Predicting Viscoplastic Anisotropy in Alpine Glaciers: A Flowline approach including Dynamic Recrystallization

*Contact : antonin.hilzheber@univ-grenoble-alpes.fr

Hilzheber A. Thiriet M. Chauve T. Montagnat M. Gimbert F. - IGE, Univ. Grenoble Alpes, CNRS, INRAE, IRD, Grenoble INP - Grenoble, France*

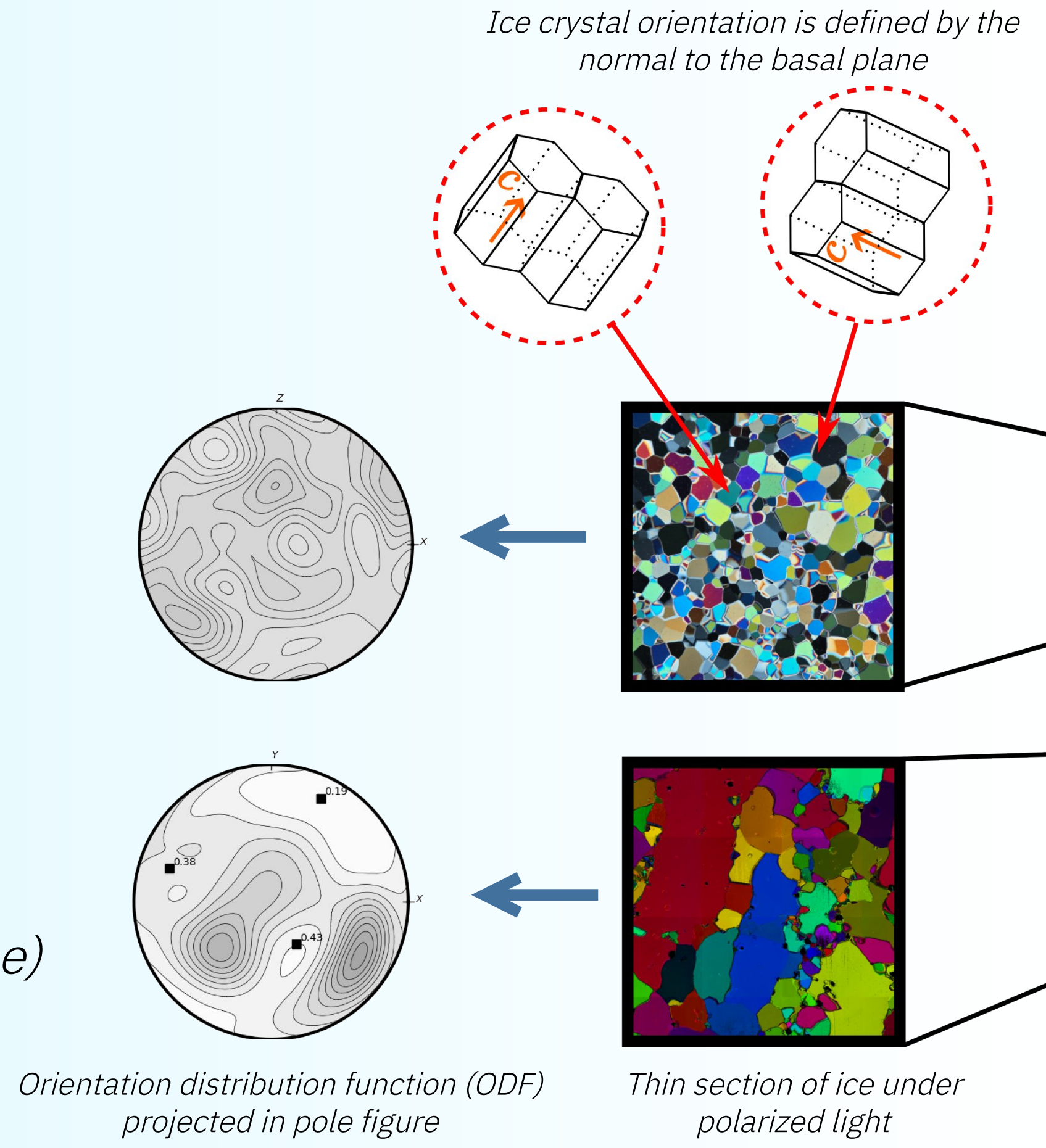
1.Context

Ice → **polycrystalline** material

Crystal orientation distribution (or **texture**) control its mechanical anisotropy :

Random orientations
→ **isotropic** viscoplastic behavior
(*Glen's flow law, n=3)*

