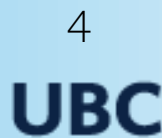
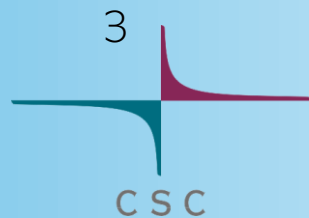
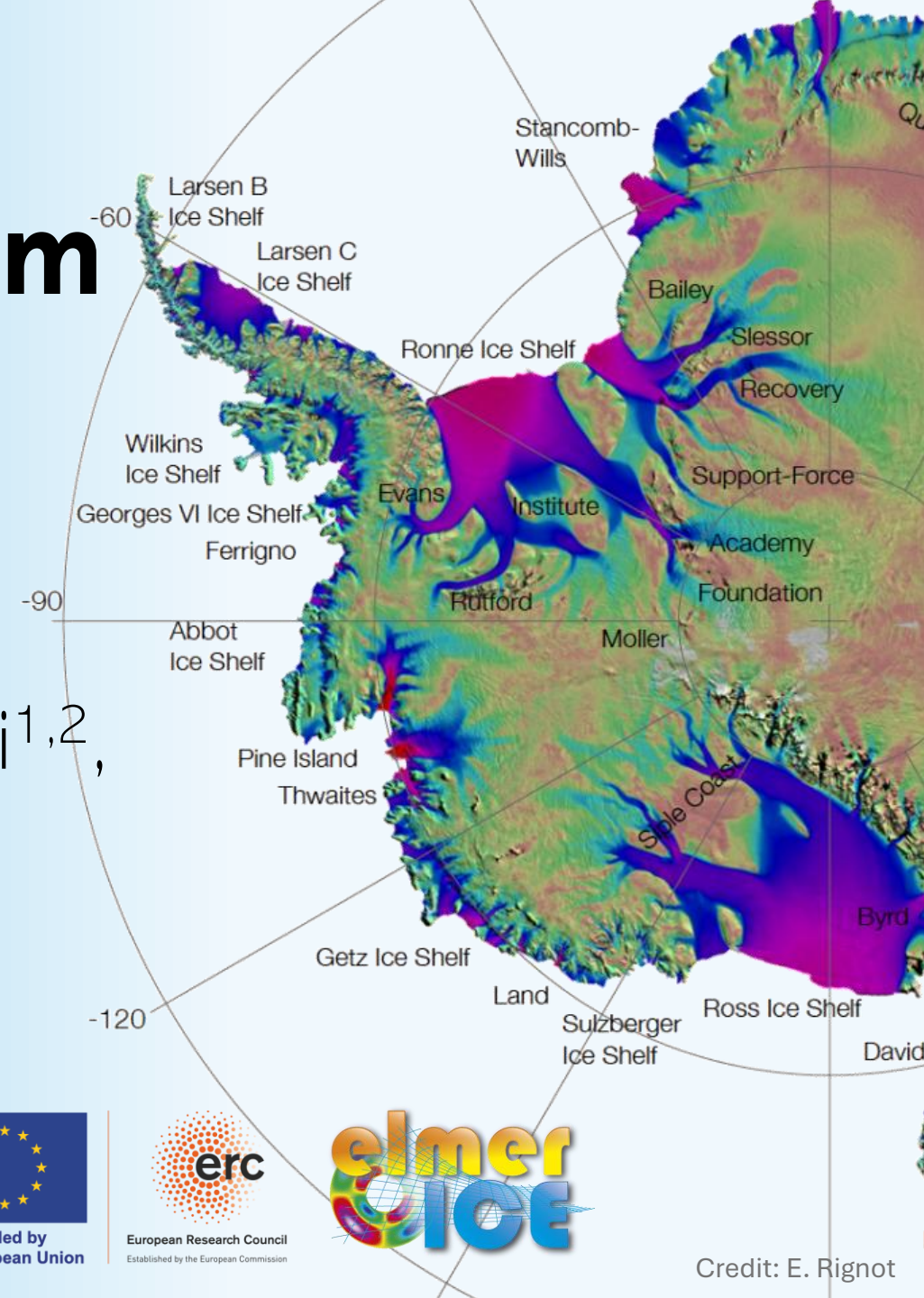


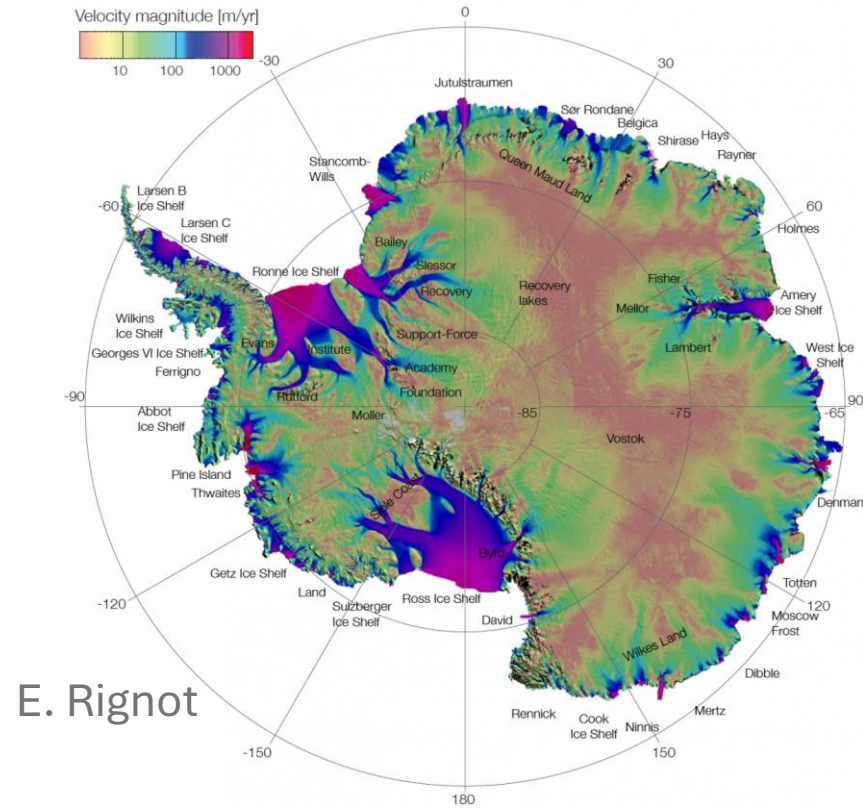
# Temporal ice-stream dynamics resulting from subtemperate sliding instabilities

Tilly Woods<sup>1</sup>, Silas Lloyd<sup>1</sup>, Elisa Mantelli<sup>1,2</sup>,  
Thomas Zwinger<sup>3</sup>, Christian Schoof<sup>4</sup>

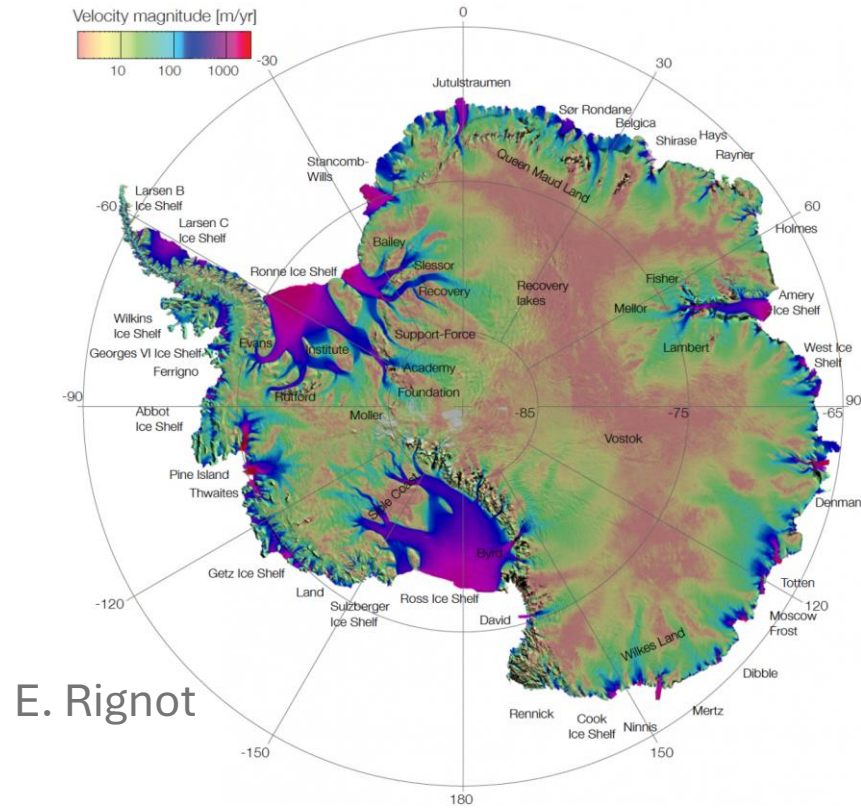


Credit: E. Rignot

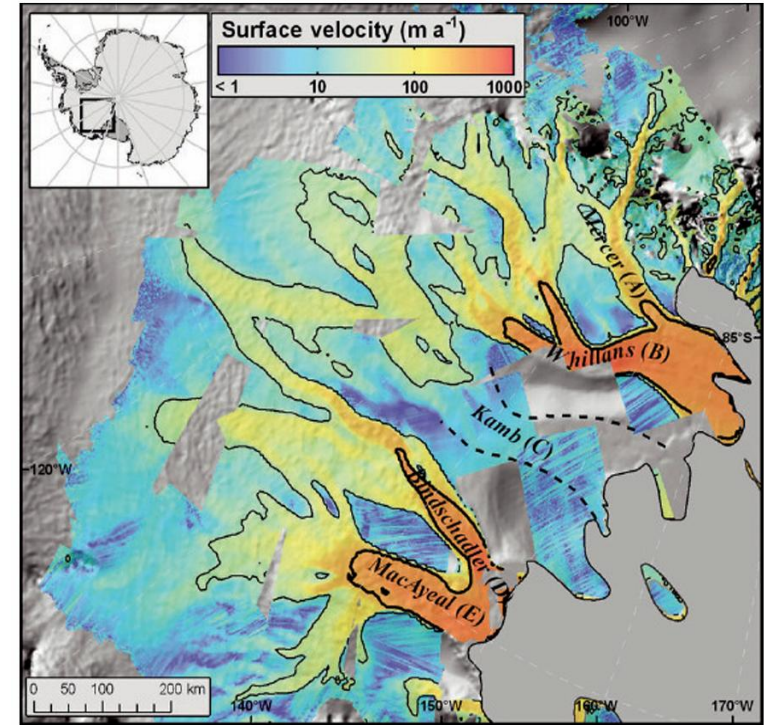
**Ice streams** account for 80% ice mass loss from Antarctica (Bamber *et al* 2000)



**Ice streams** account for 80% ice mass loss from Antarctica (Bamber *et al* 2000)

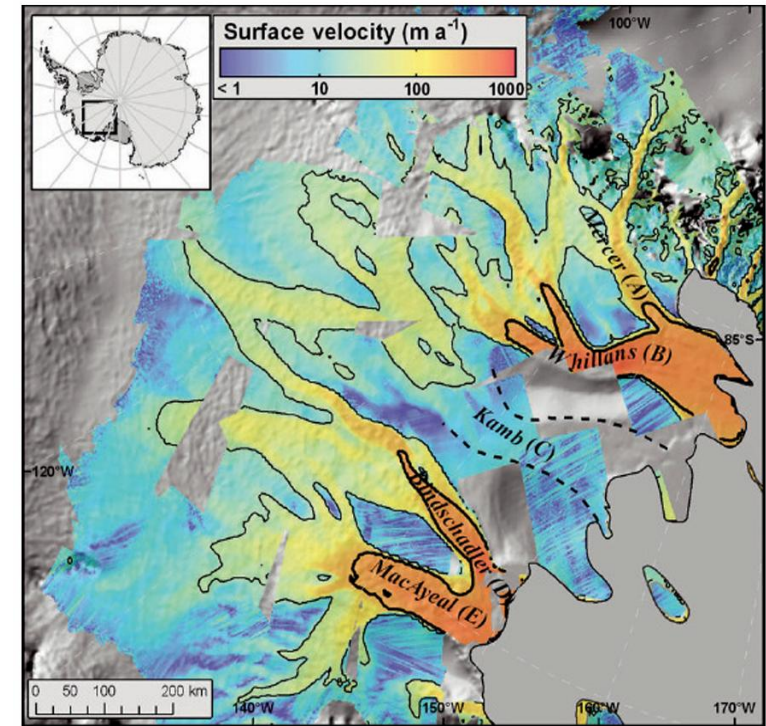
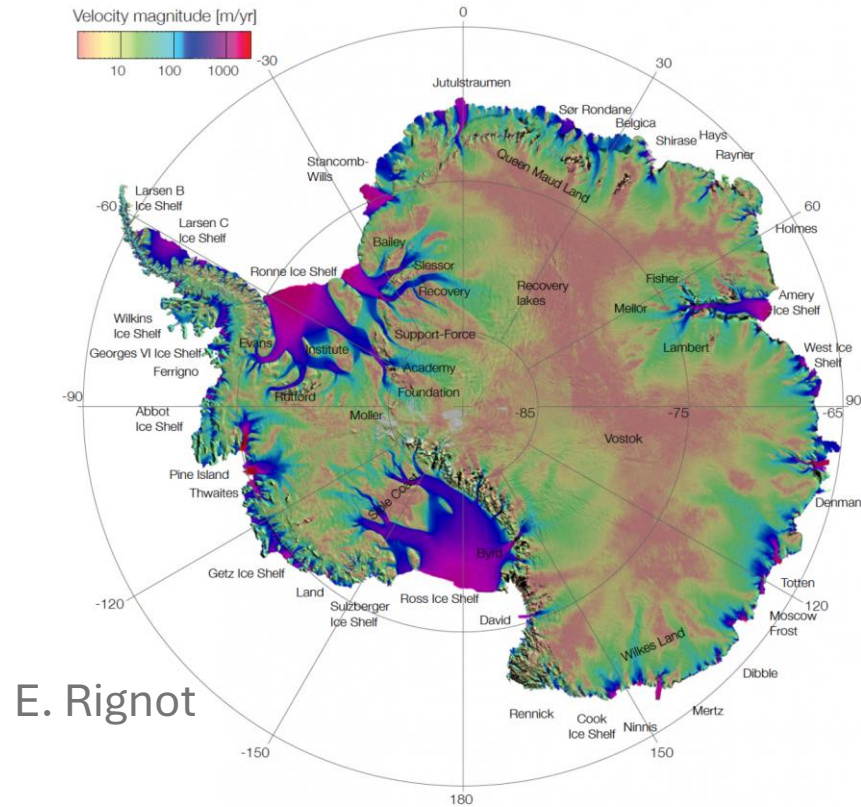


Complex *spatial* and *temporal* dynamics



**Ice streams** account for 80% ice mass loss from Antarctica (Bamber *et al* 2000)

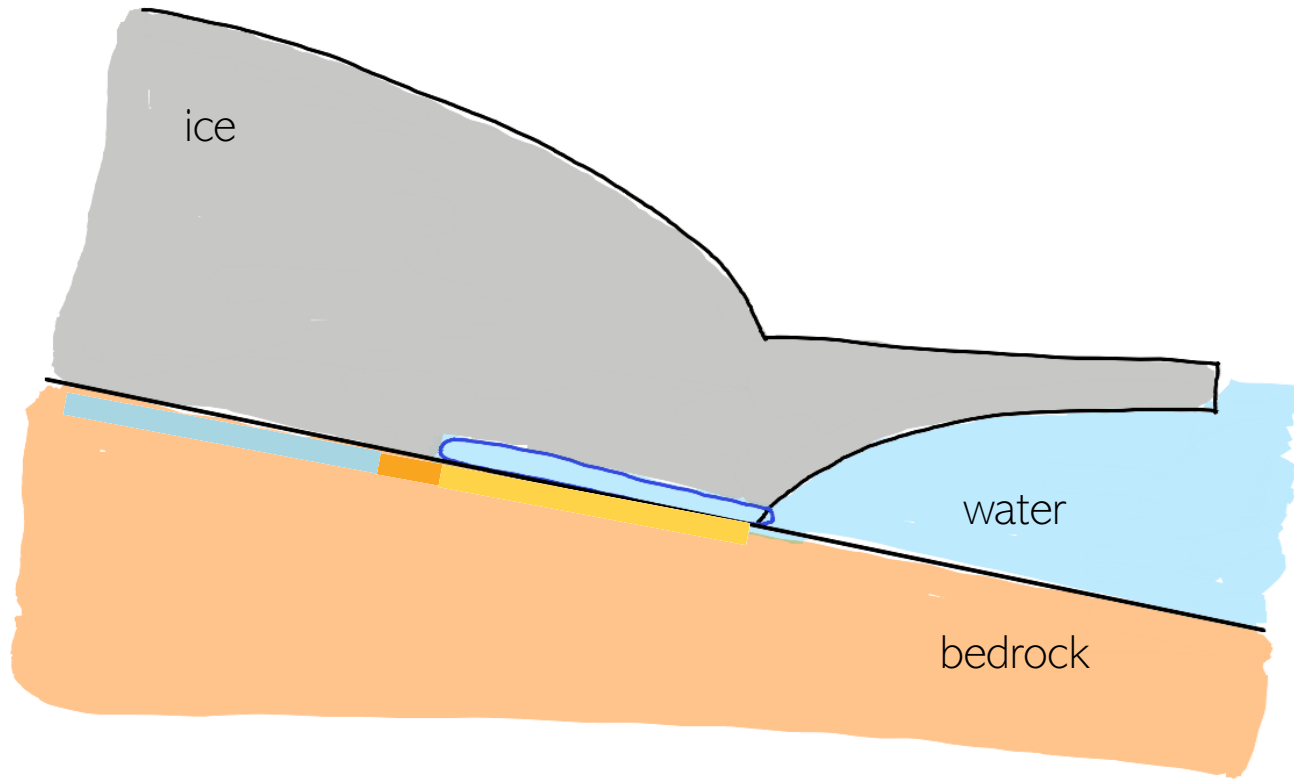
Complex *spatial* and *temporal* dynamics



What controls where and when ice streams form (and turn off)?

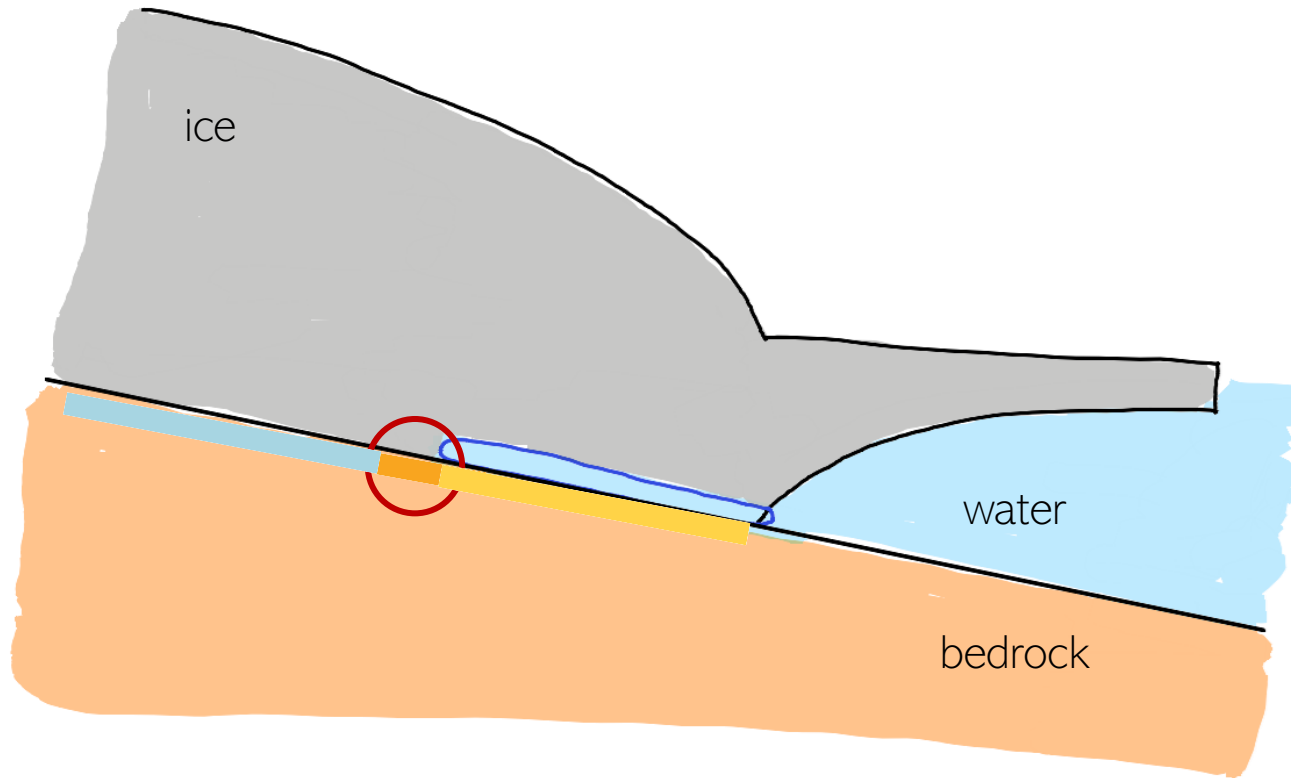
# Subtemperate sliding

Sliding starts *below* the melting point.



