USA, South Dakota, Brookings, Wednesday, January 22, 2014, 5:48 PM, -32F

The Substance of Uncertainty in Systemic Approach: Statistical Learning for Time Series of Geospheres: Natural Scientist's Point of View

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EGU26
GENERAL ASSEMBLY

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On campus of CCNY, NY, Aug 18, 2025
in Dr. David B. Steinman Hall

An Abstract

In the natural sciences, statistical learning for the geosphere time series (atmosphere, hydrosphere, cryosphere, lithosphere and biosphere) addresses the substance of uncertainty by treating the Earth as a coupled, multi-scale cyber system (J. Krcho, 1970). The time series are sequences of observation in time that reflect results of interaction subsystems of the Geosphere obtained as sequences of observation in time. The systemic approach shifts analysis from one dimensional time series to discover, describe and model complex interactions and cybernetics feedback loops across scales.

Statistical learning allows extraction of Hilbert Spaces from observations with results defining the Geosystems as fuzzy time-spatial structures (Zadeh) with dimensionality and quantitative characteristics of variability reflected from data. The substance of uncertainty may be defined as an ability for models to describe variability in data. From a natural scientist's perspective, uncertainty is not merely "noise" but a property of the systemic approach to modeling the Earth system's complexity.

The Hydrosphere is the most dynamic of the Geospheres and connects with all of them. The Hydrosphere may be described with nine interacting fuzzy elements: water of seas & oceans, stream runoff shell, water of closed lakes, atmospheric water, water of glaciers, water of permafrost rocks, connate groundwater, water trapped in rocks & minerals of lithosphere, and water of biosphere. The stream runoff shell includes* terrestrial stream network.

Model definition (Minsky, 1969) here includes a concept and kinds of coordinate system; a hierarchy of watersheds in the Hydrosphere; representation results of analysis of empirical data; representation of some* knowledge and new concepts.

Visualization results will be presented following the above concepts with interpretation on the example of two watersheds (USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN, USGS 06191500 Yellowstone River at Corwin Springs MT). Besides six models of these two mesoscale watersheds based on statistical learning of three types of fuzzy structures, the concept will be illustrated with reference to hydrological maps based on time spatial structure obtained by Statistical Learning with use of empirical data from the Great Lakes watershed of North America.

These models for watershed as an element of cyber model for Geosphere and results obtained them illustrates that the systemic approach with statistical learning on empirical data may be successful to find interactions for other Geospheres and bigger natural systems.

The Scientific Hydrology growled out from multiscale cartography surface and groundwater interaction for evaluating regional and global water resources (a Report by Gilbrich and Struckmeier "50 Years of Hydro(geo)logical Cartography", 2014 UNESCO CGWM IAN BGR) unfortunately as parallel branch to Stochastic Hydrology (Klemeš, Koutsoyiannis). The modern cartography of water resources taking root in concepts from Horton, Strahler, Kudelin, using statistical learning for quantitative description time spatial variability, is the scientific branch of Hydrology. Union of those two branches with joint efforts from scientists and engineers is certainly coming.

Topics

1- Key idea: Statistical learning in the Systemic Approach reveals the structure of Earth's complexity rather than smoothing it away.

• Earth system time series are not isolated measurements; they are expressions of interacting subsystems across scales. • To understand variability, we must treat the Earth as a coupled, multi-scale cybernetic system. • Uncertainty is not noise — it is a property of a model of an any subsystem in the Geosphere

2- Conceptual Framework: Cybernetic Geosphere.

• Earth as a system of interacting feedback loops. • Time series as outputs of subsystem interactions. • Systemic analysis moves beyond 1-D signals to multi-scale interactions

3- Why the Hydrosphere?

• Most dynamic of all geospheres represented as nine fuzzy elements: 1. Seas & oceans. 2. Stream-runoff shell. 3. Closed lakes. 4. Atmospheric water. 5. Glaciers. 6. Permafrost. 7. Connate groundwater. 8. Water in lithospheric rocks/minerals. 9. Water in the biosphere. • Connects to atmosphere, pedosphere, lithosphere & biosphere. • Acts as a carrier of information across scales

4 - The Stream-Runoff Shell.

• Includes the terrestrial river network. • Functions as a dynamic, branching information-transport system. • Has hierarchical identification & defining boundaries. • Ideal for statistical learning of time-spatial variability.
• Allows to have empirical time spatial data

5- Representing Knowledge & Model (Minsky, 1968).

A model includes: • Three components: natural object, model & researcher. • Choice of coordinate system. • Representation of empirical data. • Integration of existing knowledge (frame). • Introduction of new concepts (model) derived from statistical learning: a reflection of system complexity, a measurable property & a constraint on model expressiveness.

6- Statistical Learning as a Systemic Tool.

• Extracts Hilbert spaces from empirical data. • Dimensionality & variability emerge from the data. • Defines geosystems as fuzzy time-spatial structures (for Statistical Learning after Zadeh)

7- Case Studies.

Two mesoscale watersheds: • USGS 04010500 — Pigeon River at Middle Falls, MN/ • USGS 06191500 — Yellowstone River at Corwin Springs, MT.
We present: • Six models [101x12], [101x12D], [12Dx101] & [114x12], [114x12D], [12Dx114]. • Based on three types of fuzzy structures. • Derived from statistical learning on empirical, recalculated & transposed recalculated hydrological data

8- Great Lakes US Basin Example.

• Hydrological maps constructed from learned time-spatial structures. • Demonstrates scalability from mesoscale to regional systems. • Shows how fuzzy structures reveal hidden interactions

9- Hydrology: Scientific Hydrology.

• Rooted in multiscale cartography. • Surface–groundwater interactions in the Systemic Approach. • Regional/global water-resource assessment
• Report 50 Years of Hydro(geo)logical Cartography, Gilbrich & Struckmeier, 2014. Scientific Hydrology & Stochastic Hydrology are two traditions. These traditions evolved in parallel, but not together..

10- Hydrology: Stochastic Hydrology.

• Variability, randomness, long-memory processes. • (Klemeš; Koutsoyiannis). Scientific Hydrology & Stochastic Hydrology are two traditions. These traditions evolved in parallel, but not together.

11- Toward a Unified Structural Hydrology.

• In Search for Hydrology as a Natural Science. • The Theory in Hydrology & General System Theory. • A call to unify Science.

12- Conclusion.

• Presented part of modern natural science with name of Structural Hydrology • Statistical learning provides a systemic, cybernetic view of the Geosphere • Fuzzy time-spatial structures capture the essence of variability
• Watersheds serve as powerful units for modeling Geospheric interactions • This approach opens a path toward unified hydrological science.

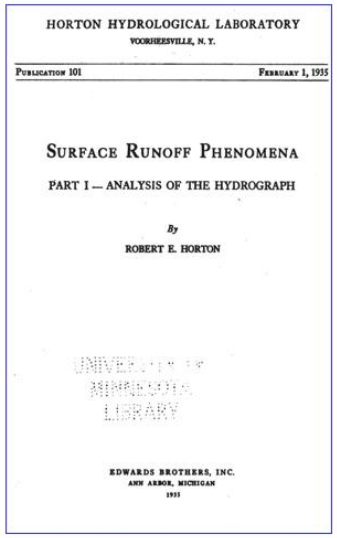
1 – Key idea: Statistical Learning in the Systemic

Approach reveals the structure of Earth's complexity rather than smoothing it away

- Earth system time series are not isolated measurements, they are expressions of interacting subsystems across scales
- To understand variability, we must treat the Earth as a coupled, multi-scale cybernetic system
- Uncertainty is not noise — it is a property of a model of an any subsystem in the Geosphere

One dimensional analysis & the way of visualization of the discharge data by founder & AI now

The concept of hydrograph analysis by R. F. Horton (1935)



HORTON HYDROLOGICAL LABORATORY
VOORHEESVILLE, N. Y.

PUBLICATION 101 — FEBRUARY 1, 1935

SURFACE RUNOFF PHENOMENA

PART I — ANALYSIS OF THE HYDROGRAPH

By

ROBERT E. HORTON

UNIVERSITY OF MINNESOTA LIBRARY

EDWARDS BROTHERS, INC.
ANN ARBOR, MICHIGAN
1935

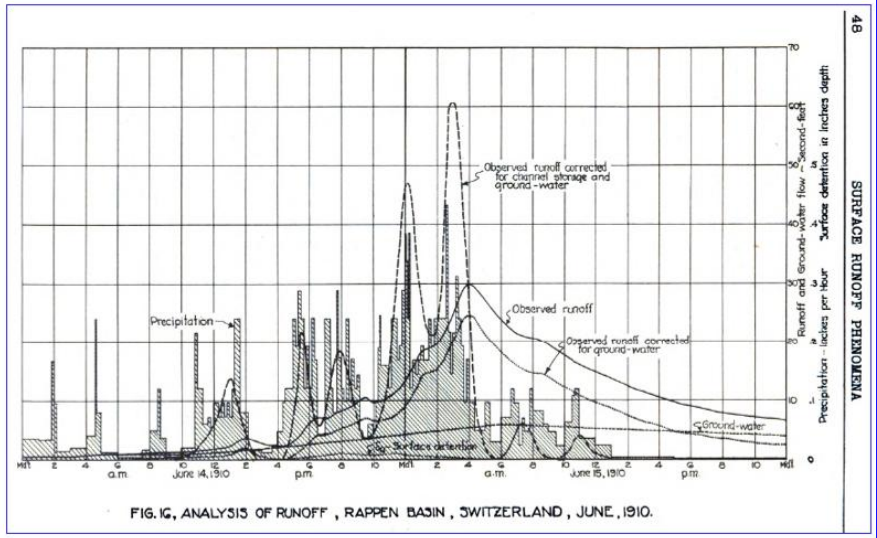
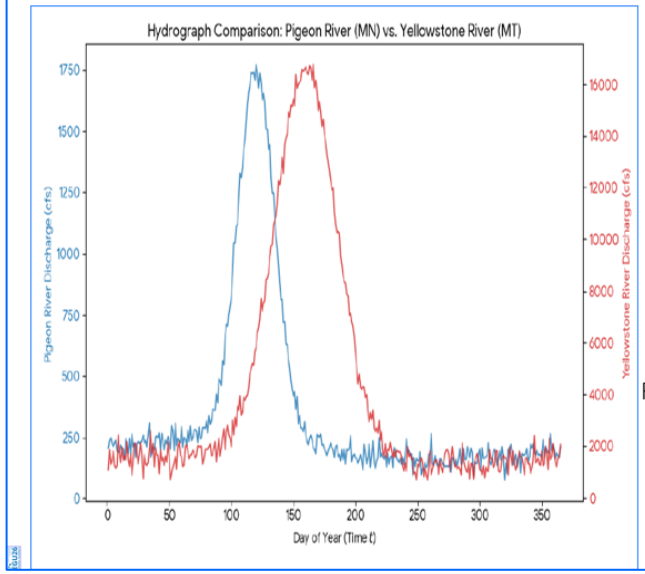


FIG. IG., ANALYSIS OF RUNOFF, RAPPEN BASIN, SWITZERLAND, JUNE, 1910.

