

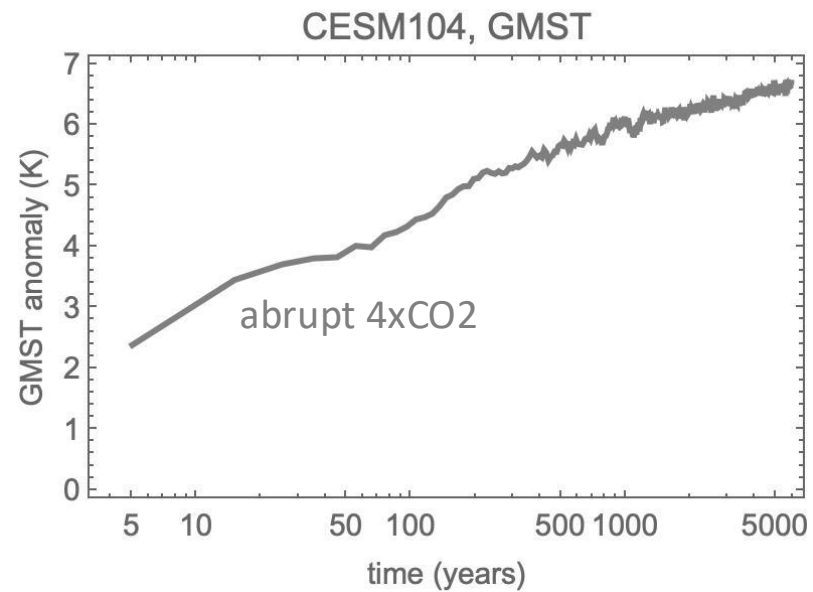
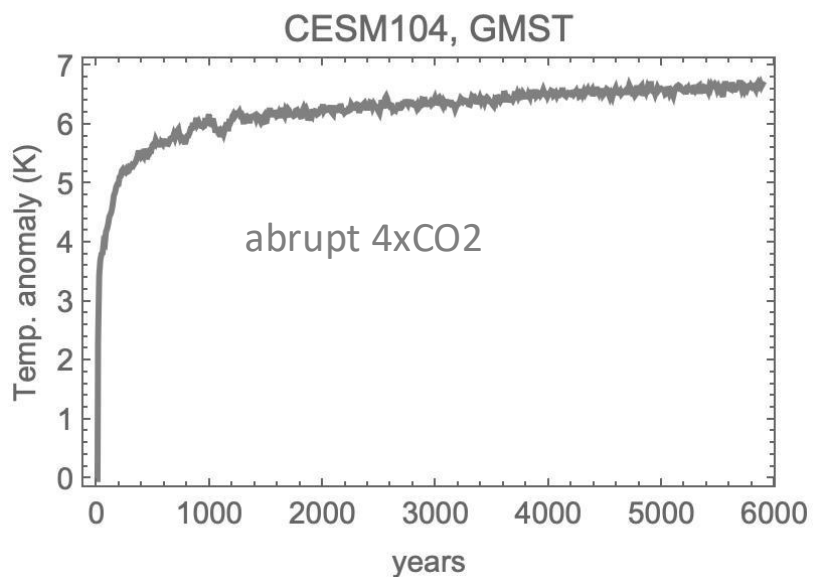
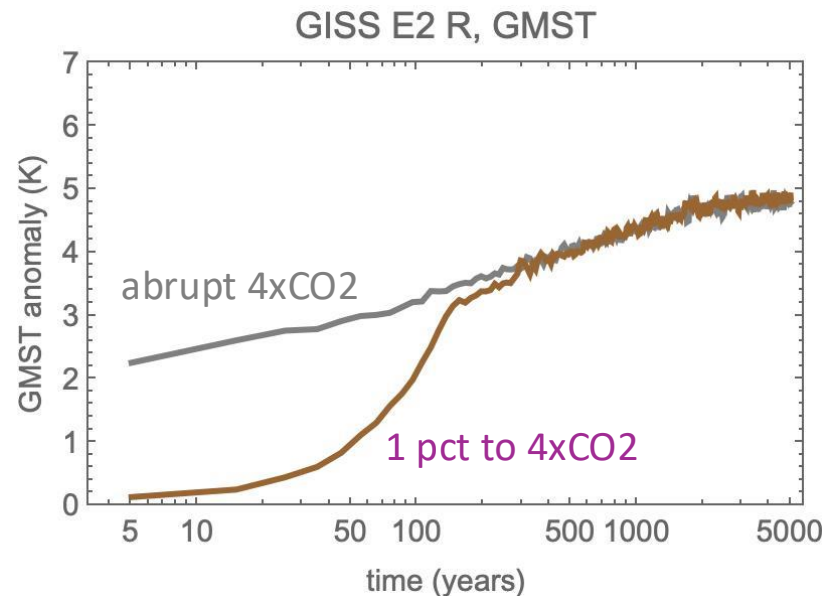
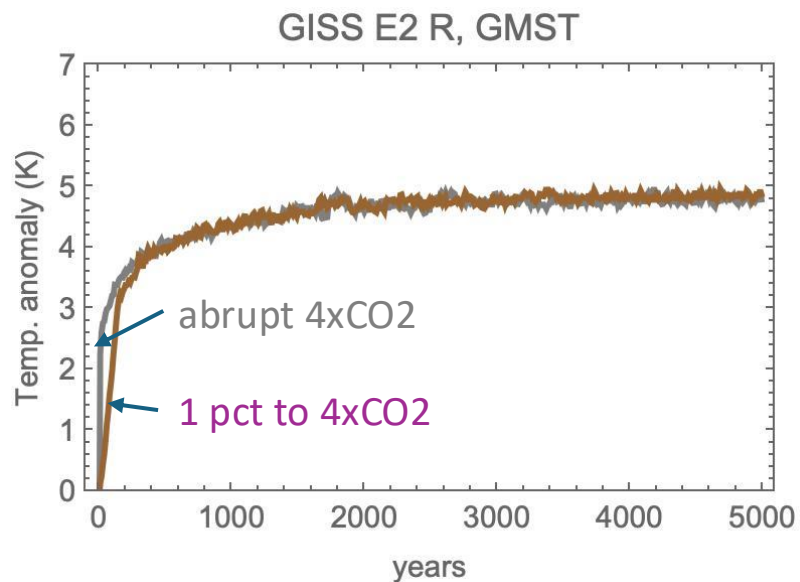
# Parsimonious models emulating millennium-long AOGCM simulations

Some promising results from analysis of millennium-long runs from the LongRunMip ensemble.

## Summary

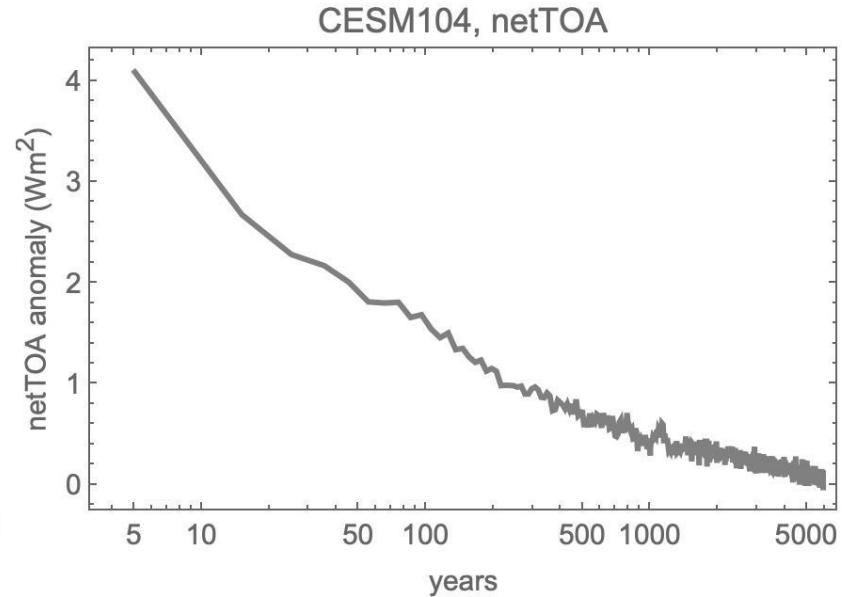
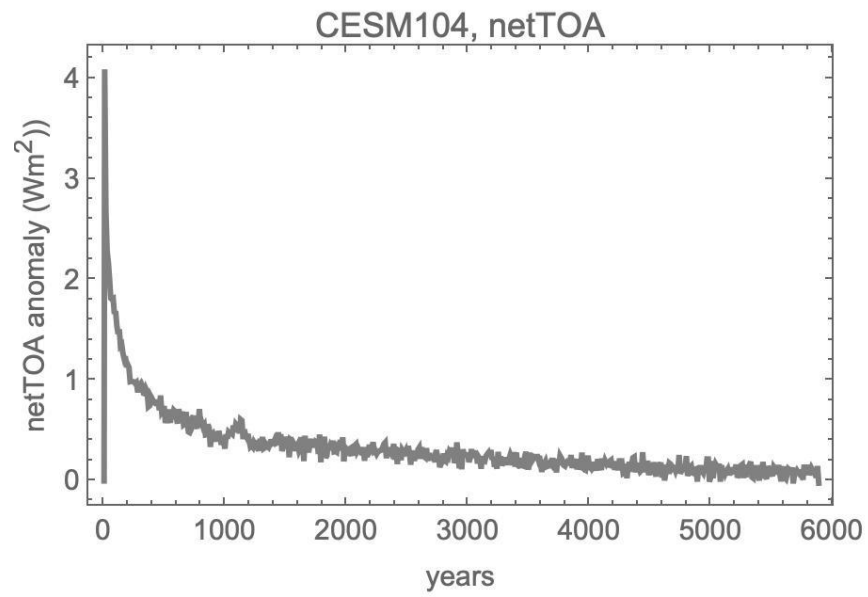
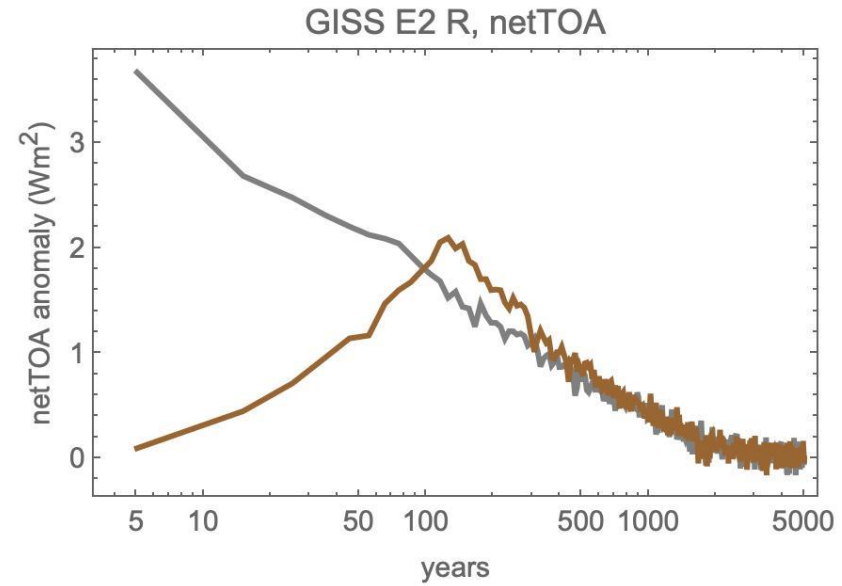
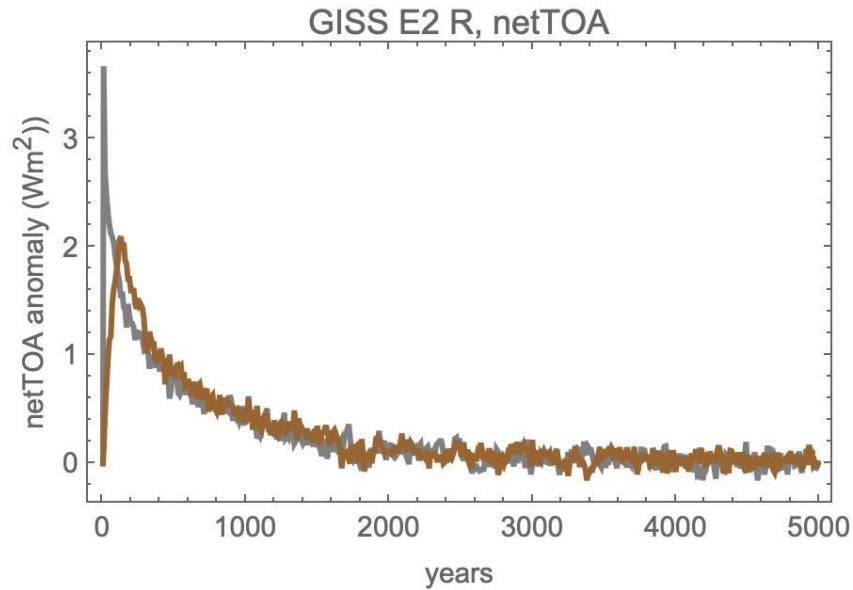
- An impulse response model in the form of a superposition of three exponential responses with model parameters (3 amplitudes and 3 decay times) determined by a millennium-long abrupt 4xCO<sub>2</sub> run, may accurately model the global mean surface temperature (GMST) for arbitrary forcing scenarios.
- Gregory plots, using AOGCM data for TOA radiation fluxes and allowing for temperature-dependent feedback parameters, may be used to produce emulators for the energy balance valid for arbitrary forcing scenarios.
- Applicability needs to be verified by access to more millennium-long simulations of AOGCMs under a variety of forcing scenarios.

