



Investigating Hybrid Spherical Harmonic and Neural Network Models for Efficient Functional Approximation of the Global Gravity Field Representation

Jhon Restrepo-Berrío¹; Jürgen Kusche²; Anne Springer²; Marc Rußwurm^{1,2}
¹Wageningen University and Research, ²University of Bonn

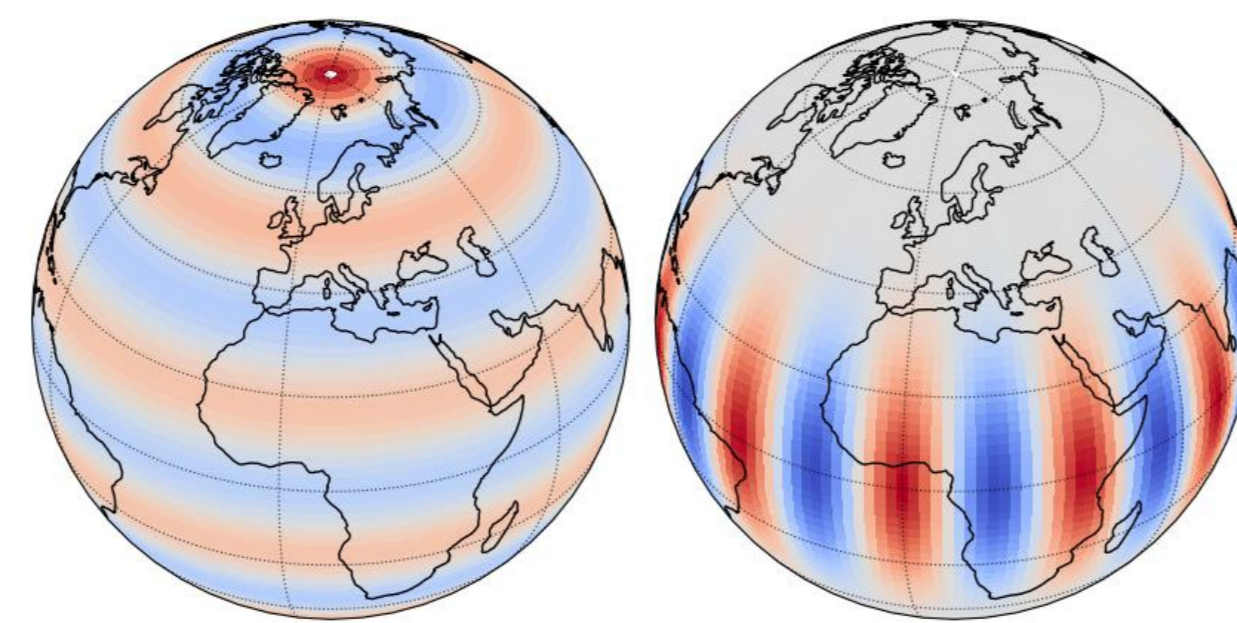


1. Introduction

- Spherical Harmonics (SH) modelling of Earth's Gravity field suffers **computational** limitations and **convergence** constraints [1].
- Linear SH coefficients are global parameters and cannot easily adapt to spatially **heterogeneous data** with **regionally varying** signal characteristics.
- This paper investigates hybrid** methods of *analytical* SH and *numerical* Neural Networks as **data-driven** strategies that offer a promising alternative to analytical modelling.