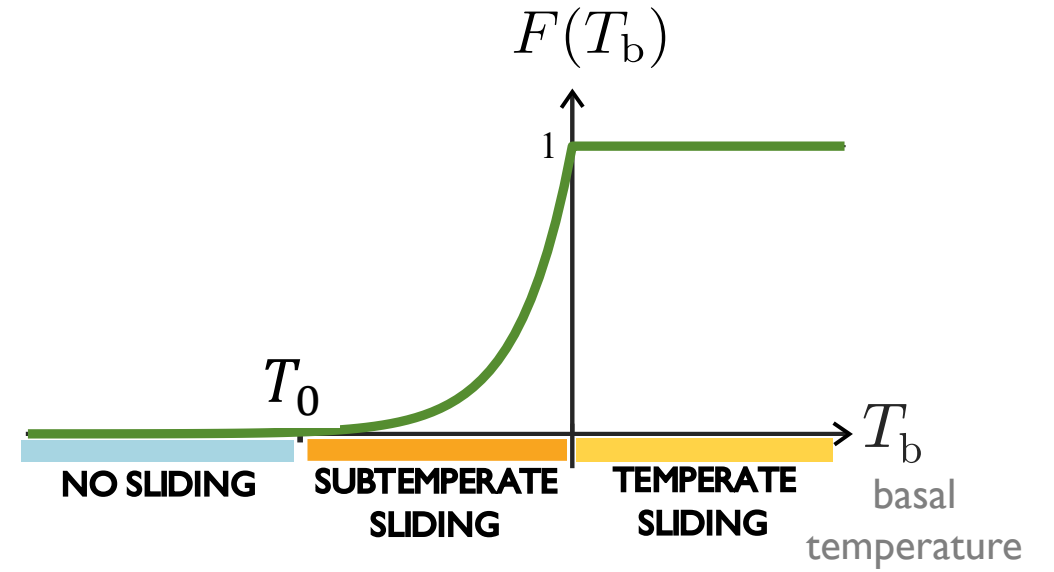
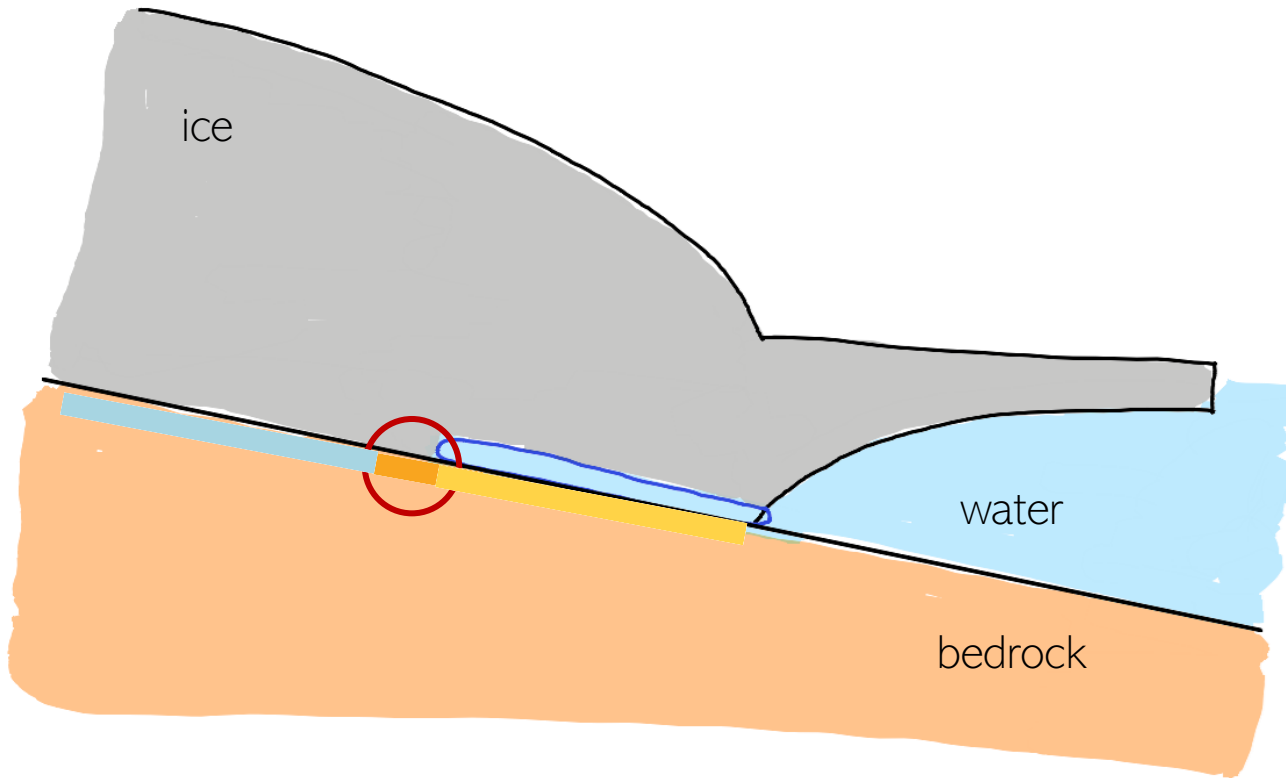
# Subtemperate sliding

Sliding starts *below* the melting point.



# Subtemperate sliding

Sliding starts *below* the melting point.



sliding speed

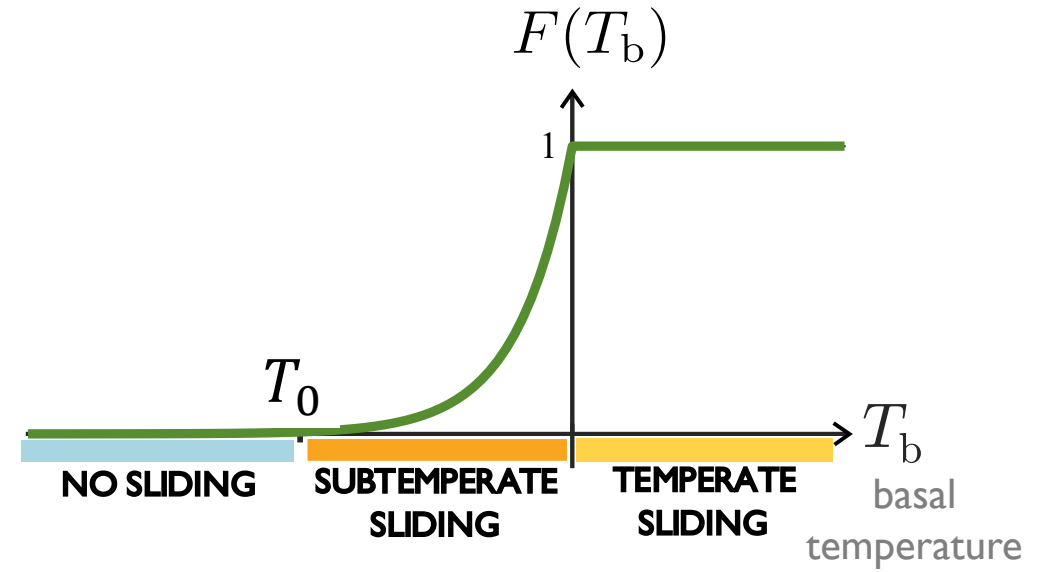
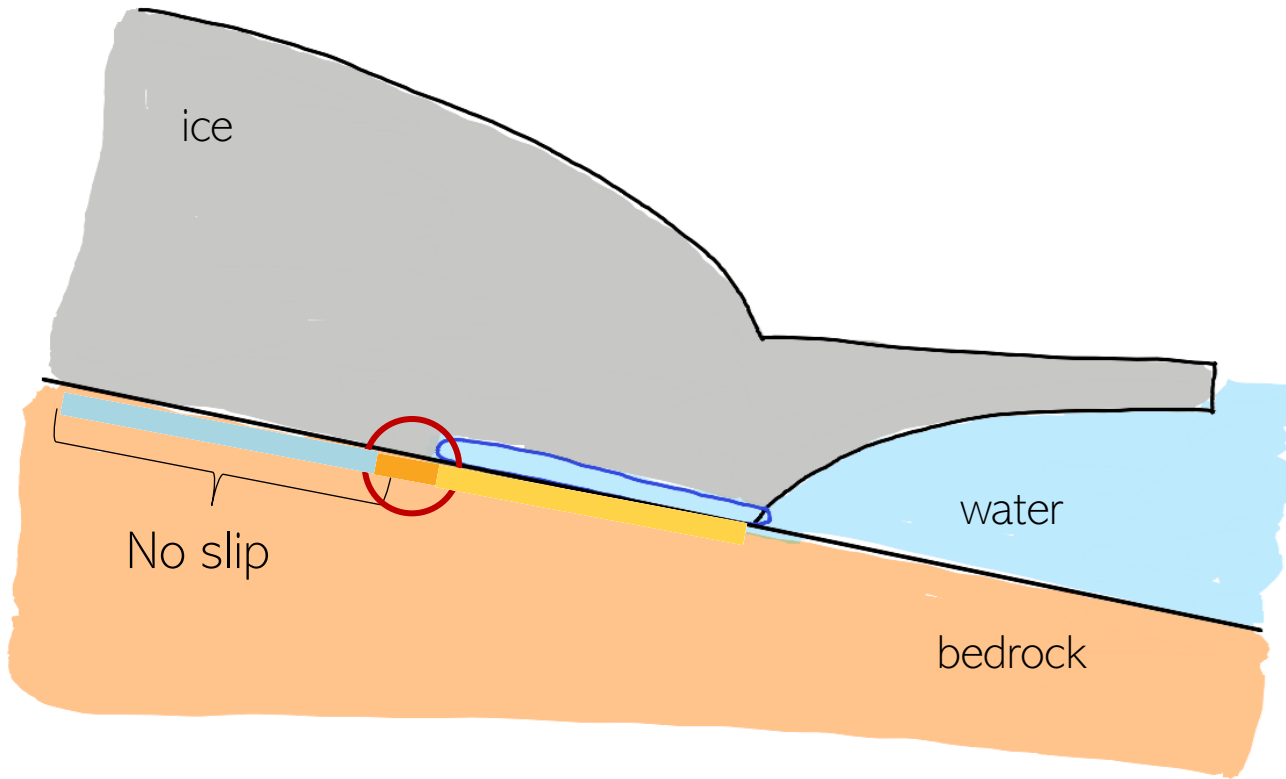
$$\frac{u_b}{\tau_b} = \gamma^{-1} F(T_b)$$

basal shear stress

friction coefficient

# Subtemperate sliding

Sliding starts *below* the melting point.



$$\frac{u_b}{\tau_b} = \gamma^{-1} F(T_b)$$

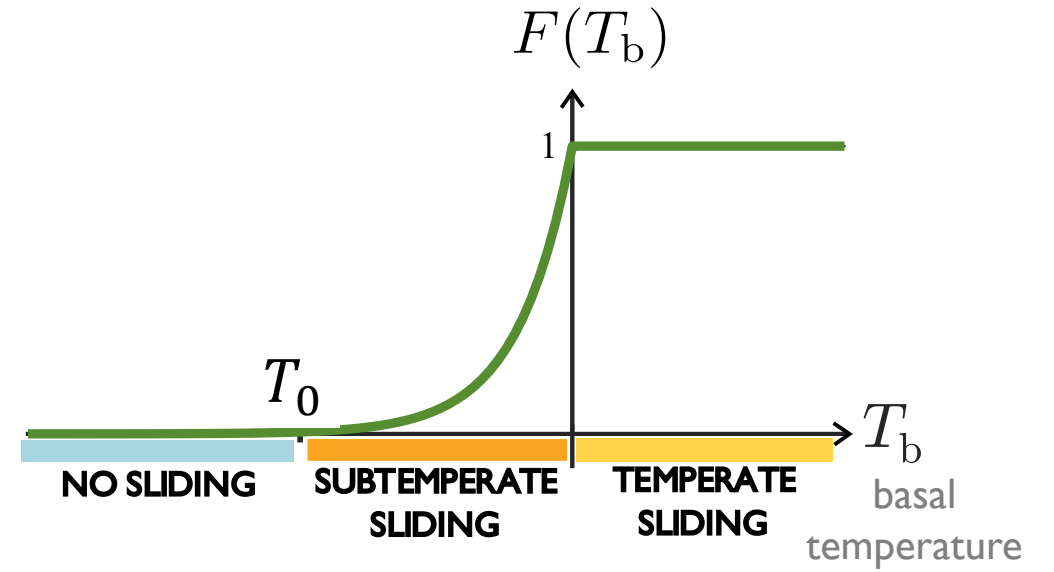
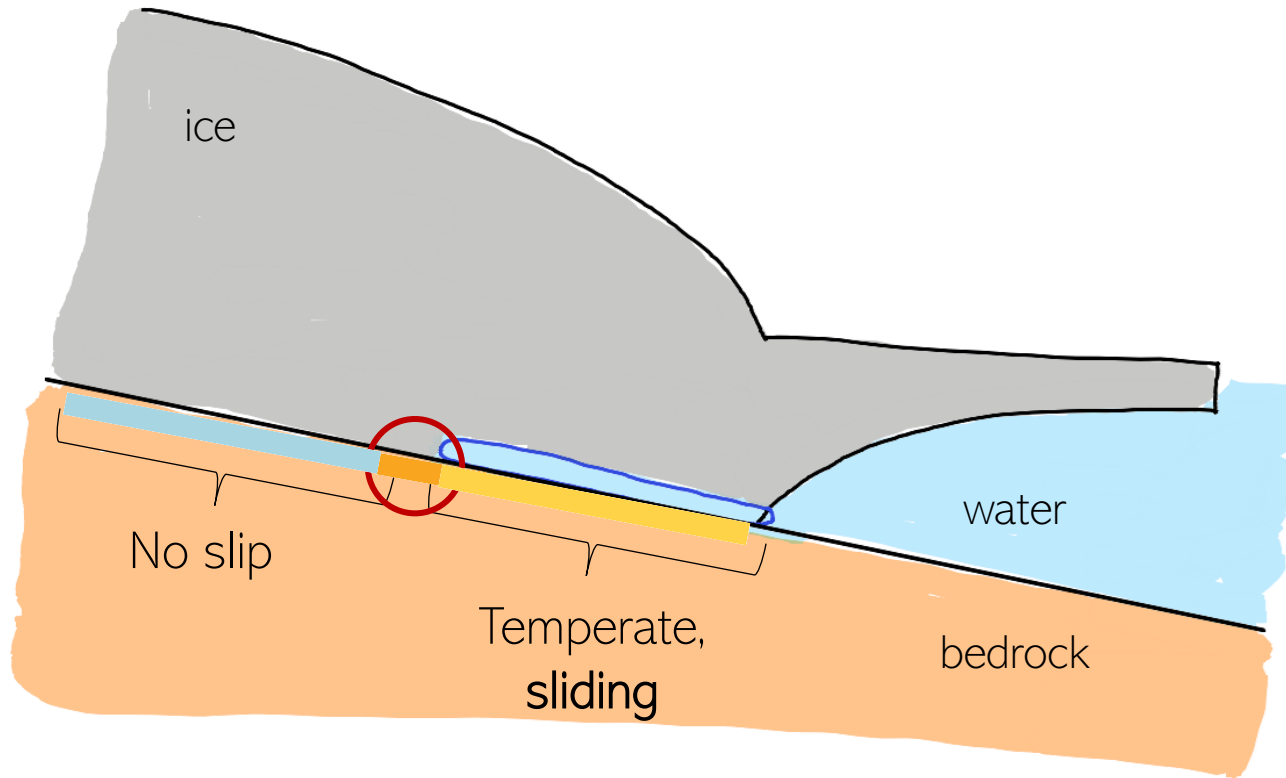
sliding speed

basal shear stress

friction coefficient

# Subtemperate sliding

Sliding starts *below* the melting point.



$$\frac{u_b}{\tau_b} = \gamma^{-1} F(T_b)$$

sliding speed

basal shear stress

friction coefficient

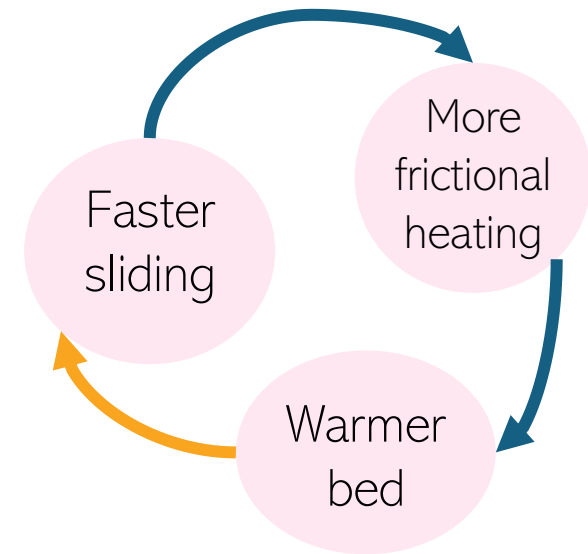
Subtemperate sliding region is  
*unconditionally unstable*

Mantelli and Schoof, *Proc. R. Soc. A*, 2019b

# Subtemperate sliding region is *unconditionally unstable*

Mantelli and Schoof, *Proc. R. Soc. A*, 2019b

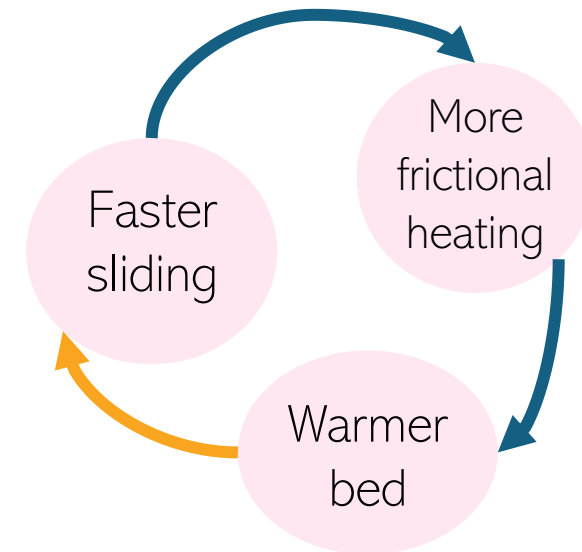
Thermo-frictional  
feedback:



# Subtemperate sliding region is *unconditionally unstable*

Mantelli and Schoof, *Proc. R. Soc. A*, 2019b

Thermo-frictional  
feedback:



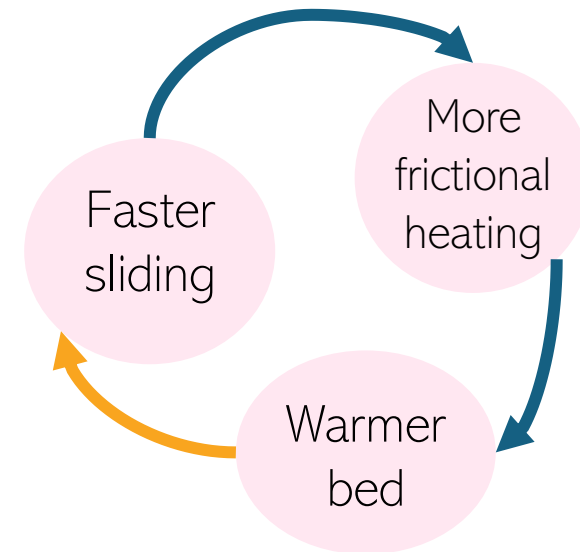
How do small-scale instabilities of the subtemperate region play out at the scale of a whole ice sheet?