# Global mean surface air temperature (GMST)



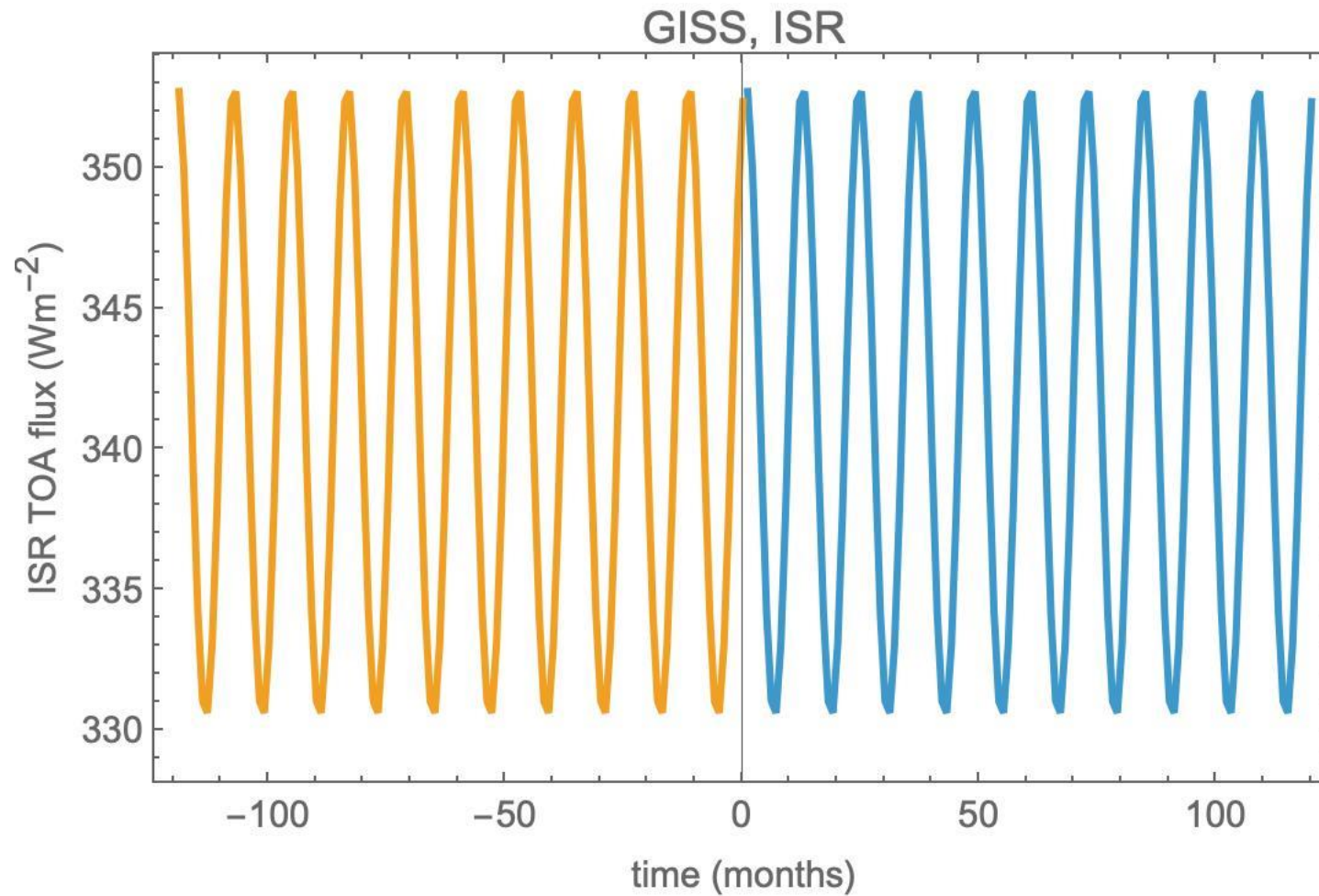
No 1 pct to 4xCO2 run  
for CESM104 exists in  
LongRunMip

# Net top-of-atmosphere (netTOA) incoming radiation

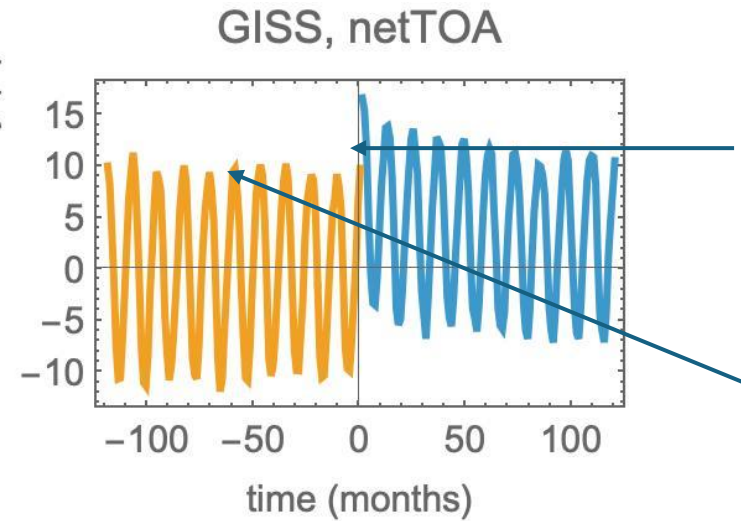
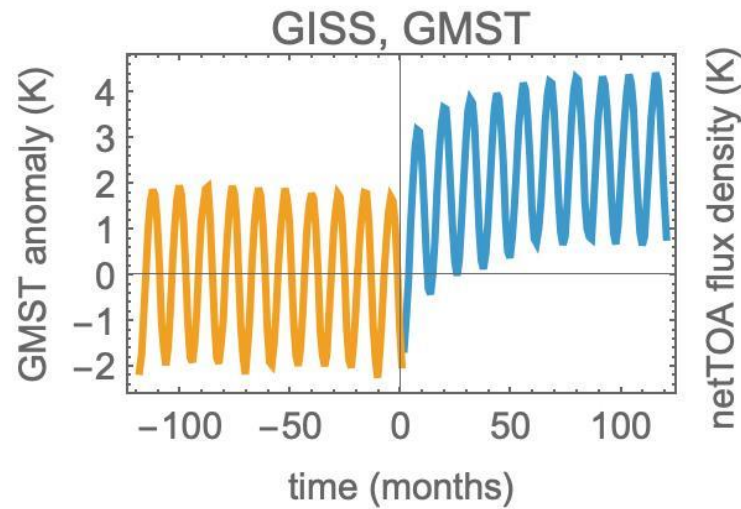


# Incoming shortwave radiation (ISR)

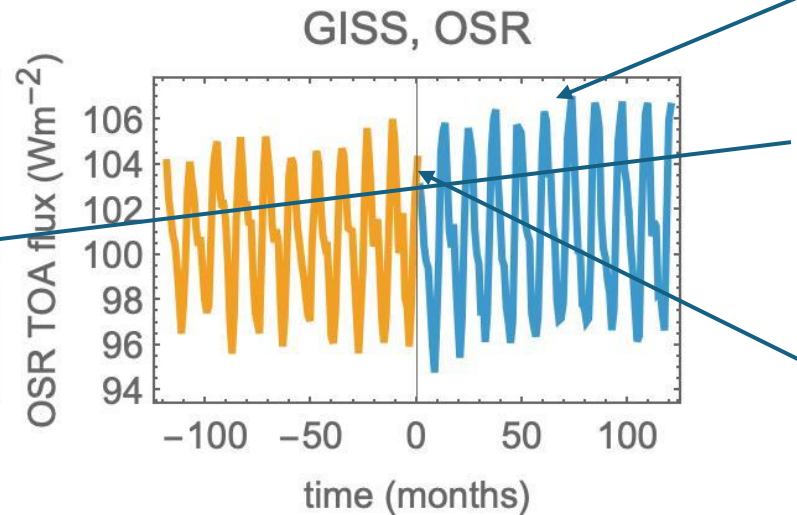
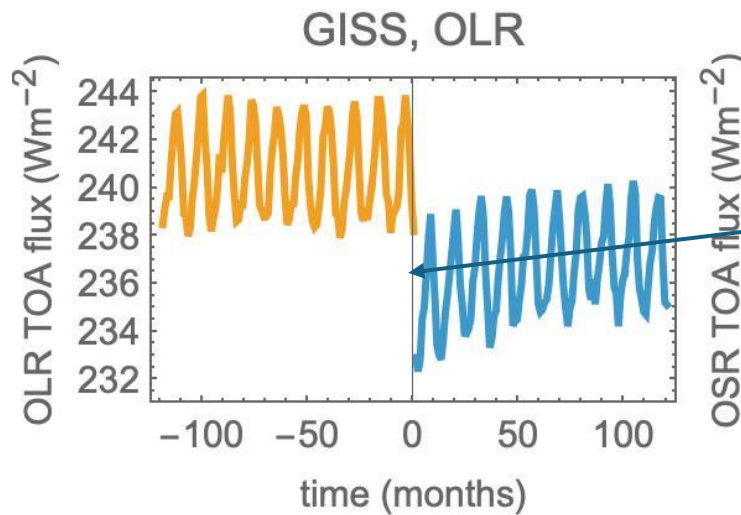
Seasonal oscillation due to eccentricity of Earth orbit