Characteristics of the two watersheds by Gemini



The following data points reflect the empirical representation of these two distinct subsystems:

Pigeon River at Middle Falls, MN (USGS 04010500): Drainage Area: 609 square miles. Hydrologic Unit Code (HUC): 04010101. Context: Located in the Northern Lakes and Forests ecoregion, characterized by humid continental climates and significant snowmelt runoff in early spring.

Yellowstone River at Corwin Springs, MT (USGS 06191500): Drainage Area: 2,615 square miles. Hydrologic Unit Code (HUC): 10070002. Annual Runoff Ratio: Approximately 54.6%. Dynamics: Exhibits a high-volume snow-driven hydrograph with peaks typically occurring in June. Recent extreme events, such as the 2022 flood (peaking at 43,300 cfs), illustrate the "fuzzy" nature of these systems where record-breaking variability challenges traditional static models.

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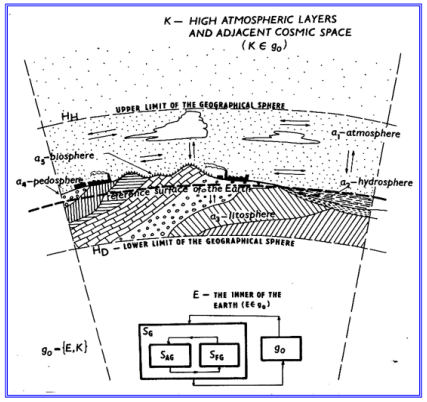
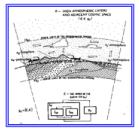
- Earth as a system of interacting feedback loops

A Cybernetic Model for Geosphere of the Earth



Vertical slice of the Geographical Sphere with two independent elements: System of Anthropological Geography (S_{AG}) & System of Physical Geography (S_{FG}). Arrows indicate vertical & horizontal components of matter, energy & information circulation *(after Krcho, 1978)*

Components of Physical Geography Sphere



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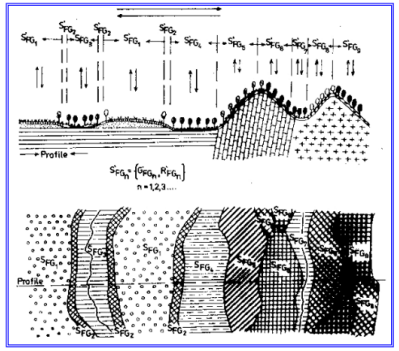

The System of Physical Geography Sphere (S_{FG}) with five independent elements: a_1- atmosphere, a_2- hydrosphere, a_3- lithosphere, a_4- pedosphere, a_5- biosphere. On cross-section & map shown subsystems for Sa_3, Sa_4 & Sa_5.

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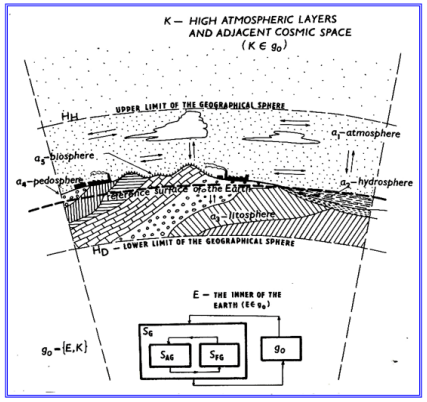

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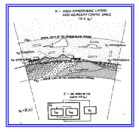

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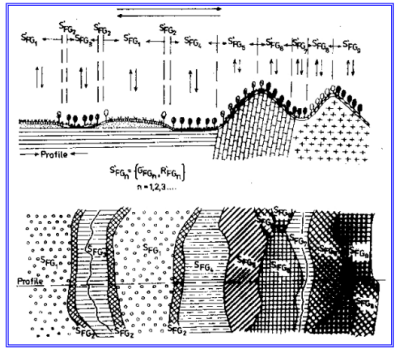

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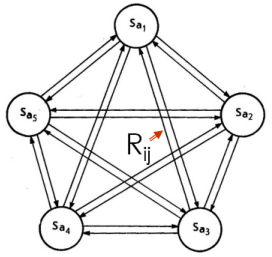
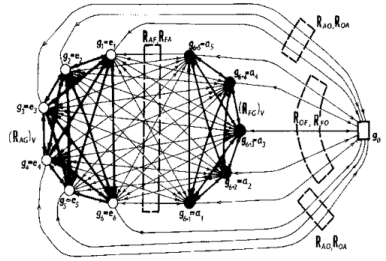
Structures & Matrixes of Relations *(after Krcho, 1978)*



{Rij} is a matrix of relations between components of the landscape



The number of characteristics for elements of landscape is unlimited & the number is unlimited for dependences too

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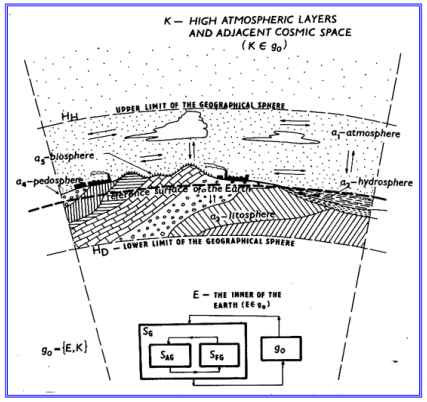

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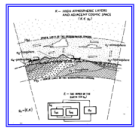

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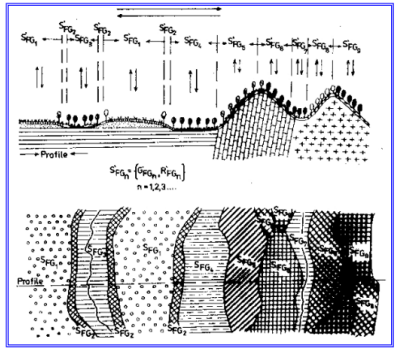

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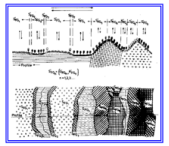
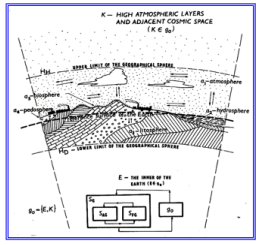
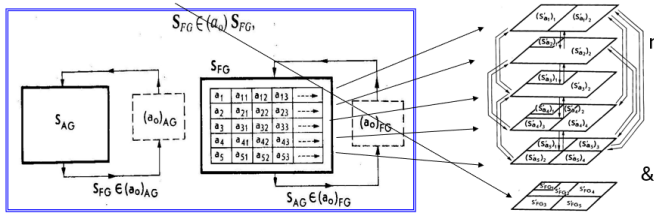


Every element $a_1 – a_5$ of S_{FG} is a System Sa_i & consists of units: $a_1(a_{11}, a_{12}, a_{13} \ldots)$, $a_2(a_{21}, a_{22} \ldots)$, … a_5 & those units may be considered as a subsystems Sa_{ij}.





Every **Sa_i** & **Sa_{ij}** may be characterized by matrix of input {**Wi**}, matrix of output {**Qi**}, & matrix of states {**Hi**}. *(after Krcho, 1978)*

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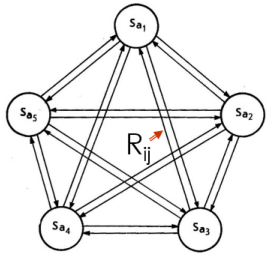

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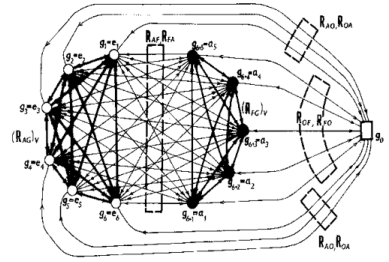

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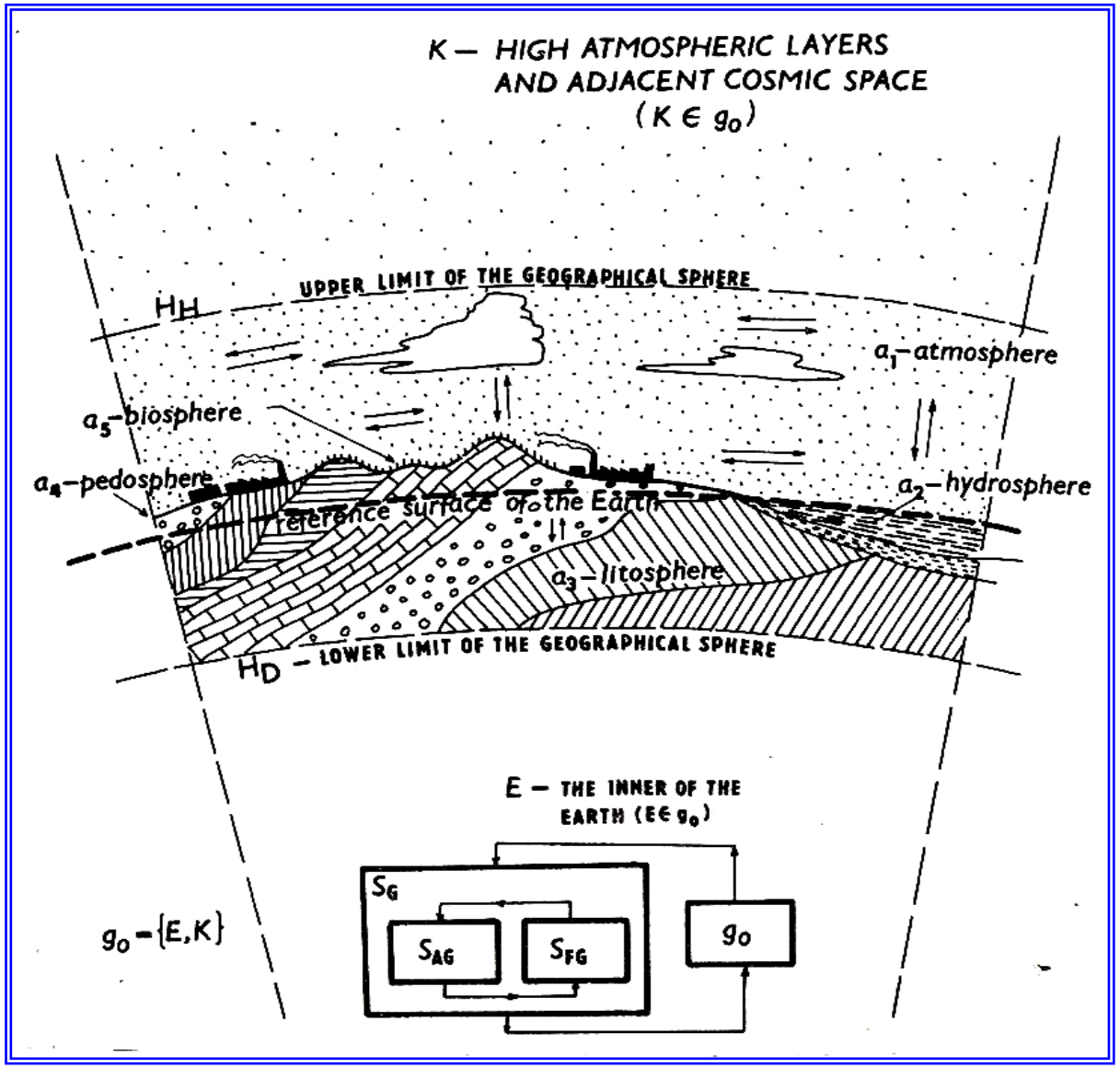