# Subtemperate sliding region is *unconditionally unstable*

Mantelli and Schoof, *Proc. R. Soc. A*, 2019b

Linear stability analysis  $\Rightarrow$   
travelling wave solutions

Thermo-frictional  
feedback:



How do small-scale instabilities of the subtemperate region play out at the scale of a whole ice sheet?

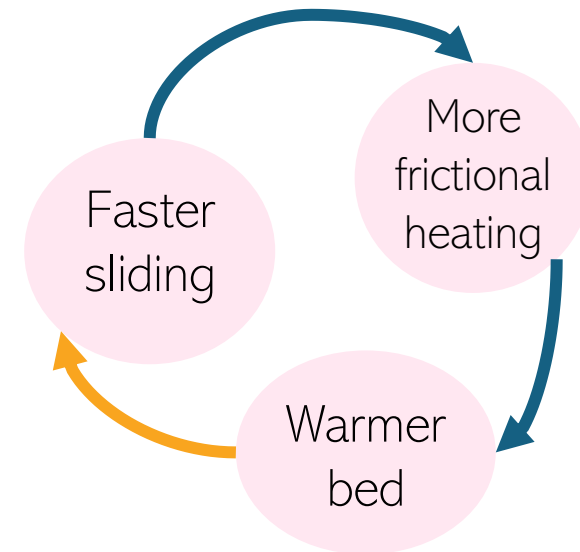
# Subtemperate sliding region is *unconditionally unstable*

Mantelli and Schoof, *Proc. R. Soc. A*, 2019b

Linear stability analysis  $\Rightarrow$   
travelling wave solutions

Surge-like oscillations?

Thermo-frictional  
feedback:



How do small-scale instabilities of the subtemperate region play out at the scale of a whole ice sheet?

# Subtemperate sliding region is *unconditionally unstable*

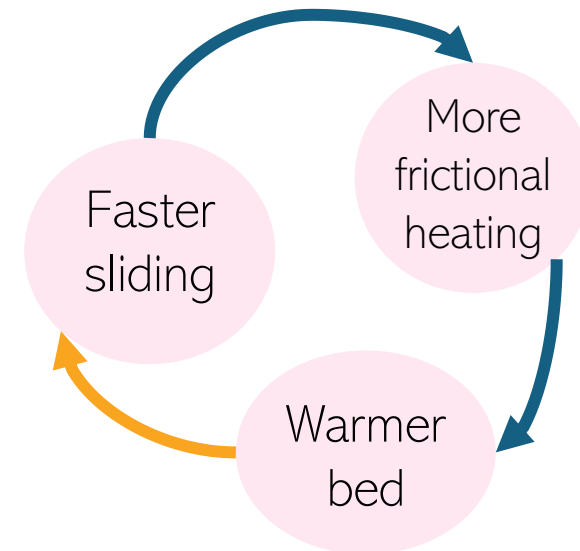
Mantelli and Schoof, *Proc. R. Soc. A*, 2019b

Stokes model is necessary

Linear stability analysis  $\Rightarrow$  travelling wave solutions

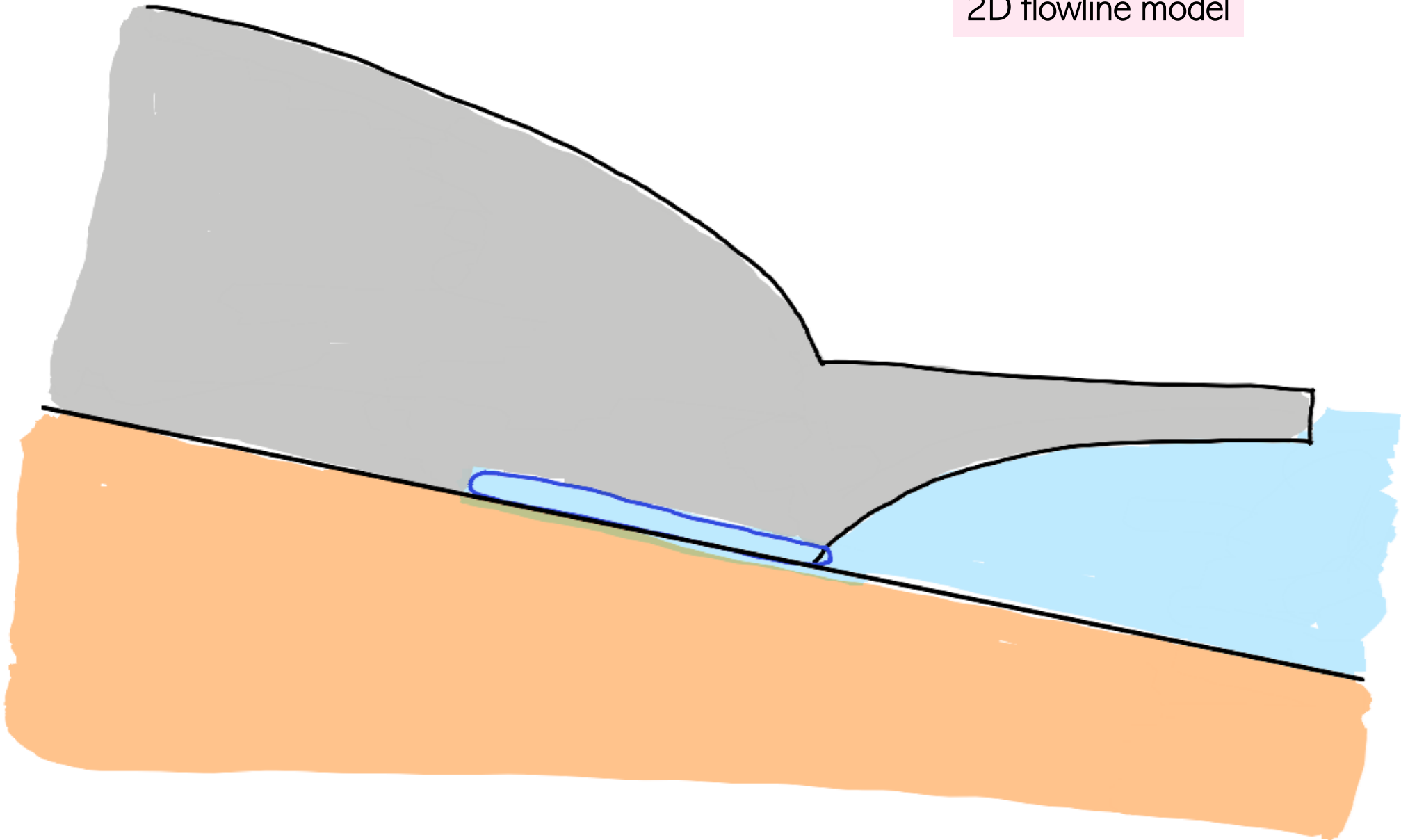
Surge-like oscillations?

Thermo-frictional feedback:

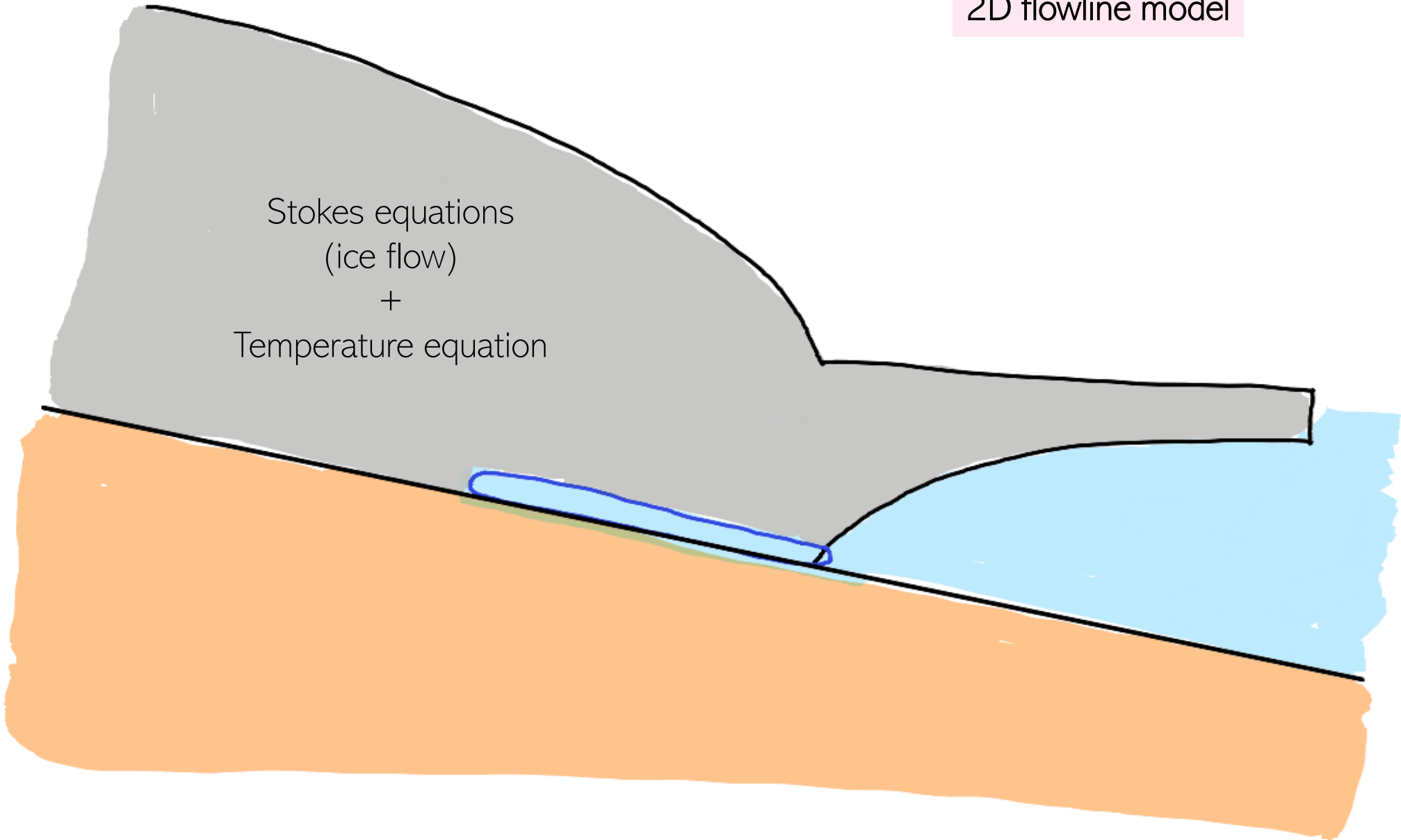


How do small-scale instabilities of the subtemperate region play out at the scale of a whole ice sheet?

2D flowline model



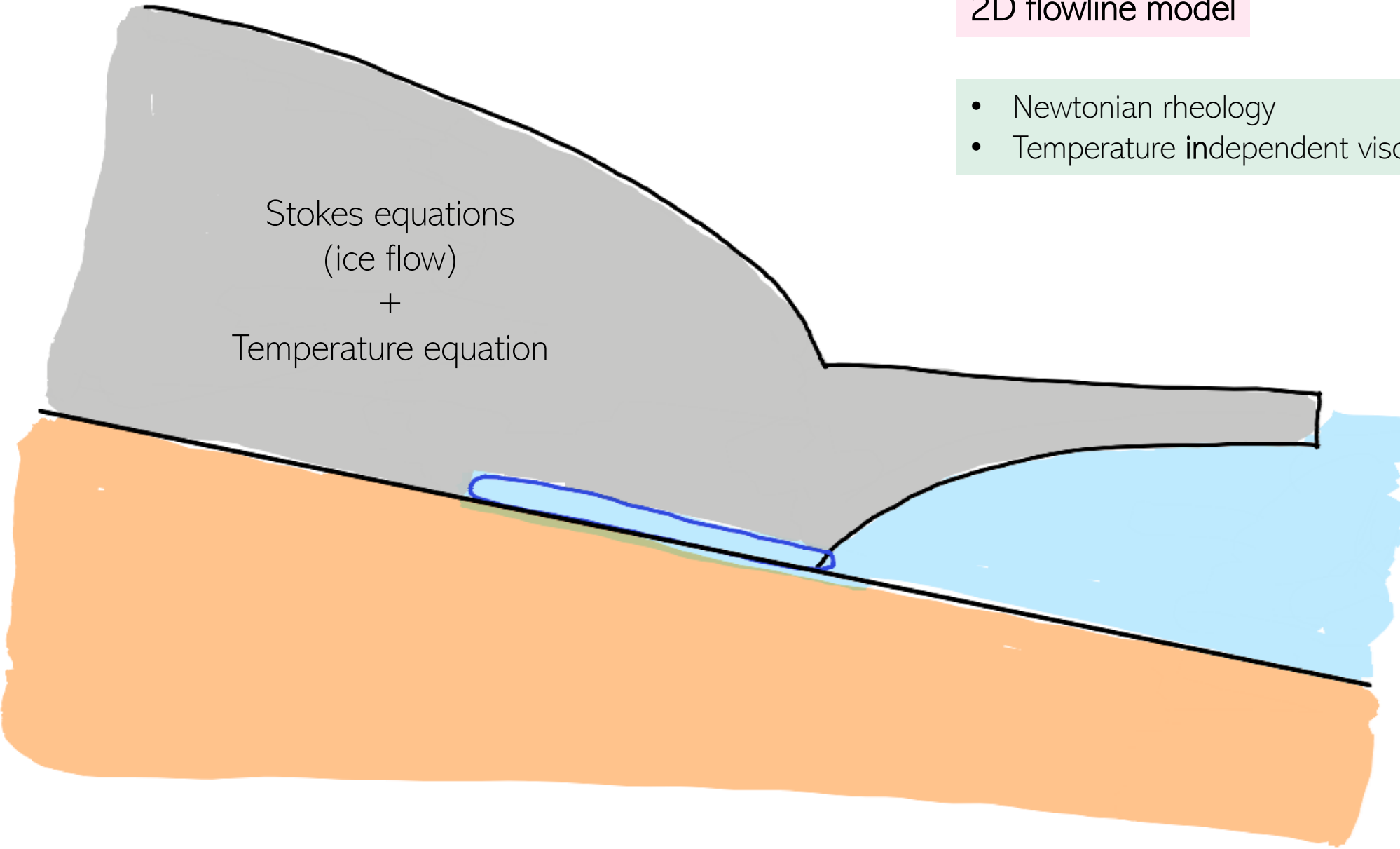
2D flowline model



## 2D flowline model

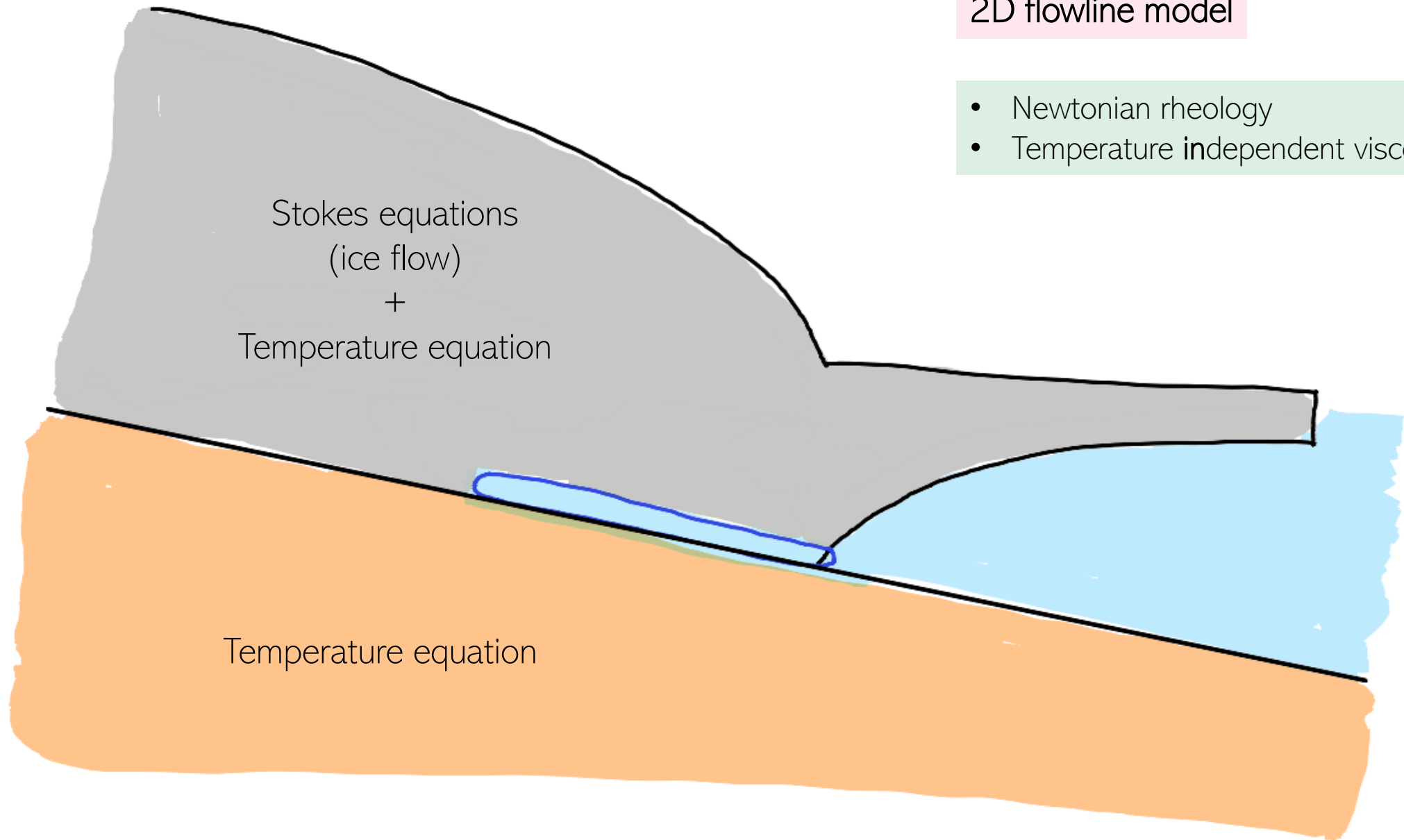
- Newtonian rheology
- Temperature independent viscosity