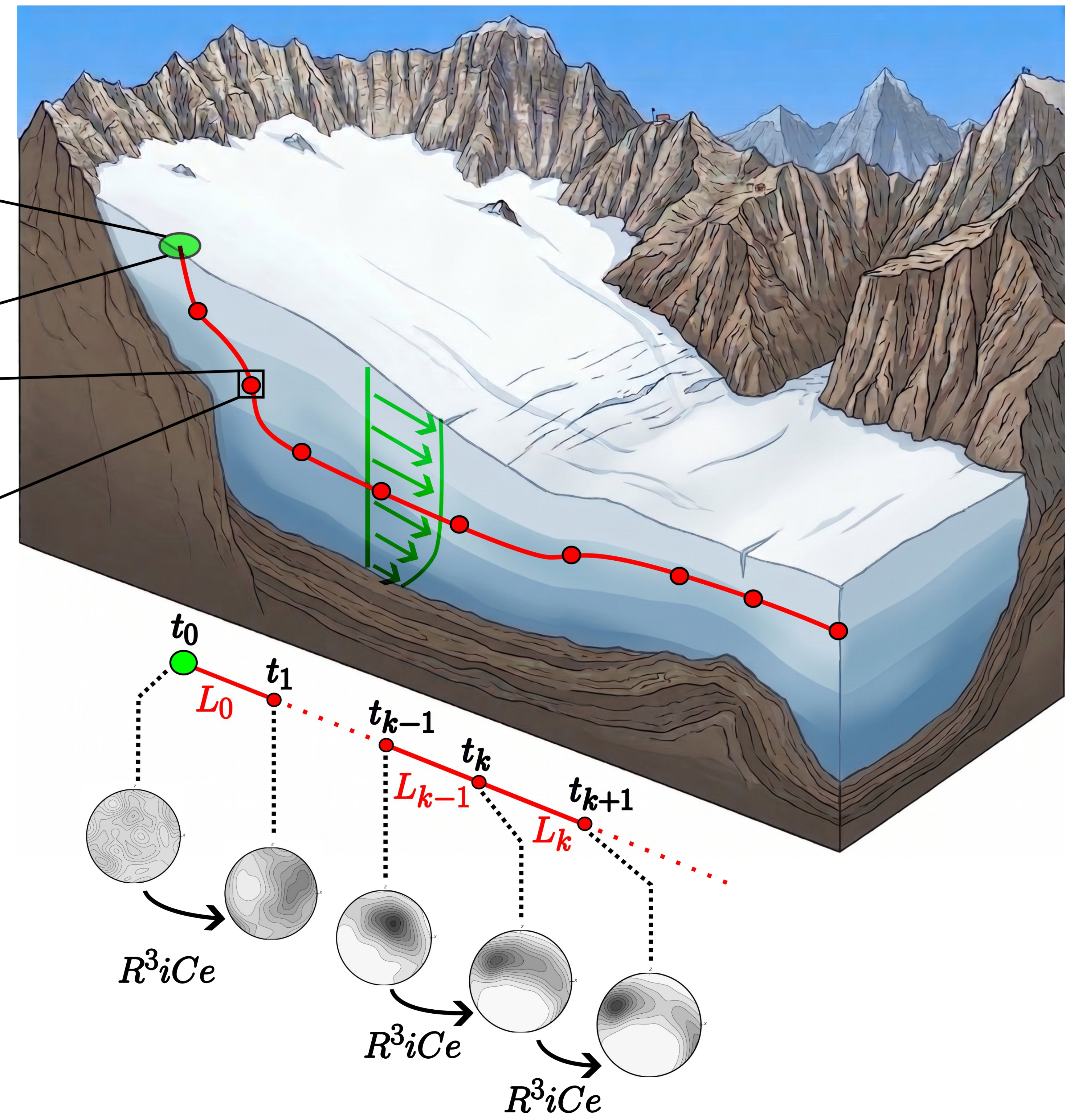
Preferred orientations
→ **anisotropic** viscoplastic behavior
(direction-dependent softening
or hardening relative to the isotropic case)



Ice crystal orientation is defined by the normal to the basal plane

Orientation distribution function (ODF) projected in pole figure

Thin section of ice under polarized light



In a glacier flow, texture evolves through **lattice rotation** and **dynamic recrystallization (DRX)**

→ Development of local viscoplastic anisotropy, which can in turn affect glacier flow