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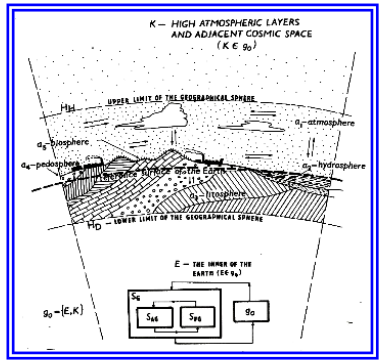

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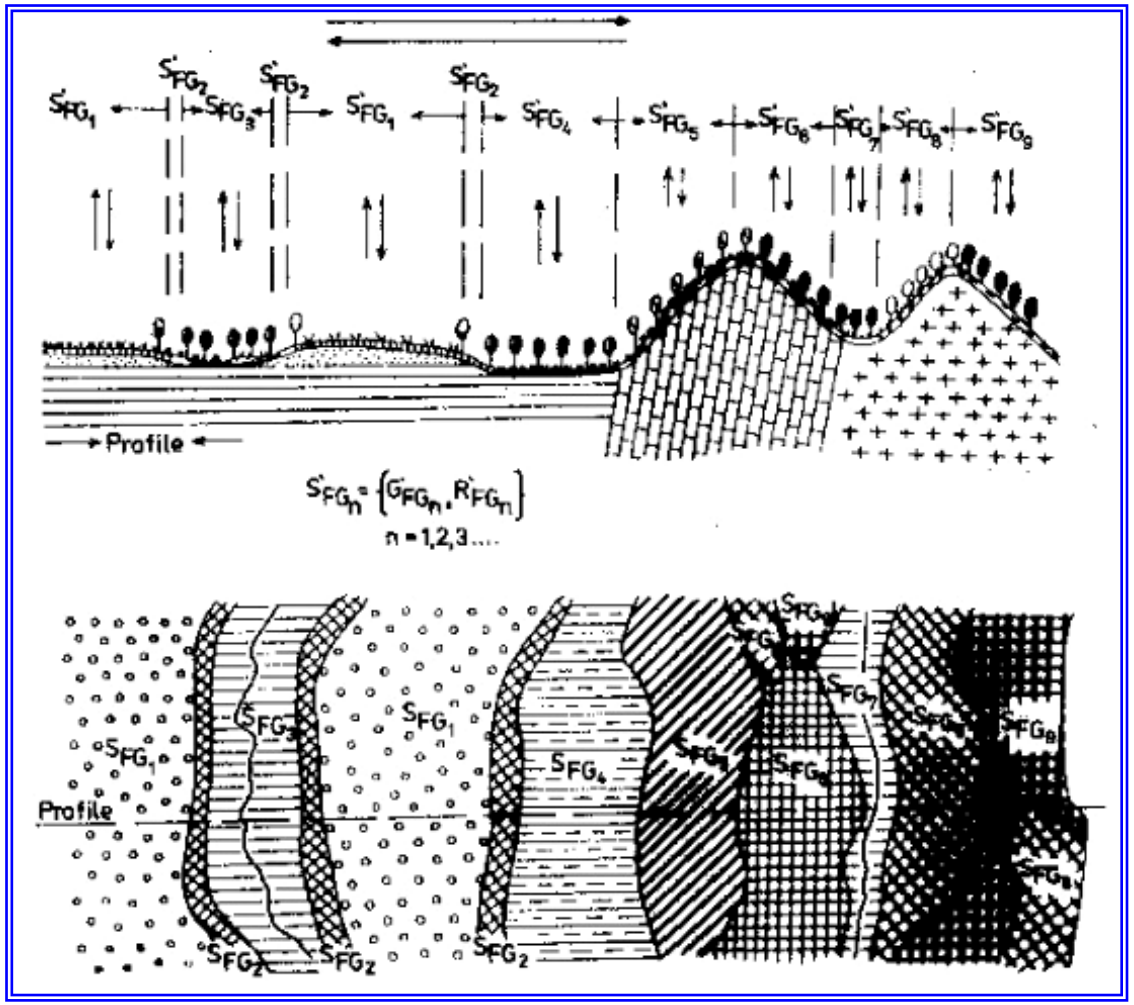

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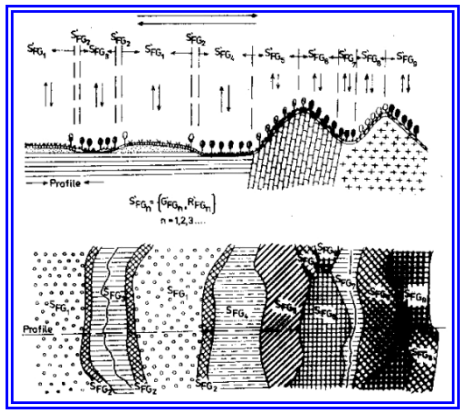

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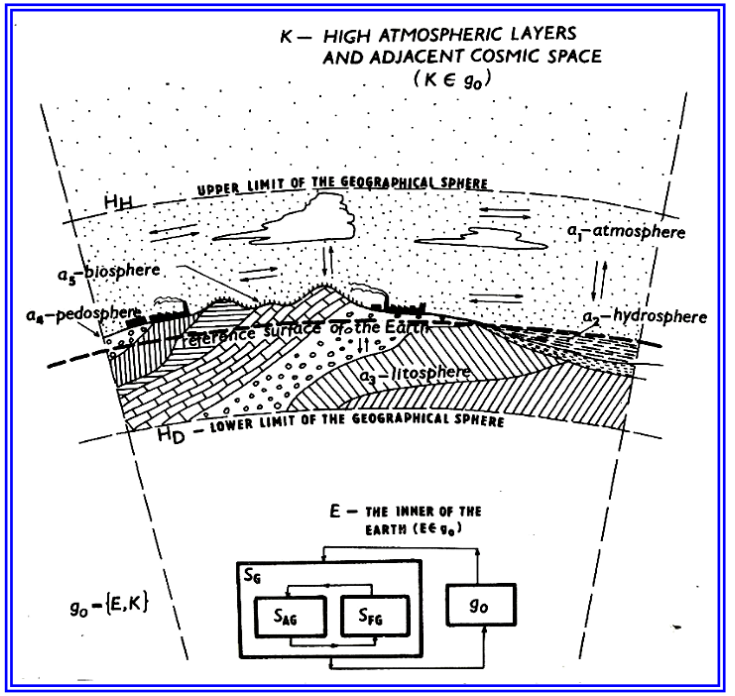

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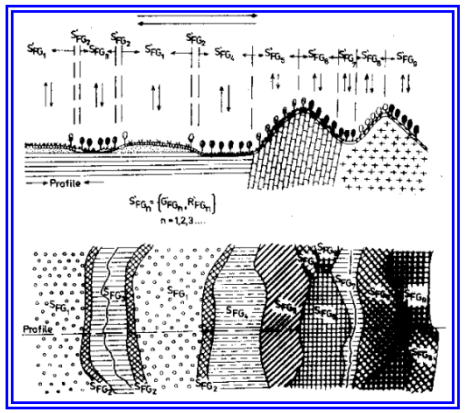

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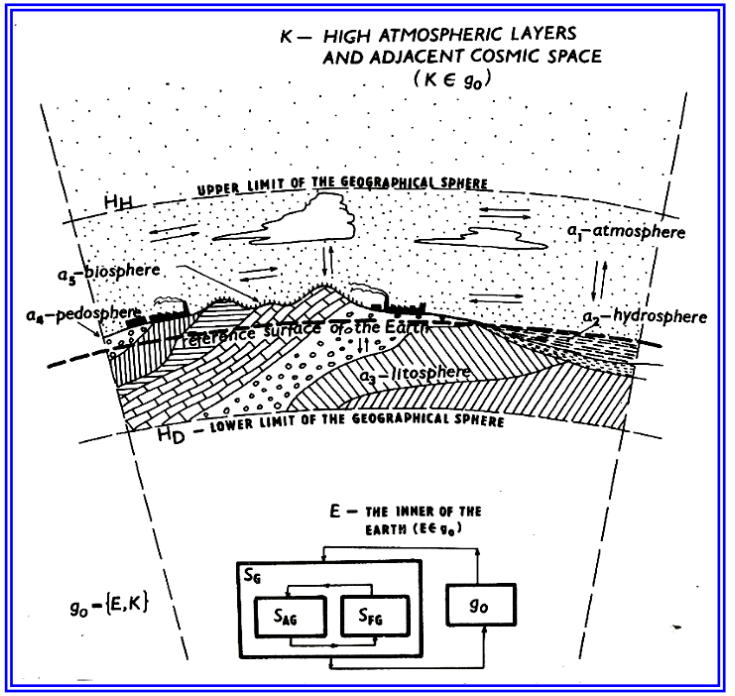

