense stationary and uncorrelated
  - All systematic influences are accounted for → noise is random and normally distributed
  - Individual 3-hourly short-arcs are uncorrelated
- The different observation types and short arcs are accumulated at the normal equation level and then solved:

$$N = \sum_{j=1}^{3}\sum_{m=1}^{M} A_{m,j}^T \Sigma_{m,j}^{-1} A_{m,j}, \qquad n = \sum_{j=1}^{3}\sum_{m=1}^{M} A_{m,j}^T \Sigma_{m,j}^{-1} \Delta l_{m,j}, \qquad j \in [SST, POD_C, POD_D]$$

$$\Delta\hat{x} = N^{-1} n$$

- The quadratic sum of the residuals for each frequency and observation component is formed using residuals for each arc and separating them in the spectral domain.

$$e_{n,i}^2 = \sum_{m=1}^{M} \hat{e}_m^T \Sigma_m^{-1} F_{n,i} \Sigma_m^{-1} \hat{e}_m, \qquad \hat{e}_m = \Delta l_m - A_m \Delta\hat{x}, \qquad i \in [KBR, LRI, CMN, POD_C, POD_D]$$

$$F_{n,kbr} = \begin{bmatrix} S_1F_nS_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad F_{n,lri} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & S_2F_nS_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad F_{n,cmn} = \begin{bmatrix} S_1F_nS_1^T & S_1F_nS_2^T & 0 & 0 \\ S_2F_nS_1^T & S_2F_nS_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

- Using the redundancies of each frequency and the respective variance, the cofactor matrix can be estimated.

$$R_{n,i} = \sum_{m=1}^{M} F_{n,i}\Sigma_m^{-1}(I - A_m(A_m^T\Sigma_m^{-1}A_m)^{-1}A_m^T)$$

$$a_{n,i}^2 = \frac{e_{n,i}^2}{\mathrm{trace}(R_{n,i})} \qquad Q_{i,m} = \sum_{n_f=0}^{N_m-1} a_{n,i}^2 F_{n,i}$$

- This matrix is then scaled arc-wise to obtain the covariance matrix of each observation component.

$$\Sigma_{m,i} = \hat{\sigma}_{m,i}^2 Q_{i,m} \qquad \hat{\sigma}_{m,i}^2 = \hat{\alpha}_{m,i}^2 \sigma_{m,i}^2 \qquad \hat{\alpha}_{m,i}^2 = \frac{e_{m,i}^2}{\mathrm{trace}(R_{m,i})}$$

- The estimation of the covariance matrix of the reduced observations is an iterative process.

$$\Sigma_{sst} = \Sigma_{kbr} + \Sigma_{lri} + \Sigma_{cmn} = \begin{bmatrix} \Sigma_{kbr} + S_1\Sigma_{cmn}S_1^T & S_1\Sigma_{cmn}S_2^T \\ S_2\Sigma_{cmn}S_1^T & \Sigma_{lri} + S_2\Sigma_{cmn}S_2^T \end{bmatrix}_m$$

$$\Sigma_m = \Sigma_{sst} + \Sigma_{pod_C} + \Sigma_{pod_D} = \begin{bmatrix} \Sigma_{sst} & 0 & 0 \\ 0 & \Sigma_{pod_C} & 0 \\ 0 & 0 & \Sigma_{pod_D} \end{bmatrix}_m$$

- m = number of short arc — $m \in [1, ..., M]$
- n = number of epoch in each specific arc m — $n \in [1, ..., N_m]$
- j = number of observation types — $j \in [SST, POD_C, POD_D]$
- i = number of observation type components — $i \in [KBR, LRI, CMN, POD_C, POD_D]$

Observation noise separation

- The covariance matrices of the SST observation components can be used to disentangle the post-fit residuals through collocation.

$$\begin{bmatrix} \hat{s}_{kbr} \\ \hat{s}_{lri} \\ \hat{s}_{cmn} \end{bmatrix} = \begin{bmatrix} \Sigma_{kbr} & 0 & 0 \\ 0 & \Sigma_{lri} & 0 \\ 0 & 0 & \Sigma_{cmn} \end{bmatrix} \begin{bmatrix} I & 0 \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Sigma_{kbr} + S_1\Sigma_{cmn}S_1^T & F_1\Sigma_{cmn}S_2^T \\ S_2\Sigma_{cmn}S_1^T & \Sigma_{lri} + S_2\Sigma_{cmn}F_2^T \end{bmatrix}^{-1} \begin{bmatrix} \hat{e}_{kbr} \\ \hat{e}_{lri} \end{bmatrix}$$

- Using least-squares prediction, the SST residuals (both containing the common component) are split into the individual parts.
- The separation is not perfect, so there might be some leakage between the noise sources.
- The better the stochastic modeling, the better the disentanglement.