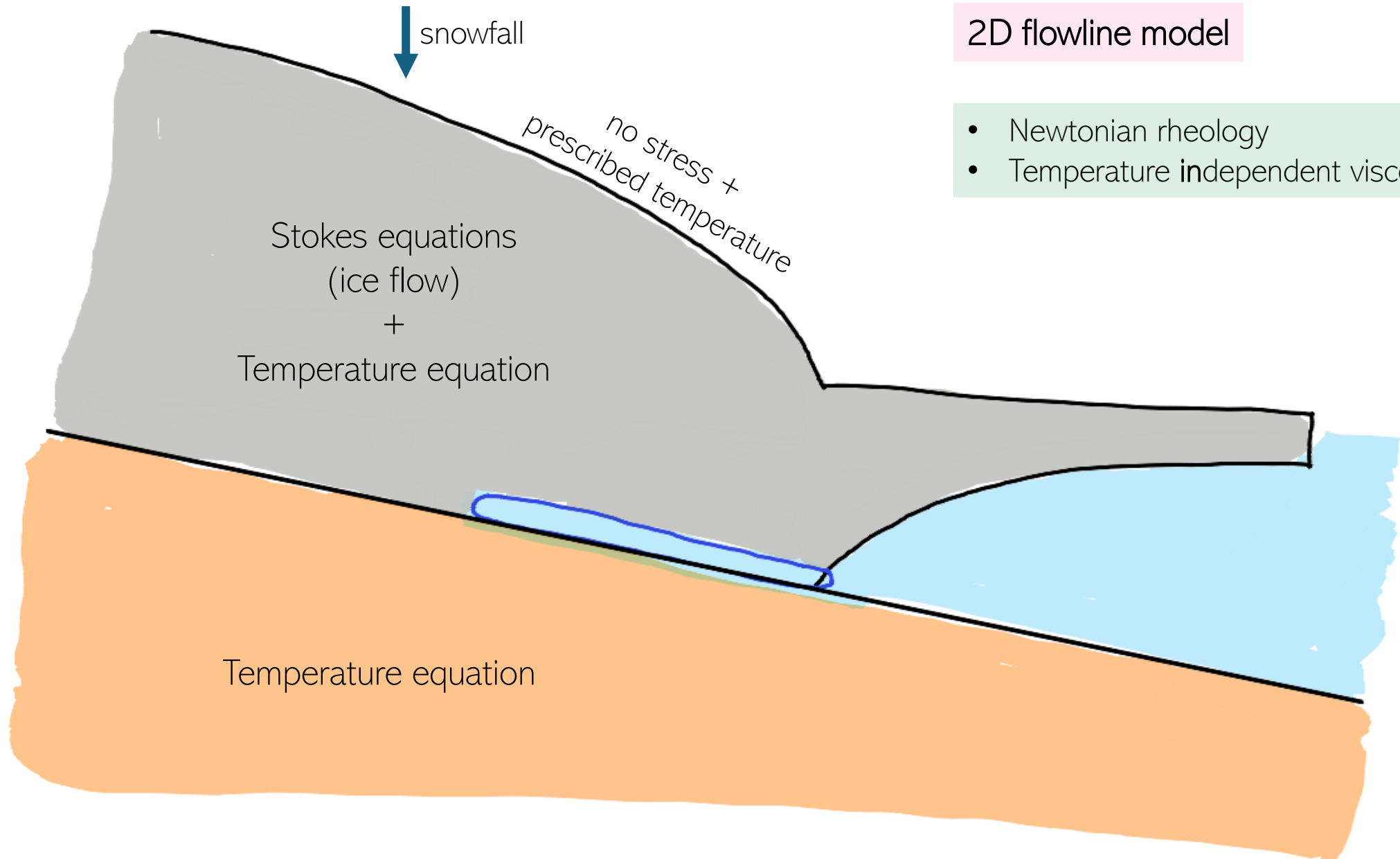
Stokes equations  
(ice flow)  
+  
Temperature equation



## 2D flowline model

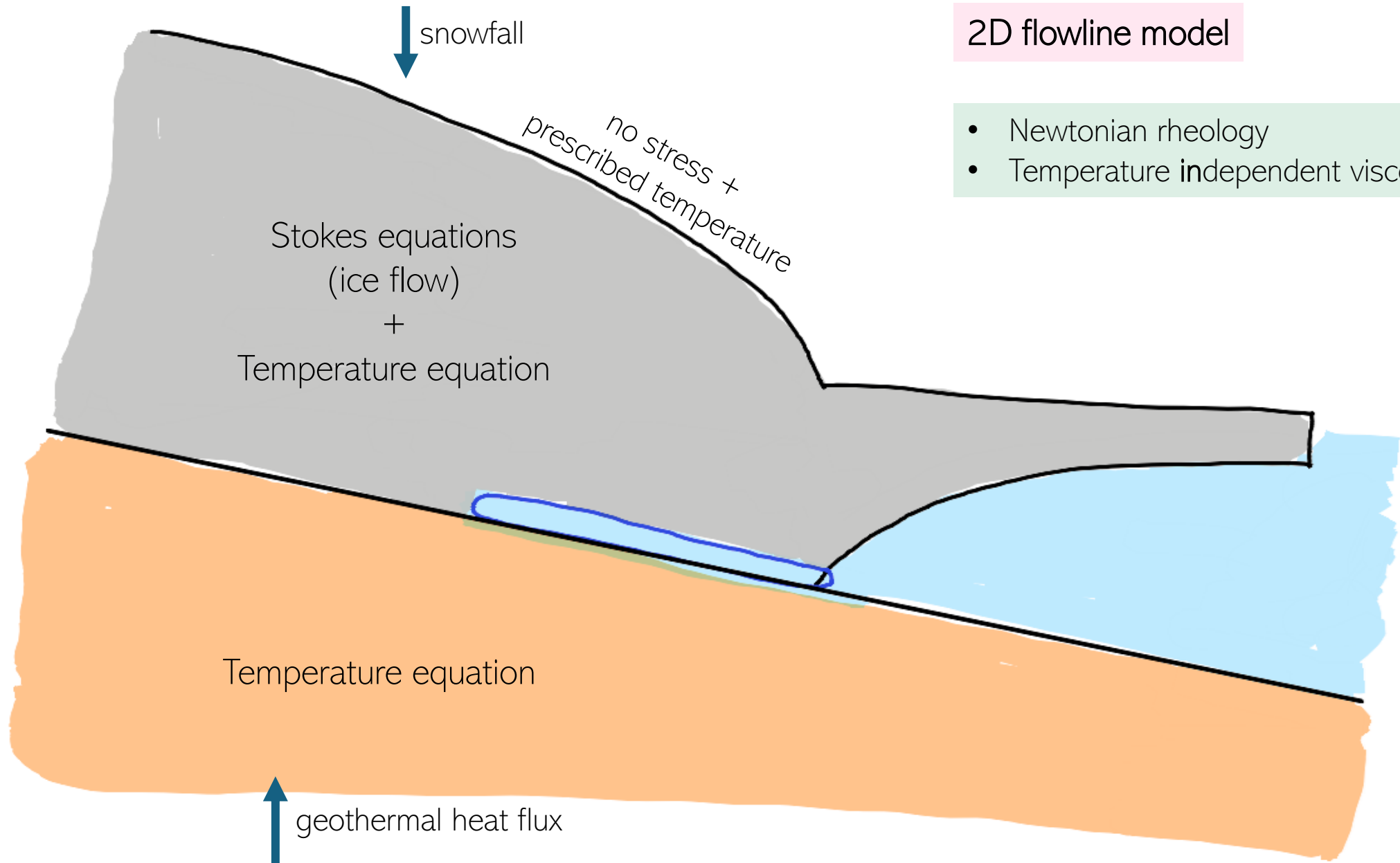
- Newtonian rheology
- Temperature independent viscosity





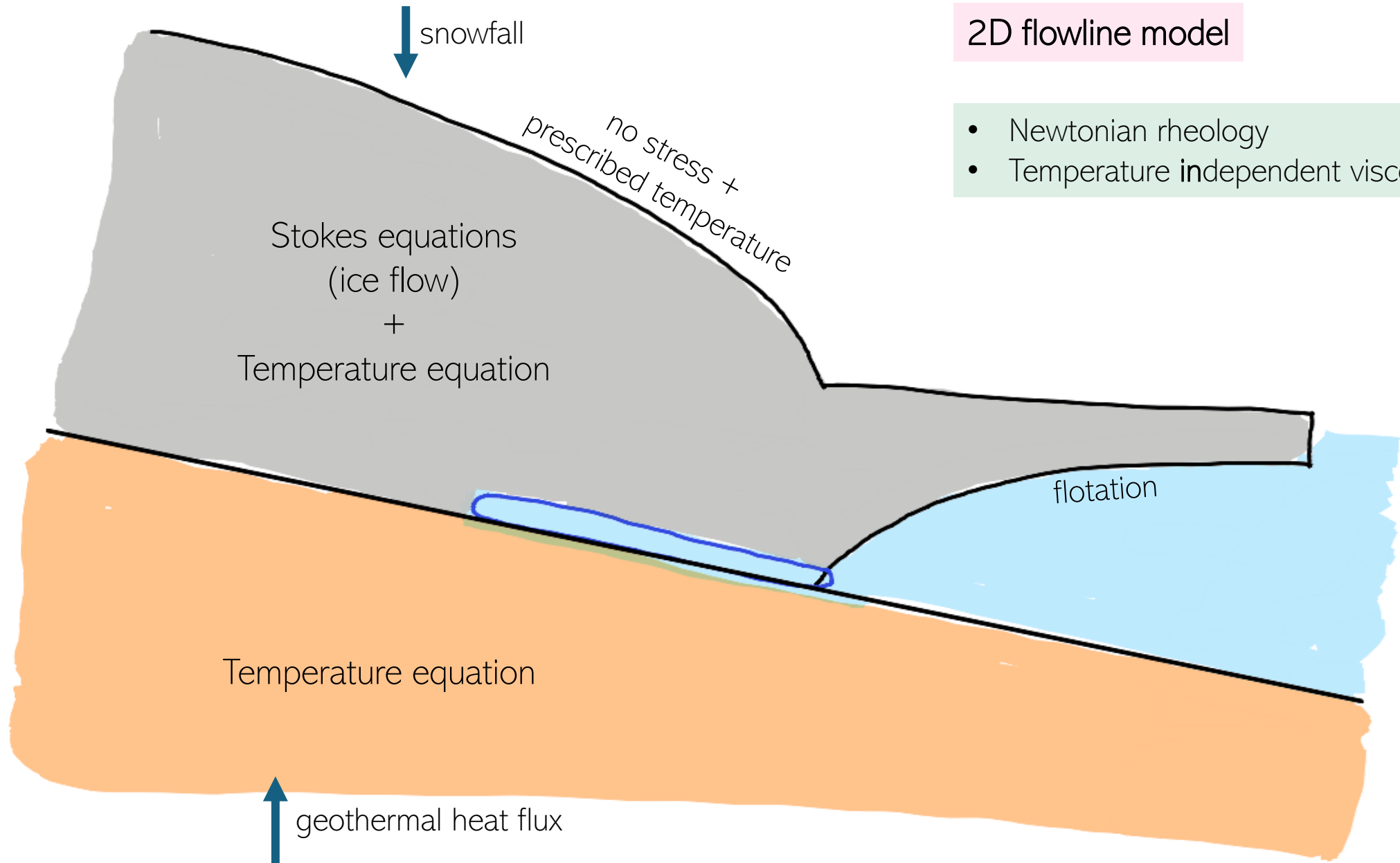
2D flowline model

- Newtonian rheology
- Temperature independent viscosity



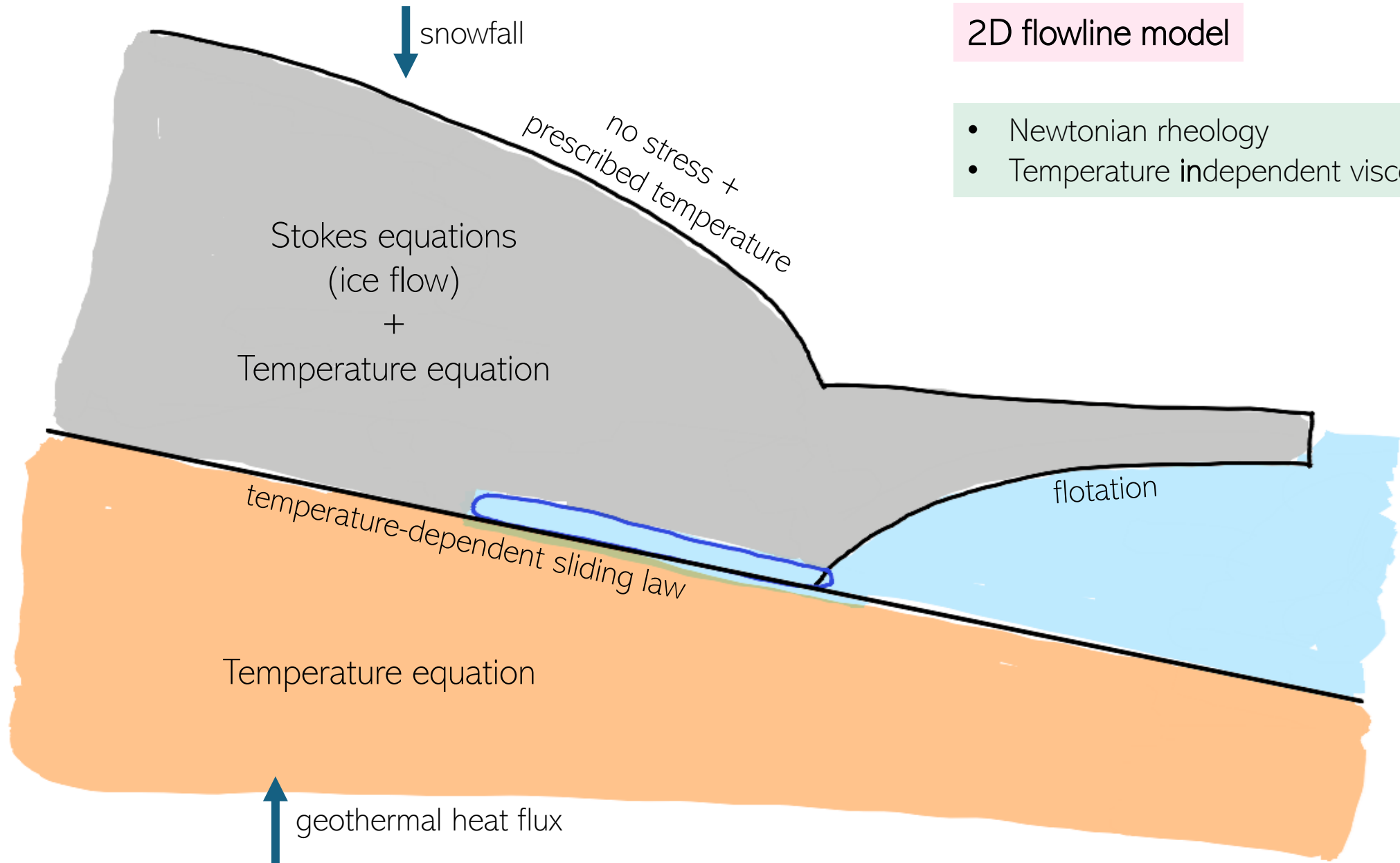
2D flowline model

- Newtonian rheology
- Temperature independent viscosity



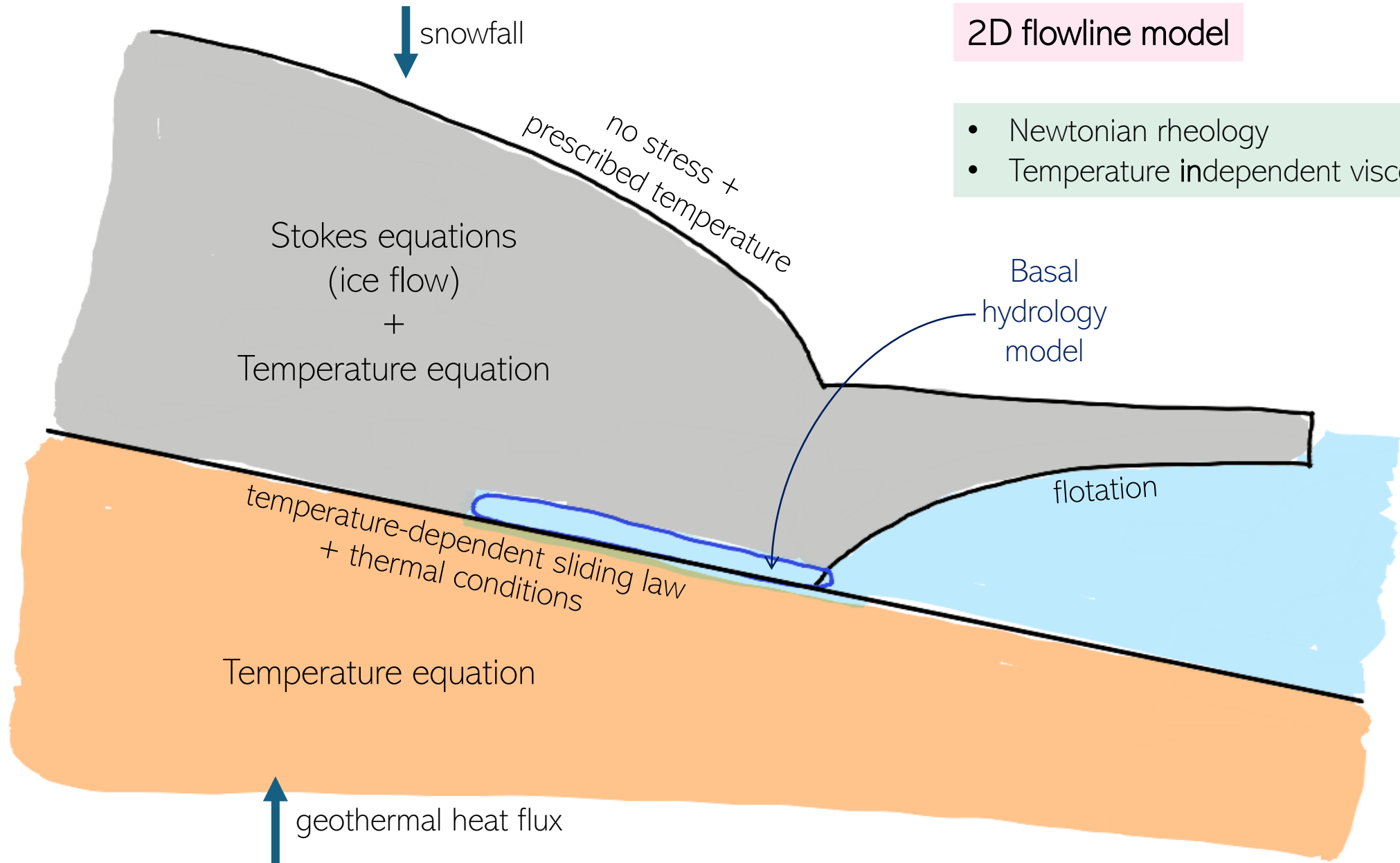
## 2D flowline model

- Newtonian rheology
- Temperature independent viscosity



2D flowline model

- Newtonian rheology
- Temperature independent viscosity

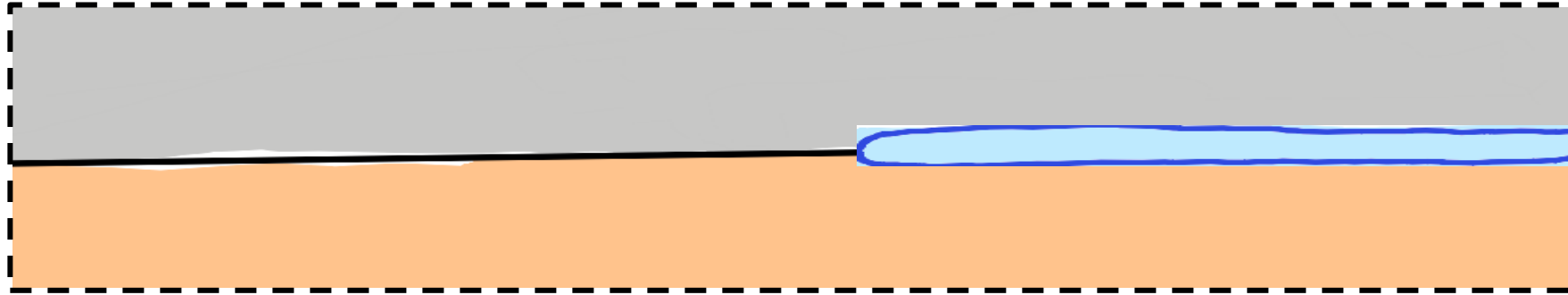


2D flowline model

- Newtonian rheology
- Temperature independent viscosity

No slip

Subtemperate sliding



**COLD**

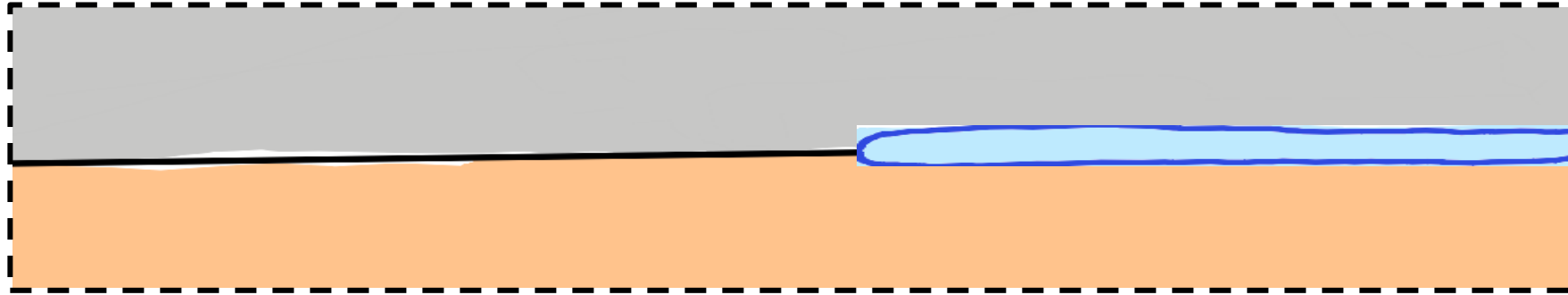
Basal  
energy  
balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