# Jump in GMST, net TOA, outgoing longwave radiation (OLR), and outgoing shortwave radiation (OSR) due to abrupt 4xCO<sub>2</sub> at t=0



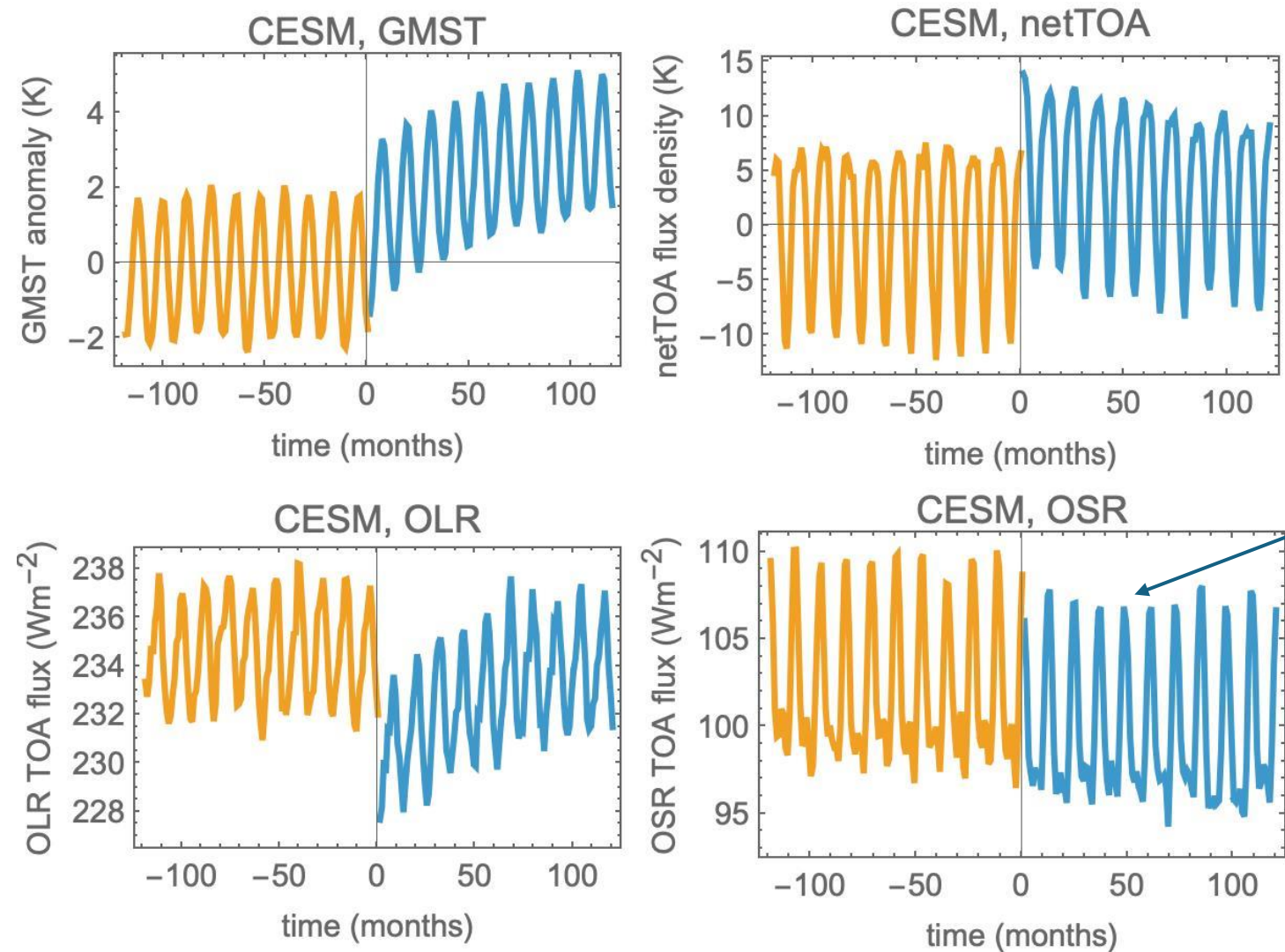
In principle, the radiative forcing is  $F_{4xCO_2} = \Delta netTOA$ , but there is big uncertainty due to natural variability from December to January as shown in the control run.



$\Delta OSR$  increases on decadal time scale

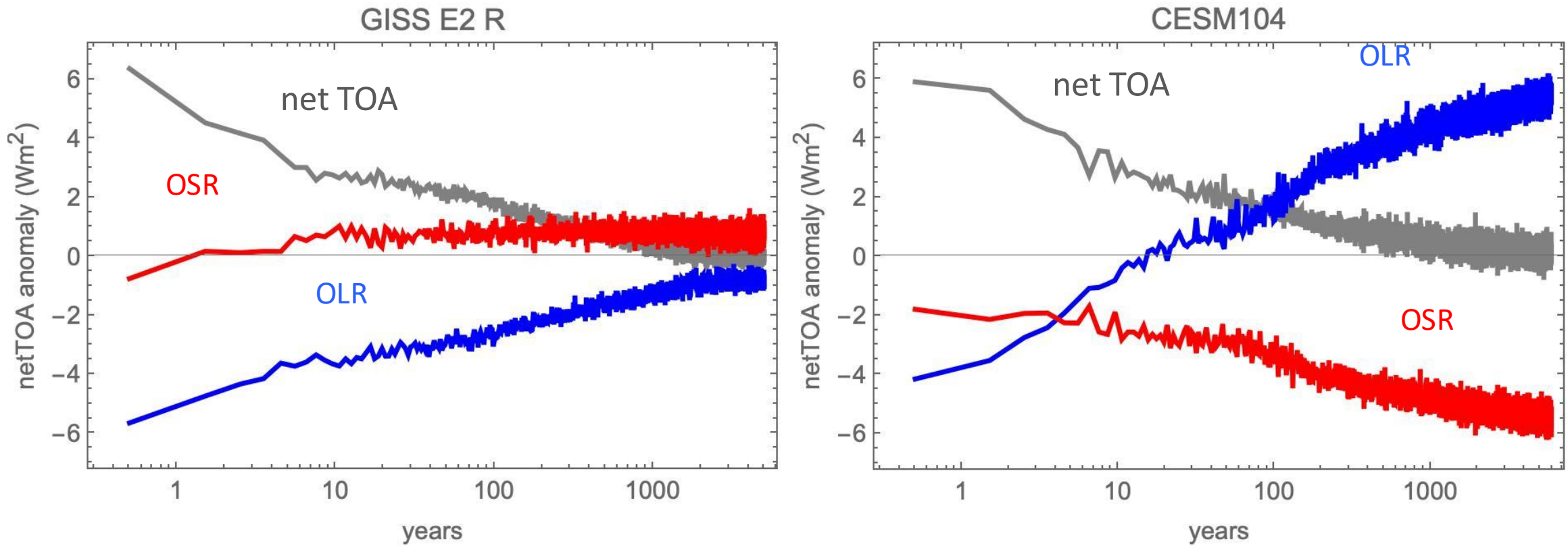
The main contribution to the forcing comes directly from CO<sub>2</sub> as a jump in longwave radiation;  $-\Delta OLR$ , but a smaller contribution comes from a jump in the albedo;  $\Delta OSR$

Similar plots for CESM104. Note that OSR declines over the decade after abrupt  $\text{CO}_2$ , in contrast to GISS, where it increases.



$\Delta\text{OSR}$  decreases on decadal time scale

Annual averages of OSR (albedo) evolve very different in the two models. In CESM, the decreasing albedo must be compensated by increasing OLR, which requires a higher feedback temperature increase and consequently a higher climate sensitivity.



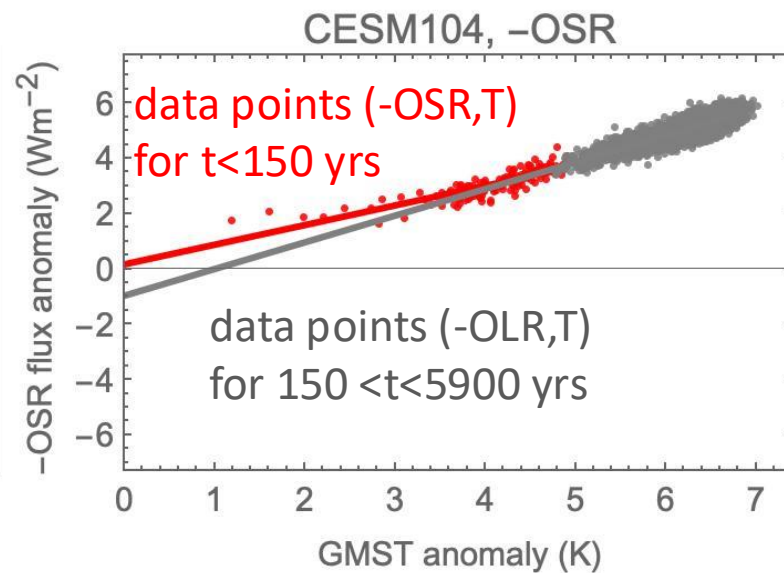
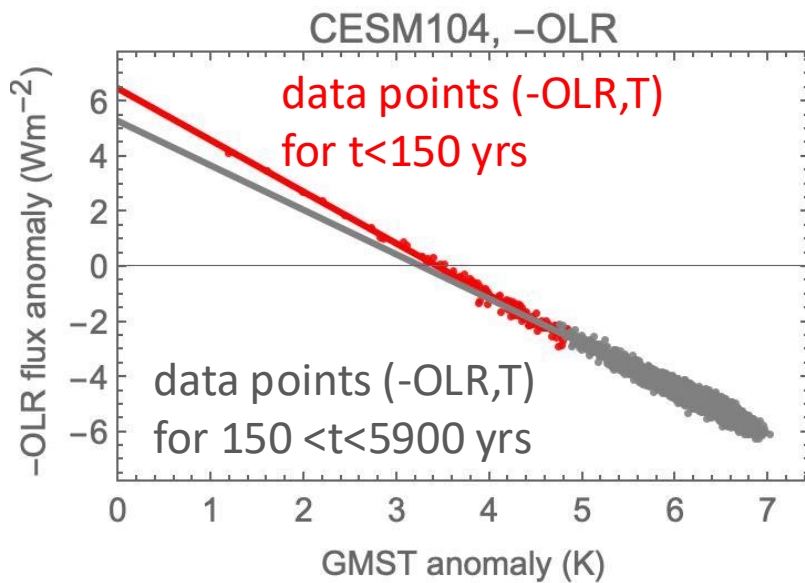
$$\text{net TOA} = -(\text{OSR} + \text{OLR})$$

Gregory plots: netTOA (and its components, -OLR and -OSR) plotted versus temperature anomaly T.