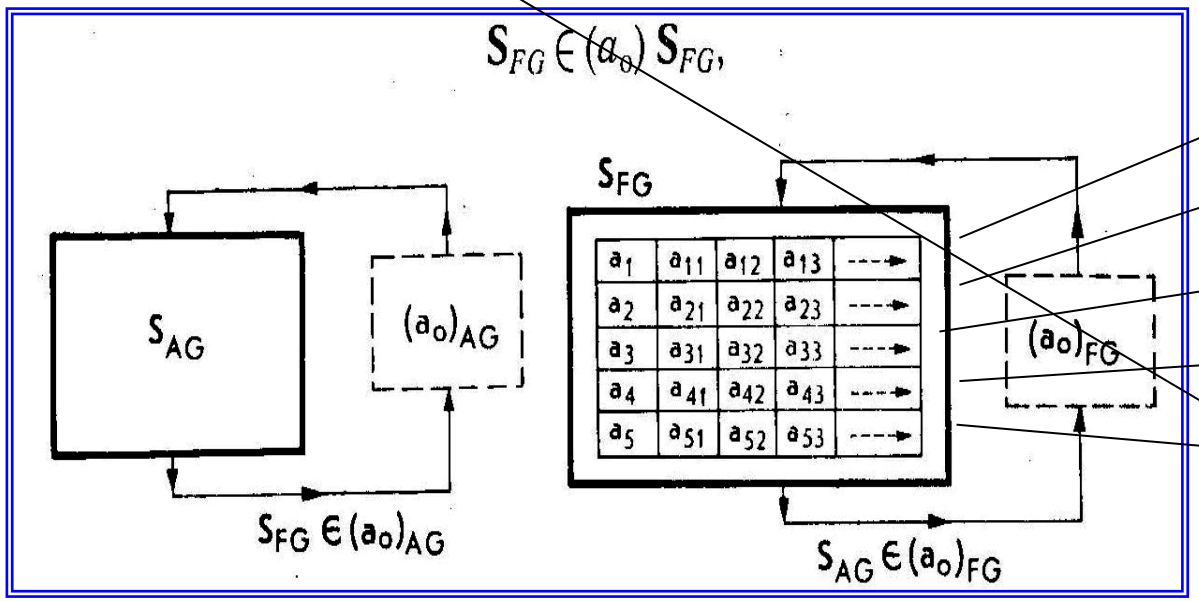

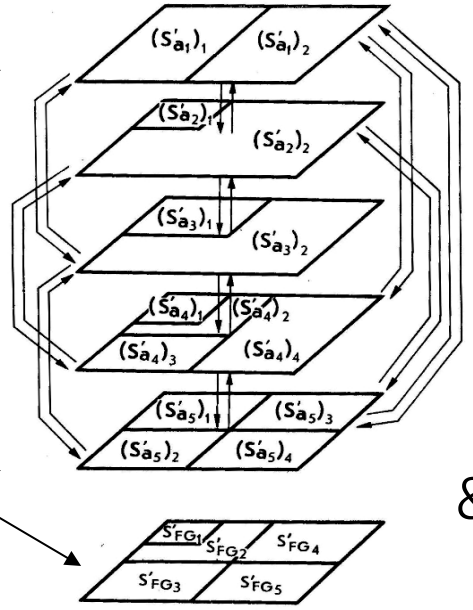

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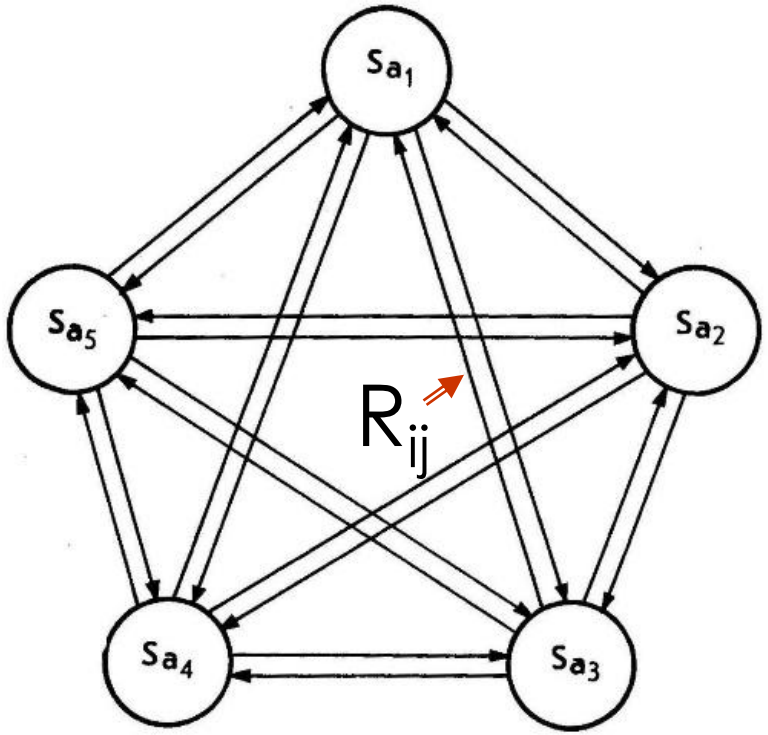

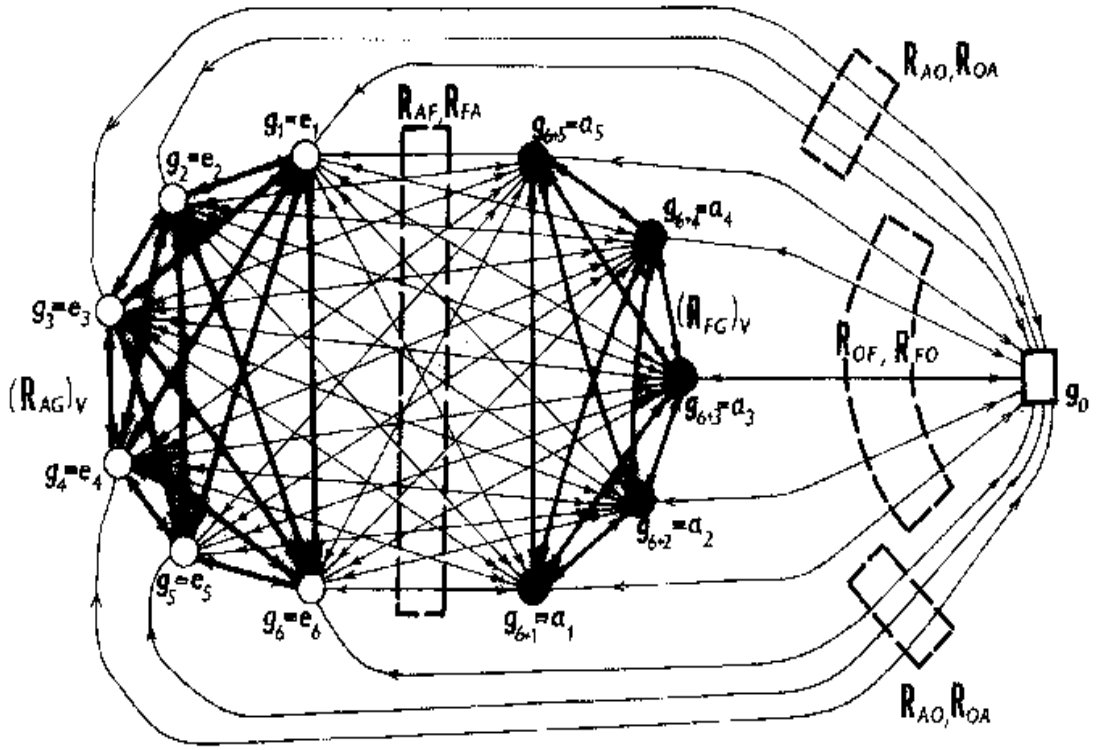

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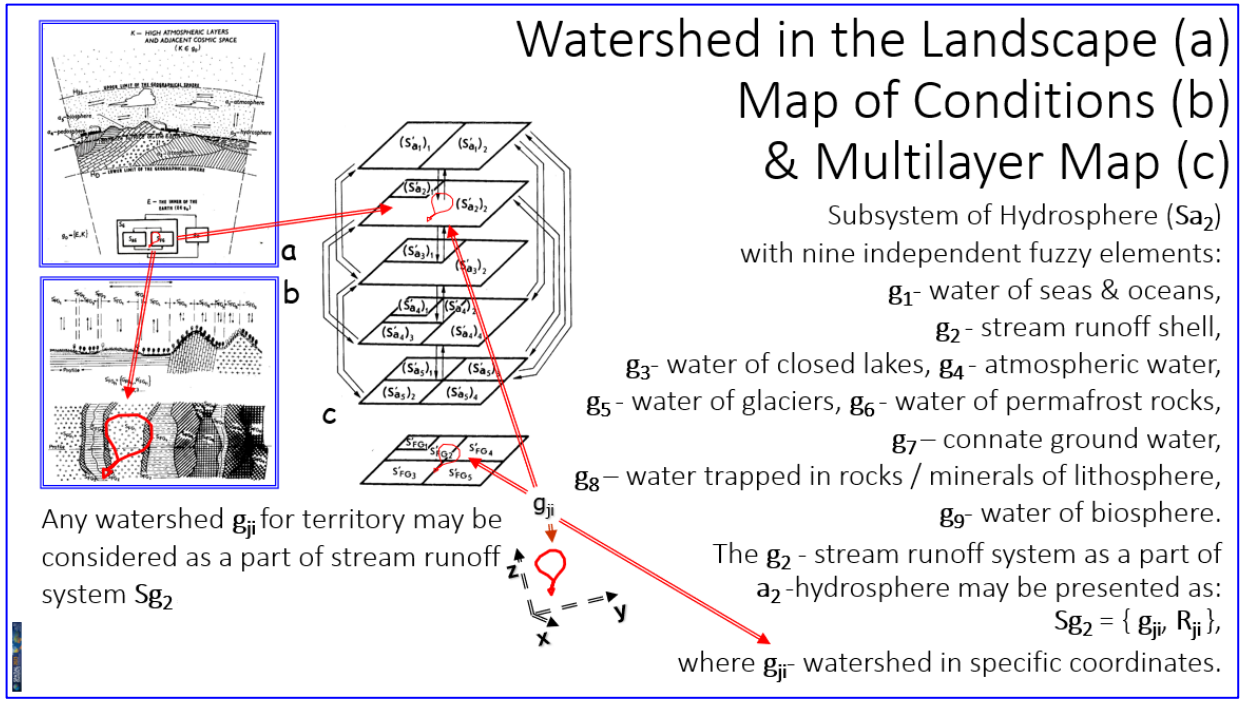
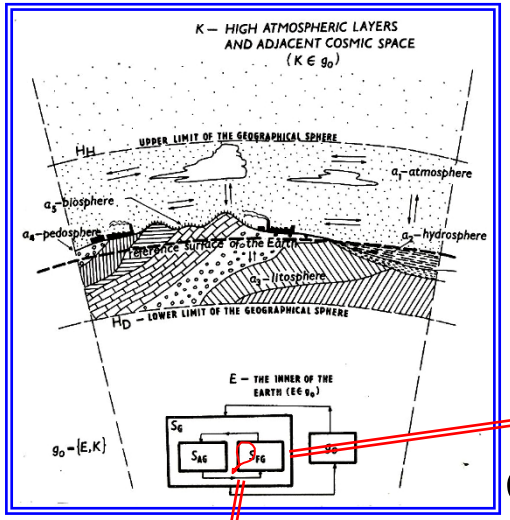
- Most dynamic of all geospheres represented as nine fuzzy elements:
  1. Seas & oceans. **2**. Stream-runoff shell. **3**. Closed lakes. **4**. Atmospheric water. **5**. Glaciers. **6**. Permafrost. **7**. Connate groundwater. **8**. Water in lithospheric rocks/minerals. **9**. Water in the biosphere
- Connects to atmosphere, pedosphere, lithosphere & biosphere
- Acts as a carrier of information across scales



Watershed in the Landscape (a) Map of Conditions (b) & Multilayer Map (c)