No slip

Subtemperate sliding

Temperate sliding



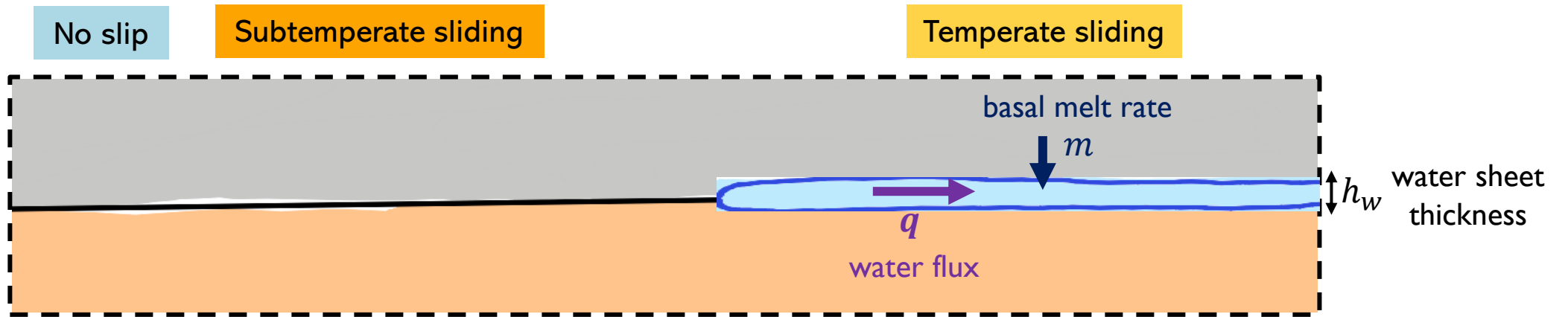
**COLD**

**TEMPERATE**

Basal  
energy  
balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

$$T_b = T_m$$



**COLD**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

**TEMPERATE**

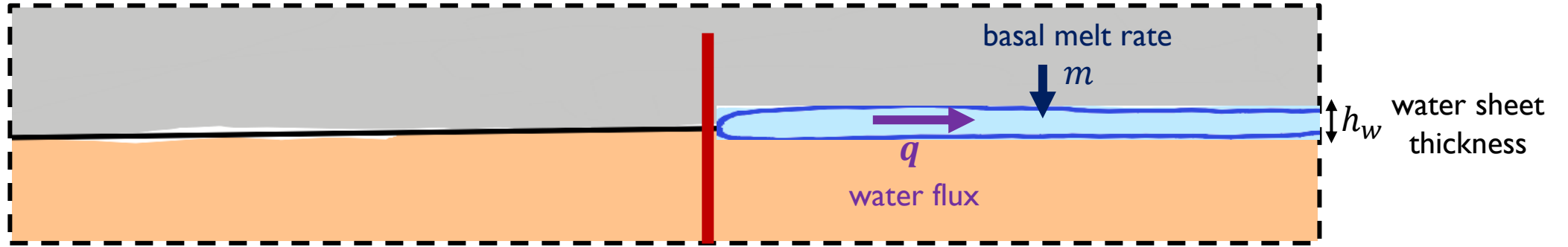
$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance

No slip

Subtemperate sliding

Temperate sliding



**COLD**

*Free boundary*

**TEMPERATE**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

$$T_b = T_m$$

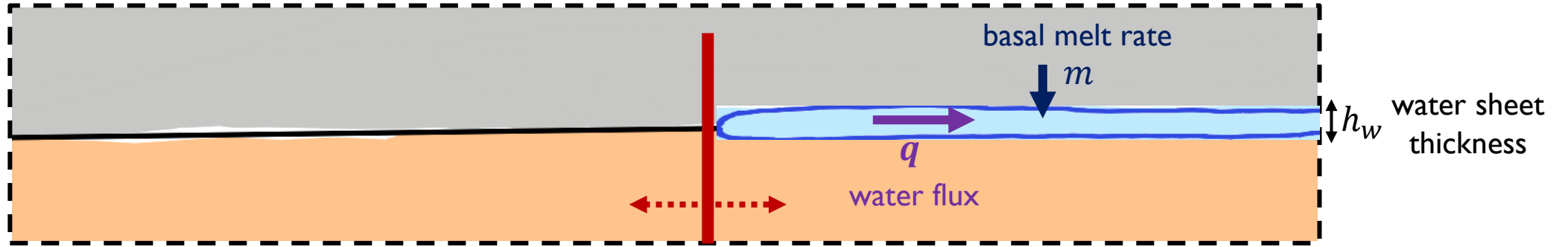
+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$

from basal energy balance

No slip

Subtemperate sliding

Temperate sliding



**COLD**

*Free boundary*

**TEMPERATE**

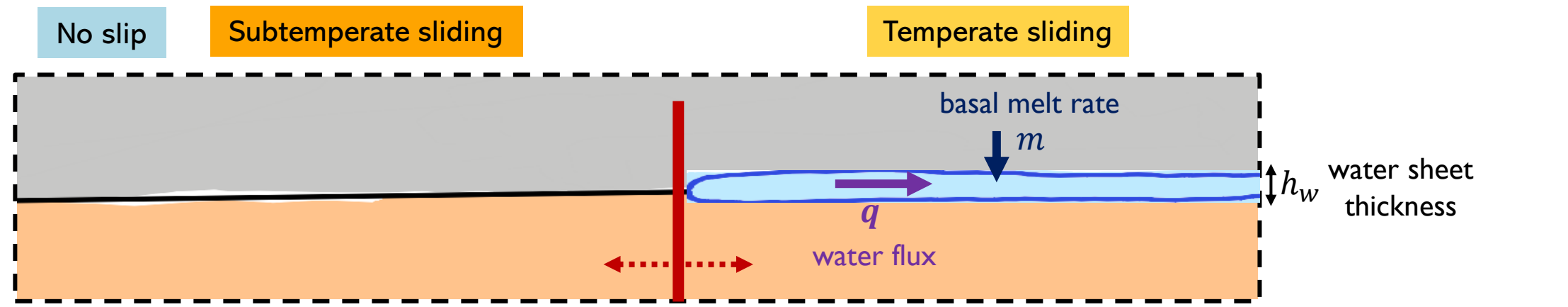
Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$

from basal energy balance



**COLD**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

*Free boundary*

**TEMPERATE**

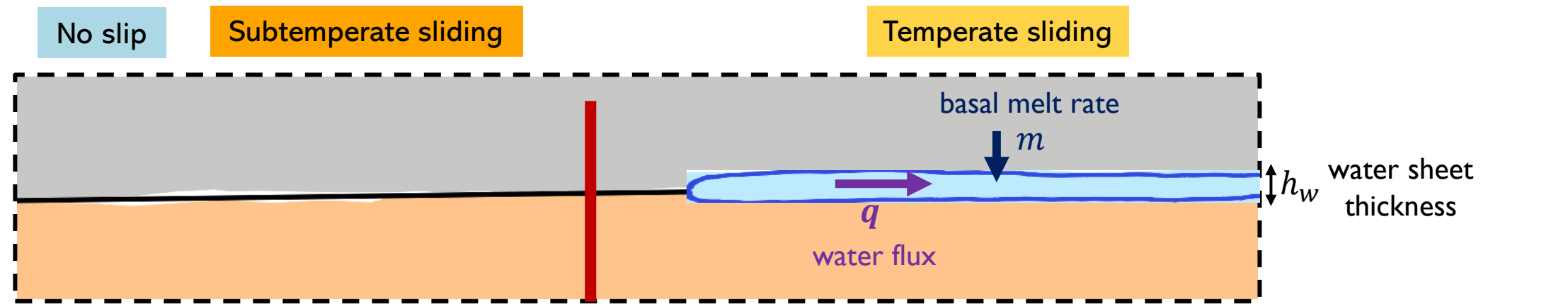
$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance



**1. Growing temperate region**

Bed must become temperate *before* water can be present



**COLD**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

*Free boundary*

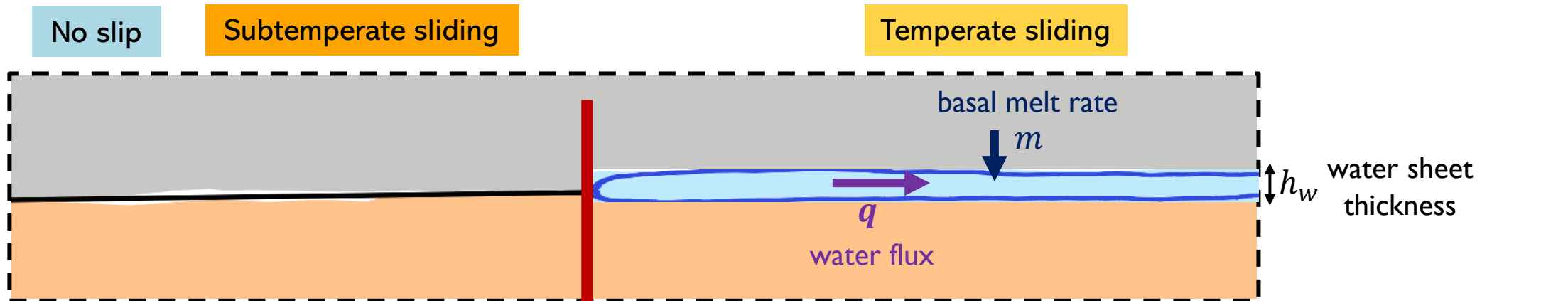
**TEMPERATE**

$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance



**1. Growing temperate region**  
 Bed must become temperate *before* water can be present



**COLD**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

*Free boundary*

**TEMPERATE**

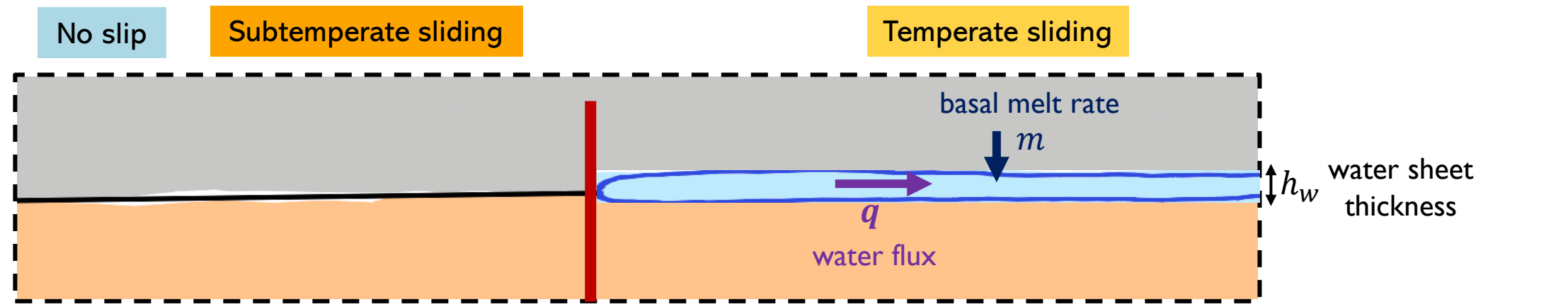
$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance



**1. Growing temperate region**

Bed must become temperate *before* water can be present



**COLD**

**TEMPERATE**

Free  
boundary

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance

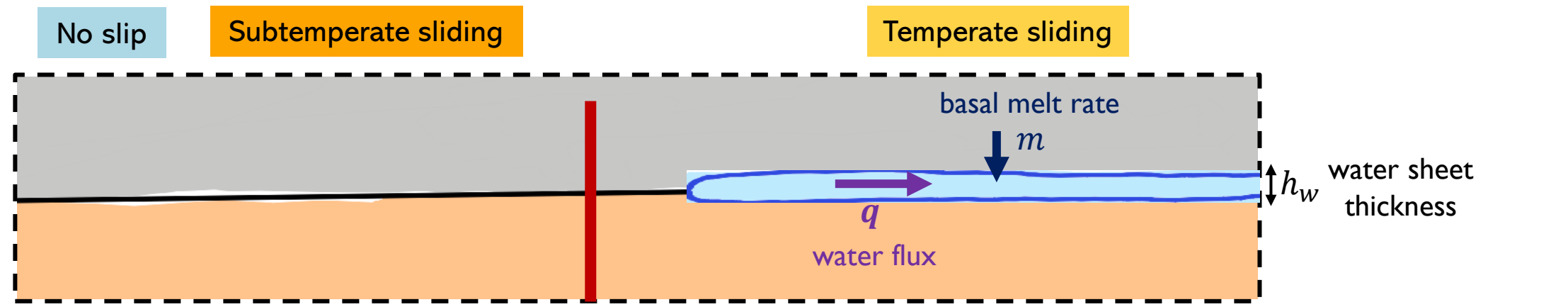


**1. Growing temperate region**

Bed must become temperate *before* water can be present

**2. Shrinking temperate region**

All water must freeze *before* bed becomes cold



**COLD**

**TEMPERATE**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

*Free boundary*

$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$

from basal energy balance



**1. Growing temperate region**  
Bed must become temperate before water can be present

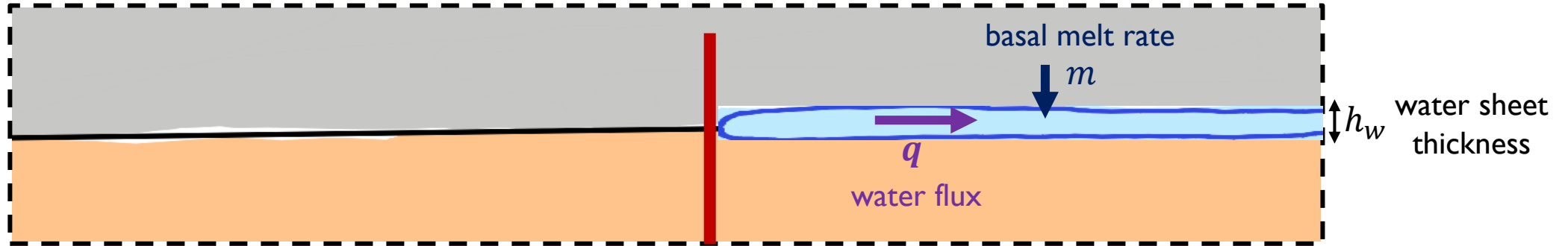


**2. Shrinking temperate region**  
All water must freeze before bed becomes cold

No slip

Subtemperate sliding

Temperate sliding



**COLD**

*Free boundary*

**TEMPERATE**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance

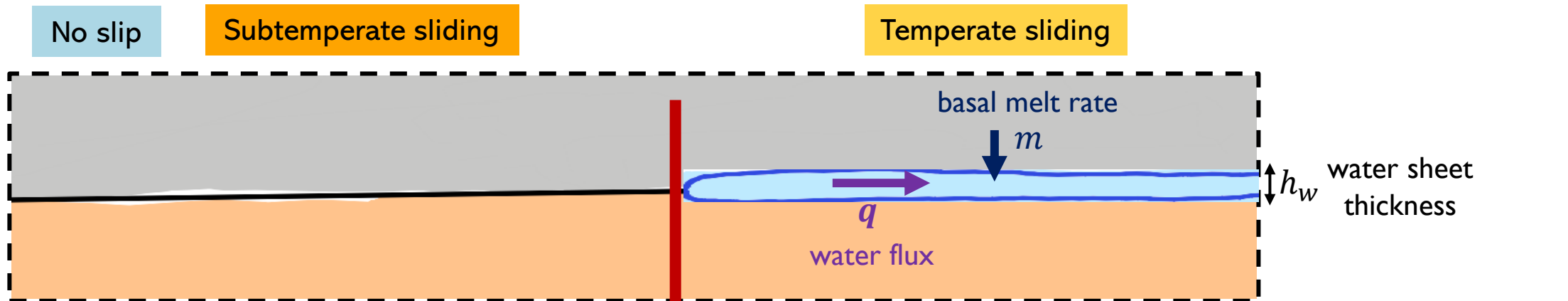


1. *Growing* temperate region

Bed must become temperate *before* water can be present

2. *Shrinking* temperate region

All water must freeze *before* bed becomes cold



**COLD**

Basal energy balance:

$$\left[ k \frac{\partial T}{\partial z} \right]_{-}^{+} + \underbrace{\tau_b u_b}_{\text{frictional heating}} = 0$$

*Free boundary*

**TEMPERATE**

$$T_b = T_m$$

+ Hydrology model:  $\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m$  from basal energy balance



**1. Growing temperate region**

Bed must become temperate *before* water can be present

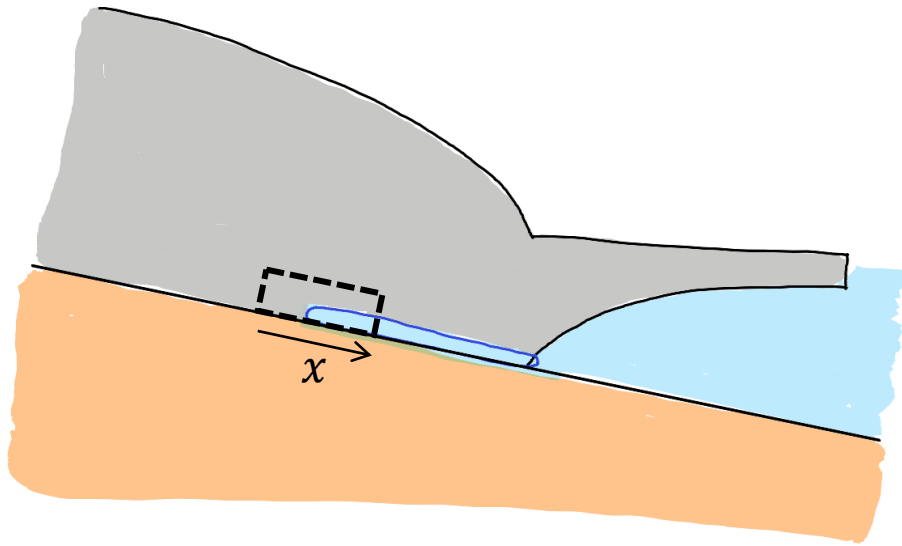


**2. Shrinking temperate region**

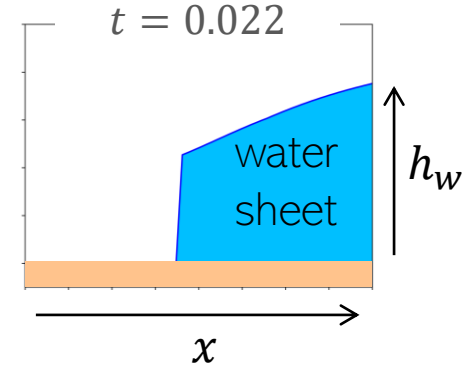
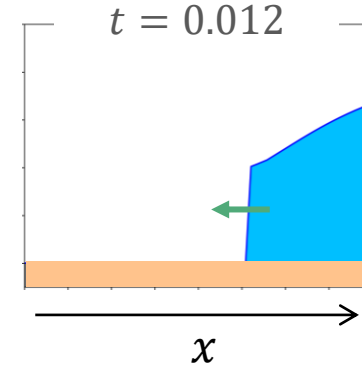
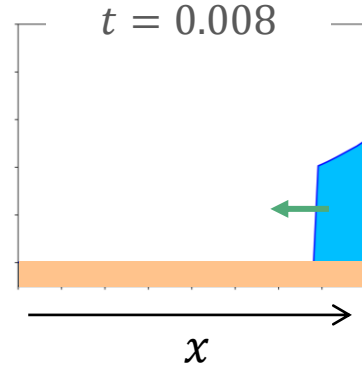
All water must freeze *before* bed becomes cold