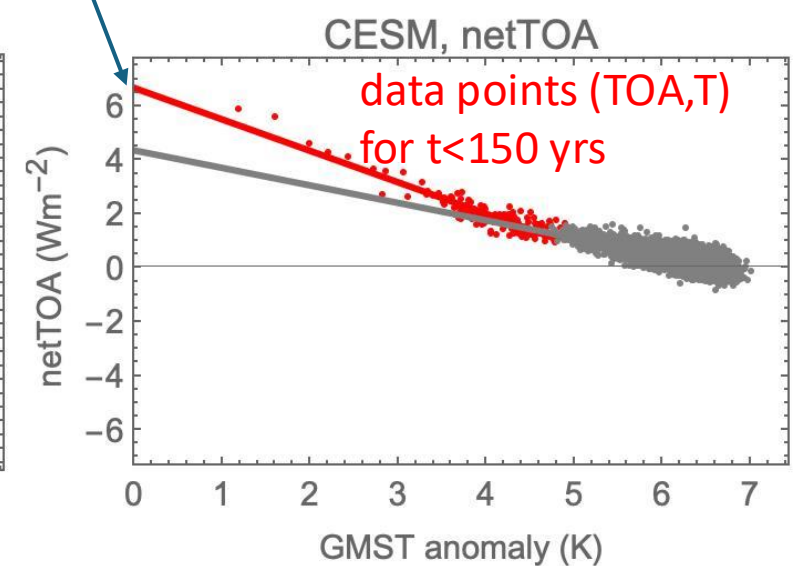
$$\text{OLR} = \lambda_{\text{OLR}} T$$

$$\text{OSR} = \lambda_{\text{OSR}} T$$

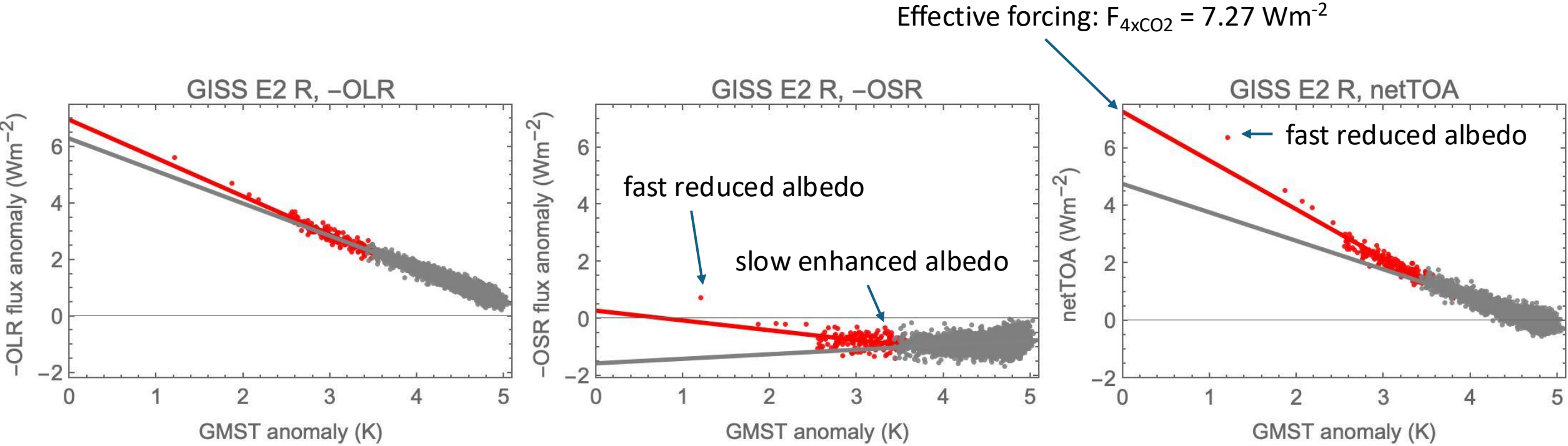
$$\text{netTOA} = -(\lambda_{\text{OLR}} + \lambda_{\text{OSR}}) T$$



Effective forcing:  $F_{4\times\text{CO}_2} = 6.69 \text{ Wm}^{-2}$



Gregory plots for GISS. The main difference from CESM is the decreasing –OSR (increasing albedo) during the first centuries. This is the main explanation of the lower climate sensitivity.



# The «physical» three-box model

Time-independent feedback parameter and other coefficients

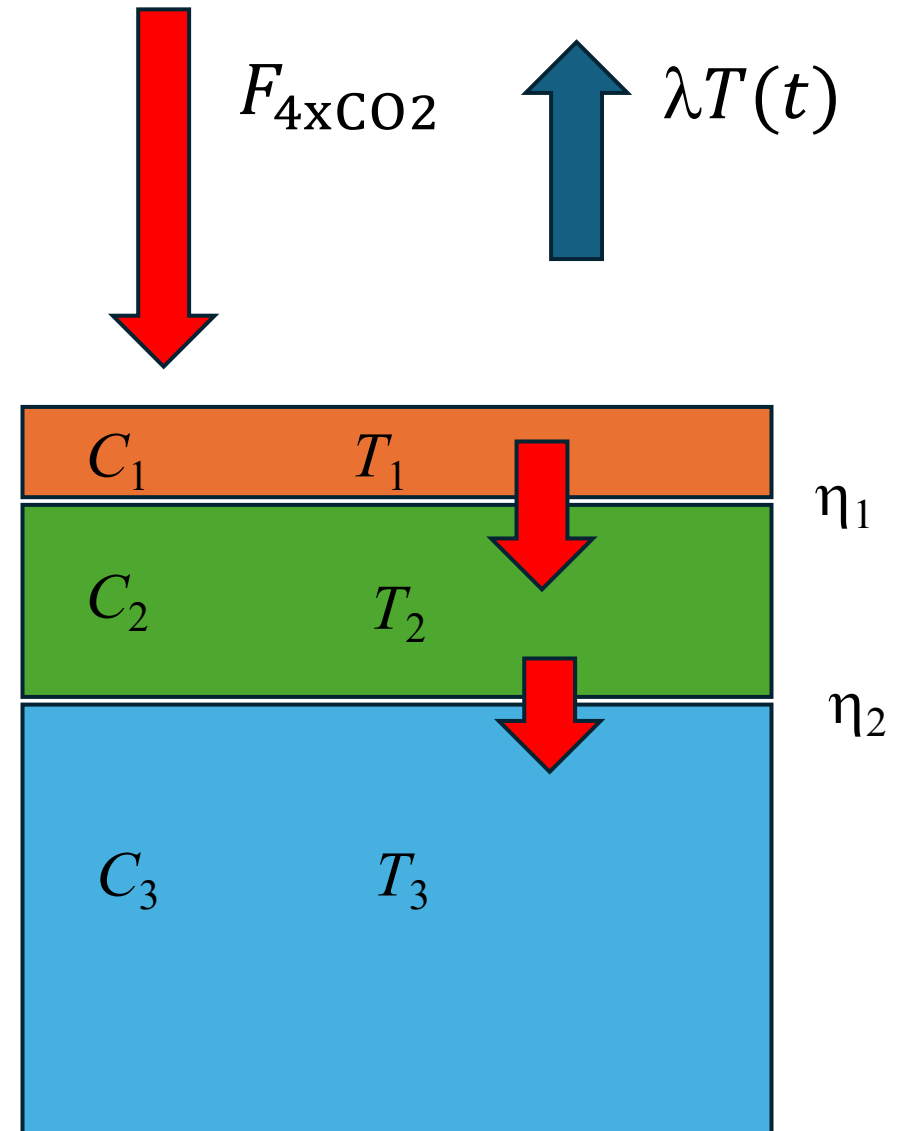
$$C_1 \frac{d}{dt} T(t) = F_{4xCO_2} - \lambda T(t) - \eta_1 [T(t) - T_2(t)]$$

$$C_2 \frac{d}{dt} T_2(t) = \eta_1 [T(t) - T_2(t)] - \eta_2 [T_2(t) - T_3(t)]$$

$$C_3 \frac{d}{dt} T_3(t) = \eta_2 [T_2(t) - T_3(t)]$$

At equilibrium when  $t \rightarrow \infty$  ;

$$F_{4xCO_2} - \lambda T(\infty) = 0$$



# The «statistical» three-box model

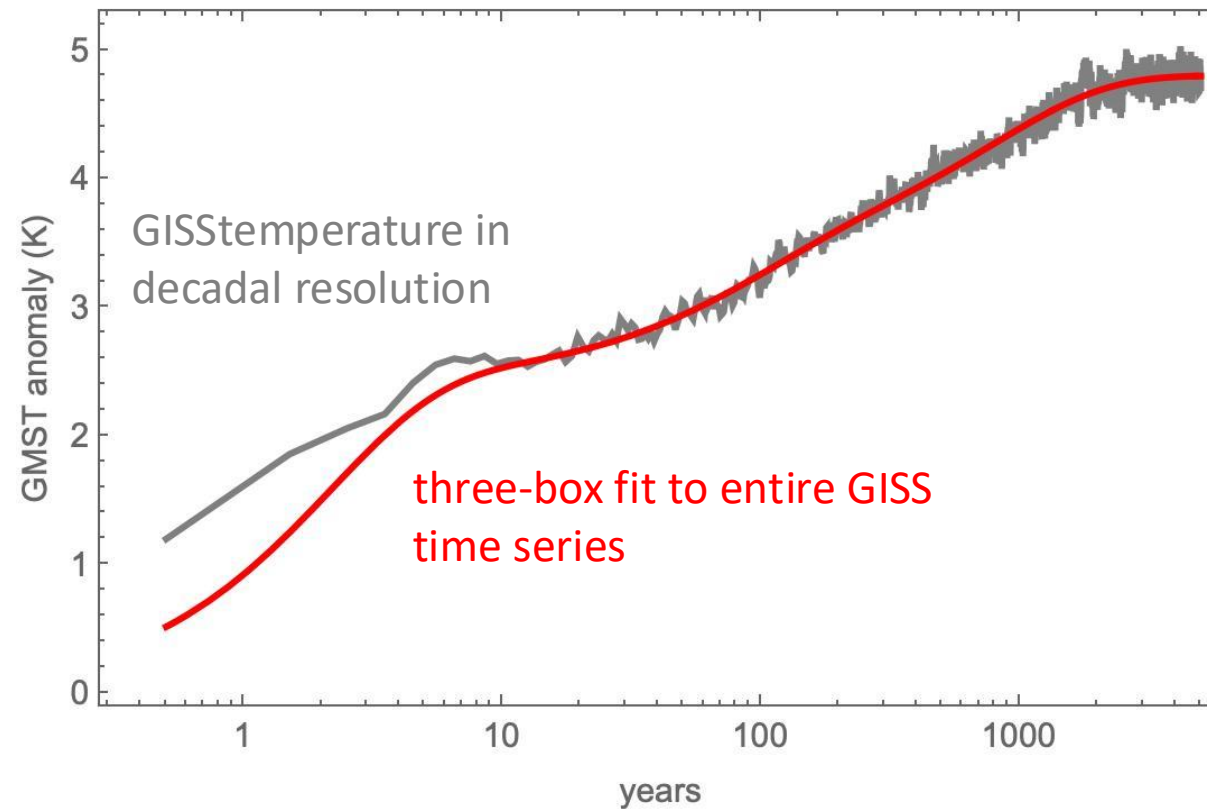
Rypdal and Fredriksen (2017)\* showed that the physical three-box model With a fixed feedback parameter  $\lambda$  has solutions on the form:

$$T(t) = T_1(1 - e^{-t/\tau_1}) + T_2(1 - e^{-t/\tau_2}) + T_3(1 - e^{-t/\tau_3})$$

