Flow statistics in a disordered pillar array

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Why are flow statistics important?

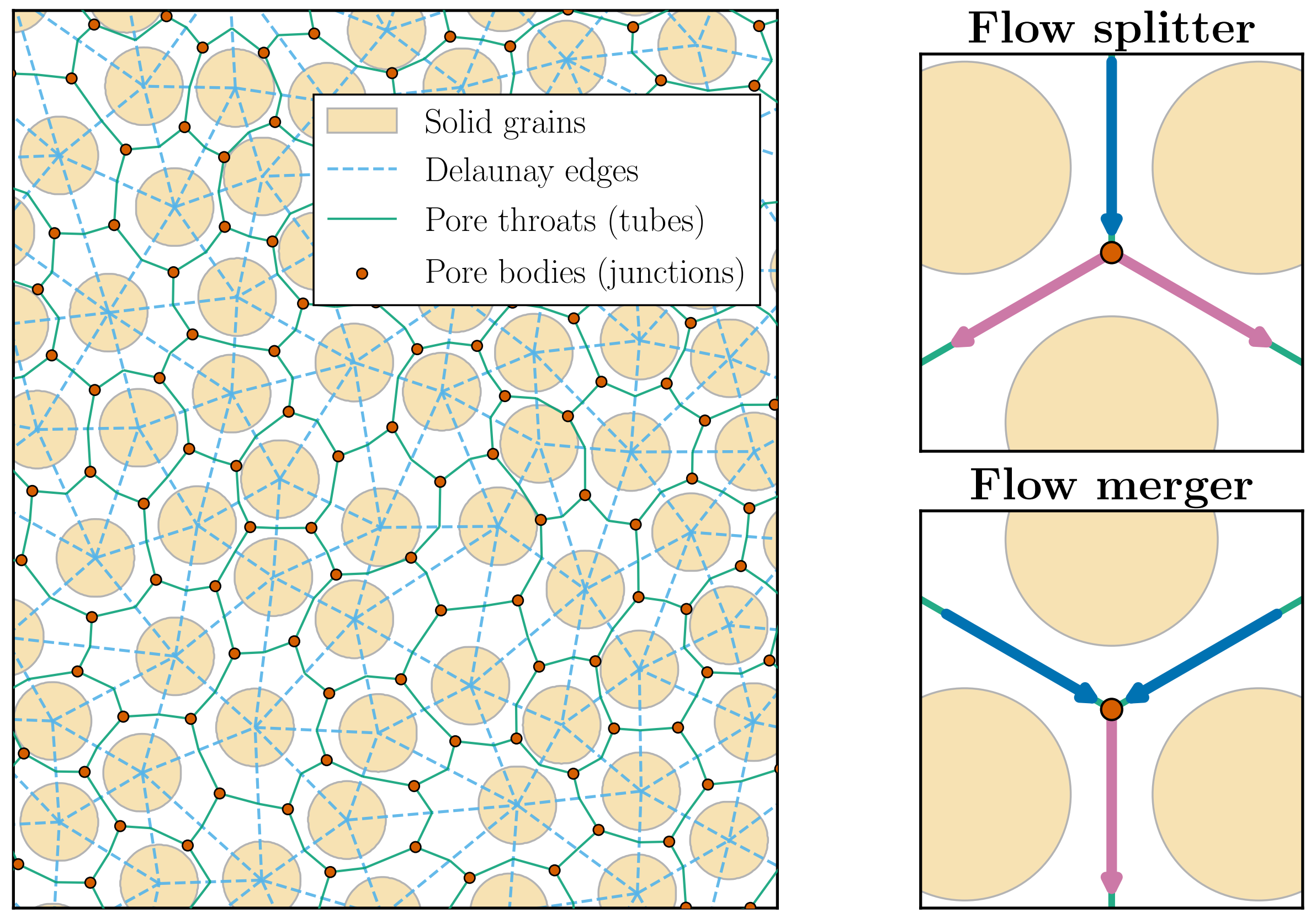
In recent decades, the success of models based on random processes (like **CTRW** models) in characterizing transport and mixing in porous media has revealed the importance of understanding flow statistics.