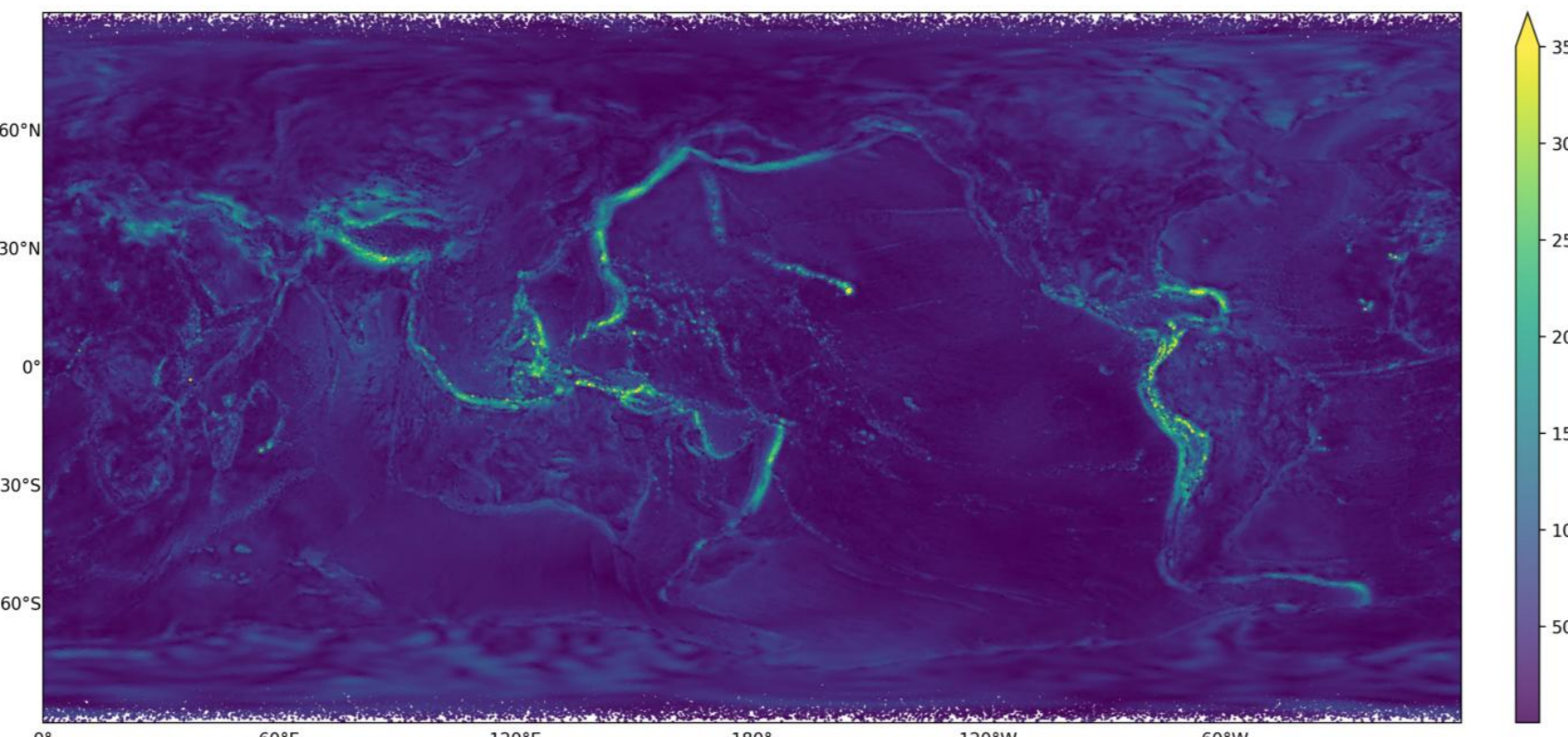