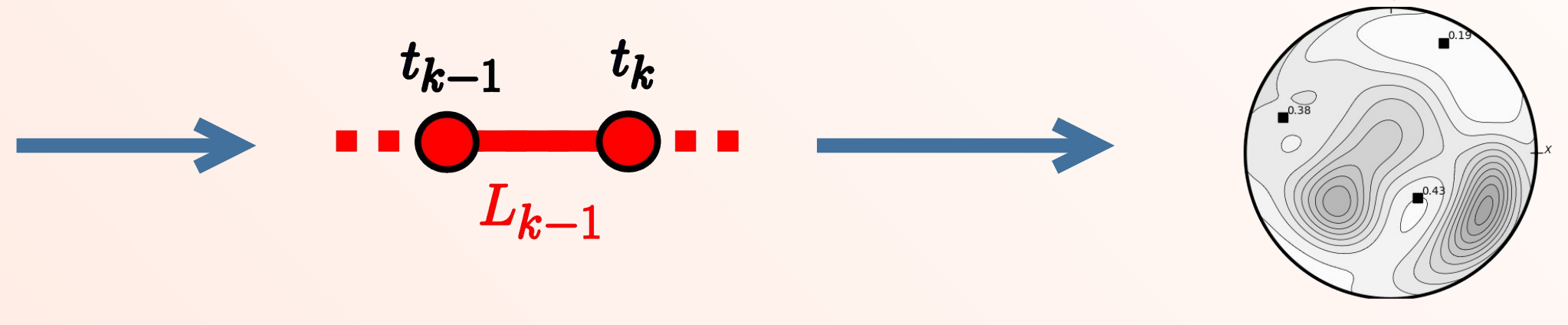
However, this feedback remains poorly constrained, especially when DRX is significant

2.Objective

Simulate ice deformation along glacier flowlines and predict the resulting texture evolution, including dynamic recrystallization

3.Method

– *Flow-line-based texture evolution :*



Velocity field from large-scale simulation

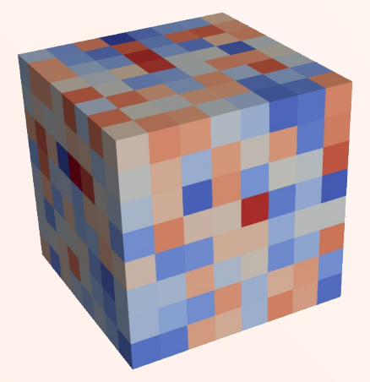
Discretized flowline + kinematic input for R³iCe

Texture simulated step by step

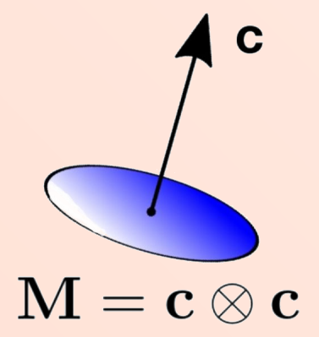
– *Texture evolution model with DRX :* ***R³iCe Model***

R³iCe *(Chauve et al. 24)* → full-field finite element model for polycrystalline ice
→ validated against creep experiments

One element → one crystallographic orientation (c-axis, **c**) →



One color → one element



$\mathbf{M} = \mathbf{c} \otimes \mathbf{c}$

Anisotropic viscoplastic law (described by local c-axis) →

$$\mathbf{S} = \eta_n^\star \left(2\alpha_1 \mathbf{D} + 2\alpha_2 \mathbf{M}^D Tr(\mathbf{MD}) + \alpha_3 (\mathbf{MD} + \mathbf{DM})^D\right)$$

$$\eta_n^\star = 2\eta_n \left(\alpha_1 tr(\mathbf{D}^2) + \alpha_2 tr(\mathbf{MD})^2 + \alpha_3 tr(\mathbf{MD}^2)\right)^{\frac{1-n}{2n}}$$