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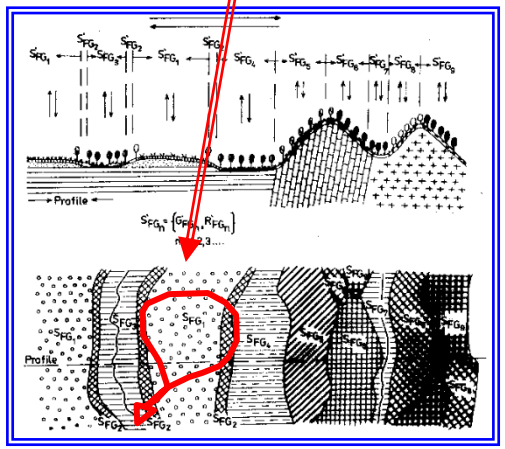

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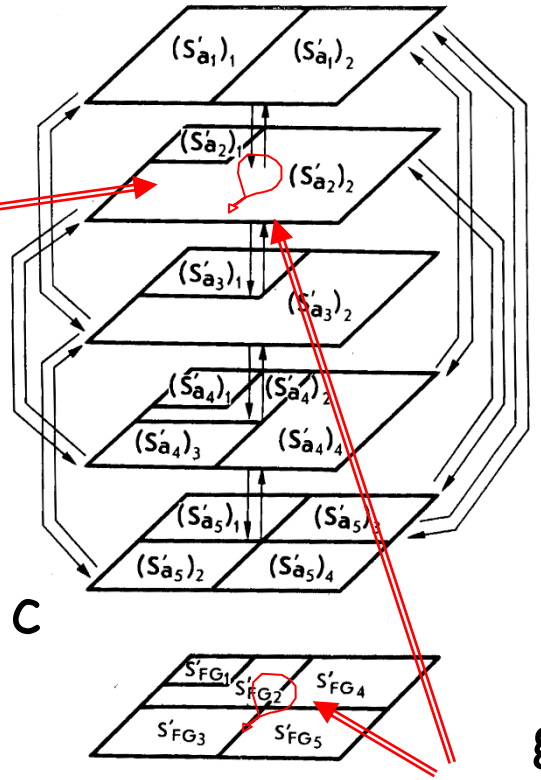
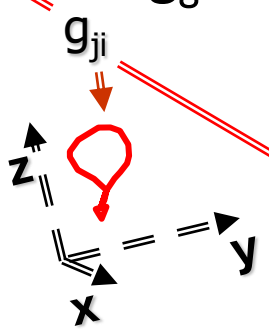


c

Any watershed g_{ji} for territory may be considered as a part of stream runoff system Sg_2



Subsystem of Hydrosphere (Sa_2) with nine independent fuzzy elements:

g_1- water of seas & oceans,

g_2 - stream runoff shell,

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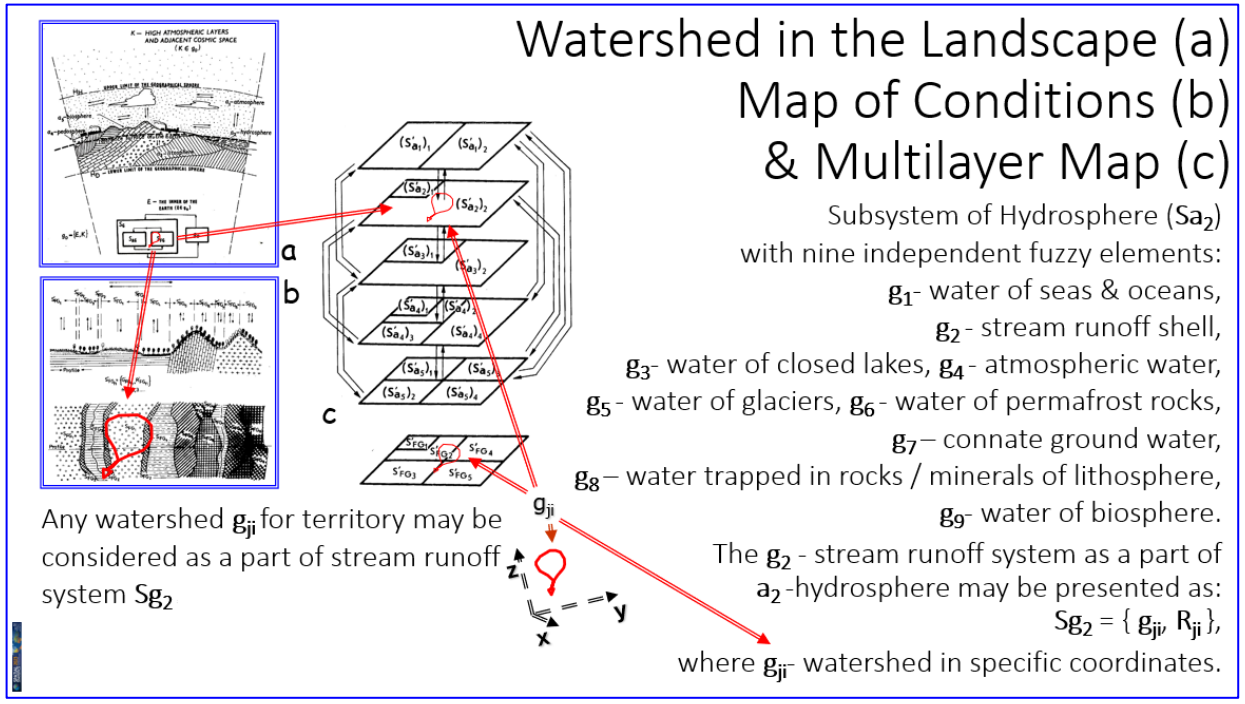
The g_2 - stream runoff system as a part of a_2 -hydrosphere may be presented as:

$$Sg_2 = \{ g_{ji}, R_{ji} \},$$

where g_{ji}- watershed in specific coordinates.

4 – The Stream-Runoff Shell

- Includes the terrestrial river network
- Functions as a dynamic, branching information-transport system
- Has hierarchical identification & defining boundaries
- Ideal for statistical learning of time-spatial variability
- Allows to have empirical time spatial data



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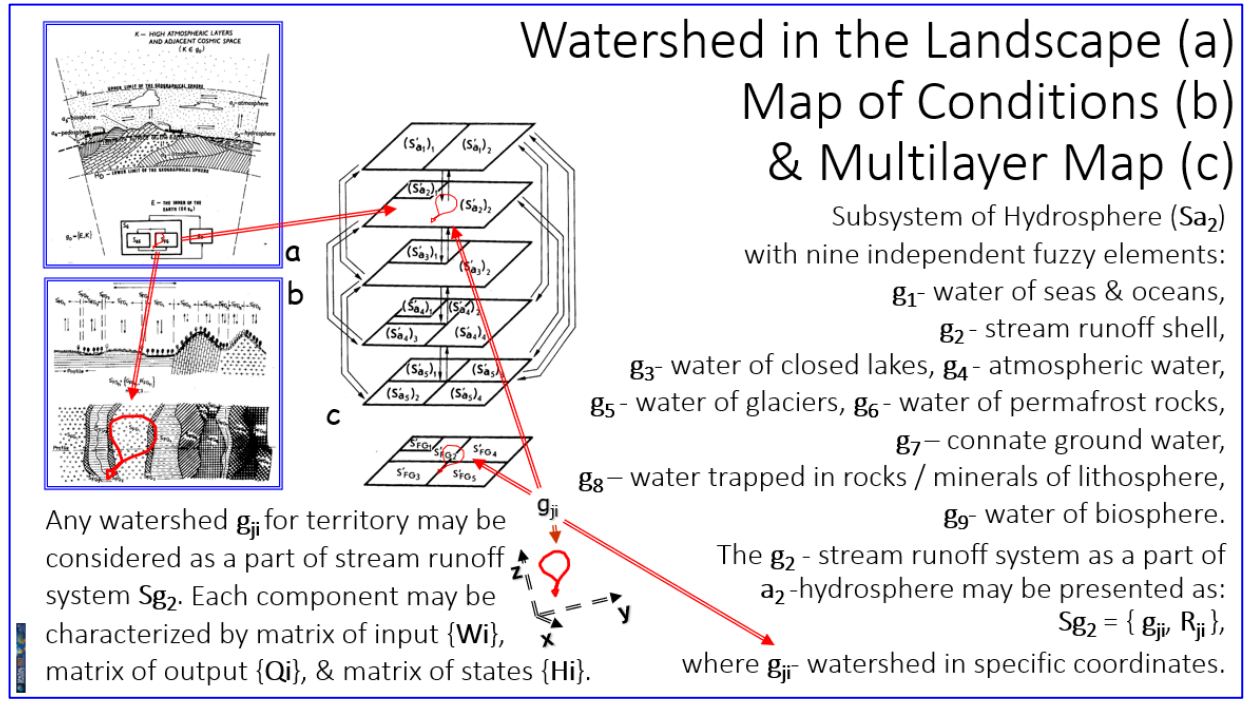

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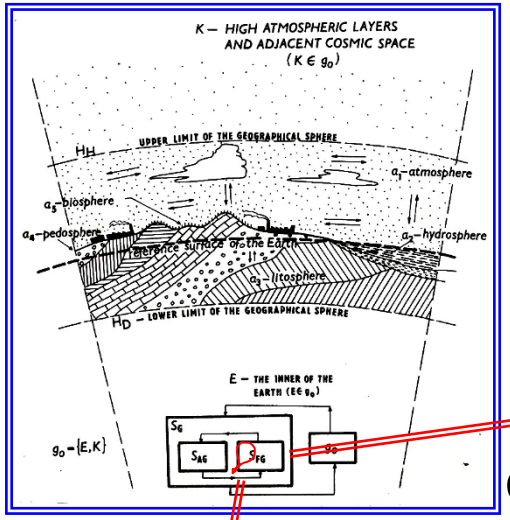

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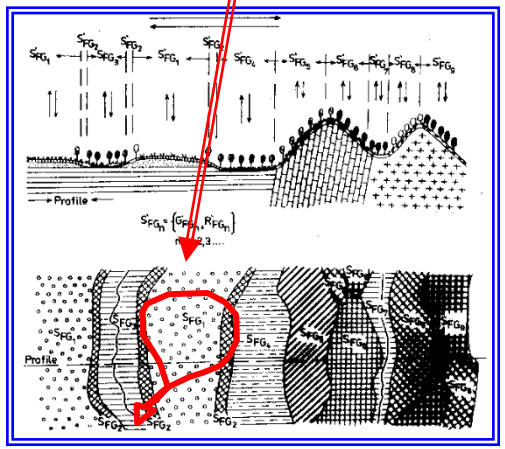