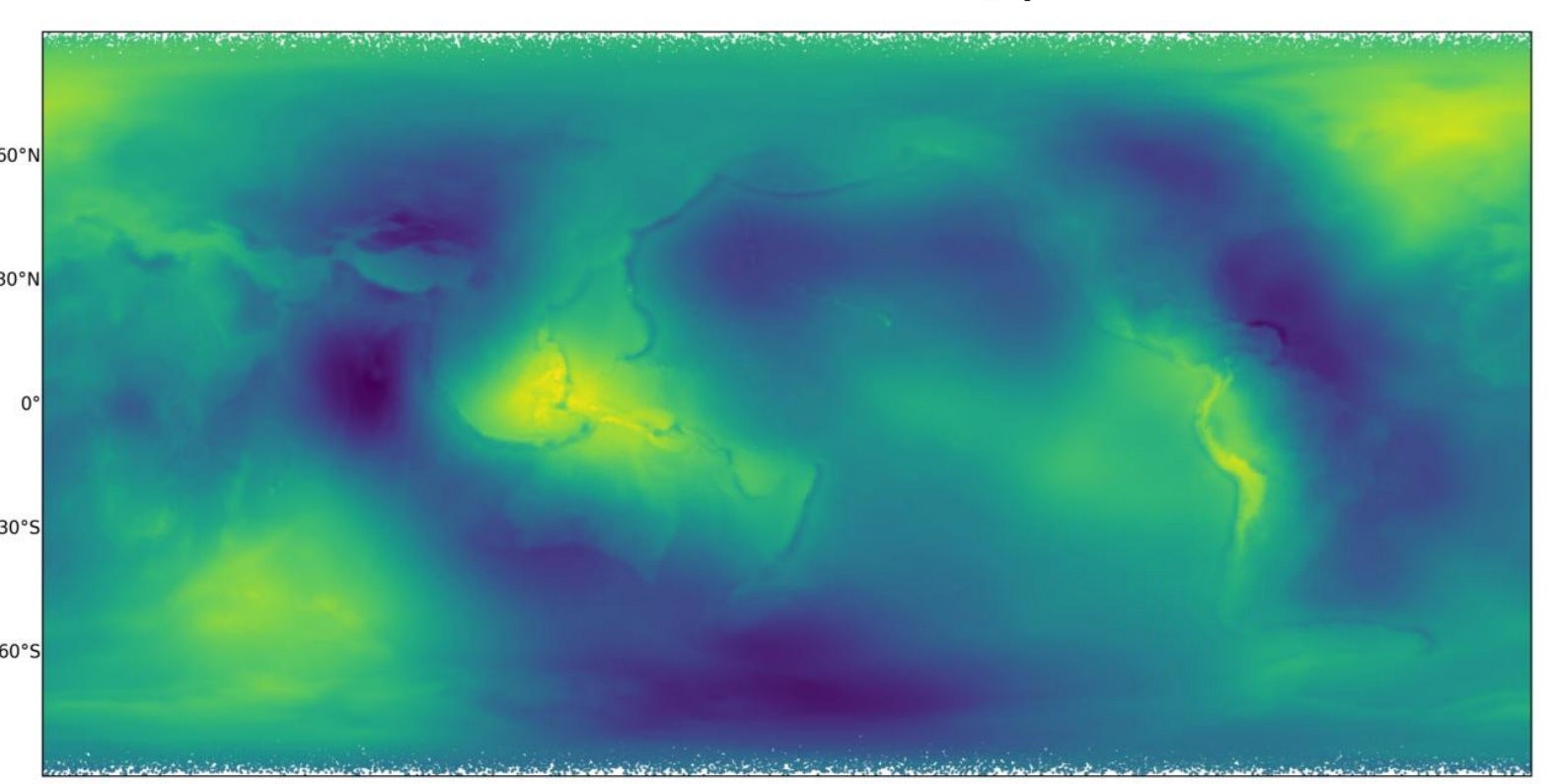
2. Experimental Data