Code

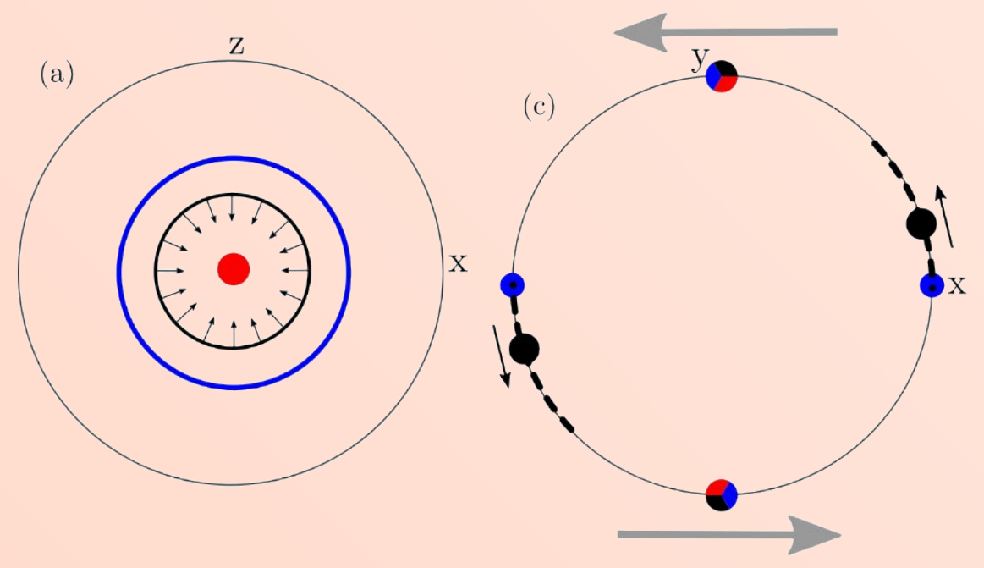
Doc

c-axis orientation evolution →

$$\frac{\partial \mathbf{c}}{\partial t} = \boxed{\mathbf{Wc} - \lambda \left[\mathbf{Dc} - \left(\mathbf{c^T D c}\right)\mathbf{c}\right]} + \boxed{\frac{1}{\Gamma_{RX}} (\mathbf{c_0} - \mathbf{c})}$$

driven by a balance between → *Lattice rotation* ↓ *local deformation* ; *DRX* ↓ *local stress*

S → local deviatoric stress ; D → local strain rate ; W → local spin ; C_0 → "ideal" orientation produced by DRX ; n → power of viscoplastic law ; α_i → viscosities



🟥 Solutions of $\mathbf{Wc} - \lambda \left[\mathbf{Dc} - \left(\mathbf{c}^T \mathbf{D c}\right)\mathbf{c}\right] = \mathbf{0}$

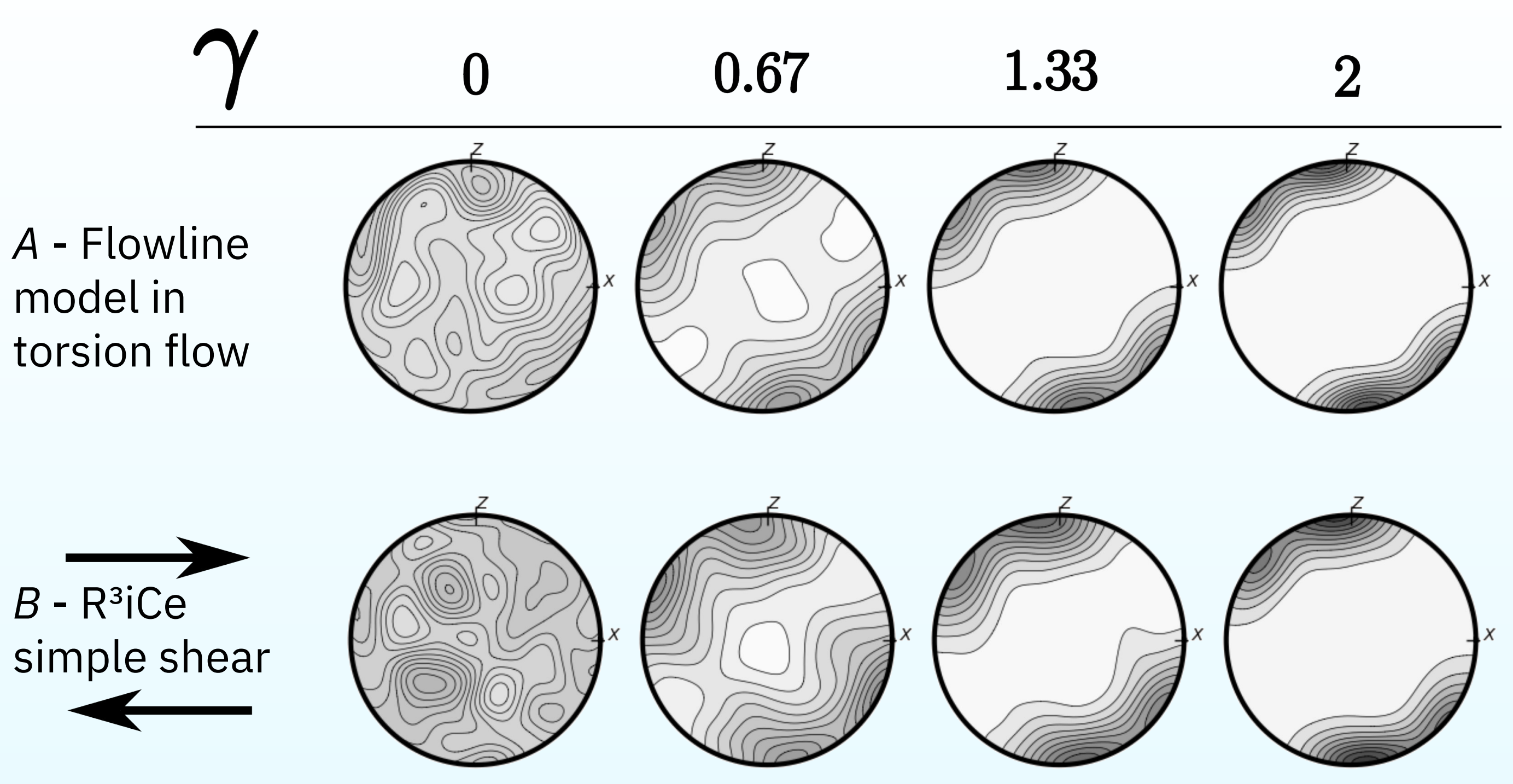
🟦 Solutions of $\frac{1}{\Gamma_{RX}} (\mathbf{c_0} - \mathbf{c}) = \mathbf{0}$