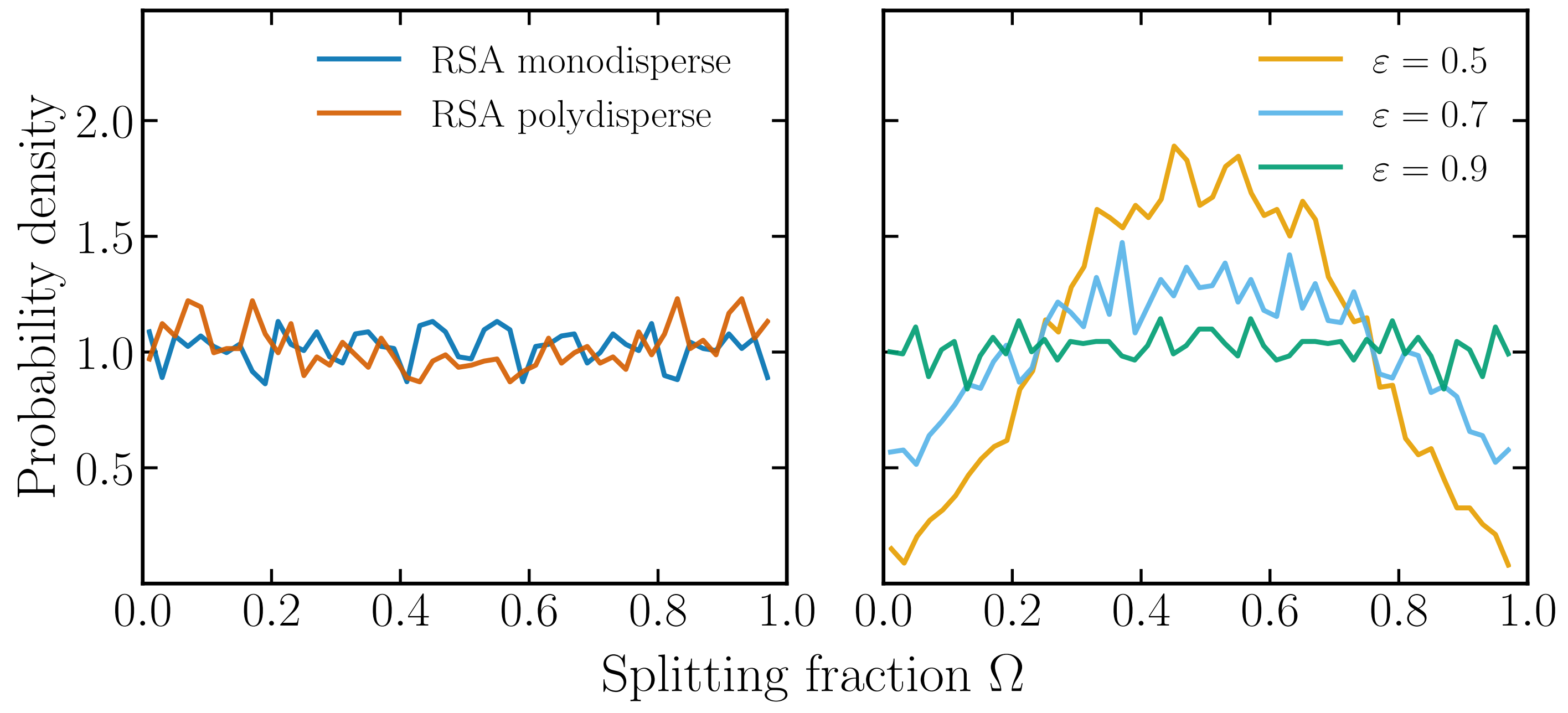
A disordered array of cylindrical pillars (equivalent to a **2D random array of non-overlapping circular obstacles**) is one of the simplest and most archetypal examples of a porous medium. Nevertheless, even in this simple case, no clear theory exists to describe the flow rate distribution.