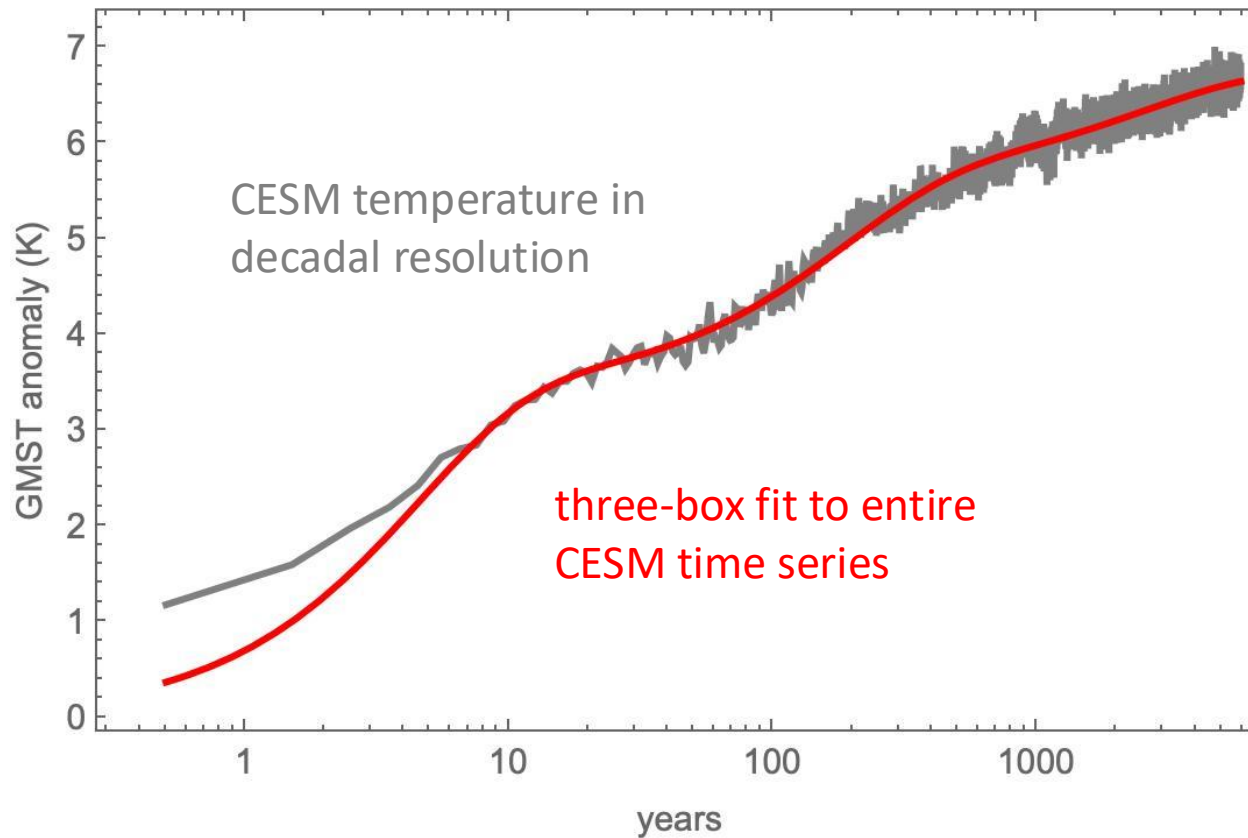
I shall hypothesize that this model can work as a statistical model for the GMST even if the feedback factor  $\lambda(t)$  is time-dependent.

\* Fredriksen, H. B., and Rypdal, M. (2017), Long-Range Persistence in Global Surface Temperatures Explained by Linear Multibox Energy Balance Models, J. Climate (30)18, 7157-7168, <http://doi.org/10.1175/JCLI-D-16-0877.1>

# Least-square fit of statistical three-box model to the entire 5000 year GISS E2 R GMSTtime series



# Least-square fit of statistical three-box model to the entire 5900 year CESM104 GMSTtime series



## The main hypotheses

The «statistical three-box Parsimonious Model» (the PM), with the model parameters estimated from the abrupt 4xCO<sub>2</sub> run, will provide a good emulator model for arbitrary forcing  $F(t)$ .

The temperature responds linearly to the forcing  $F(t)$ , i.e., there exists an impulse response function  $G(t)$  such that,

$$T(t) = \int_0^t G(t - t')F(t')dt$$

The impulse response function is related to the step response function  $R(t)$  via,

$$G(t) = \frac{d}{dt}R(t) = \frac{d}{dt} \sum_{i=1}^3 T_i (1 - e^{-t/\tau_i}) = \sum_{i=1}^3 (T_i/\tau_i) e^{-t/\tau_i}$$

How to use three-box fit for  $T(t)$  to abrupt  $4xCO_2$  to model  $T(t)$  for the 1%  $\rightarrow$   $4xCO_2$  forcing  $F(t)$ .

Plot

$$N(T) = F_{4x} - \int_0^T \lambda(T') dT' = \text{netTOA} + F_{4x} - F(T)$$

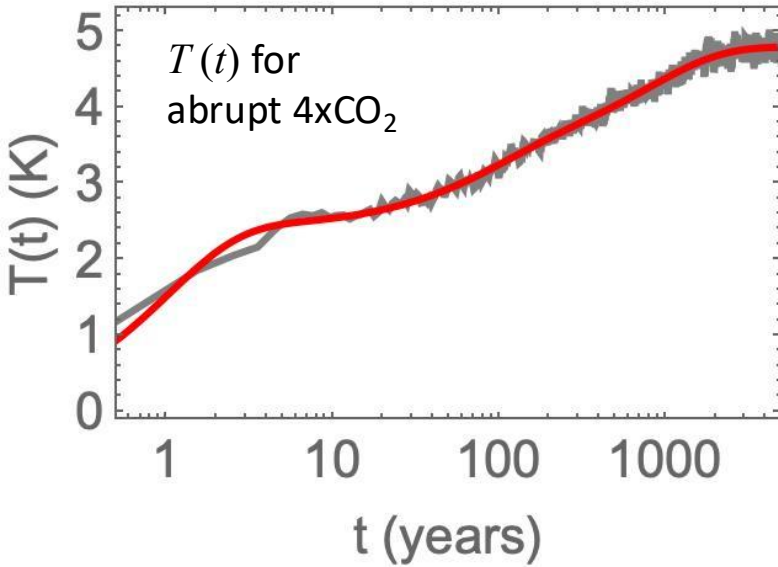
where  $\lambda(T) = -dN/dT$  is a temperature-dependent feedback parameter.

Then use the polynomial fit to the Gregory plot to find a simple expression for  $N(T)$ , and the three-box fit for  $T(t)$  to model the net top-of-the-atmosphere flux density:

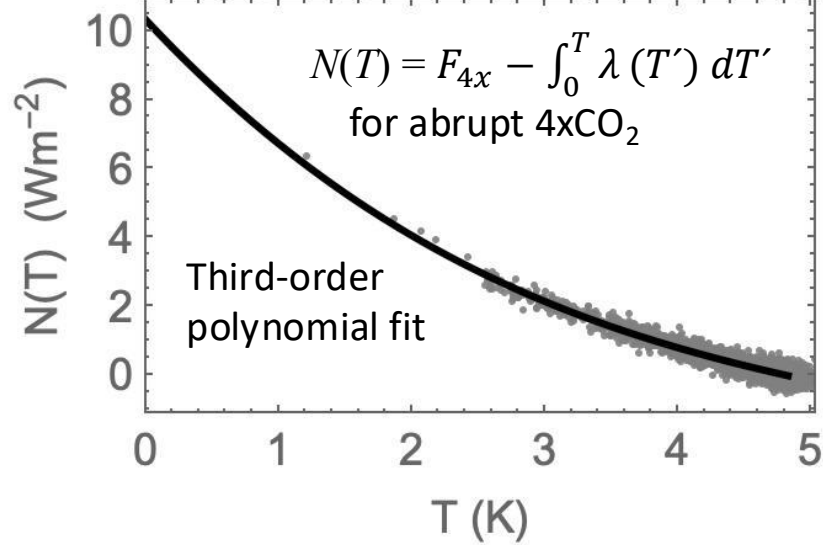
$$\text{netTOA}(t) = N(T(t)) + F(t) - F_{4x}$$

# GISS E2 R: $T(t)$ and netTOA flux for abrupt $4xCO_2$ and 1pct $\rightarrow 4xCO_2$

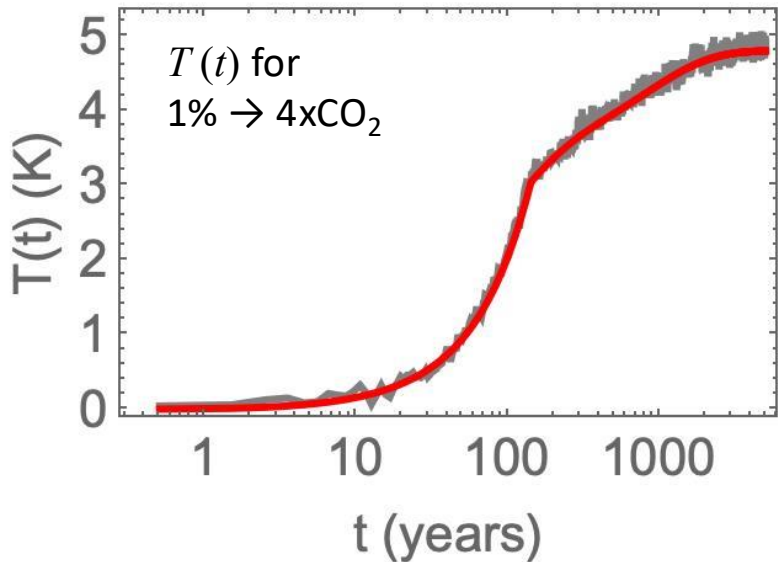
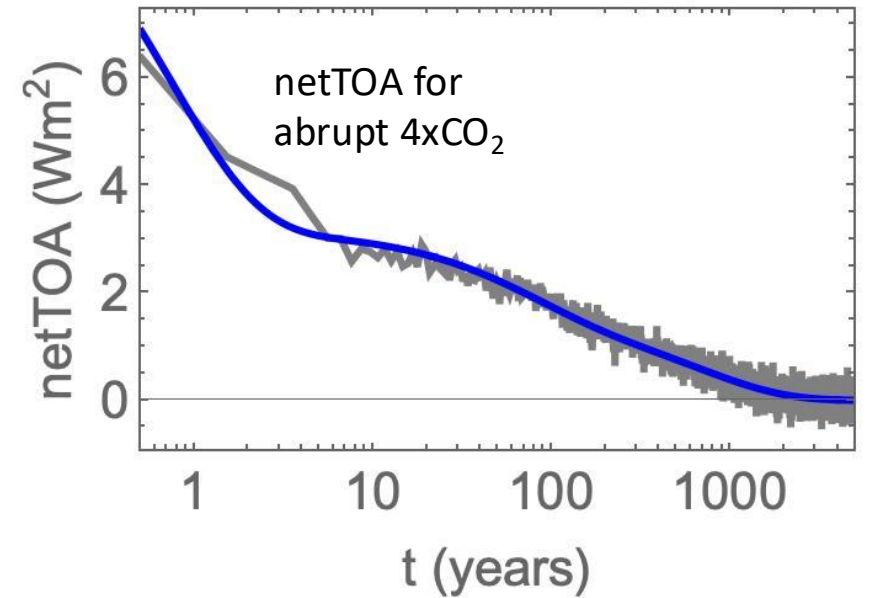
### Three-box, 5000 yr fit