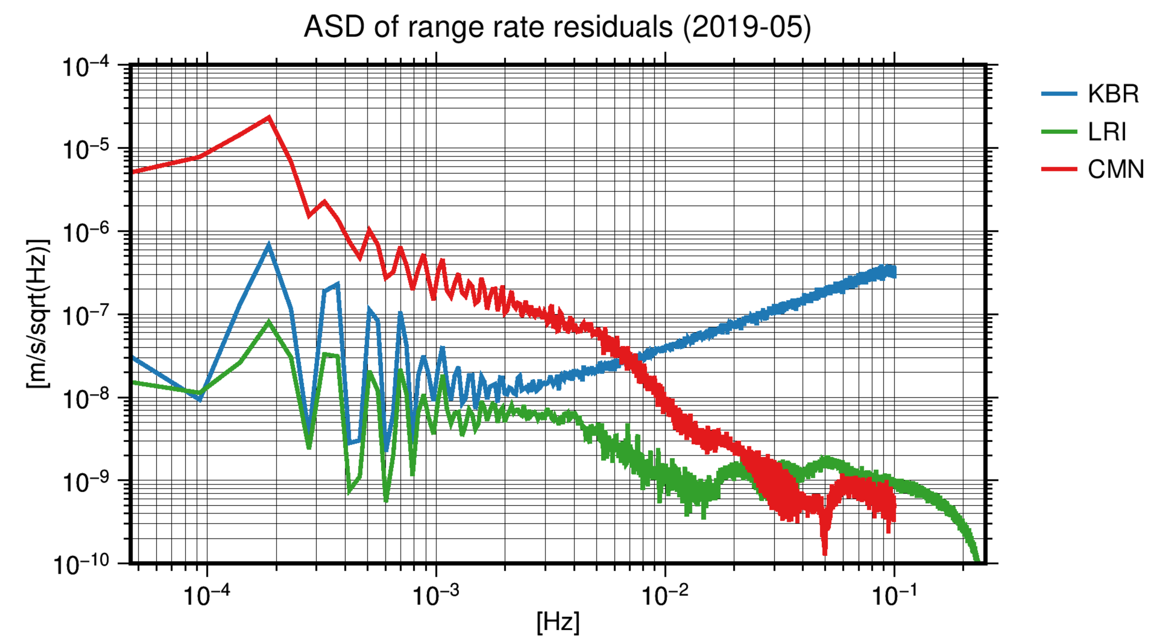


Fig 1: Amplitude spectral density of the disentangled post-fit residuals

Results

- The disentangled range-rate residuals were numerically differentiated to range accelerations using a degree 1 polynomial approximation, and the results were plotted as ground tracks in a terrestrial reference frame.
- The noise level of the KBR residuals is much higher than that of the LRI or CMN components. KBR noise is amplified at high frequencies due to differentiation to range-rates.
- The noise level of the LRI is low enough to reveal systematic errors which are related to inadequate modeling of thruster firings along the geomagnetic equator [3].
- The CMN component includes not only noise from the accelerometer and the star camera, but also uncertainties arising from the applied background models. This is particularly evident in areas of significant oceanographic variability, such as the Gulf Stream.