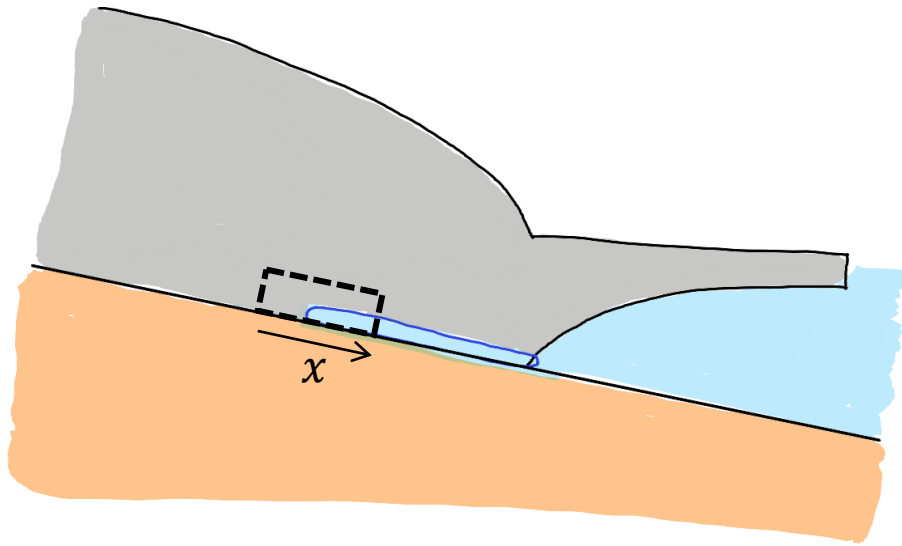
# Cold-temperate boundary migration



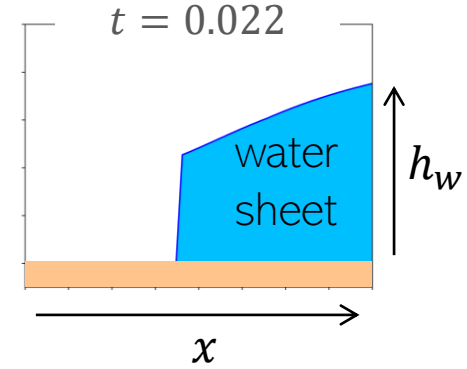
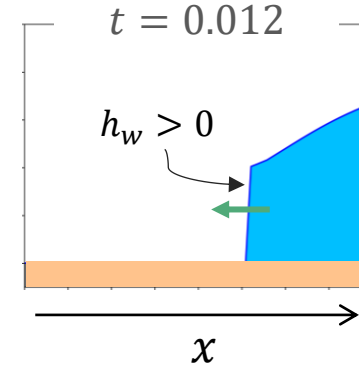
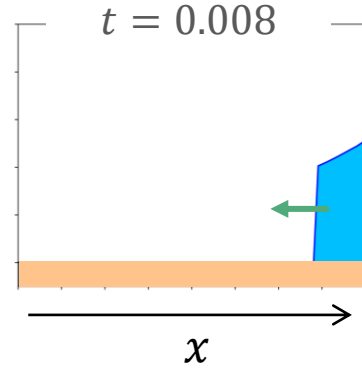
Growing  
temperate  
region



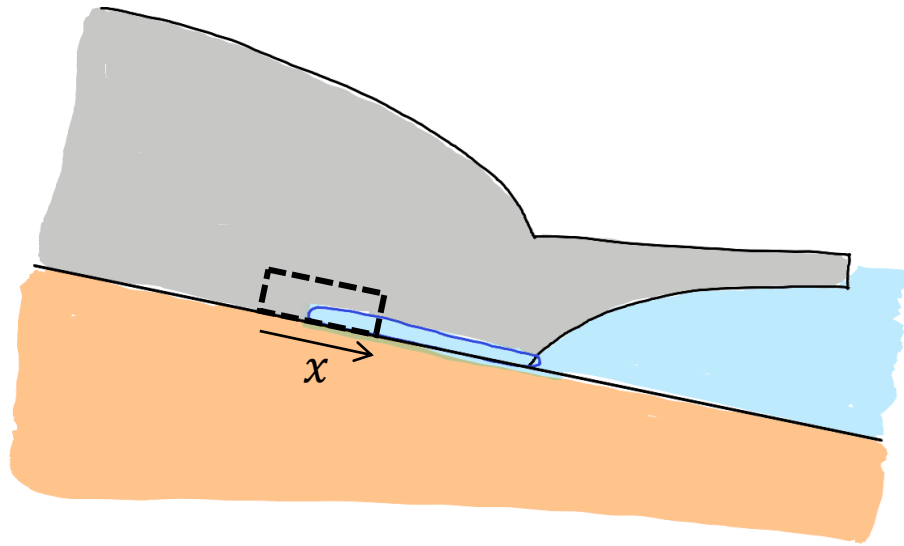
# Cold-temperate boundary migration



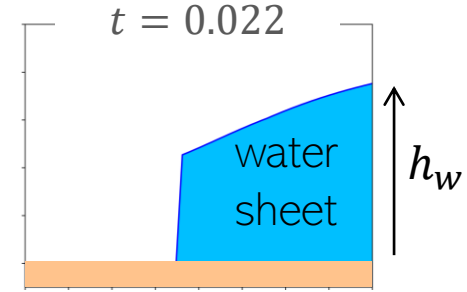
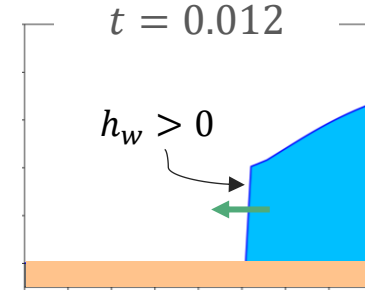
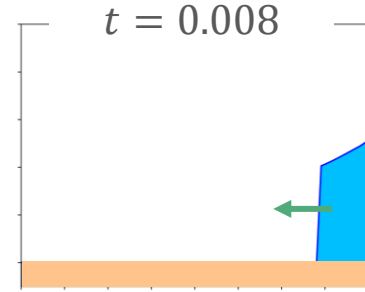
Growing  
temperate  
region



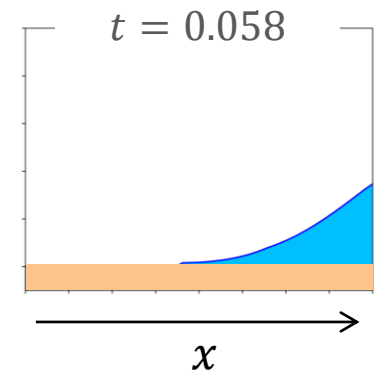
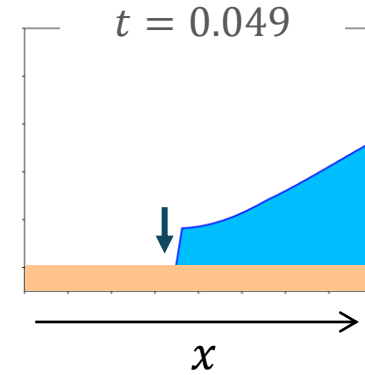
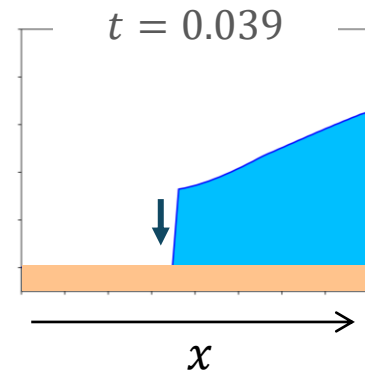
# Cold-temperate boundary migration



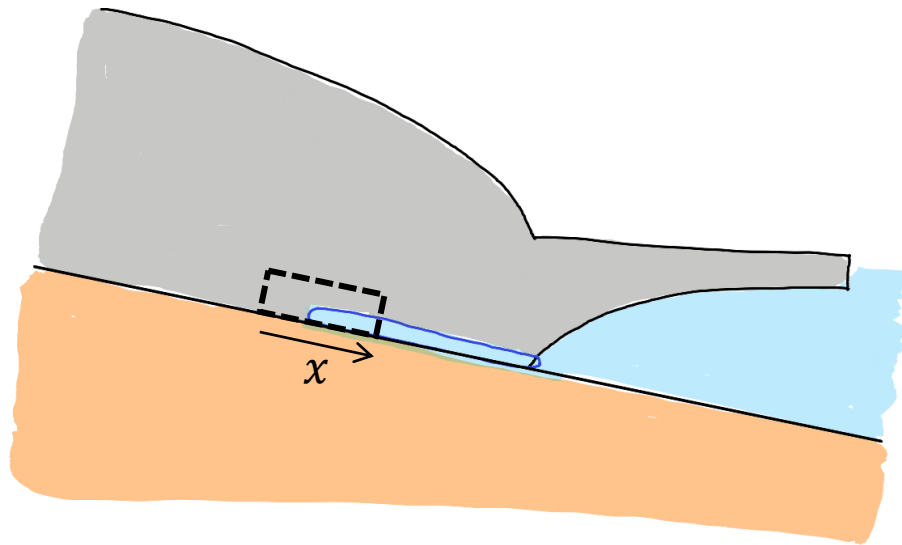
Growing  
temperate  
region



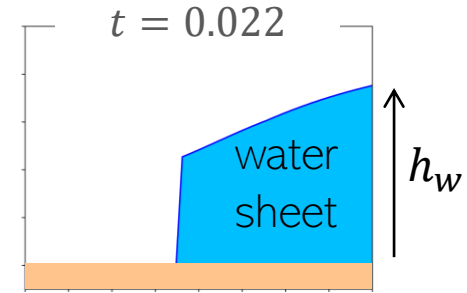
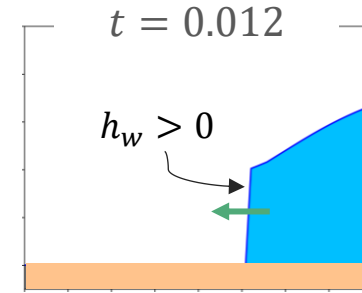
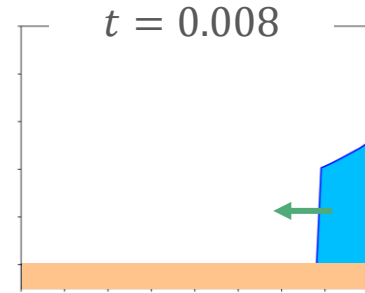
Freezing



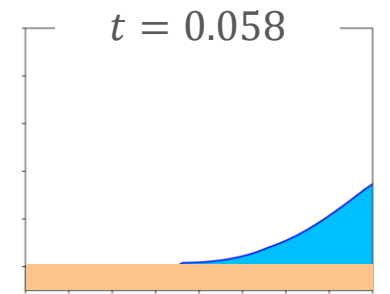
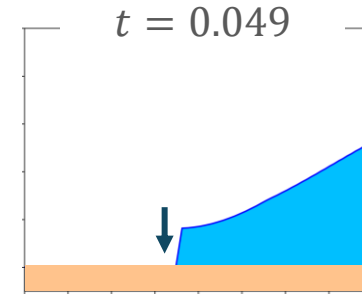
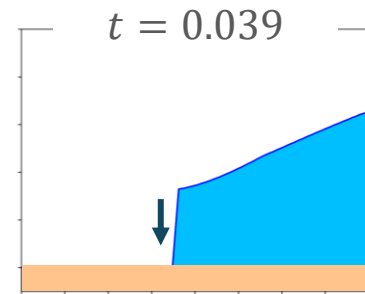
# Cold-temperate boundary migration



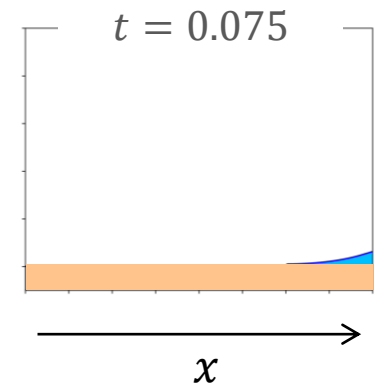
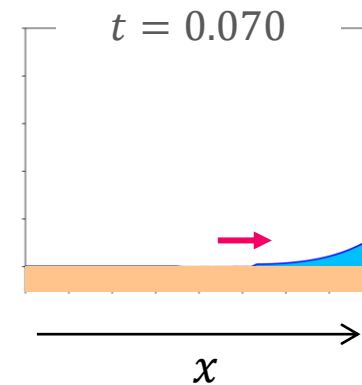
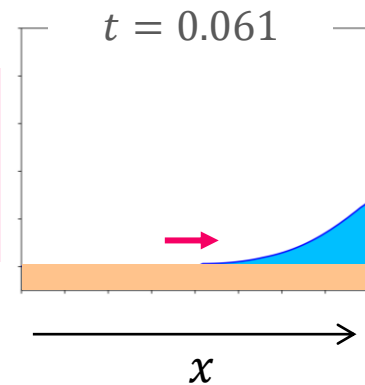
Growing temperate region



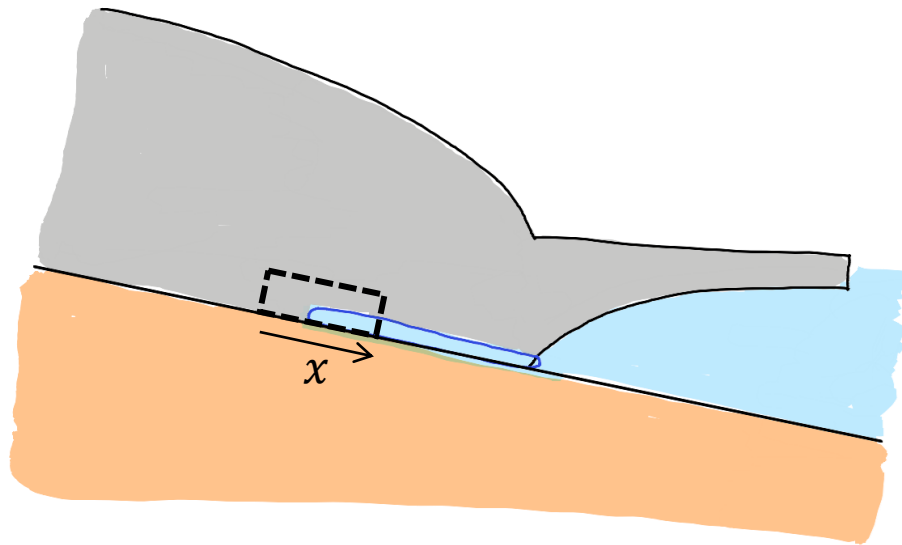
Freezing



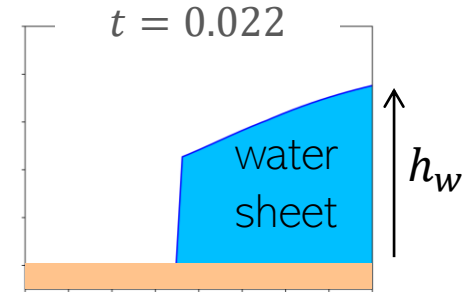
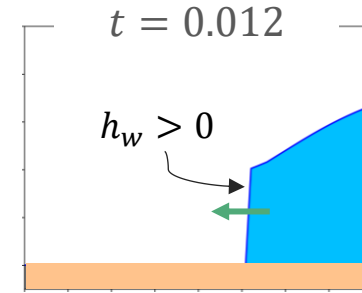
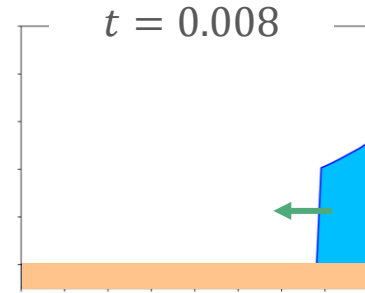
Shrinking temperate region



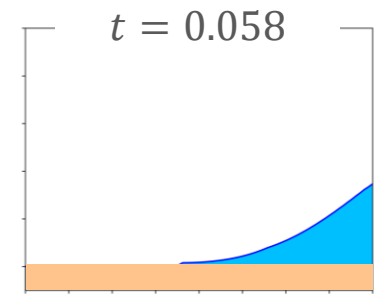
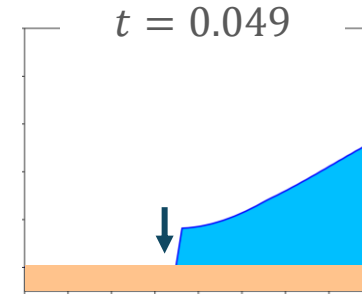
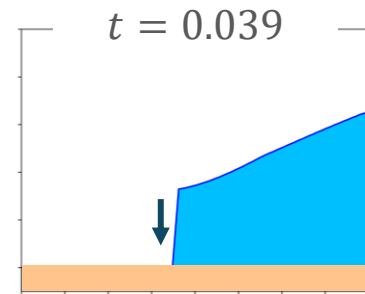
# Cold-temperate boundary migration



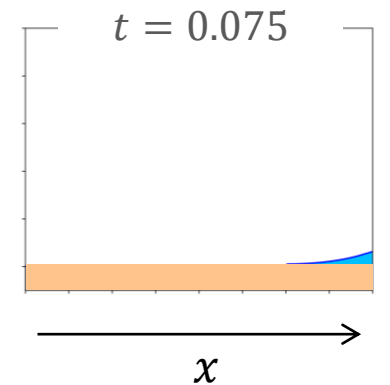
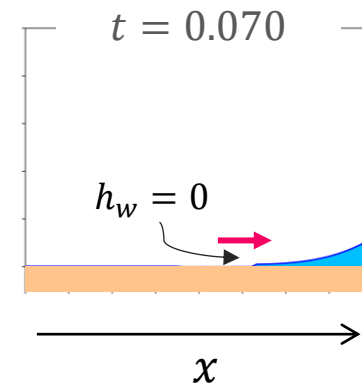
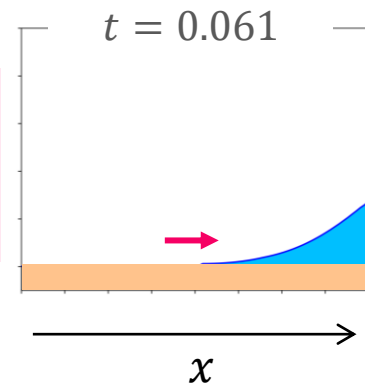
Growing  
temperate  
region