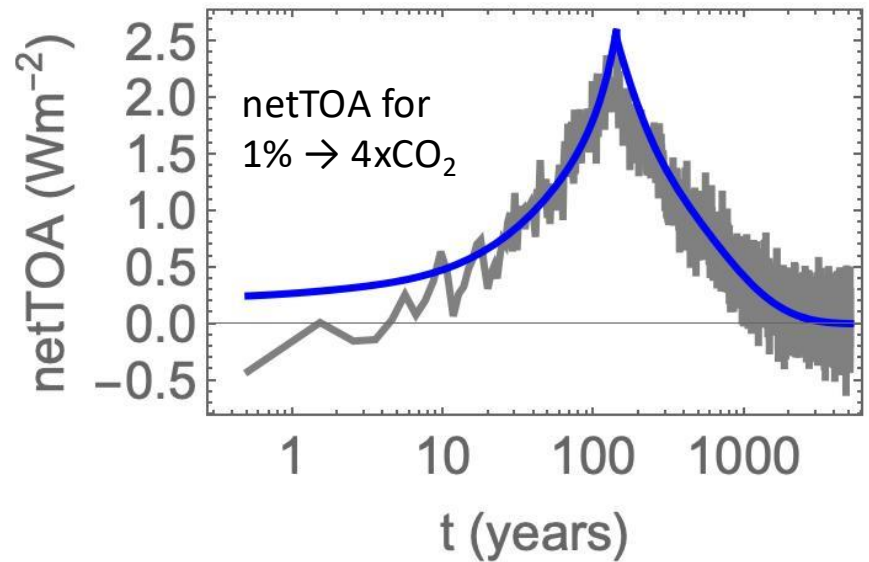
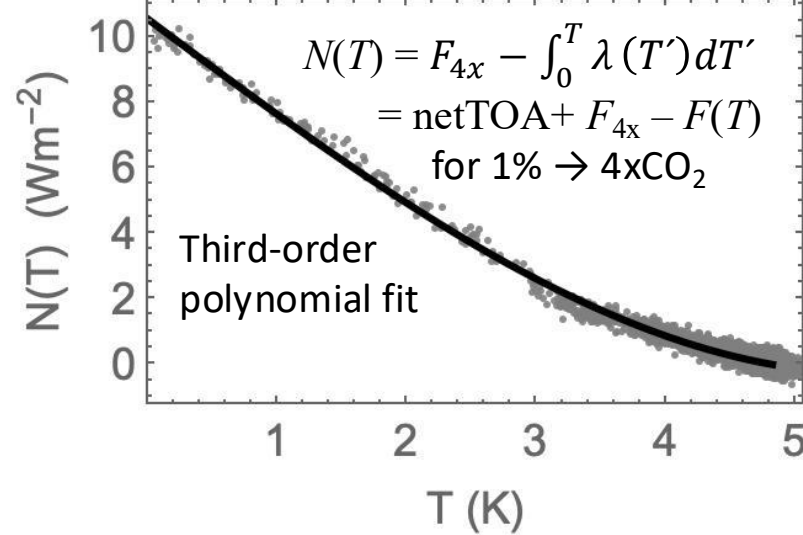
### Gregory plot, abrupt $4xCO_2$



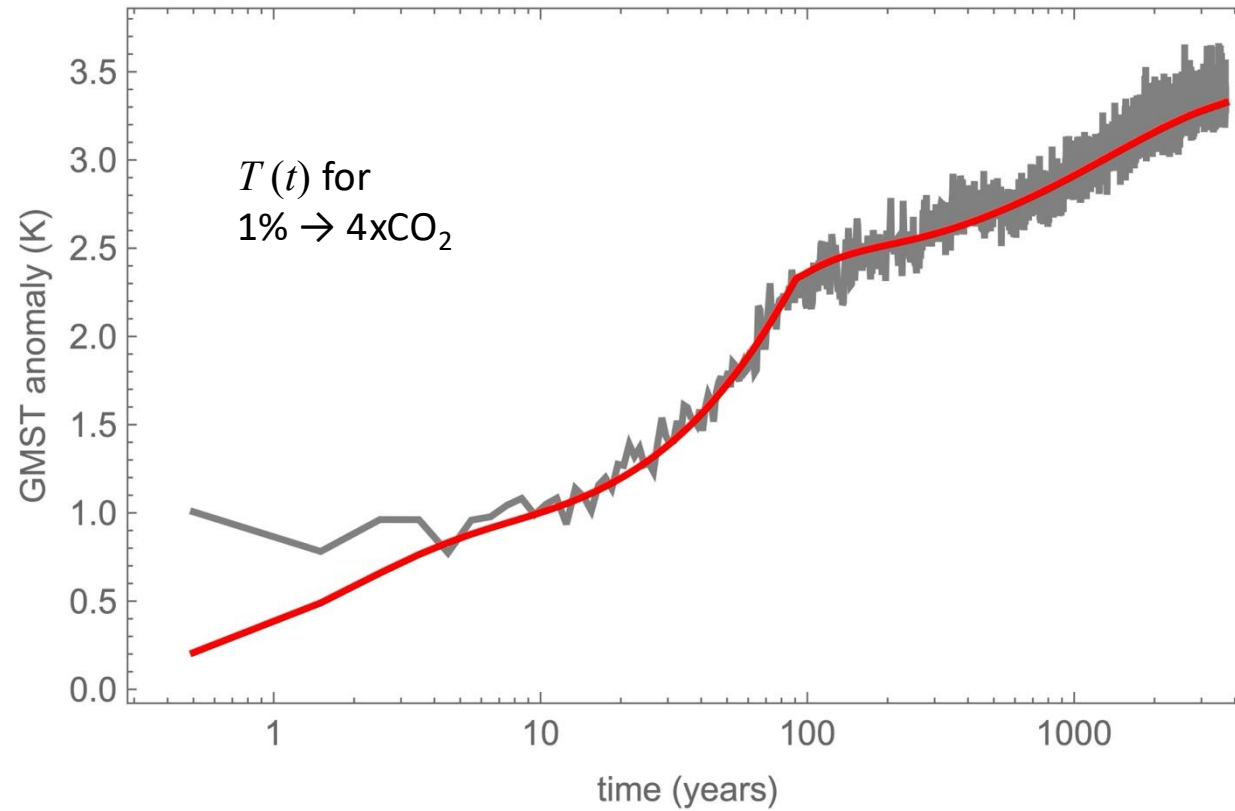
### Three-box, 5000 yr fit



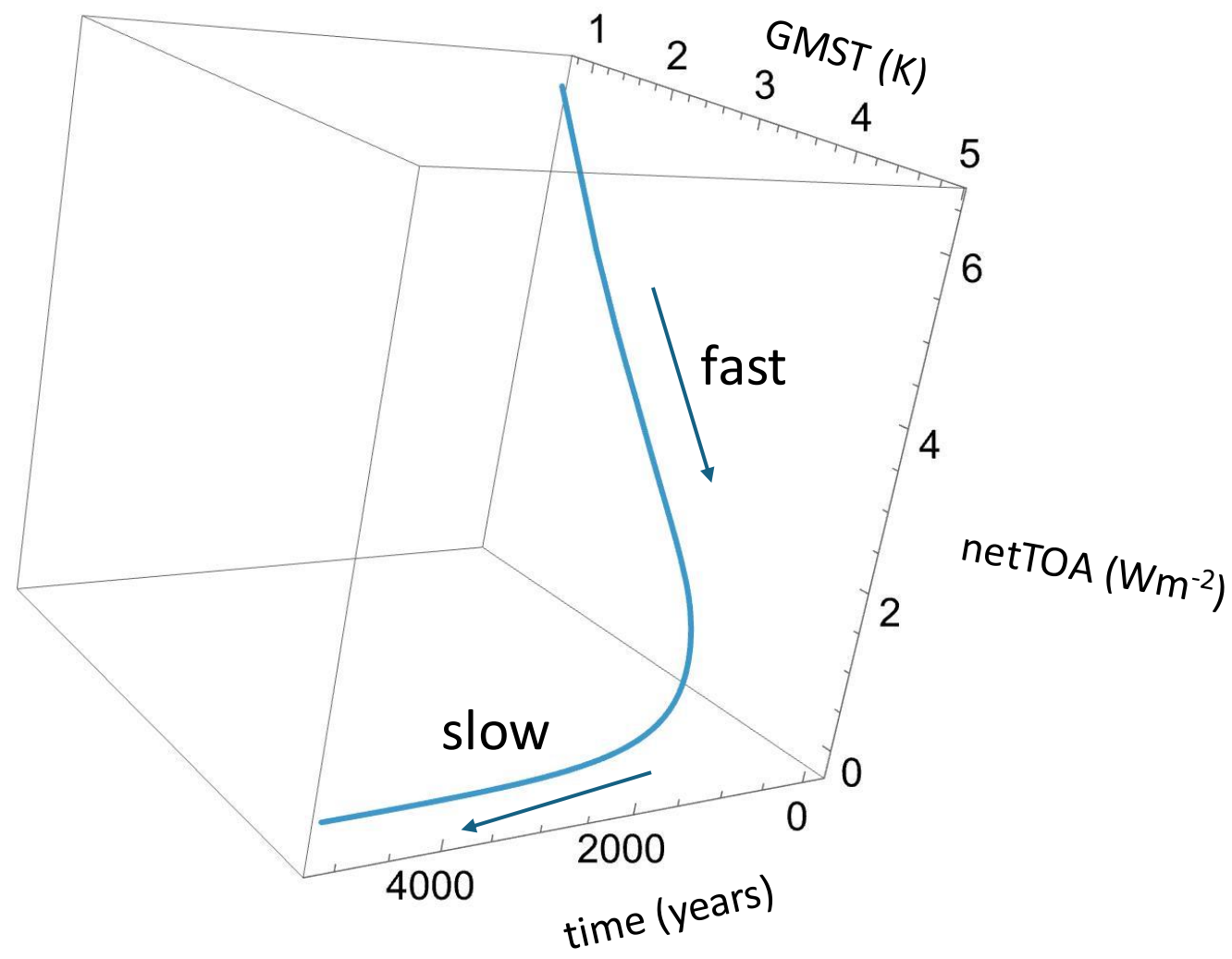
### Gregory plot, 1% $\rightarrow 4xCO_2$