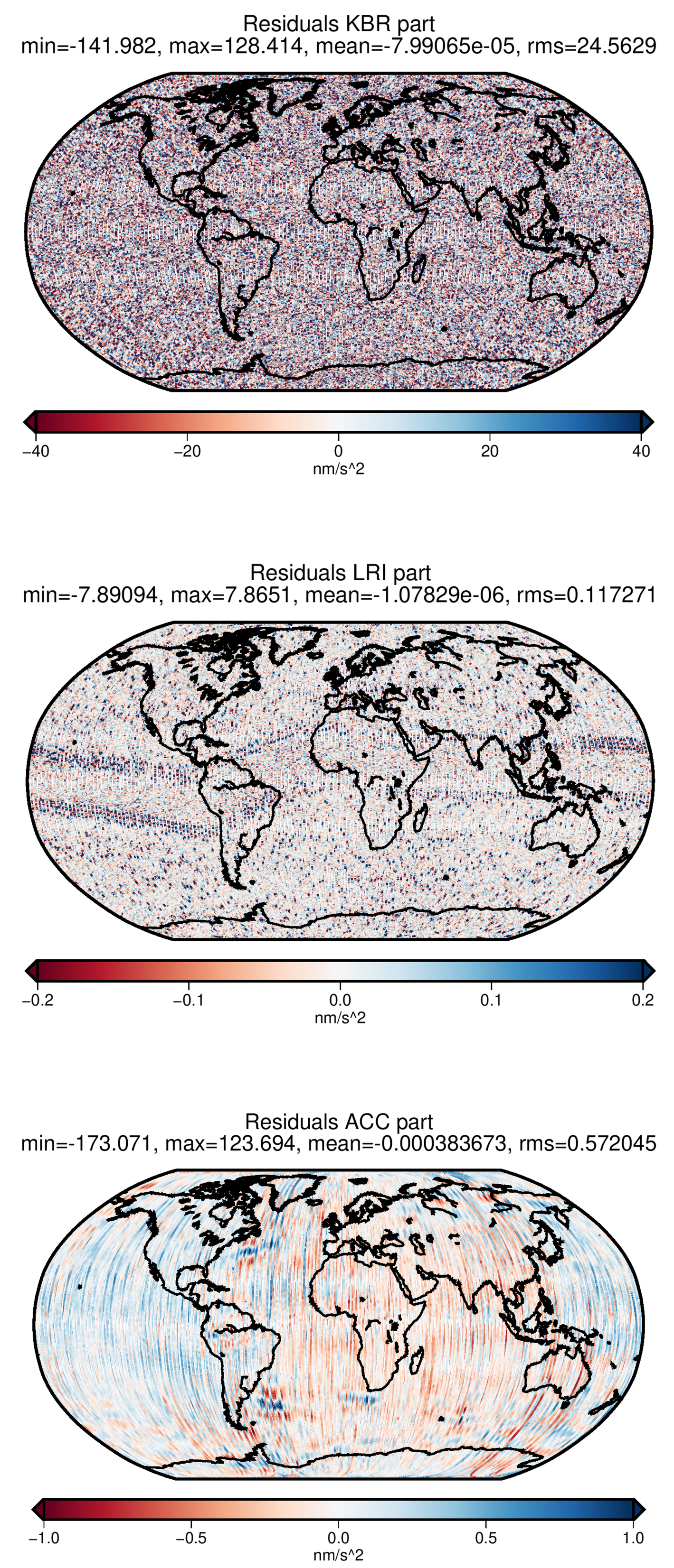


Fig 2: Ground tracks of the differentiated residual components

Static gravity field GOCO2025s

- The stochastic modeling using both SST observations was also employed in the determination of the satellite-only global gravity field model GOCO2025s [4].
- Fig 3. shows the impact of GRACE-FO in GOCO2025s.

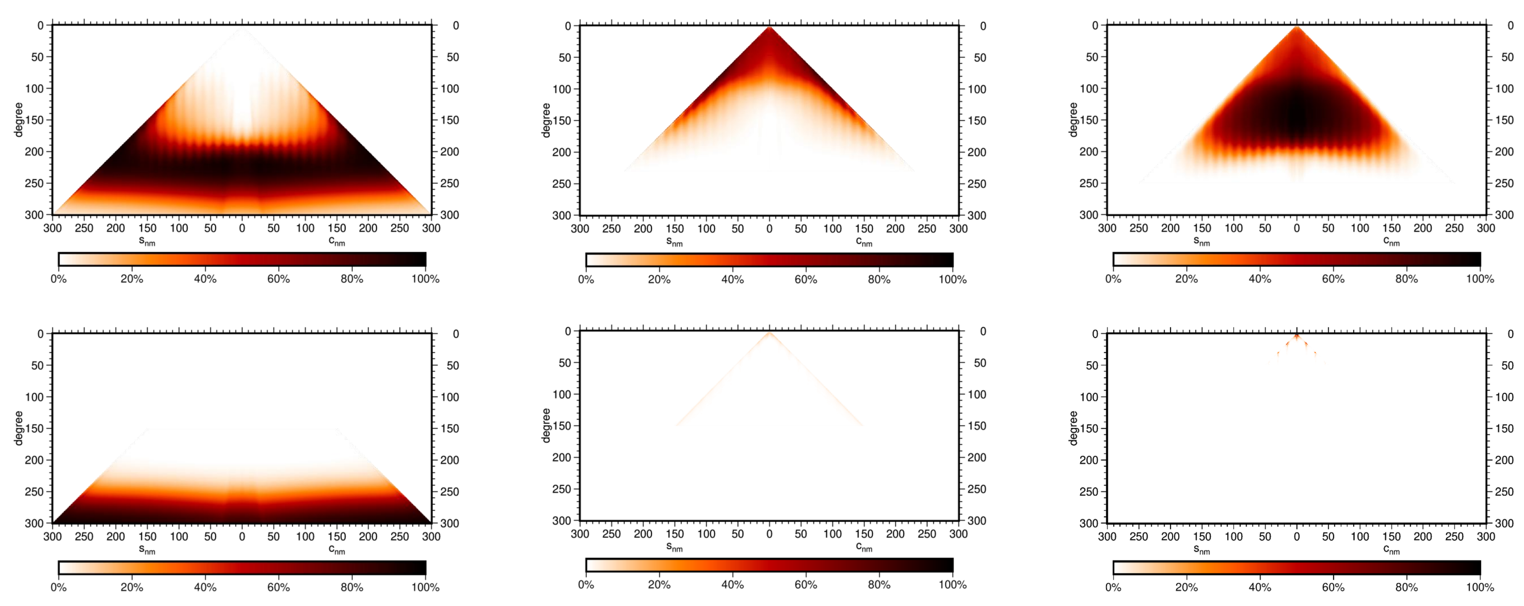


Fig 3: Contribution of the individual components of GOCO2025s

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Contact

Felix Öhlinger

felix.oehlinger@tugraz.at

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