A pore-network approach

We approximate pore throats as interconnected 2D **tubes** governed by **Poiseuille flow**. Previous literature [1] indicates that a pore network is a valid representation of a porous medium, and that pore-network calculations accurately replicate direct fluid dynamics simulations.

When building a model for flow statistics based on this pore-network picture, prior works [1] assume that all throats are i.i.d. and fail to capture the flow statistics in the presence of **local flow correlations**.





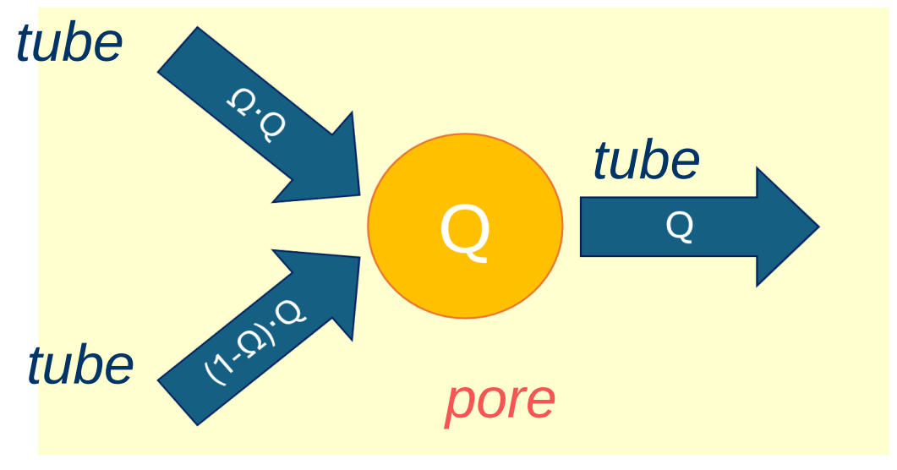
The *Y* model

Our theoretical model, the ***Y* model**, is built upon three fundamental properties of 2D disordered arrays of disks:

- The *Y* shape of every junction: tubes are not independent, they meet at pores (junctions). Every pore connects exactly three tubes (a natural consequence of Voronoi tessellation in disordered media).

- Uniform splitting fractions: for sufficiently disordered systems, the fraction **Ω** of flow directed into each branch at a splitting node behaves as a uniformly distributed random variable between 0 and 1.

- Topological balance: for any sufficiently large system, the total number of flow splitters equals the number of flow mergers.