We generate 5,250,000 data points from **EGM2008** at $L_{max} = 2190$ of Earth gravity and potential [2].



a) Train+validation, potential

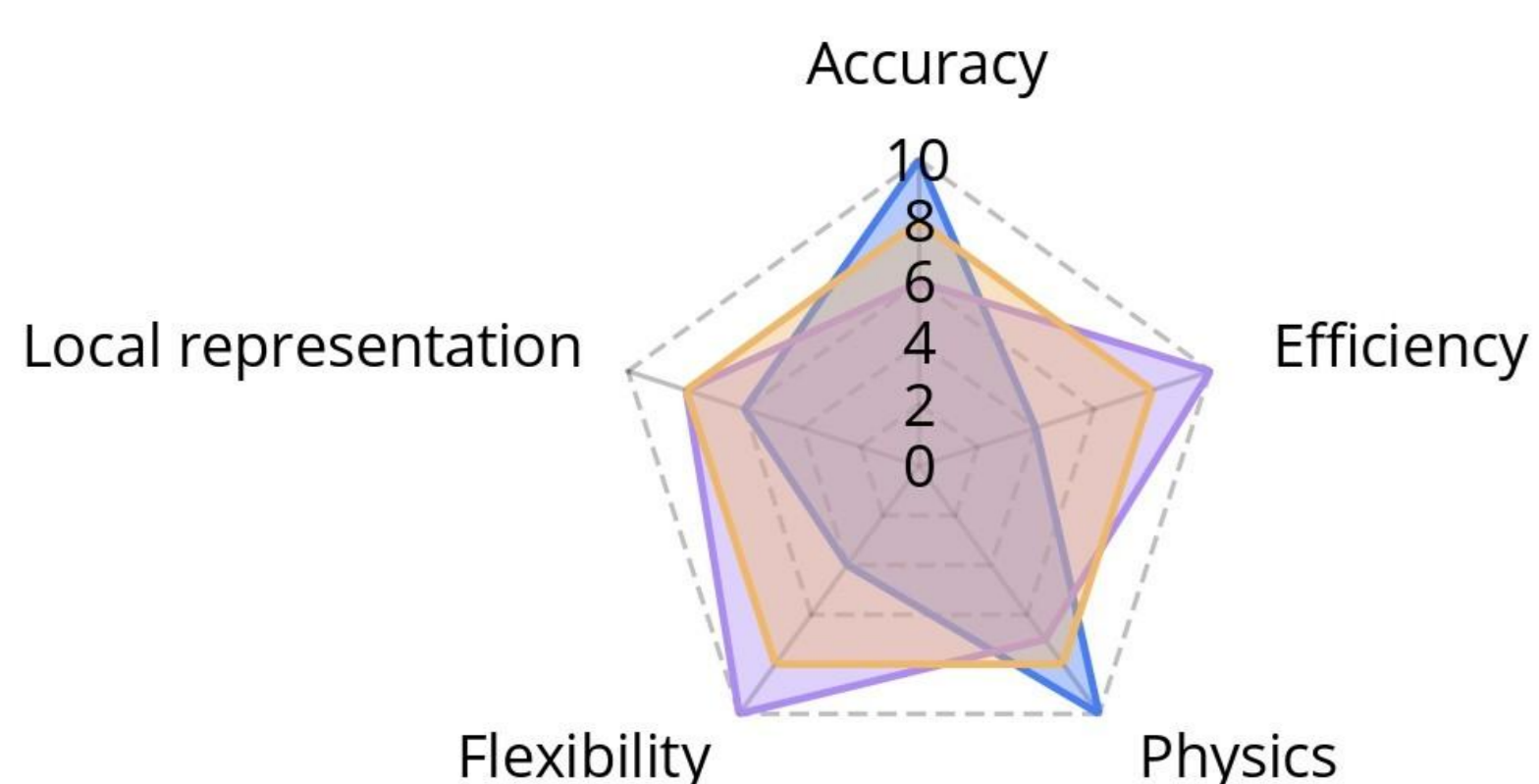