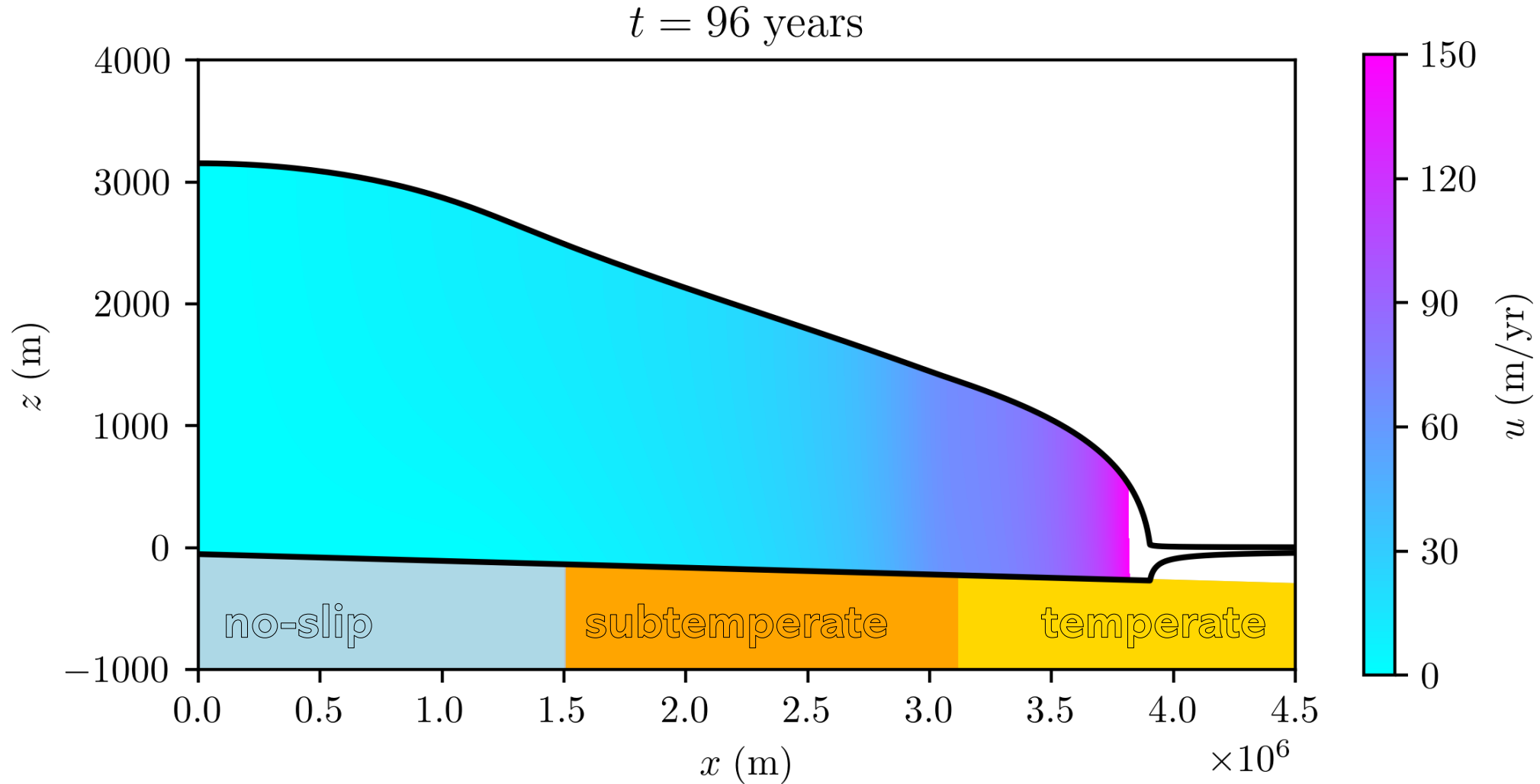
Freezing



Shrinking  
temperate  
region

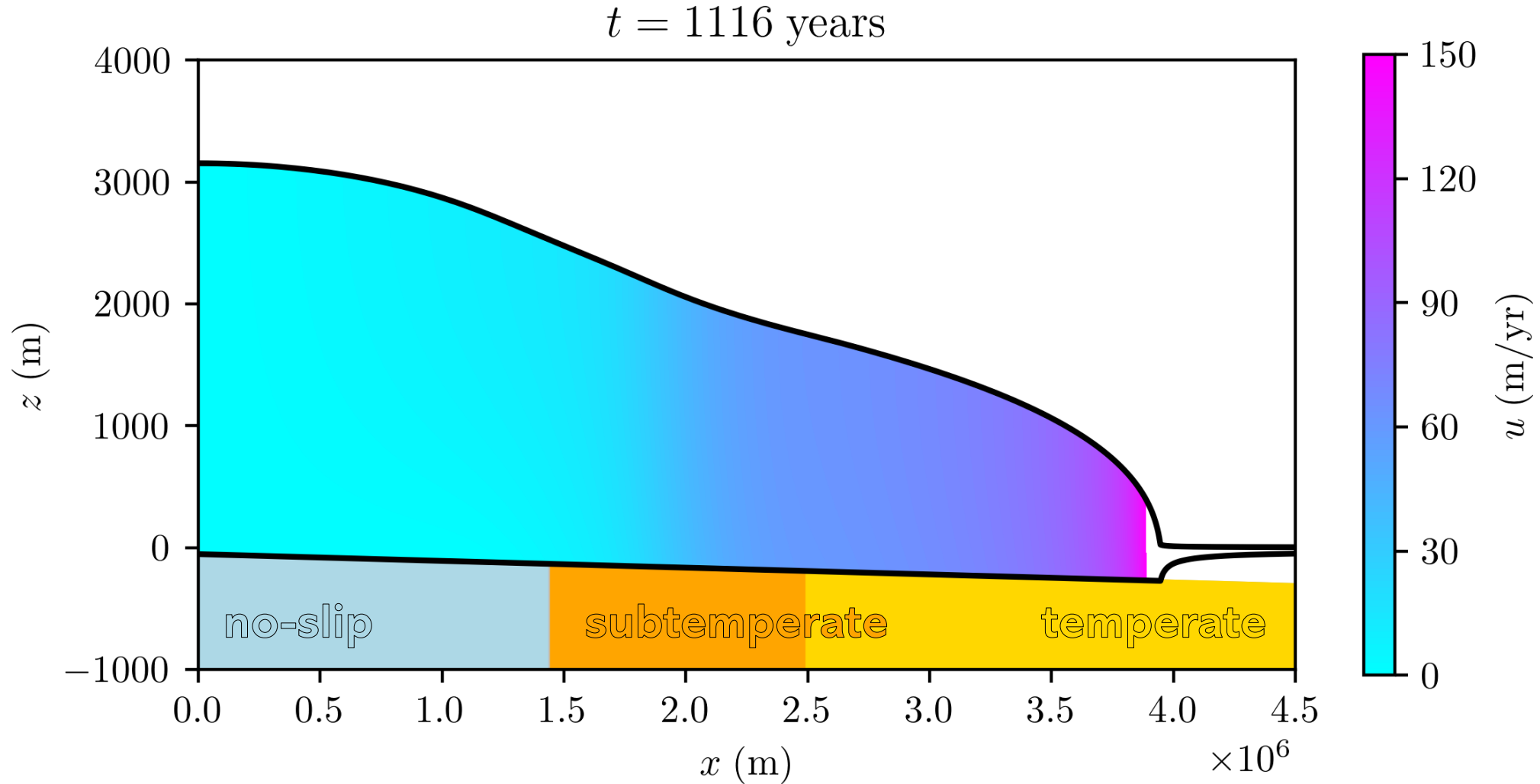


# Oscillations at ice-sheet scale...



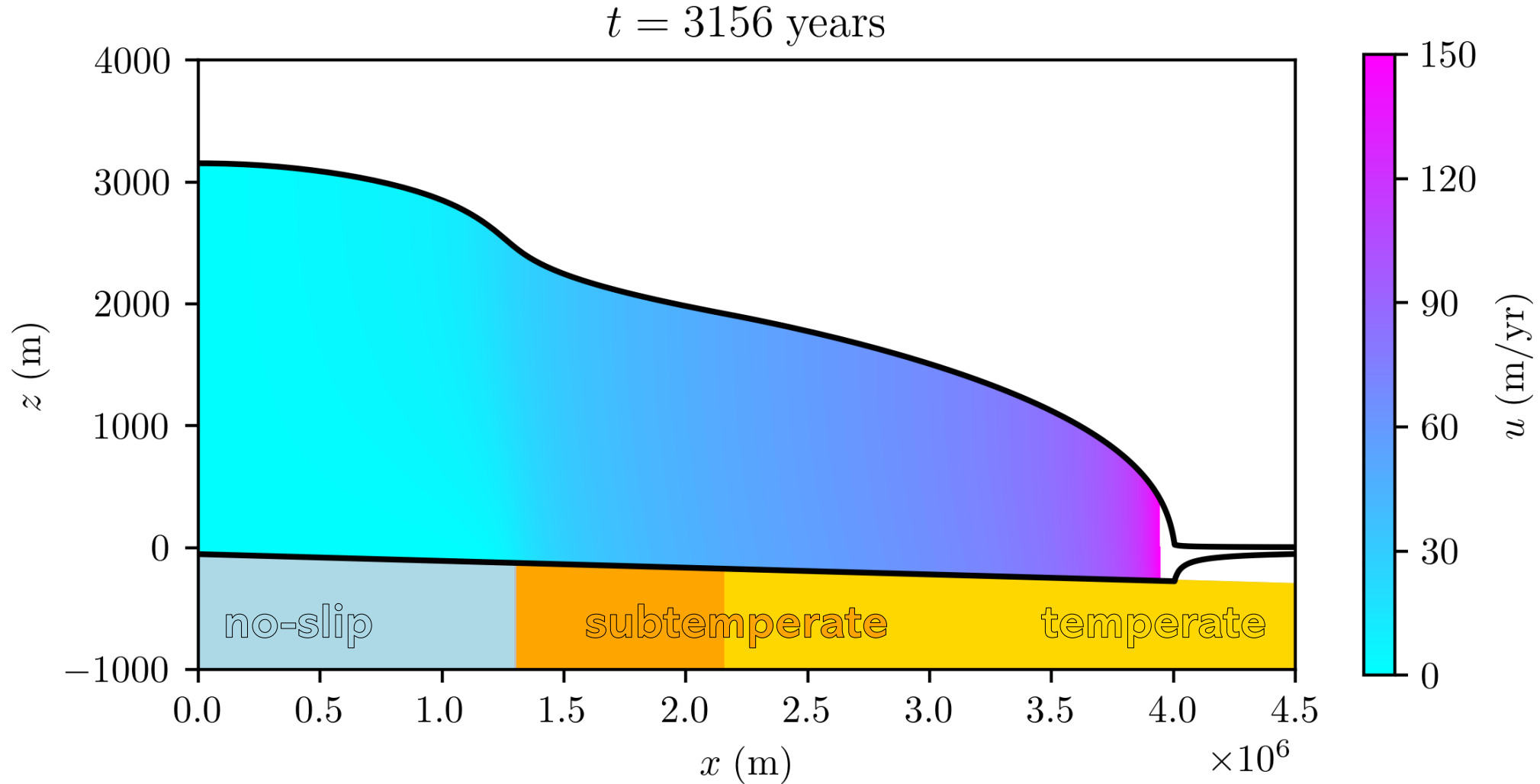
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



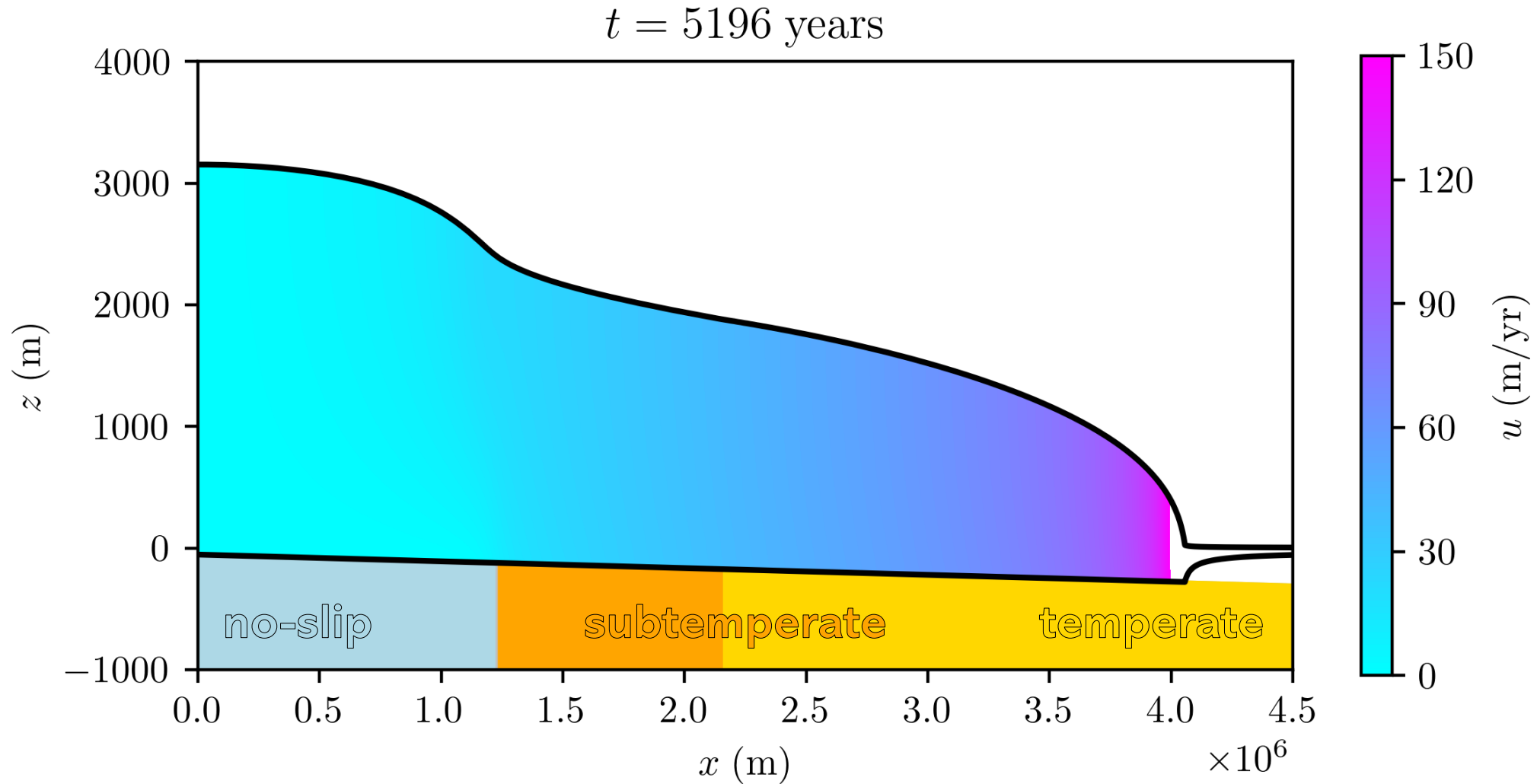
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



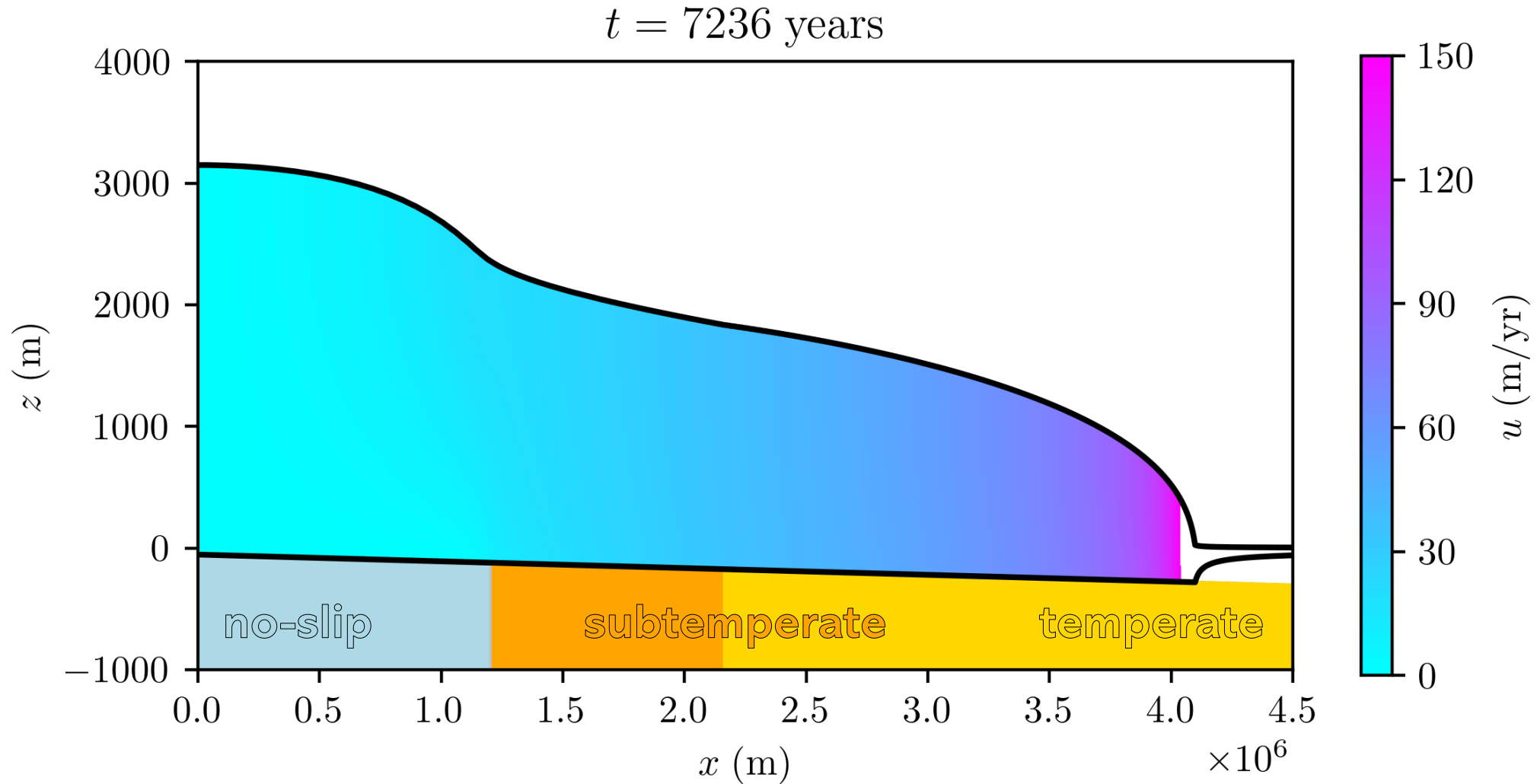
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



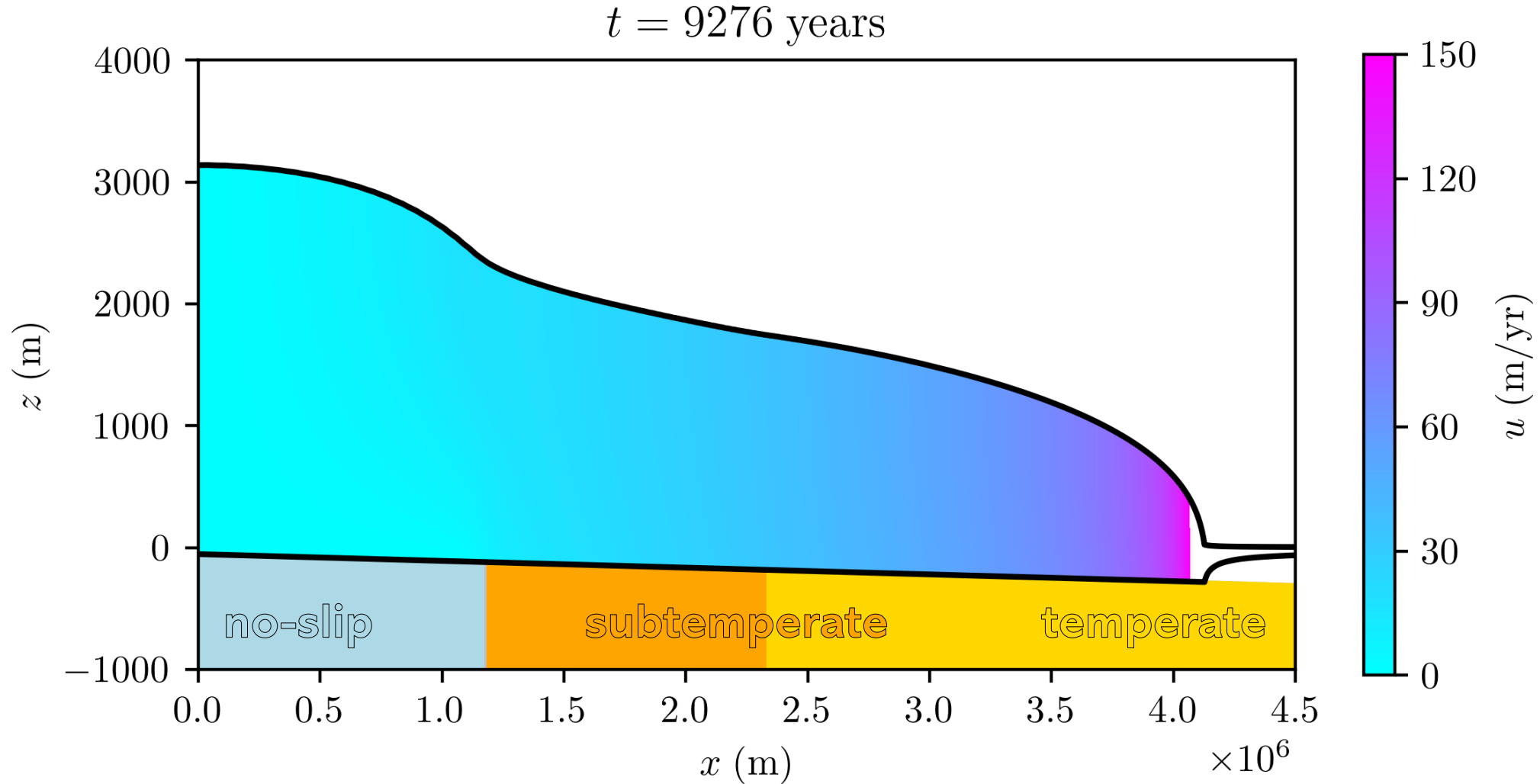
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