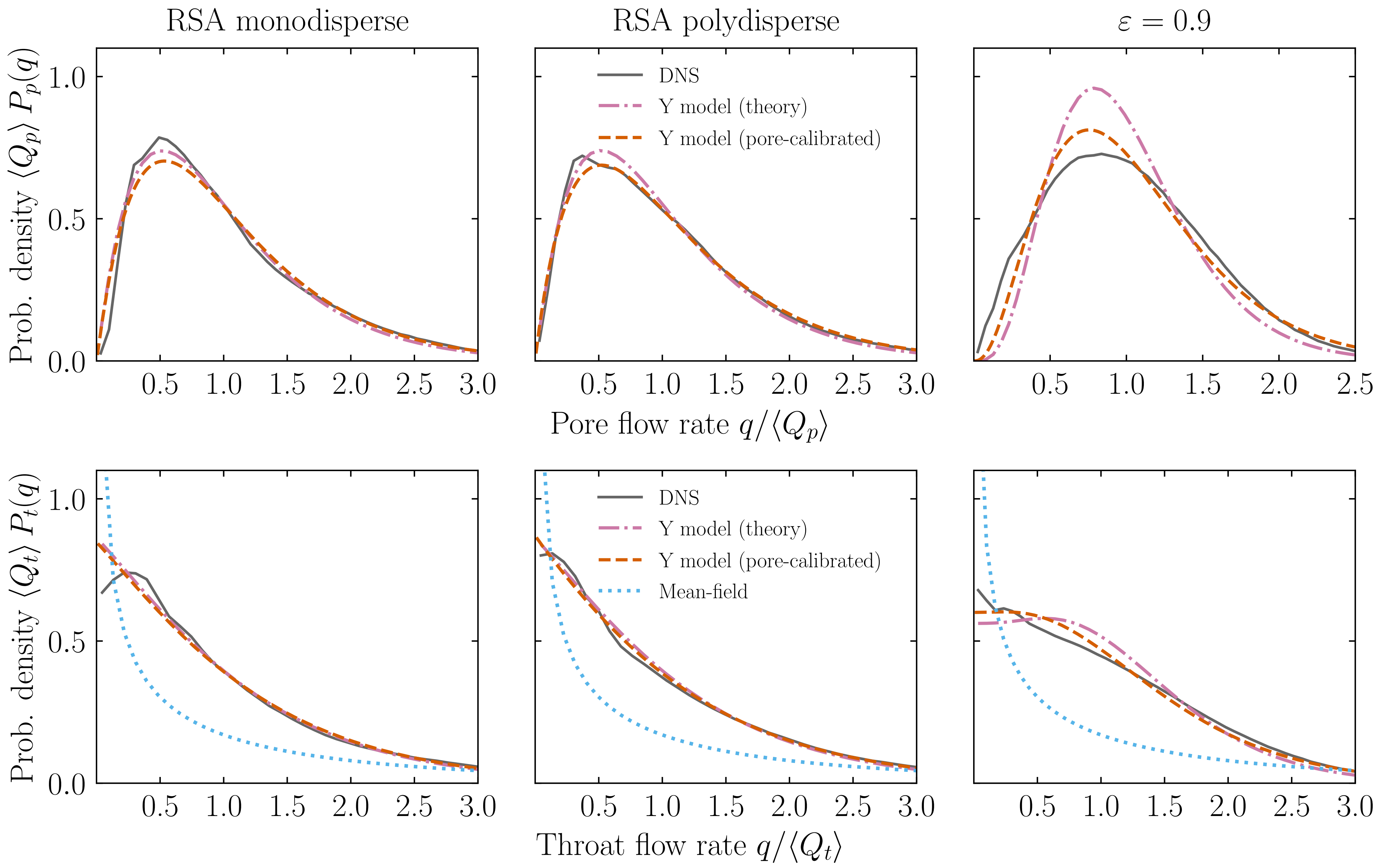


Model predictions

By applying Lukacs's theorem [2], the *Y* model predicts a **Gamma** distribution for **pore flow rates**, which in turn fully determines the **throat flow distribution**.

$$P_p(q) = \frac{q^{k-1}e^{-q/\theta}}{\Gamma(k)\theta^k}$$

$$P_t(q) = \frac{1}{\Gamma(k)\theta}\left[\frac{1}{3}\left(\frac{q}{\theta}\right)^{k-1} e^{-q/\theta} + \frac{2}{3}\Gamma\left(k-1, \frac{q}{\theta}\right)\right]$$

