Low Uncertainty Regional Climate Projections without Irrelevant Weather Details

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Abstract

ECS(**Charney report, 1979**) = [1.5-4.5]K
ECS(**IPCC AR6, 2021**) = [1.8-5.6]K

** Despite an exponential increase in modern computing power, uncertainties in conventional (GCM) climate models have increased for the first time in the latest AR6 report. The root problem is that these models spend unnecessary effort in calculating irrelevant weather details.**

Effective Surface HEBE

$$\tau^{1/2}(\underline{x})\partial_t^{1/2}T(\underline{x},t)+T(\underline{x},t)=s_{ECS}F(t)$$

τ(year) = relaxation time, characteristic time for a system to relax back to equilibrium

s(K/Wm^-2) = equilibrium climate sensitivity amount of temperature increase per Wm-2 forcing

F(Wm^-2) = uniform over the globe, IPCC effective radiative forcing

Has an analytically solvable Green's function using Fourier transform!

Methodology

Priors

T -> Fitting effective surface HEBE to the temperature's internal variability spectra

s -> Apply memory correction to the transient climate sensitivity (TCS)

Maximum Likelihood Estimation

Use Bayesian inference to update the priors against ERA5 2mT reanalysis dataset

Monte Carlo Simulation

Use the covariance and data matrix obtained from the MLE to define a multivariate normal distribution. Then draw pairs of variables from this posterior distribution to perform a Monte Carlo simulation, where the drawn parameters are convolved with the analytically solved Green's function

Climate Projection to 2100

A low-uncertainty temperature projection for 1750–2100 is generated, which accuracy is validated through hindcasts against instrumental and reanalysis datasets. Parametric uncertainty is smaller than internal variability across most spatial pixels, indicating a substantial improvement over GCM-based models. These results confirm the relevance of HEBE and fractional calculus framework in climate modelling

Alternative : Energy-Balance Models

$$\left(\tau(\underline{x})\partial_t\right)^h T(\underline{x},t)-s(\underline{x})l(\underline{x})(-\nabla^2)^q T(\underline{x},t)+T(\underline{x},t)=s(\underline{x})F(\underline{x},t)$$

Storage — Transport ($\nabla\cdot\underline{E}$, Column Energy Flux) — Black Body Radiation — Internal and external Forcing

Stochastic response if forced by internal variability. Deterministic response when forced externally.

Classical (Integer Order PDE)

h = 1 **q = 1 [Sellers 1969] q = 2 [Budyko 1969]**

HEBE (Half-Order-Energy-Balance Equation, TOA form)

h = 1/2 **q = 1/2**

****HEBE is a linear space-time, fractional PDE with spatially varying coefficients ****

*** It can be made nonlinear (e.g. with temperature-albedo feedbacks) to develop tipping points***

Why Fractional Calculus, Why Half-Order?

Heat Equation: $\partial_t \Leftrightarrow \partial_{xx} \rightarrow \partial_t^{1/2} \Leftrightarrow \partial_x$

3D->2D (Surface of Earth): $\partial_z \rightarrow \sqrt{-\nabla_h^2} := \nabla_s$