⬛ Solutions of $\mathbf{Wc} - \lambda \left[\mathbf{Dc} - \left(\mathbf{c}^T \mathbf{D c}\right)\mathbf{c}\right] + \frac{1}{\Gamma_{RX}} (\mathbf{c_0} - \mathbf{c}) = \mathbf{0}$

↑ Evolution of the solutions as Γ_{RX} increases

*a) uniaxial compression along **y** b) simple shear in the **xOz** plane*

4.Validation

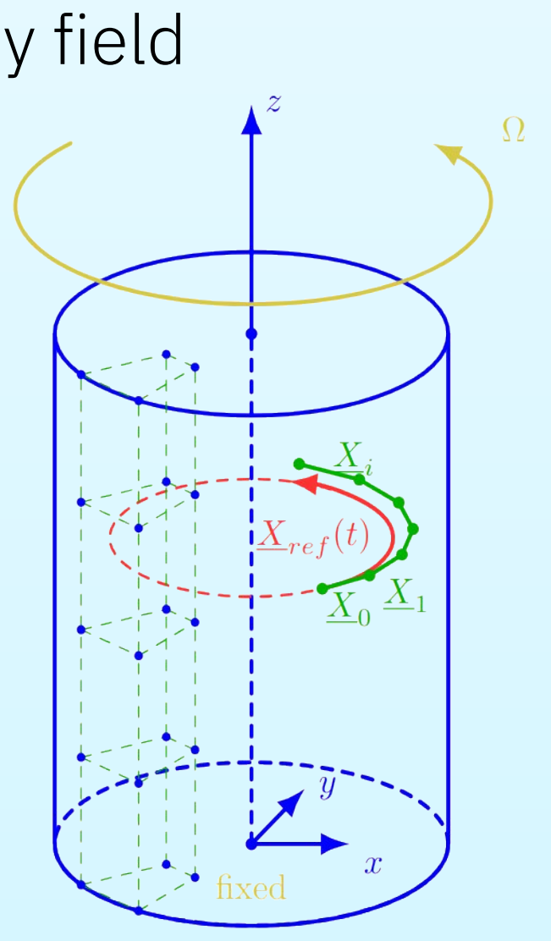


Line B : R³iCe able to simulate DRX texture evolution in simple shear (Line B) (chauve et al.24)

Line A : Flowline approach using an analytical torsion velocity field
→ Texture evolution along trajectory
→ Compared in cylindrical basis to reference (line B)

$$v_\theta(r, z) = \frac{\Omega r z}{H}$$

velocity field is defined in a cylindrical coordinate system and used to validate the model.