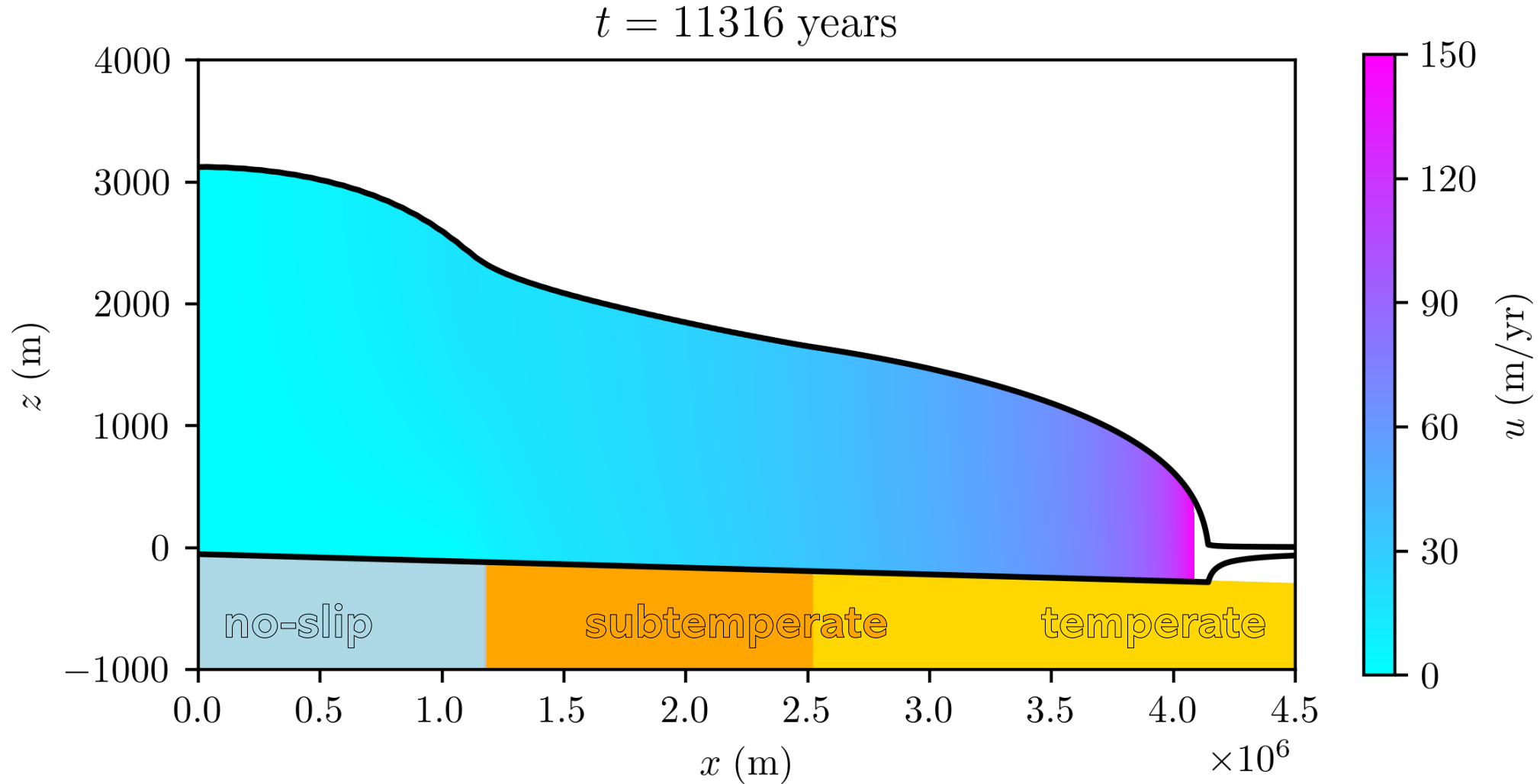
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



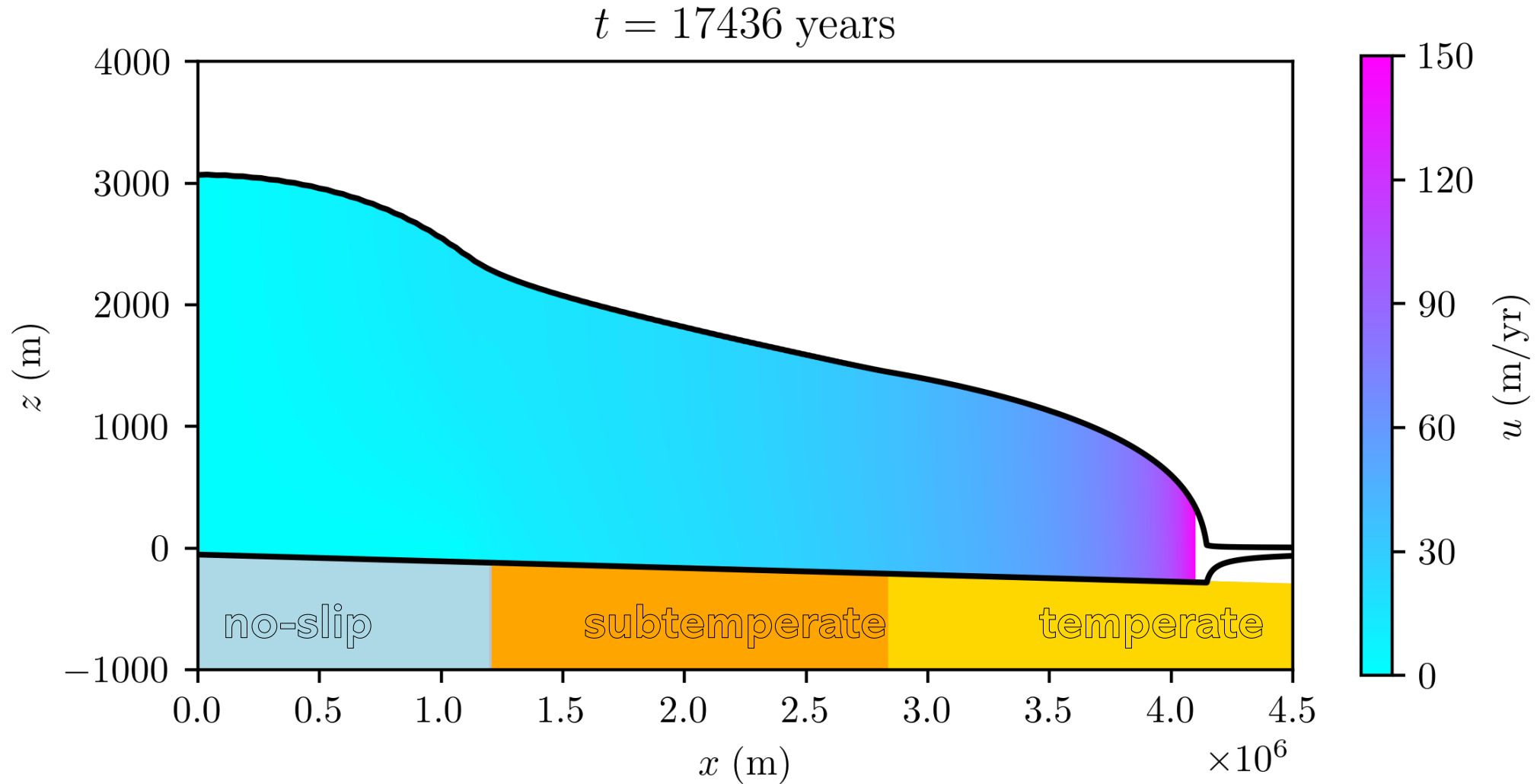
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



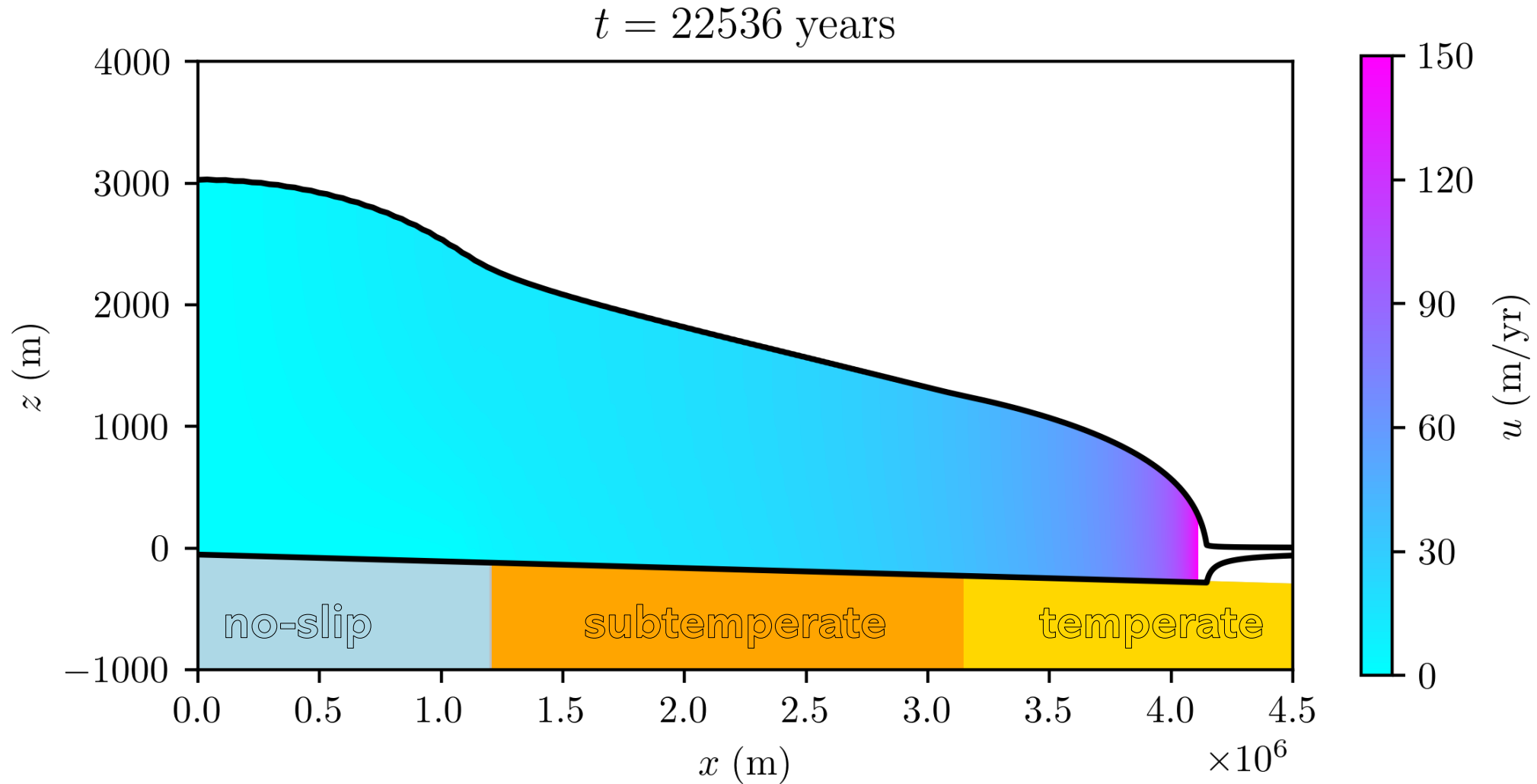
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



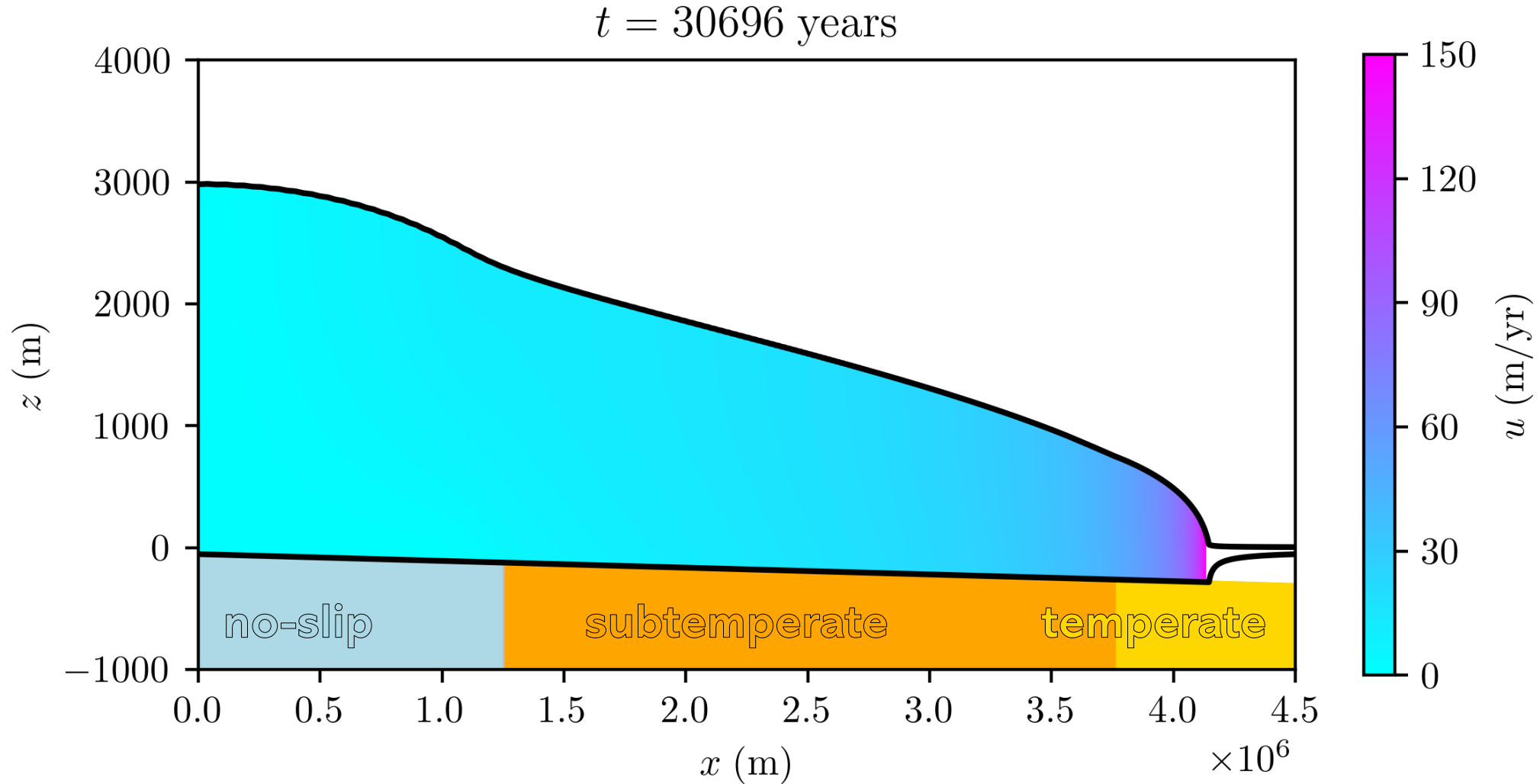
...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



...due to *subtemperate sliding* instabilities.

# Oscillations at ice-sheet scale...



...due to *subtemperate sliding* instabilities.

# Take-home messages

We've shown:

*Subtemperate sliding* instabilities  $\Rightarrow$  oscillations at ice-sheet scale

Key step:

Choice of Newtonian rheology, temperature-**in**dependent friction isolates thermo-frictional feedback

Relevance to ice-sheet modellers:

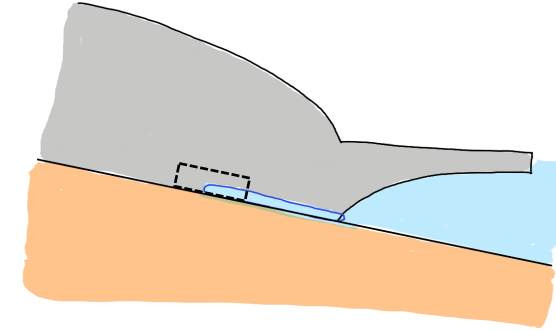
Cold-temperate boundary displays dynamics (surging behaviour) that *can only be captured using a Stokes model* – relevant beyond ice streams

# Temporal ice-stream dynamics resulting from subtemperate sliding instabilities

Thermo-frictional feedback isolated by choice of rheology (Newtonian, temperature-independent friction)

⇒ Oscillations at ice-sheet scale due to subtemperate sliding instabilities

Novel numerical implementation of cold-temperate boundary migration



## Outlook

- Fine-tuning and analysis – stay tuned
- Interactions between temporal and *spatial* instabilities → 3D

LMU, Germany:

Tilly Woods

Silas Lloyd

Elisa Mantelli

CSC, Finland:

Thomas Zwinger

UBC, Canada:

Christian Schoof



UBC



Tilly Woods, [tilly.woods@lmu.de](mailto:tilly.woods@lmu.de)

# Bonus slides

# Model equations

Stokes equations:

$$\nabla \cdot (\mu [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]) - \nabla p + \rho \mathbf{g} = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

Temperature equation in ice:

$$\rho_i c_i \frac{\partial T}{\partial t} + \rho_i c_i \mathbf{u} \cdot \nabla T - k_i \nabla^2 T = \frac{\mu}{2} [\nabla \mathbf{u} + (\nabla \mathbf{u})^T] : [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$$

Temperature equation in bedrock:

$$\rho_r c_r \frac{\partial T}{\partial t} - k_r \nabla^2 T = 0$$

Hydrology model (in temperate region):

$$\frac{\partial h_w}{\partial t} + \nabla \cdot \mathbf{q} = m \quad \mathbf{q} = -\kappa(N) \nabla \phi \quad (\text{water flux})$$

$$\kappa(N) = D_0 N^{-k_0} \quad (\text{hydraulic conductivity})$$

$$\phi = \Phi_0 - N \quad (\text{hydraulic potential})$$

$$N(h_w) = \frac{N_0 (h_r - h_w)}{h_w} \quad (\text{effective pressure})$$

$$\Phi_0 = \sigma_n + \rho_w g B$$

$$k_0 = \frac{4}{3}$$

## Boundary conditions

At the surface,  $\mathbf{z} = \mathbf{z}_s$ ,

$$T = T_s \quad (\text{prescribed surface temperature})$$

$$\boldsymbol{\tau} \cdot \mathbf{n} - p \mathbf{n} = 0 \quad (\text{no stress})$$

$$\frac{\partial z_s}{\partial t} + u \frac{\partial z_s}{\partial x} + v \frac{\partial z_s}{\partial y} = w + a \quad (\text{kinematic condition, prescribed accumulation } a)$$

At the bed,  $\mathbf{z} = \mathbf{z}_b$ ,

$$\left\{ \begin{array}{ll} \left[ k \frac{\partial T}{\partial z} \right]_-^+ + \tau_b u_b = 0 & \text{if cold} \quad (\text{basal energy balance}) \\ T_b = T_m & (\text{melting temp}) \\ m = \frac{1}{\rho_w \mathcal{L}} \left( \left[ k \frac{\partial T}{\partial z} \right]_-^+ + \tau_b u_b \right) & \text{if temperate} \quad (\text{melt rate}) \end{array} \right.$$

$$\tau_b = C e^{-(T-T_m)/T_0} u_b \quad (\text{temperature-dependent friction law})$$

$$\mathbf{u} \cdot \mathbf{n} = 0 \quad (\text{no normal flow})$$

Deep in the bedrock,

$$-k_r \frac{\partial T}{\partial z} = q_{\text{geo}} \quad (\text{geothermal heat flux})$$