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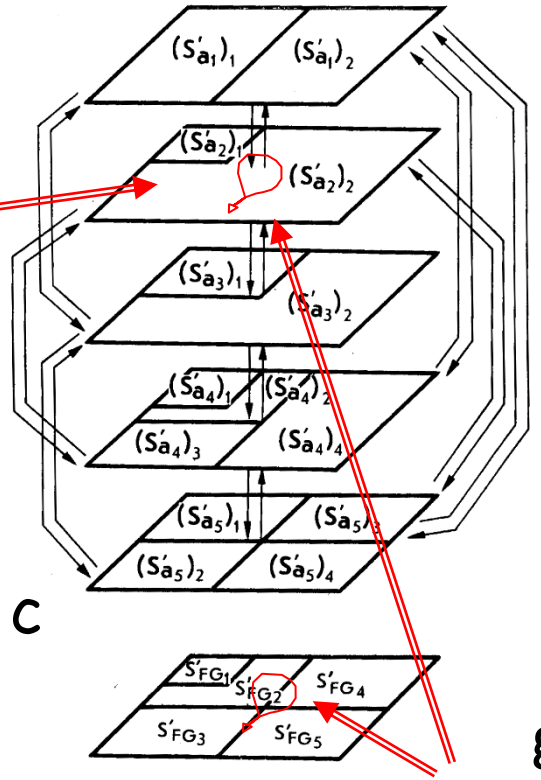

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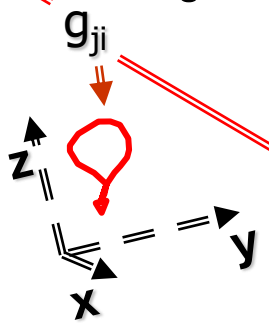

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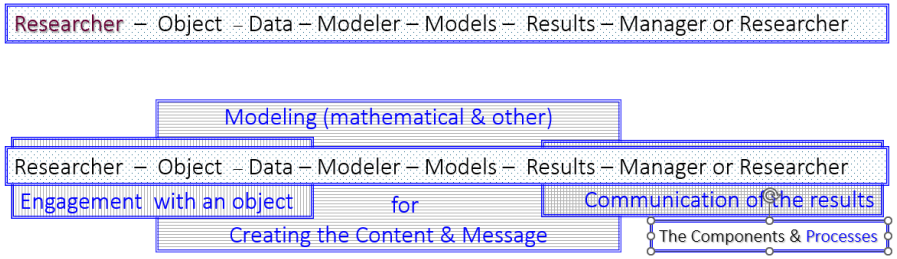
- Three components: natural object, model & researcher

Researcher Is the Principal Component in the System



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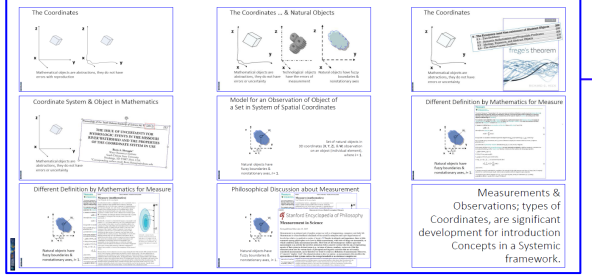
Engagement with an object for Communication of the results

Creating the Content & Message

The Components & Processes

Coordinates, Measurements & Observations



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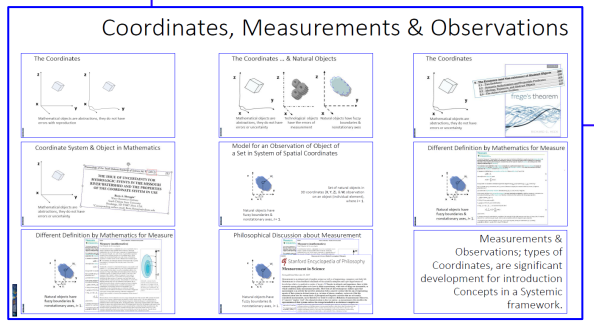

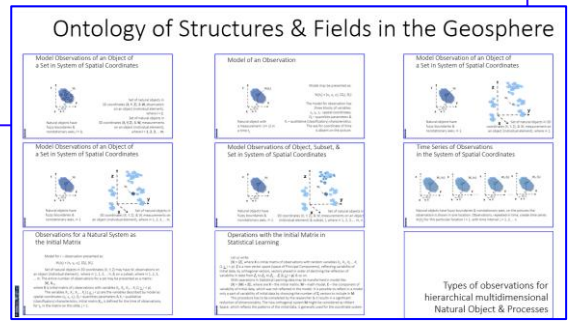

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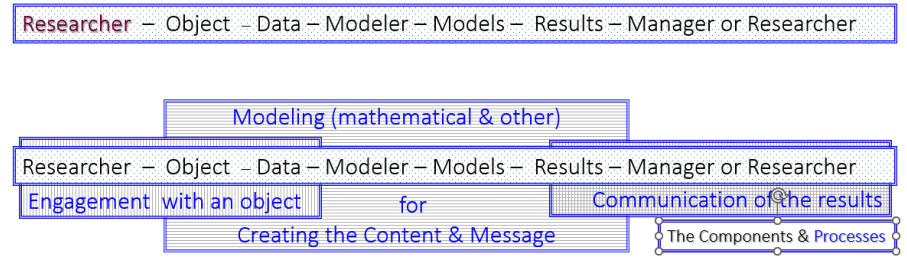

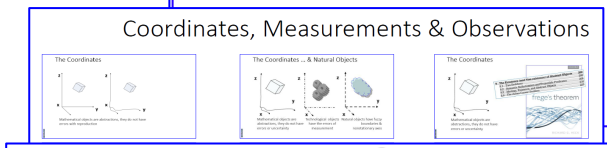

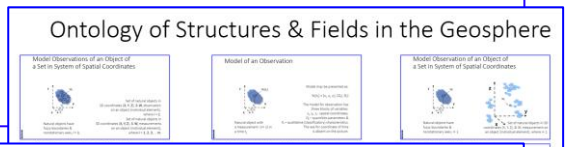

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From Scientific Concepts & Theories to Engineering, & now back to Cybernetic

"… new ideas were already starting to grow before computers created a more urgent need. Before programming languages, mathematicians such as Emil Post, Kurt Gödel, Alonzo Church, & Alan Turing already had many related ideas. In the 1940s these ideas began to spread … In the same period, there were similar movements in psychology, as Sigmund Freud, Konrad Lorenz, Nikolaas Tinbergen, & Jean Piaget also tried to imagine advanced architectures for "mental computation". In the same period, in neurology, there were my own early mentors Nicholas Rashevsky, Warren McCulloch & Walter Pitts, Norbert Wiener, & their followers & all those new ideas began to coalesce under the name "cybernetics".

Unfortunately, that new domain was mainly dominated by continuous mathematics & feedback theory. This made cybernetics slow to evolve more symbolic computational viewpoints, & the new field of Artificial Intelligence headed off to develop distinctly different kinds of psychological models."

From "A TALK WITH Marvin Minsky [Edge 2.26.98]"

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A model includes:

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- Representation of empirical data
- Integration of existing knowledge (frame)
- Introduction of new concepts

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New concepts derived from statistical learning:

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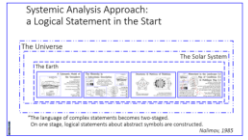

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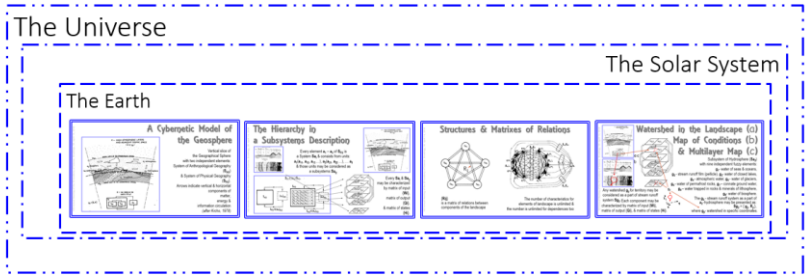
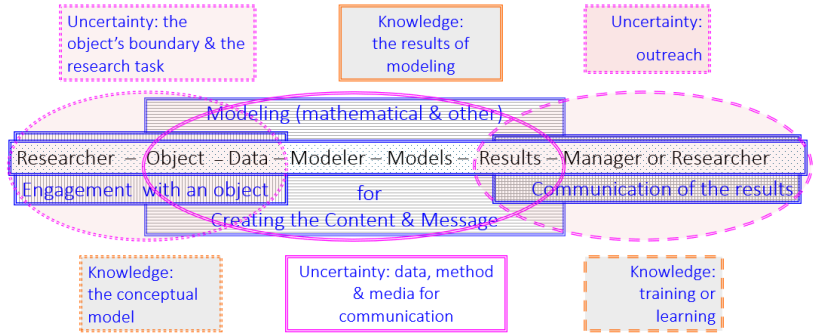
Systemic Analysis Approach: a Logical Statement in the Start



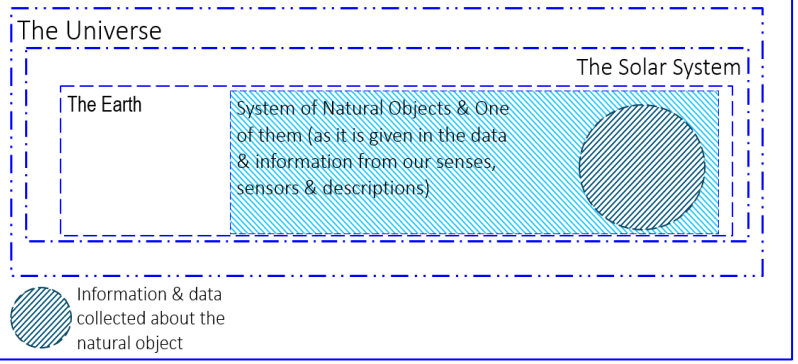
"The language of complex statements becomes two-staged.
On one stage, logical statements about abstract symbols are constructed.