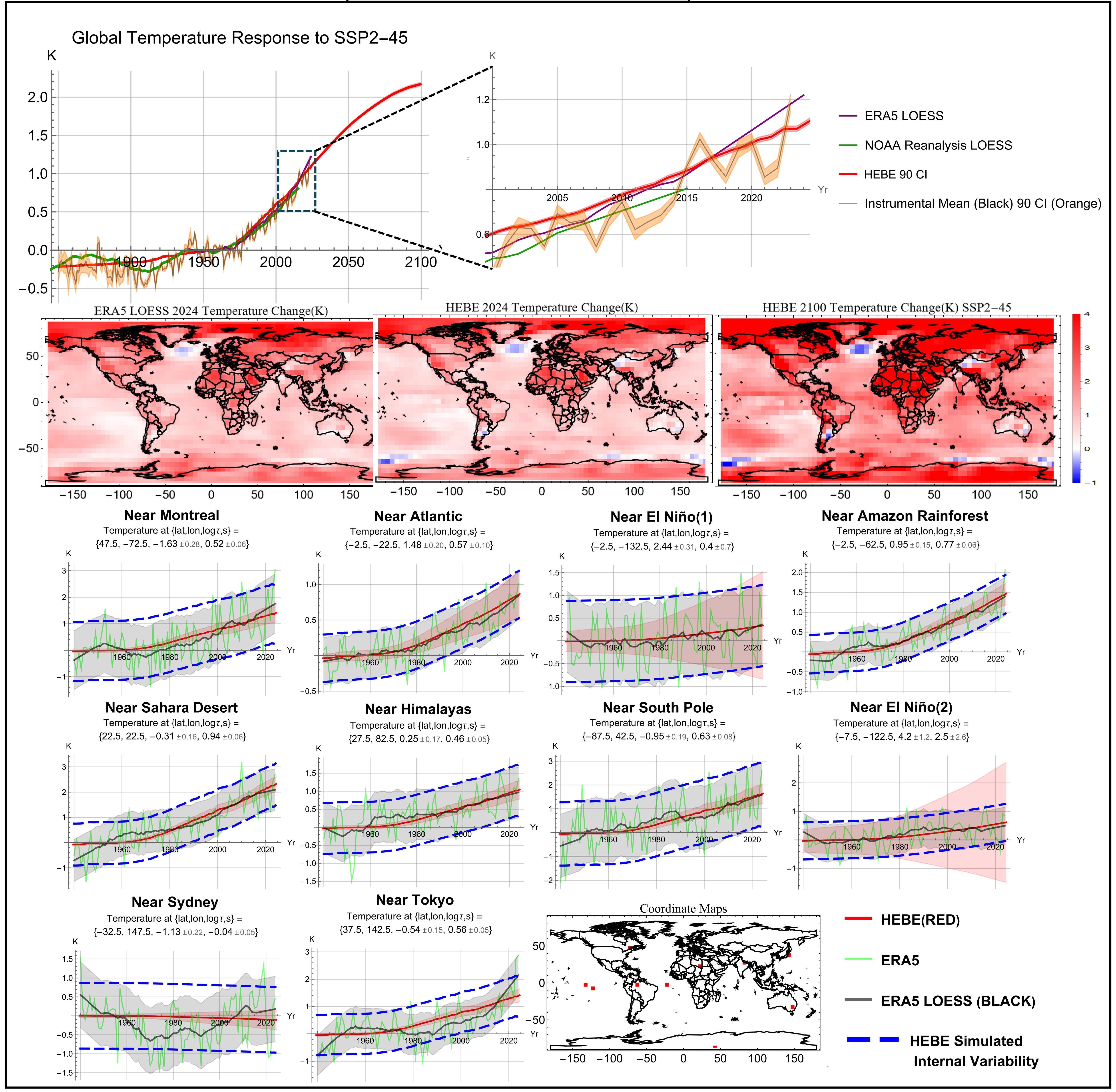
Under correct boundary conditions for the 3D heat equation, both time and space have fractional operators

- Fractional Derivative is a global operator, evaluating it requires the function's behavior over an entire interval. Unlike classical derivative, it is not an instantaneous rate of change