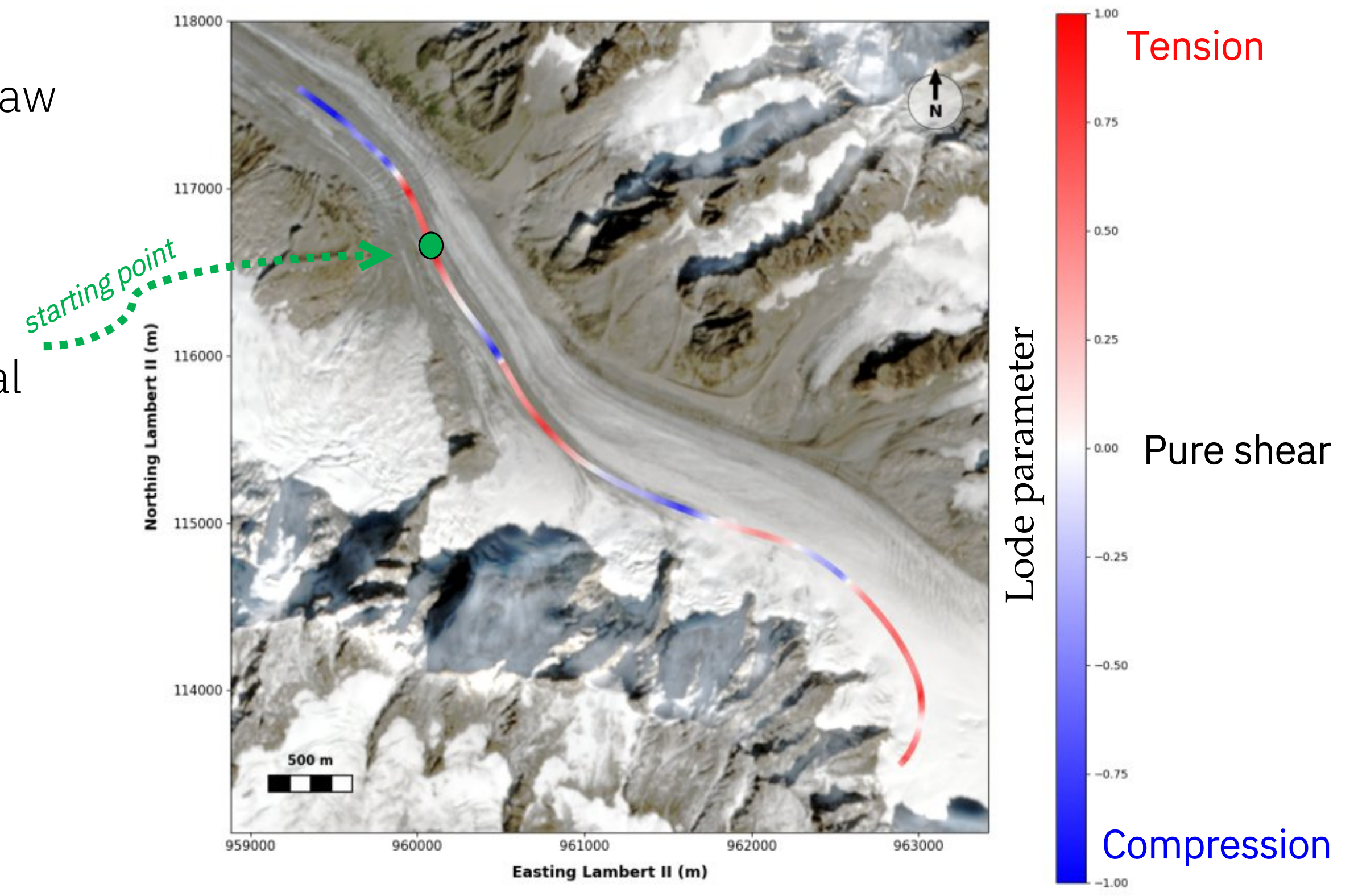


The flowline approach is **validated** using torsion–shear equivalence and successfully reproduces DRX-driven texture evolution