Nalimov, 1985

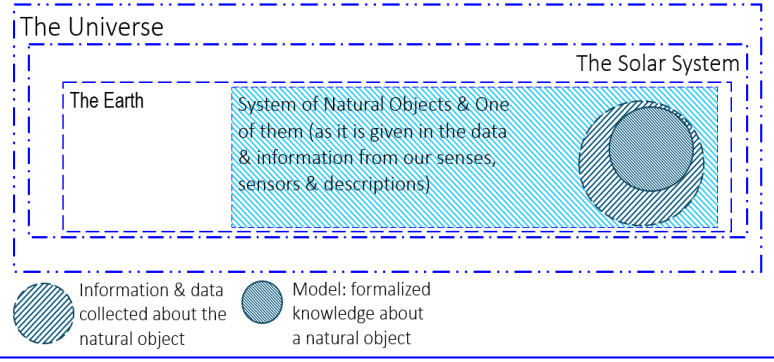
The Knowledge & Uncertainty for Components in the Systemic Analysis



Systemic Analysis Approach: Natural Object as it Is Given by Data



Systemic Analysis Approach: the Model for an Object



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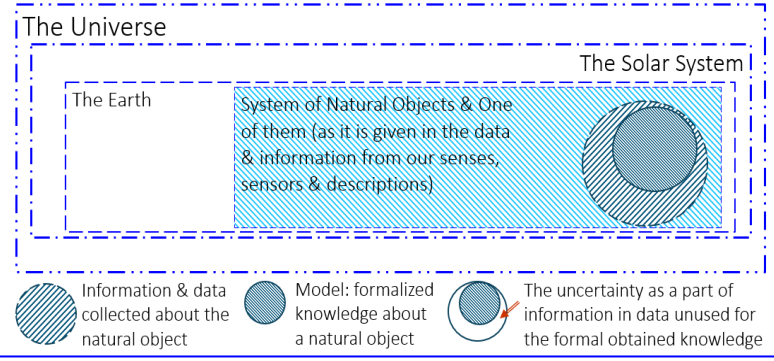

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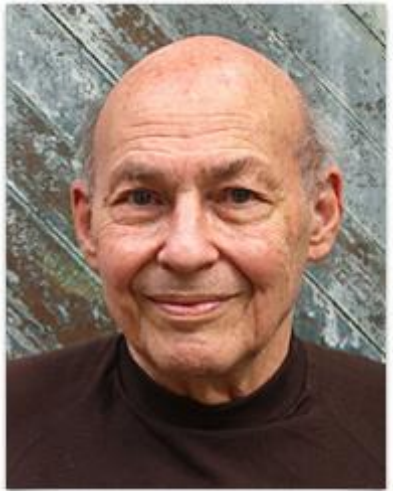

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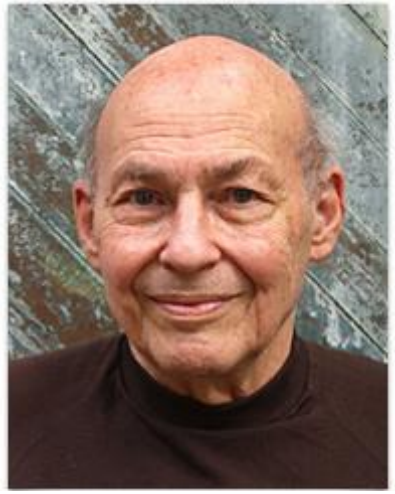

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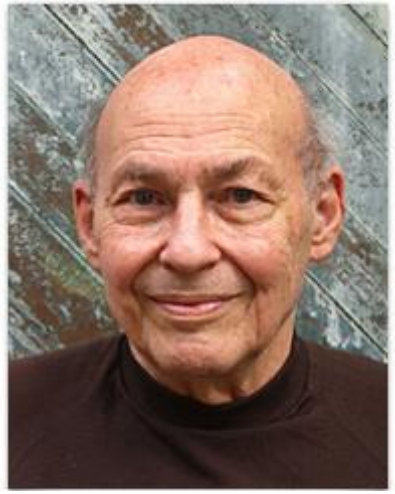

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The New Concept: about Frames (Structures)

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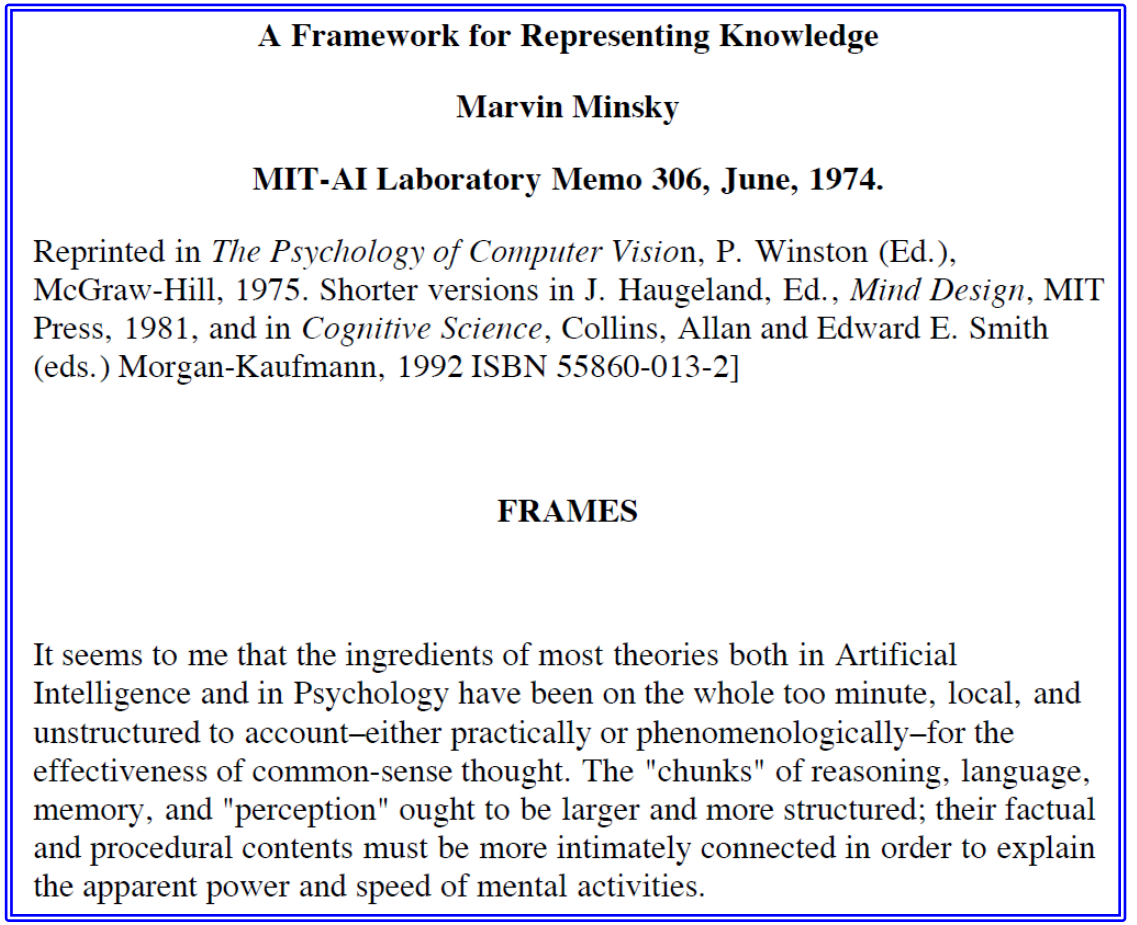


A Framework for Representing Knowledge

Marvin Minsky

MIT-AI Laboratory Memo 306, June, 1974.

Reprinted in *The Psychology of Computer Vision*, P. Winston (Ed.), McGraw-Hill, 1975. Shorter versions in J. Haugeland, Ed., *Mind Design*, MIT Press, 1981, and in *Cognitive Science*, Collins, Allan and Edward E. Smith (eds.) Morgan-Kaufmann, 1992 ISBN 55860-013-2]

FRAMES

It seems to me that the ingredients of most theories both in Artificial Intelligence and in Psychology have been on the whole too minute, local, and unstructured to account–either practically or phenomenologically–for the effectiveness of common-sense thought. The "chunks" of reasoning, language, memory, and "perception" ought to be larger and more structured; their factual and procedural contents must be more intimately connected in order to explain the apparent power and speed of mental activities.

Marvin Minsky "A Framework for Representing Knowledge" 1974, p. 1.

From Scientific Concepts & Theories to Engineering

"… new ideas were already starting to grow before computers created a more urgent need. Before programming languages, mathematicians such as Emil Post, Kurt Gödel, Alonzo Church, & Alan Turing already had many related ideas. In the 1940s these ideas began to spread … In the same period, there were similar movements in psychology, as Sigmund Freud, Konrad Lorenz, Nikolaas Tinbergen, & Jean Piaget also tried to imagine advanced architectures for "mental computation". In the same period, in neurology, there were my own early mentors Nicholas Rashevsky, Warren McCulloch & Walter Pitts, Norbert Wiener, & their followers & all those new ideas began to coalesce under the name "cybernetics".

From "A TALK WITH Marvin Minsky [Edge 2.26.98]"

From Scientific Concepts & Theories to Engineering, & now back to Cybernetic

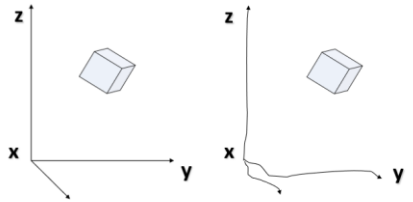
"… new ideas were already starting to grow before computers created a more urgent need. Before programming languages, mathematicians such as Emil Post, Kurt Gödel, Alonzo Church, & Alan Turing already had many related ideas. In the 1940s these ideas began to spread … In the same period, there were similar movements in psychology, as Sigmund Freud, Konrad Lorenz, Nikolaas Tinbergen, & Jean Piaget also tried to imagine advanced architectures for "mental computation". In the same period, in neurology, there were my own early mentors Nicholas Rashevsky, Warren McCulloch & Walter Pitts, Norbert Wiener, & their followers & all those new ideas began to coalesce under the name "cybernetics".

Unfortunately, that new domain was mainly dominated by continuous mathematics & feedback theory. This made cybernetics slow to evolve more symbolic computational viewpoints, & the new field of Artificial Intelligence headed off to develop distinctly different kinds of psychological models."

From "A TALK WITH Marvin Minsky [Edge 2.26.98]"

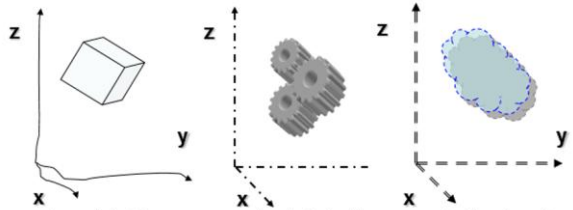
Coordinates, Measurements & Observations

The Coordinates



Mathematical objects are abstractions, they do not have errors with reproduction

The Coordinates … & Natural Objects

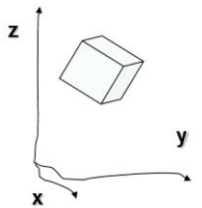


Mathematical objects are abstractions, they do not have errors or uncertainty

Technological objects have the errors of measurement

Natural objects have fuzzy boundaries & nonstationary axes

The Coordinates



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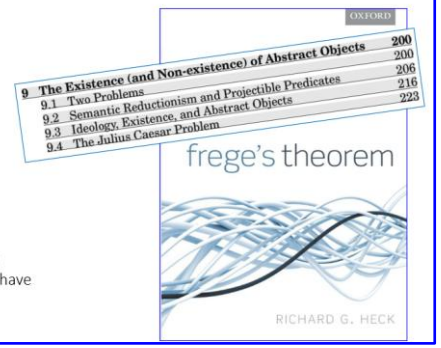
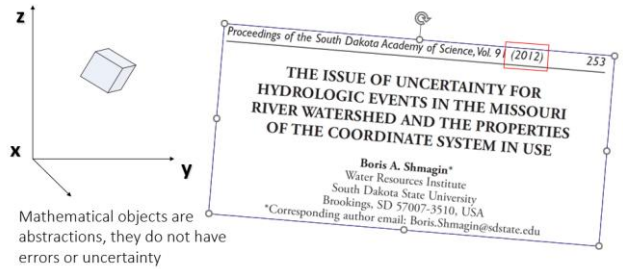
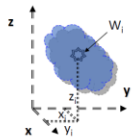


Coordinate System & Object in Mathematics



Mathematical objects are abstractions, they do not have errors or uncertainty

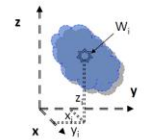
Model for an Observation of Object of a Set in System of Spatial Coordinates



Natural objects have fuzzy boundaries & nonstationary axes, $i = 1$.