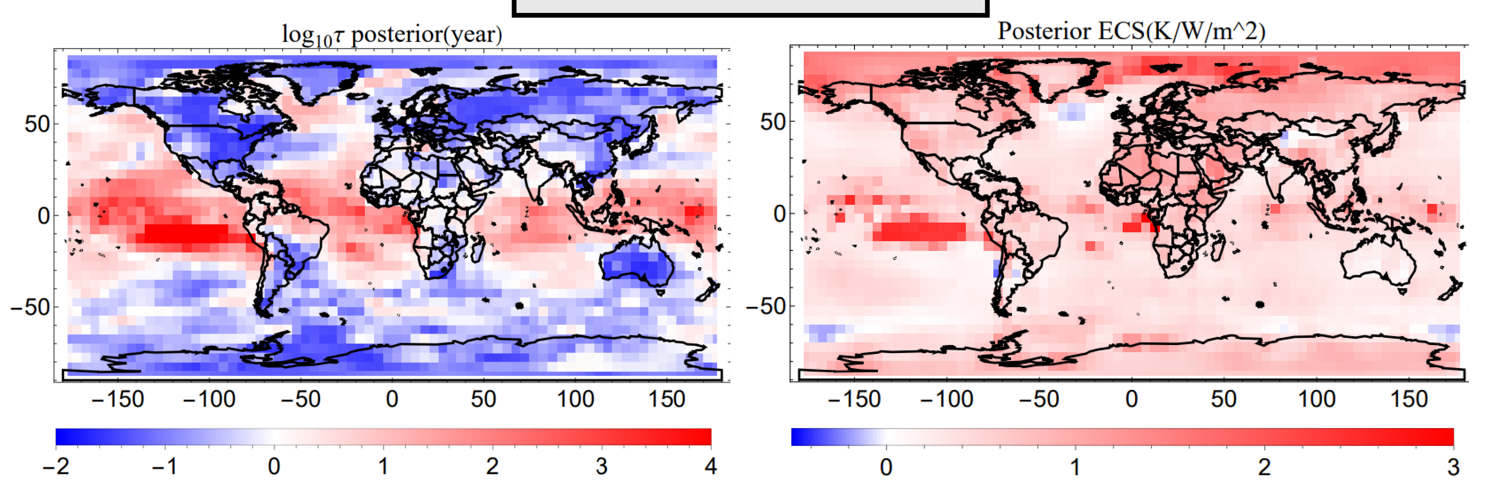
- Classical EBE models struggle at modelling energy storage process of the climate system, and fractional calculus's inherent scaling property makes it ideal for incorporating memory response into models

- HEBE structure is empirically validated by surface temperature's internal variability spectra, and column energy flux reconstructed using HEBE parameters are validated by the empirical reanalysis data