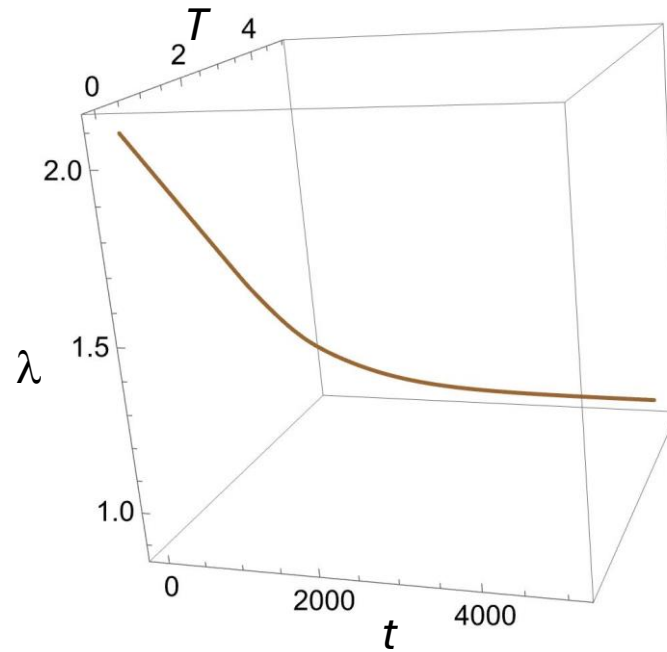
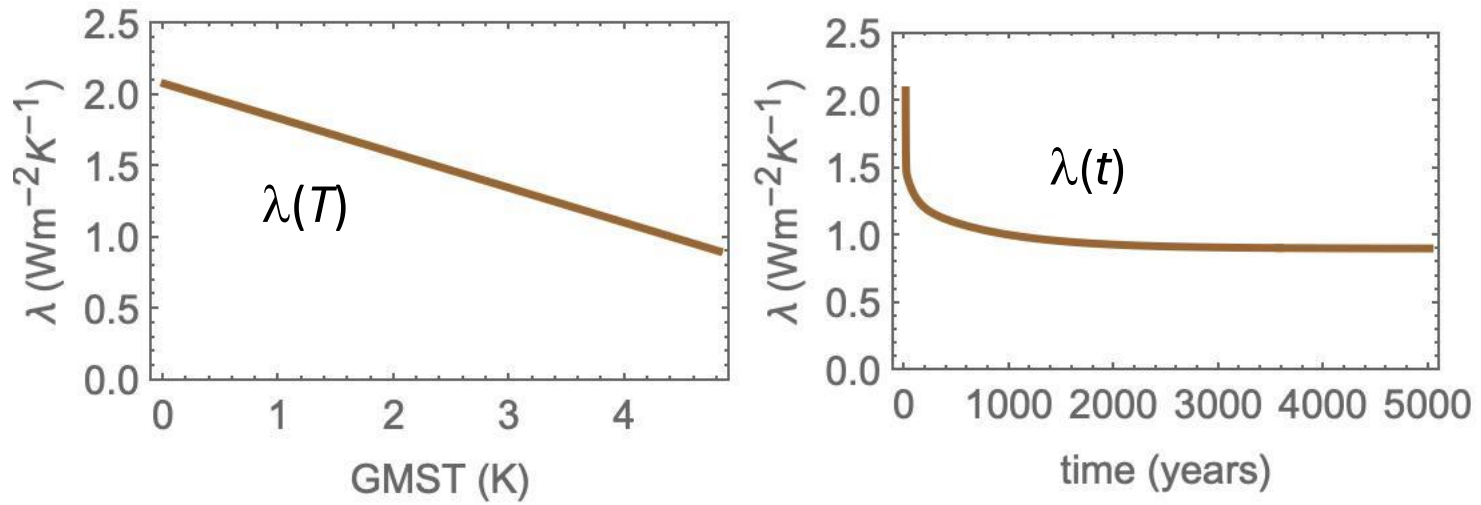
CESM3II: GMST and netTOA flux for linear CO<sub>2</sub> growth from 390 to 700 ppm during 90 years (2010-2100). Three-box model with parameters from abrupt 250 to 700 ppm has been used for fit. The discrepancy for 0 – 10 years is because the period from 250 to 390 ppm in the three-box has been modeled as an abrupt change.



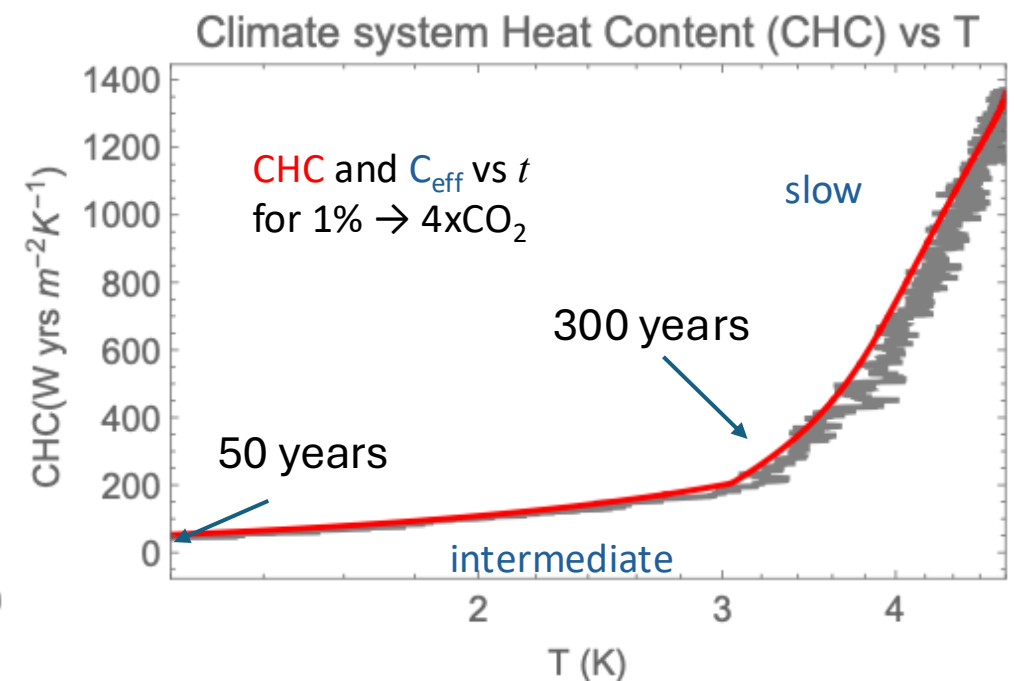
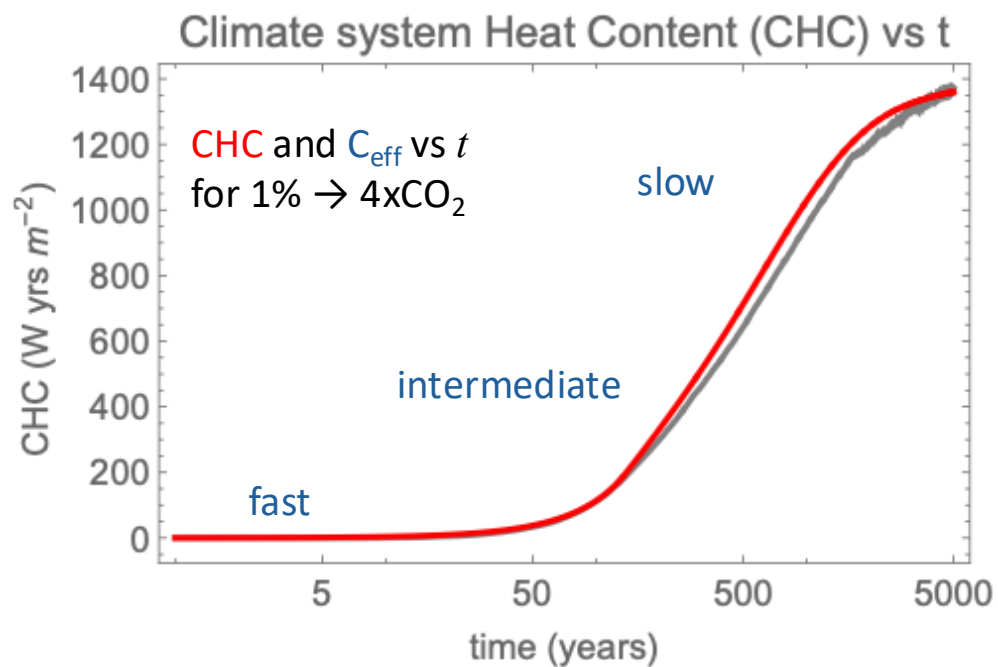
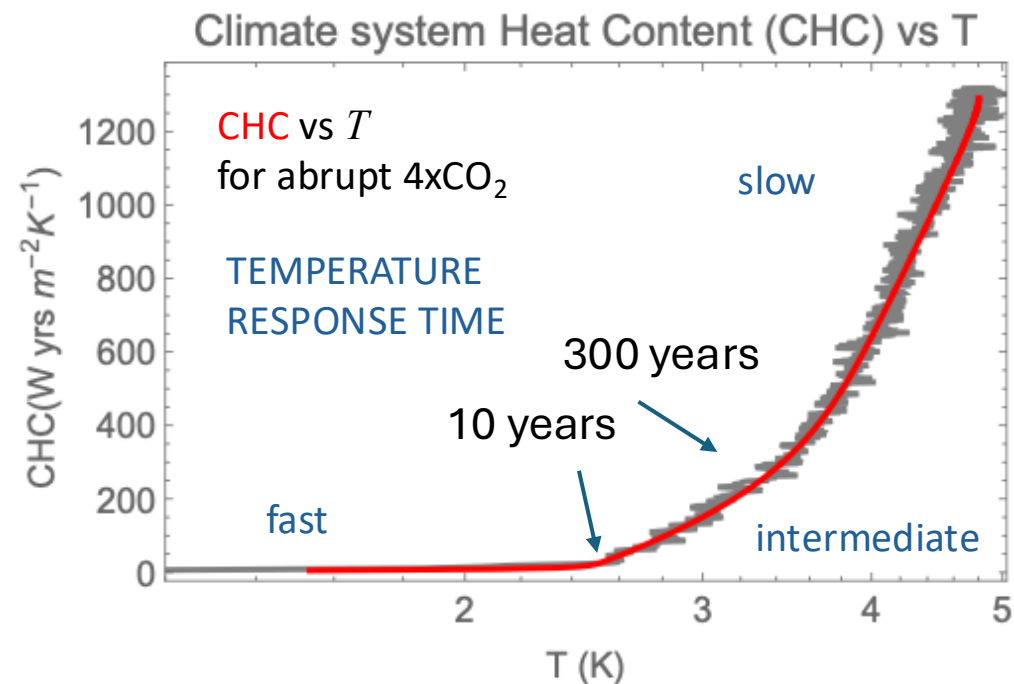
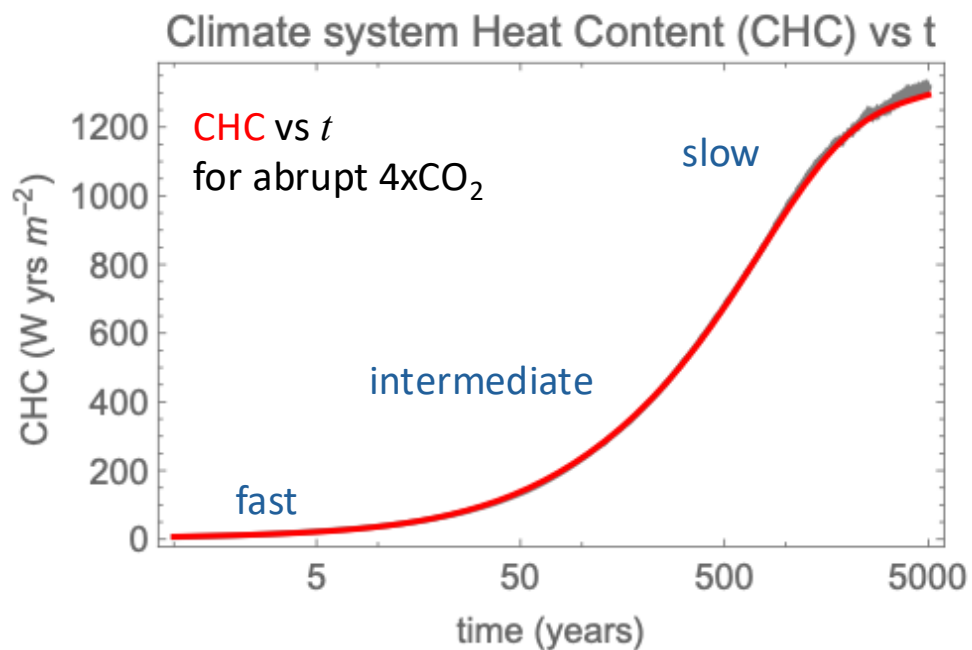
# 3D representation of the Gregory plot for GISS E2 R