b) Train+validation, acceleration



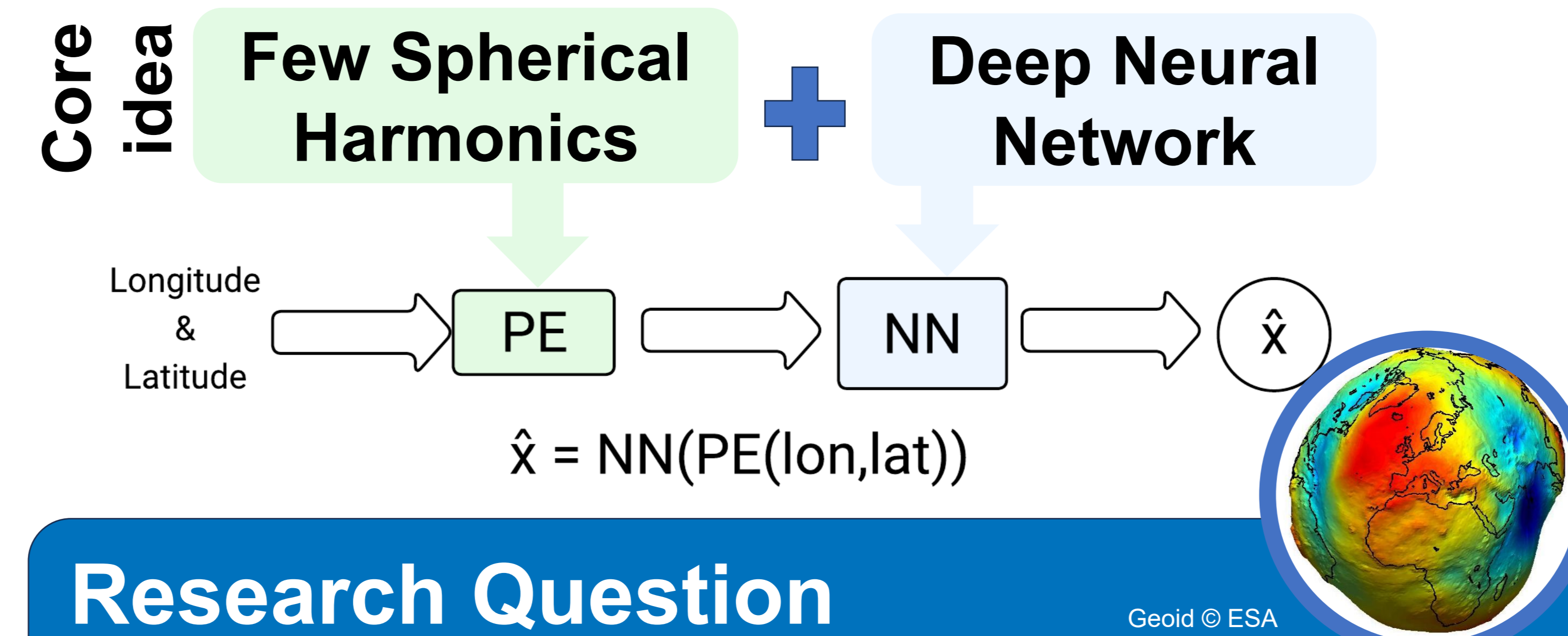
3. Related Work: Analytical SH, Numerical Neural Network and Hybrid Method