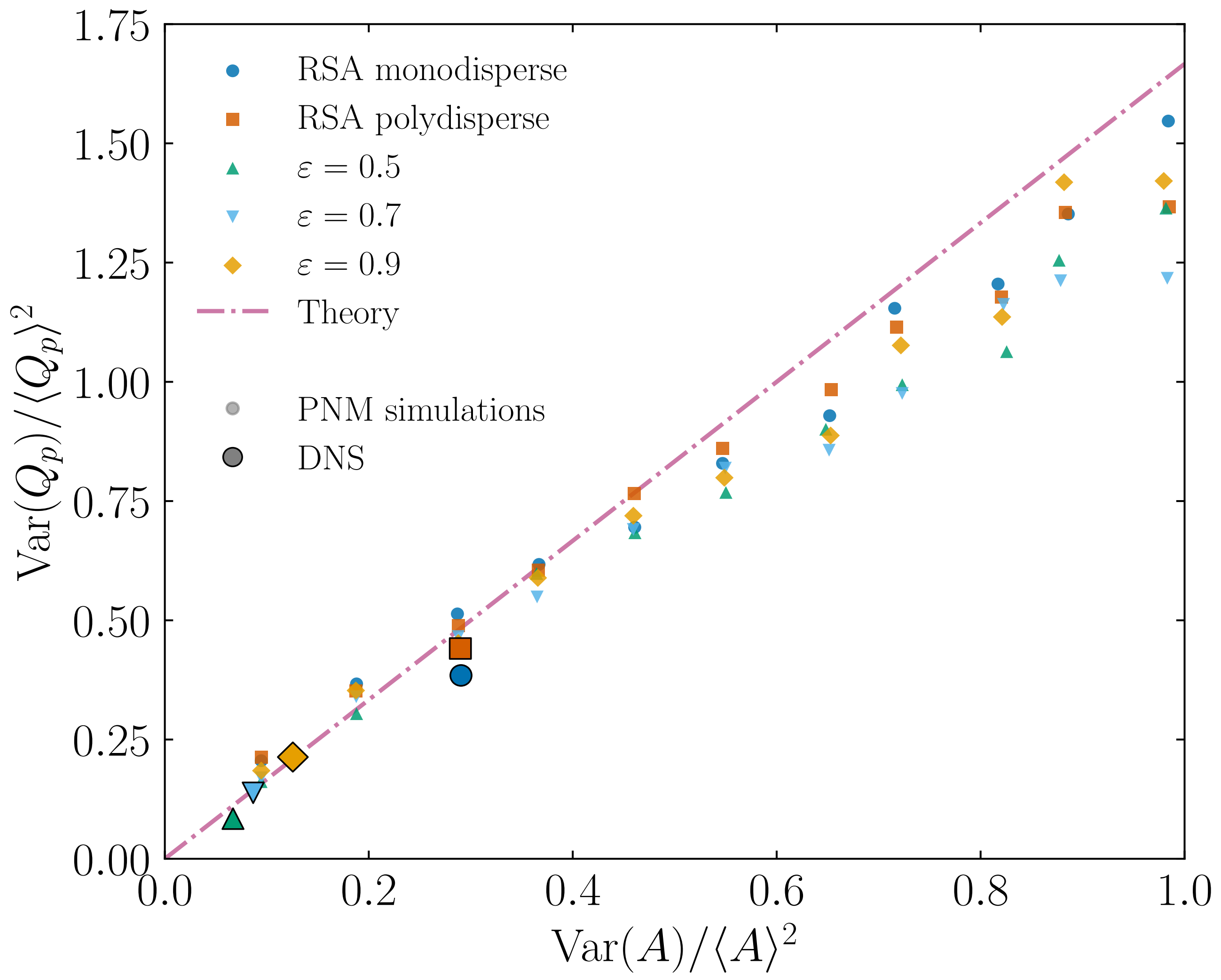


Connecting distribution parameters to geometry

Pore flow rates are strictly dictated by boundary conditions, network topology, and throat half-widths (via tube conductances).

$$\boldsymbol{q}_p = -ZCB^T\left[BCB^T\right]^{-1}\boldsymbol{s}$$

From this, we analytically derive a **linear relationship** between the squared coefficients of variation (**CV²**) of the **throat half-widths** and the pore flow rates.



$$k = \frac{\langle Q_p \rangle^2}{\mathrm{Var}(Q_p)} = \frac{1}{\mathrm{CV}^2(Q_p)}$$

$$\boxed{\mathrm{CV}^2(Q_p) = \frac{5}{3}\,\mathrm{CV}^2(A)}$$

Conclusions and future work

We have developed a simple statistical model (the *Y* model) that successfully **predicts flow statistics** in 2D disordered arrays of disks, directly linking the flow distribution parameters to **simple geometric properties** of the medium.

Since the current formulation is restricted to 2D geometries, future work will focus on extending these principles to **3D** media and more complex structures, including **unsaturated** media and fractured media.

References

1. Alim *et al.* Local pore size correlations determine flow distributions in porous media. *Phys. Rev. Lett.*, 2017
2. Lukacs, E. (1955). A characterization of the gamma distribution. The Annals of Mathematical Statistics, 26(2):319–324.

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