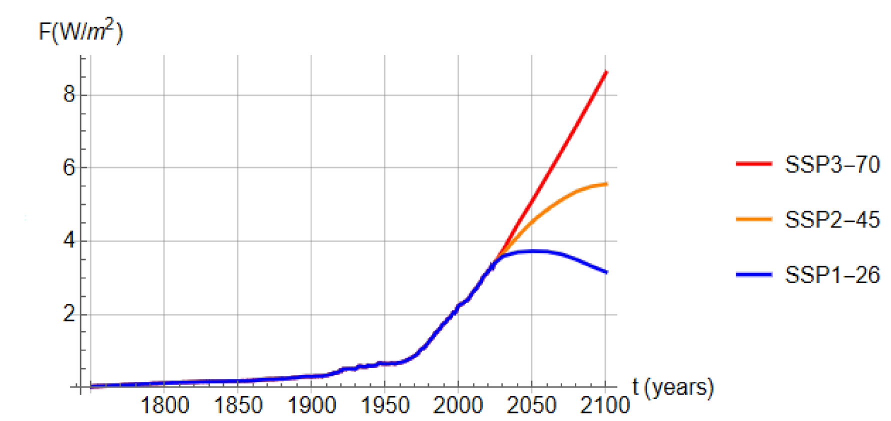
Hindcast and Projection



Parameters

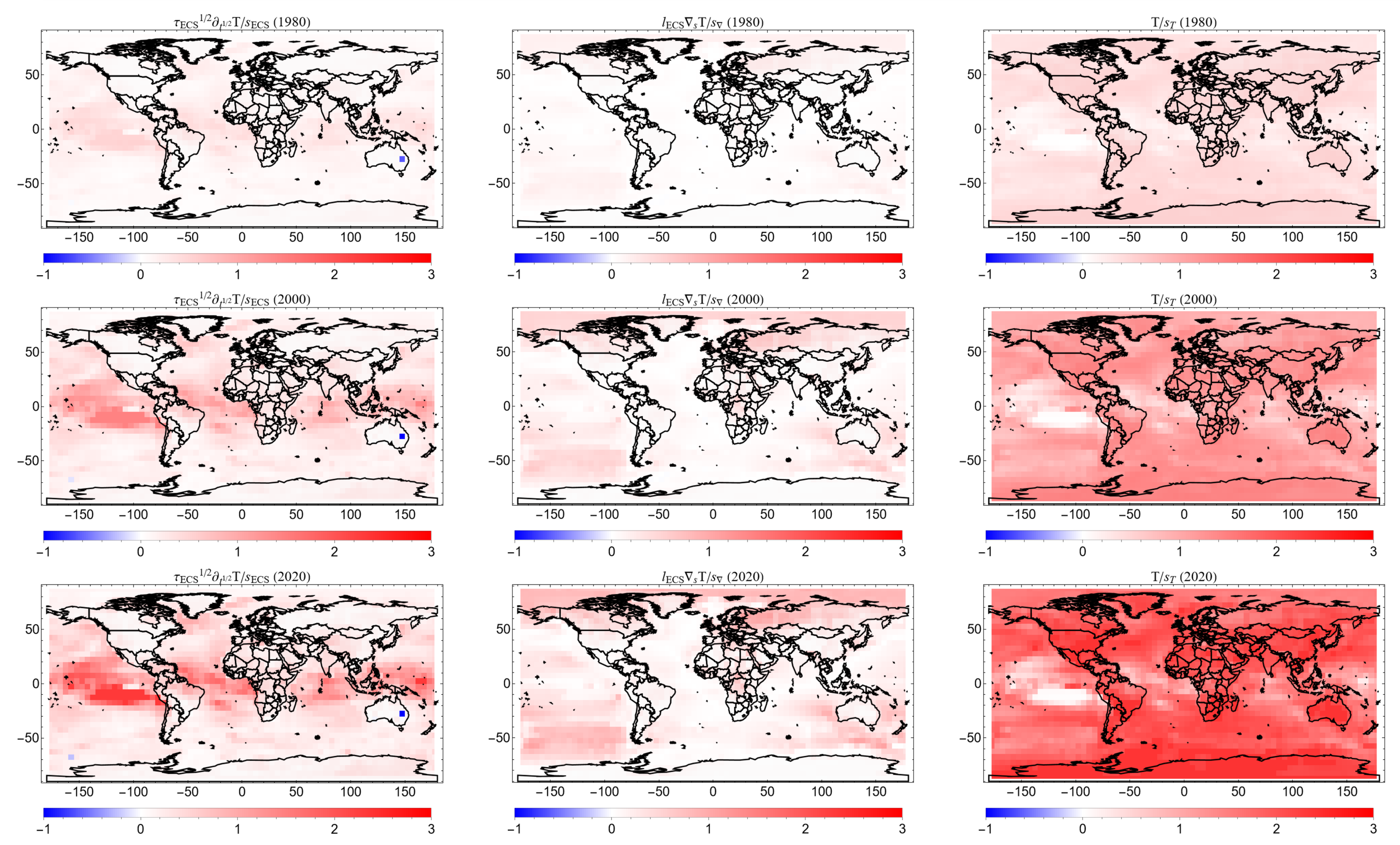


Radiative Forcing



The total deterministic forcing is the sum of green-house gases, aerosols, solar and volcano forcings. It is assumed to be spatially invariant across the entire globe and their respective contributions to the projected temperature could be added linearly due to the linear property of HEBE. A linear damping factor of 0.6 is applied onto the aerosol contribution [Procyk 2022]

Response Attribution Using Independent Pixel Approximation



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