- Analytical (SH)
- Numerical (Neural network)
- Hybrid (Analytical + neural network)

- The SH analytical method [3] uses **closed-form equations**
- The numerical neural network [4] uses **data-driven** formulations
- The hybrid method combines both



4. Proposed New Hybrid Spherical-Harmonic and Neural Network Model: SirenSH [5]

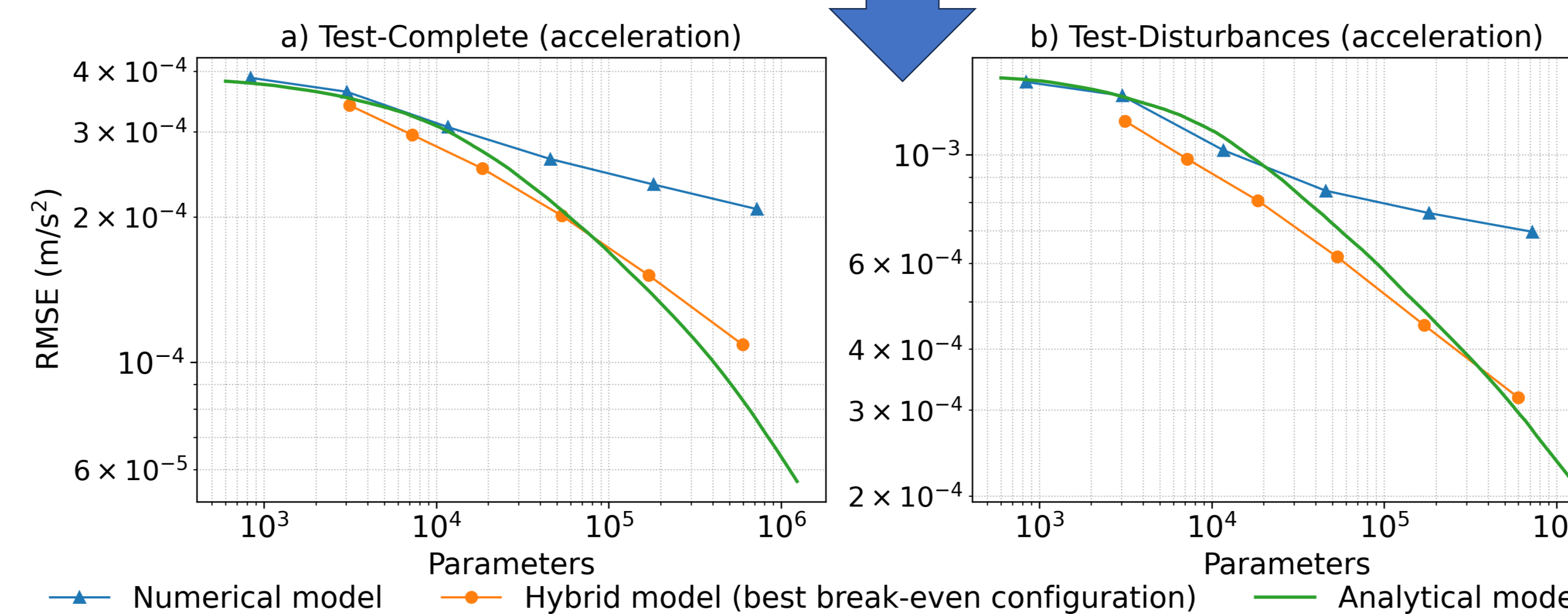


Research Question

How does a *hybrid* Spherical Harmonic Neural Network compare to *analytical* Spherical Harmonics and *numerical* neural networks in terms of approximation accuracy and parameter efficiency for gravity field representation?

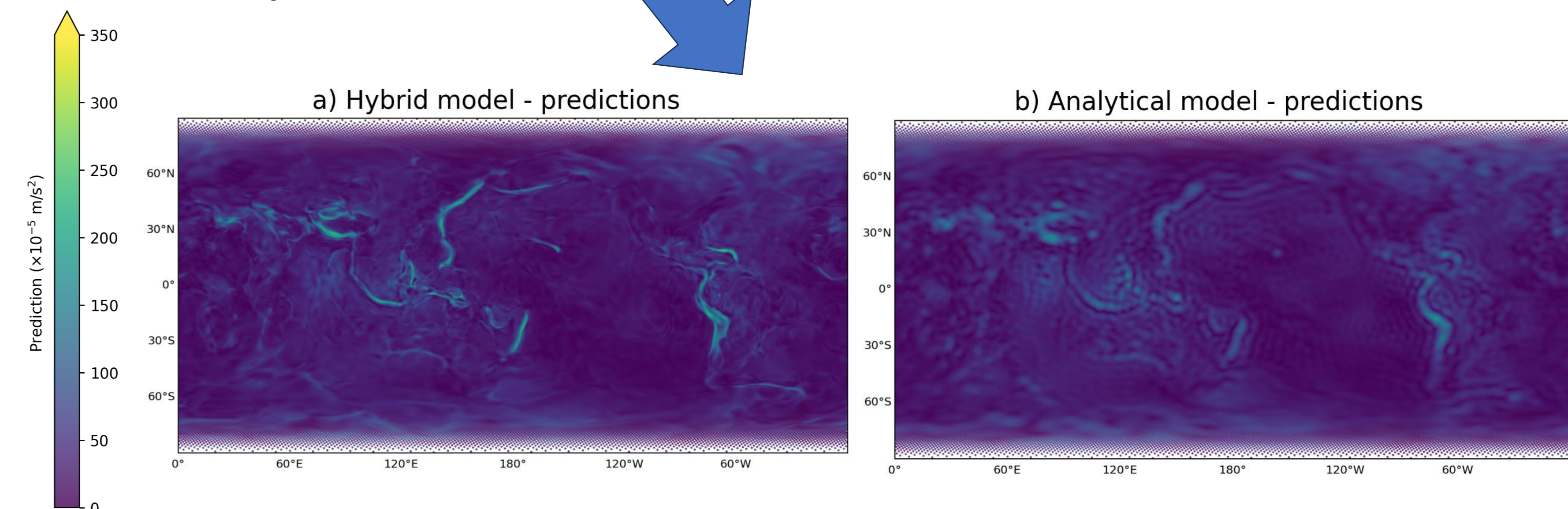
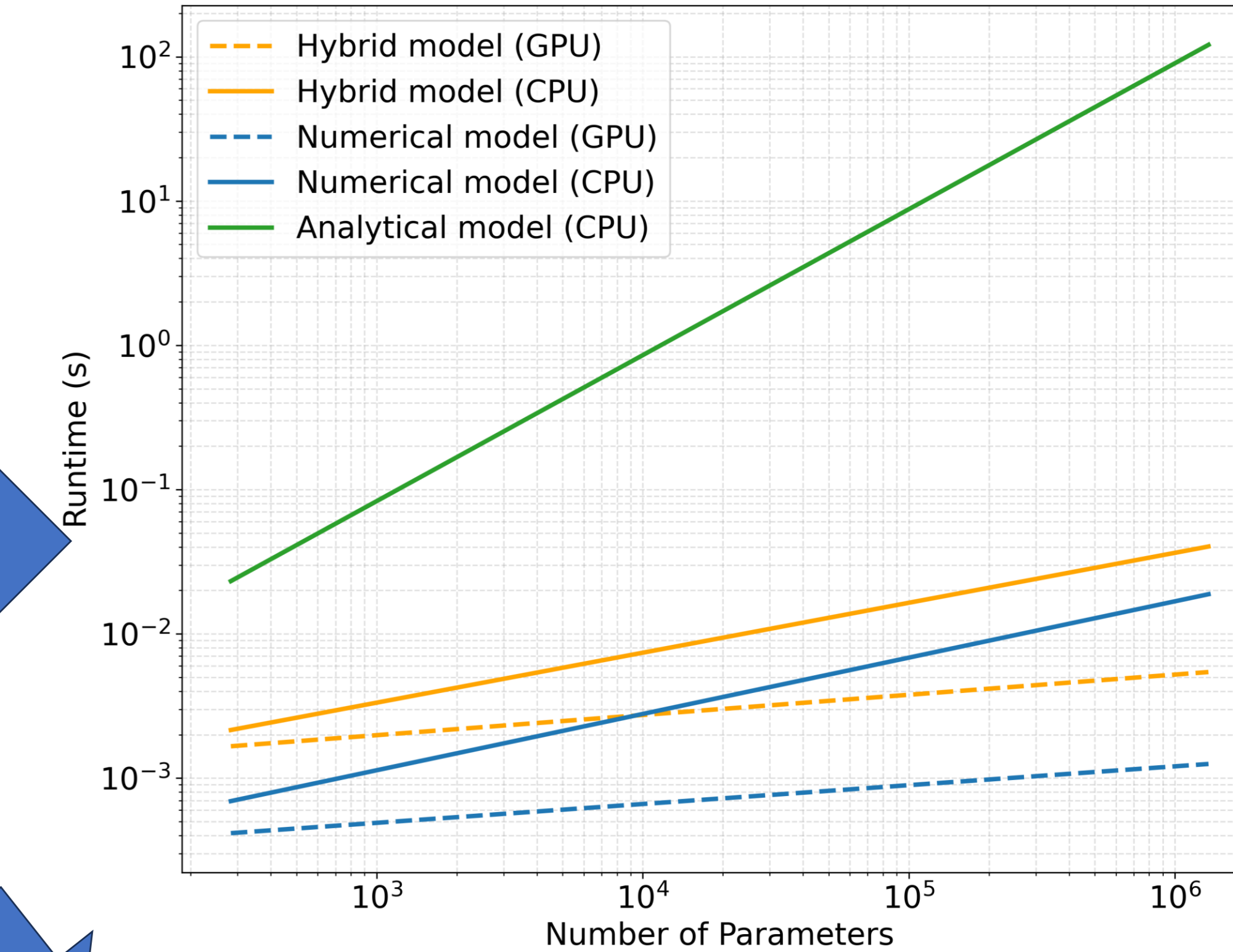
5. Results and Takeaways

Takeaway 1: The **hybrid** approach provides the **best accuracy-efficiency trade-off** for low number of parameters.



Takeaway 2: The **numerical** and **hybrid** methods are much faster than **analytical**: 0.02/0.04 seconds vs 2 minutes

Takeaway 3: The **hybrid** approach is the most accurate at high-frequency disturbances



6. Limitations

- Caution when modelling non-spherical bodies
- Limited hyperparameter search
- Numerical and hybrid methods suffer from high training times

7. Next Steps

- Learning from 3D data in the exterior
- Applying physics-informed losses
- Modelling other gravity field target (e.g., geoid height)

References

[1] Martin & Schaub, *J. Astronaut. Sci.*, 2025.
 [2] Pavlis et al., *J. Geophys. Res. Solid Earth*, 2012.
 [3] Kaula, *Theory of Satellite Geodesy*, 1966.
 [4] Martin & Schaub, *Celest. Mech. Dyn. Astron.*, 2022.
 [5] Rußwurm et al., *ICLR*, 2024.