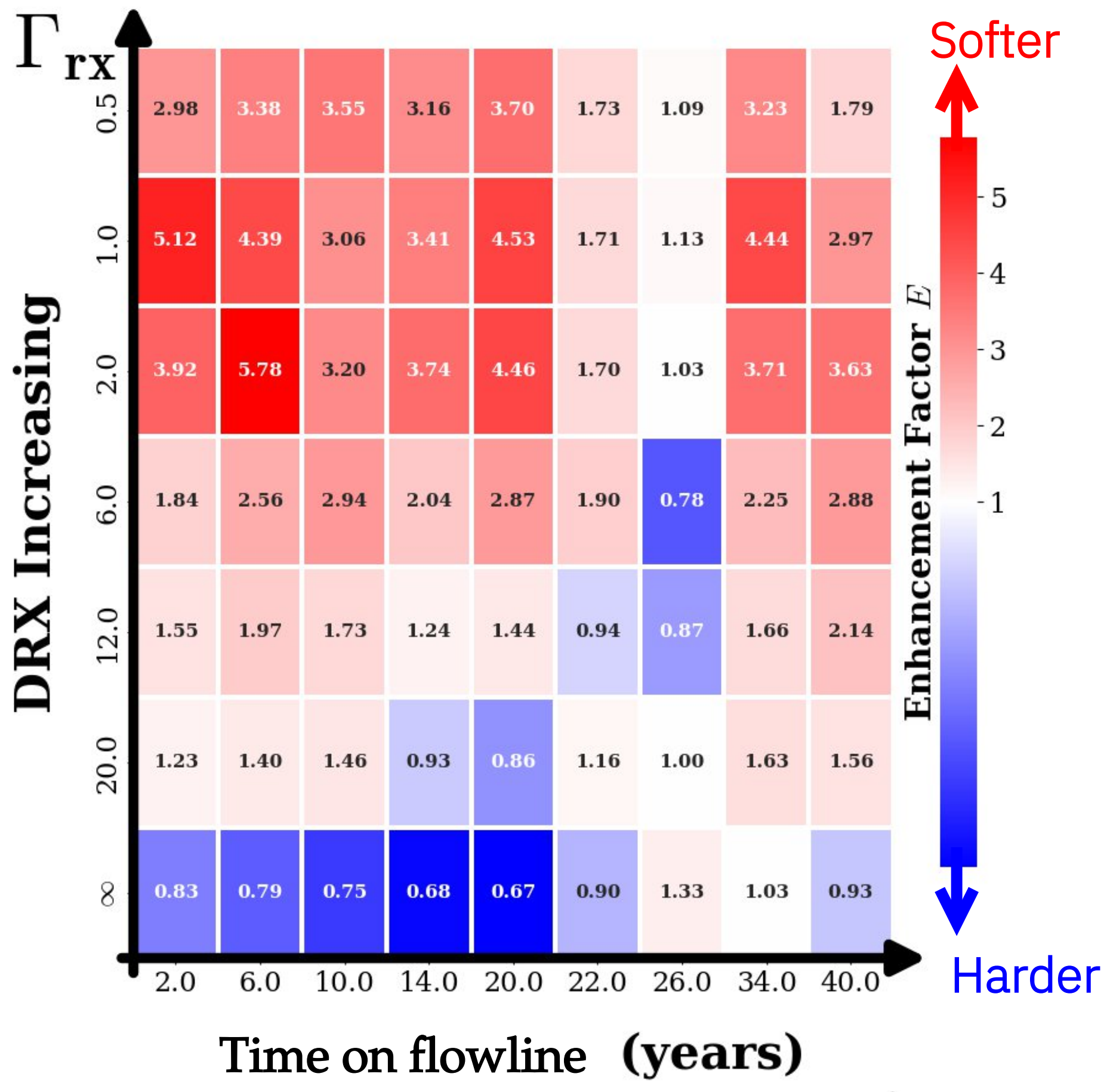
5.Application on Argentière Glacier (French Alps)

Velocity field from Elmer/Ice simulation using isotropic flow law (Gilbert et al., 2023)
↓
Discretized Flowline / mechanical loading for each time step

Varying DRX parameter Γ_{rx} →
↓
Texture evolution along flowline for various DRX amount



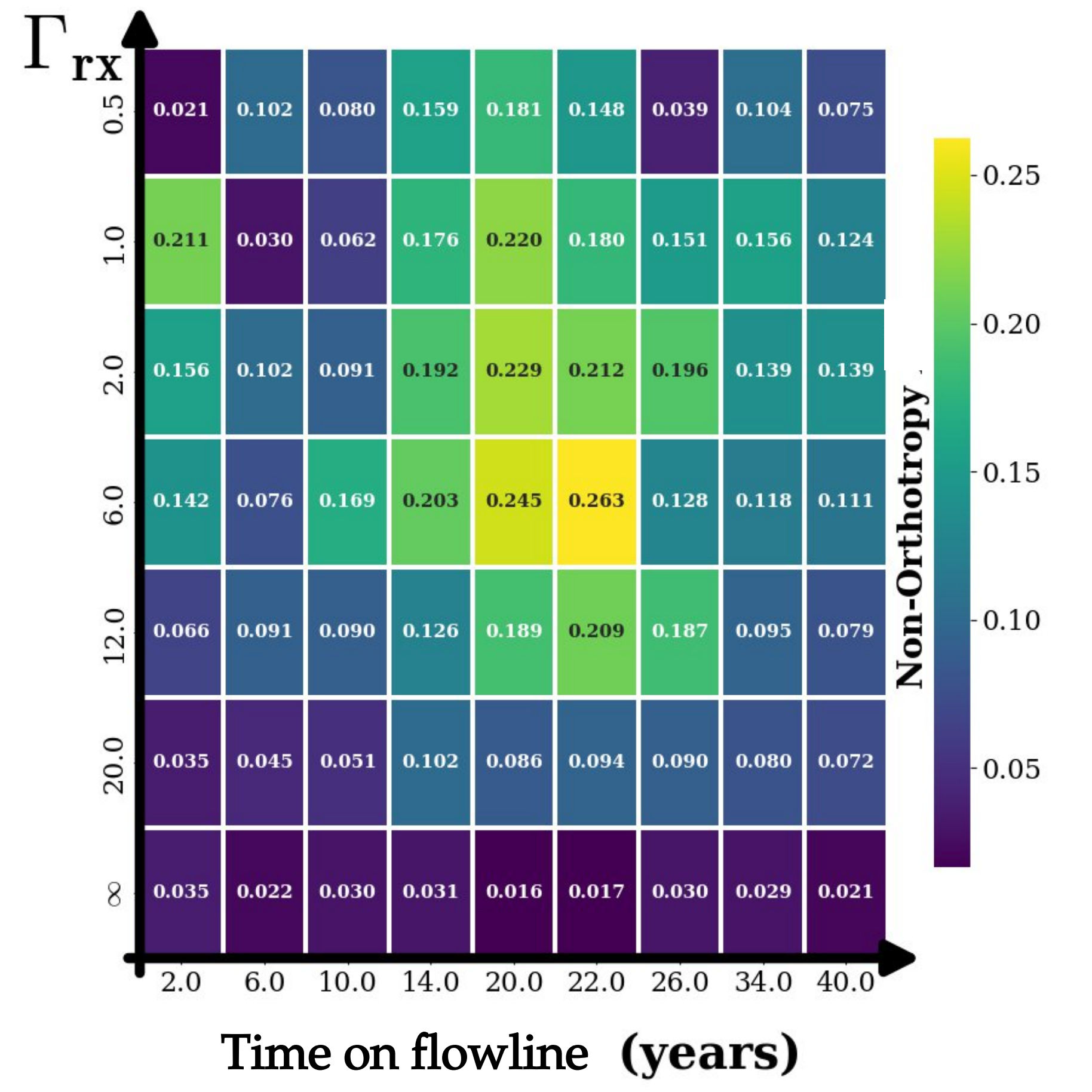
Location for interactive ODF plots along the flowline for different DRX values at each timestep