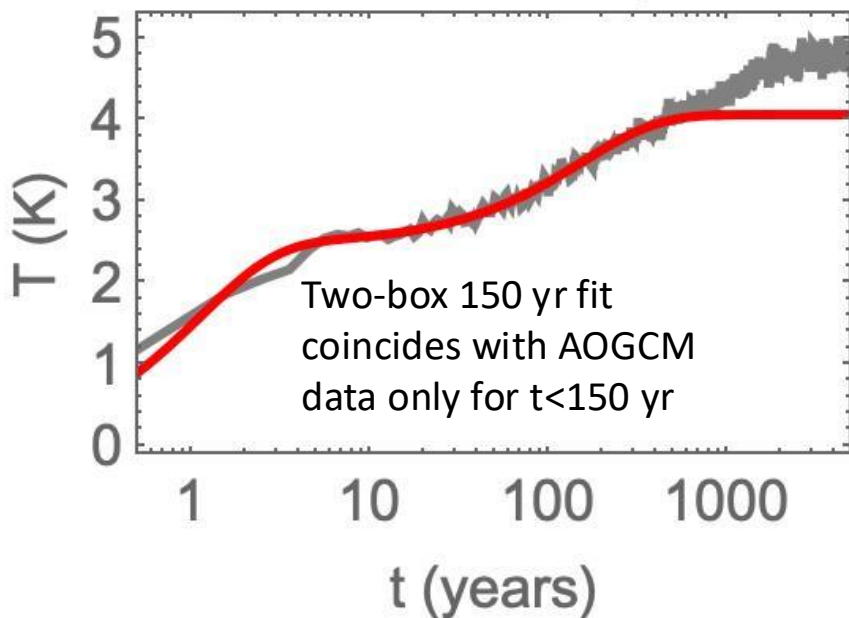
# Evolution of the feedback parameter $\lambda$ for GISS E2 R



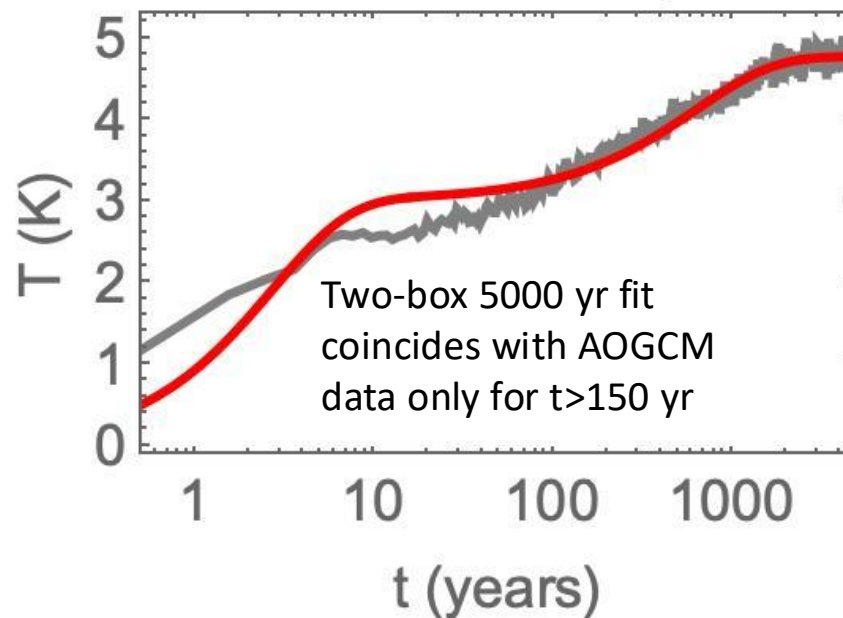
GISS E2 R



### Twobox, 150 yr fit

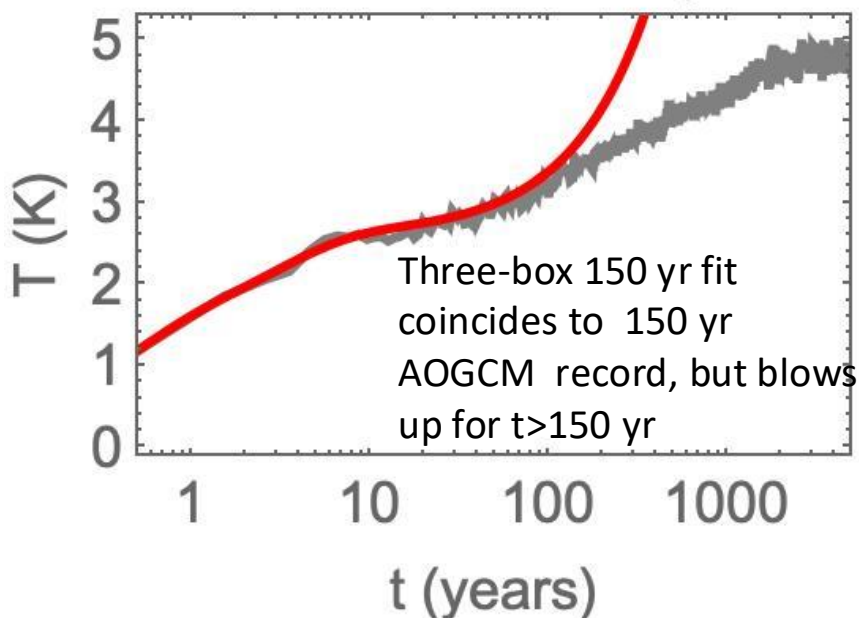


### Twobox, 5000 yr fit

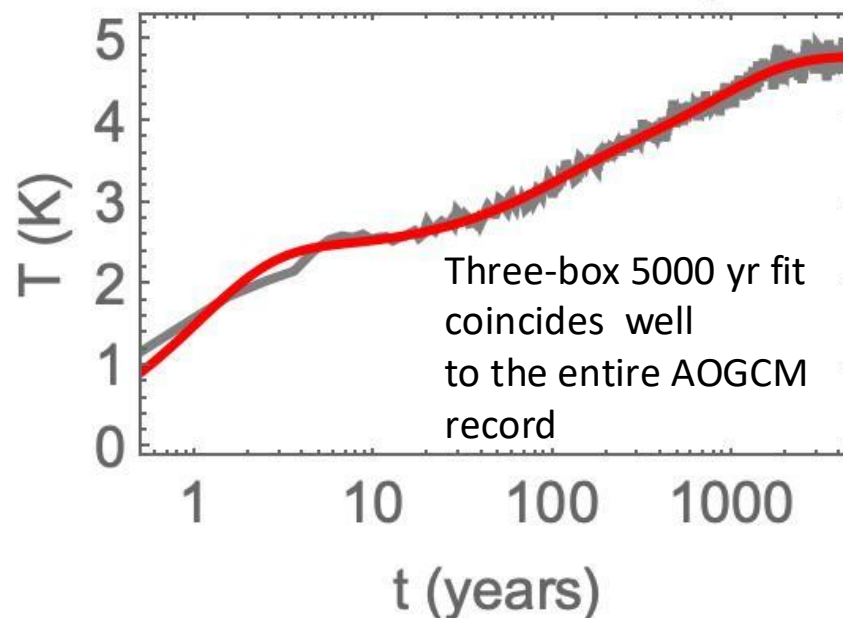


GISS E2 R

### Three-box, 150 yr fit



### Three-box, 5000 yr fit



**Only three-box model fitted to entire record yields a good fit on all time scales**

# Conclusions

- Testing the hypothesis that linear response function for GMST in the form of superposition of 3 decaying exponentials fitted to millenium-long abrupt CO<sub>2</sub>-forcing AOGCM-runs provide good emulators for arbitrary forcing scenario runs. Verified for smooth forcing growth in millennium simulations for GISS-E2-R and CCSM3II. More long runs with varying forcing for different AOGCMs are needed.
- Differences in albedo feedback among AOGCMs may explain much of the differences in ECS.
- Using GMST and netTOA data from AOGCMs to construct Geregory plots and fitting a third-order polynomial to these plots, provide accurate emulators for netTOA.
- The emulators show that CHC takes several millennia to equilbrate and that effective ocean heat capacity grows in three stages with characteristic growth times consistent with the three different response times for the GMST.
- Application of only the first 150 years of ESM data produces an emulator valid only for those 150 years, indicating overfitting.