Γ_{rx} \ Time on flowline (years)	2.0	6.0	10.0	14.0	20.0	22.0	26.0	34.0	40.0
0.5	2.98	3.38	3.55	3.16	3.70	1.73	1.09	3.23	1.79
1.0	5.12	4.39	3.06	3.41	4.53	1.71	1.13	4.44	2.97
2.0	3.92	5.78	3.20	3.74	4.46	1.70	1.03	3.71	3.63
6.0	1.84	2.56	2.94	2.04	2.87	1.90	0.78	2.25	2.88
12.0	1.55	1.97	1.73	1.24	1.44	0.94	0.87	1.66	2.14
20.0	1.23	1.40	1.46	0.93	0.86	1.16	1.00	1.63	1.56
∞	0.83	0.79	0.75	0.68	0.67	0.90	1.33	1.03	0.93

Enhancement Factor $E = \left(\frac{\tau_{eq,isotropic}}{\tau_{eq,Texture}}\right)^3$

→ DRX controls substantially viscoplastic anisotropy



Γ_{rx} \ Time on flowline (years)	2.0	6.0	10.0	14.0	20.0	22.0	26.0	34.0	40.0
0.5	0.021	0.102	0.080	0.159	0.181	0.148	0.039	0.104	0.075
1.0	0.211	0.030	0.062	0.176	0.220	0.180	0.151	0.156	0.124
2.0	0.156	0.102	0.091	0.192	0.229	0.212	0.196	0.139	0.139
6.0	0.142	0.076	0.169	0.203	0.245	0.263	0.128	0.118	0.111
12.0	0.066	0.091	0.090	0.126	0.189	0.209	0.187	0.095	0.079
20.0	0.035	0.045	0.051	0.102	0.086	0.094	0.090	0.080	0.072
∞	0.035	0.022	0.030	0.031	0.016	0.017	0.030	0.029	0.021

Non-orthotropy quantified using the 4th-order orientation tensor $\mathbf{A}^{(4)} = \langle \mathbf{c} \otimes \mathbf{c} \otimes \mathbf{c} \otimes \mathbf{c} \rangle$

→ DRX induces non-orthotropic textures

Conclusion

- The flowline texture evolution framework is validated and applied to an alpine glacier
- The amount of **DRX controls how fast textures orient** according to the local velocity gradient along the flowline
- DRX controls substantially viscoplastic anisotropy
- DRX induces **non-orthotropic textures**

Perspectives

- Validation against **field observations**
- Assess the validity of **orthotropic flow laws** for non-orthotropic fabrics
- Comparison with other texture evolution models *(Rathmann et al. 21, Richards et al. 23, Llorens et al.21)*
- **Full coupling** between texture and flow