Set of natural objects in 3D coordinates (X, Y, Z), & W_i observation on an object (individual element), where $i = 1$.

Different Definition by Mathematics for Measure



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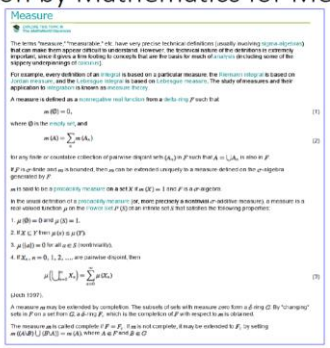
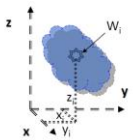


Different Definition by Mathematics for Measure



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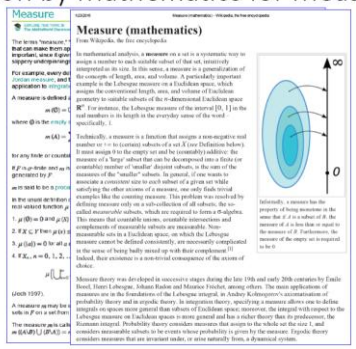
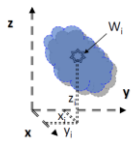


Philosophical Discussion about Measurement



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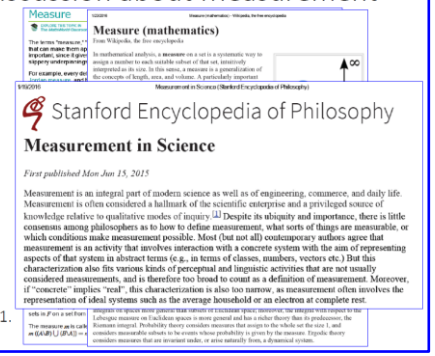
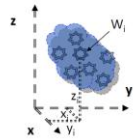


Measurements & Observations; types of Coordinates, are significant development for introduction Concepts in a Systemic framework.

Ontology of Structures & Fields in the Geosphere

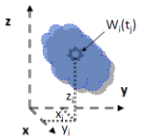
Model Observations of an Object of a Set in System of Spatial Coordinates



Natural objects have fuzzy boundaries & nonstationary axes, $i = 1$.

Set of natural objects in 3D coordinates (**X**, **Y**, **Z**), & $\mathbf{W}_i$ observation on an object (individual element), where $i = 1$.
Set of natural objects in 3D coordinates (**X, Y, Z**), & $\mathbf{W}_i$ measurements on an object (individual element), where $i = 1, 2, 3, \ldots m$.

Model of an Observation



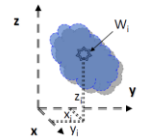
Natural object with a measurement i ($i = 1$) in a time t_j.

Model may be presented as:

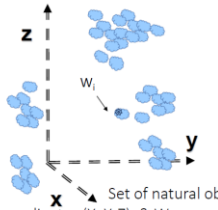
$$W_i(t_j) = [x_i, y_i, z_i], [Q_i], [K_i]$$

The model for observation has three blocks of variables:
x_i, y_i, z_i - spatial coordinates,
Q_i – quantities parameters &
K_i – qualitative (classificatory) characteristics.
The axe for coordinate of time is absent on the picture.

Model Observation of an Object of a Set in System of Spatial Coordinates

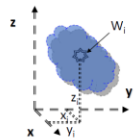


Natural objects have fuzzy boundaries & nonstationary axes, $i = 1$.



Set of natural objects in 3D coordinates (X, Y, Z), & W_i measurement on an object (individual element), where $i = 1$.

Model Observations of an Object of a Set in System of Spatial Coordinates



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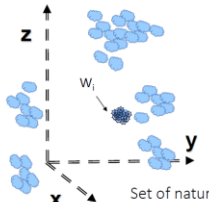
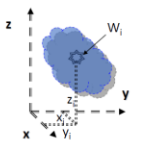
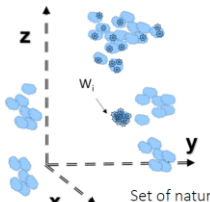


Set of natural objects in 3D coordinates (X, Y, Z), & W_i measurements on an object (individual element), where $i = 1, 2, 3, \ldots m$.

Model Observations of Object, Subset, & Set in System of Spatial Coordinates

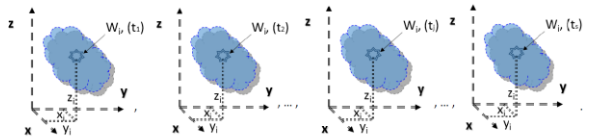


Natural objects have fuzzy boundaries & nonstationary axes, $i = 1$.



Set of natural objects in 3D coordinates (X, Y, Z), & W_i measurements on an object (individual element) & subset,), where $i = 1, 2, 3, \ldots m, n$.

Time Series of Observations in the System of Spatial Coordinates



Natural objects have fuzzy boundaries & nonstationary axes, on the pictures the observation is shown in one location. Observations, repeated in time, create time series $W_i(t_j)$ for this particular location I = 1, with time interval: j = 1, 2, ... s.

Observations for a Natural System as the Initial Matrix

Model for i – observation presented as:

$$W_i(t_j) = [x_i, y_i, z_i], [Q_i], [K_i].$$

Set of natural objects in 3D coordinates (X, Y, Z) may have W_i observations on an object (individual element), where $i = 1, 2, 3, \ldots h$, & on a subset, where $i = 1, 2, 3, \ldots m$. The entire number of observations for a set may be presented as a matrix:

$$[\mathbf{X}], \mathbf{X}_{i*j},$$

where **X** is initial matrix of j observations with variables $X_1, X_2, X_3 \ldots X_i$ ($1 \leq i < p$).

The variables $X_1, X_2, X_3 \ldots X_i$ ($1 \leq j < p$) are the variables described by model as spatial coordinates (x_i, y_i, z_i), Q_i – quantities parameters & K_i – qualitative (classificatory) characteristics. Initial matrix $\mathbf{X}_{*1}$ is defined for the time of observations, for t_j, in the matrix on this slide, j = 1.

Operations with the Initial Matrix in Statistical Learning

Let us write:

$[\mathbf{X}] = [\mathbf{Z}]$, where **X** is initial matrix of observations with random variables $X_1, X_2, X_3 \ldots X_i$ ($1 \leq i < p$). **Z** is a new vector space (space of Principal Components), reflecting variability of initial data, by orthogonal vectors, vectors placed in order of declining the reflection of variability in data from Z_1 to Z_2, to $Z_3 \ldots Z_i$ ($1 \leq i < p$) & so on.

With operations in Statistical Learning data may be transformed in model like:

$[\mathbf{X}] = [\mathbf{M}] + [\mathbf{E}]$, where are **X** – the initial matrix, **M** – math model, **E** – the component of variability of initial data, which was not reflected in the model. It is possible to reflect in a model only a part of variability of initial data by choosing the number of Z_i vectors to include in **M**.

This procedure has to be completed by the researcher & it results in a significant reduction of dimensionality. The new orthogonal system **M** might be considered as Hilbert Space, which reflects the patterns of the initial data, is generally used for the coordinate system.

Types of observations for hierarchical multidimensional Natural Object & Processes

Systemic Analysis of Natural Objects: an Initial Schema with a Researcher (Components)

Researcher – Object – Data – Modeler – Models – Results – Manager or Researcher

The Components

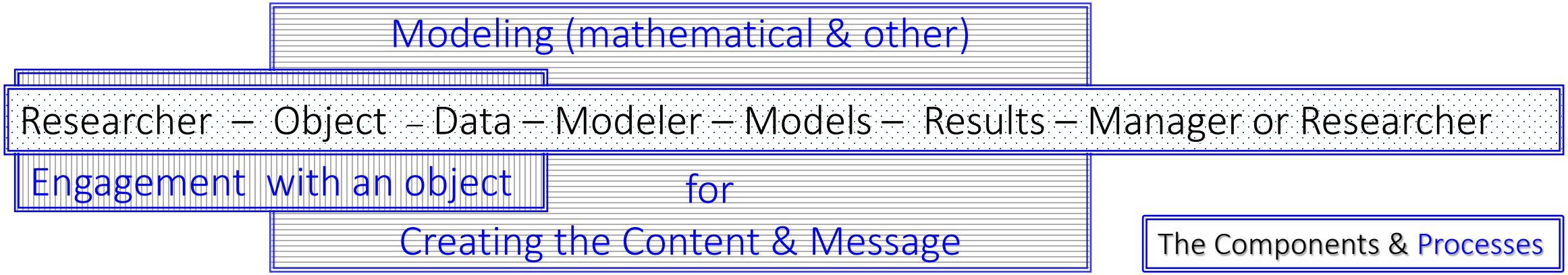
Systemic Analysis of Natural Objects: an Initial Schema with a Researcher (Process)

Researcher – Object – Data – Modeler – Models – Results – Manager or Researcher

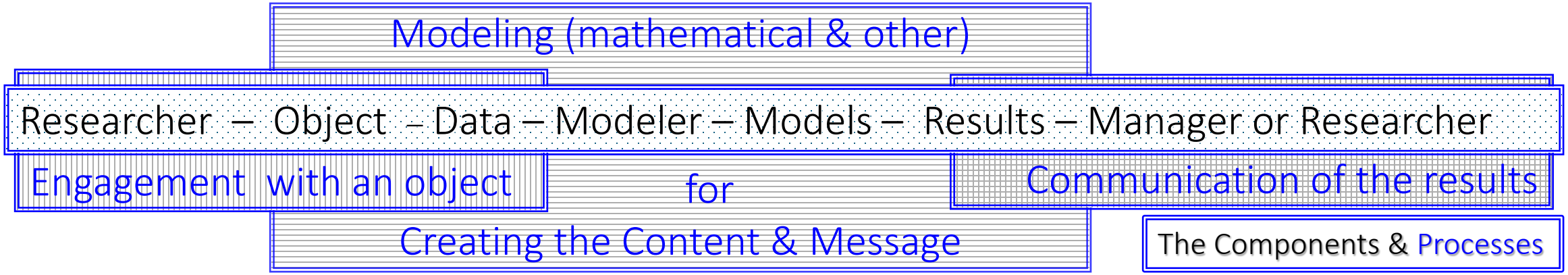
Engagement with an object

The Components & Processes

Systemic Analysis of Natural Objects: an Initial Schema with a Researcher (Processes)

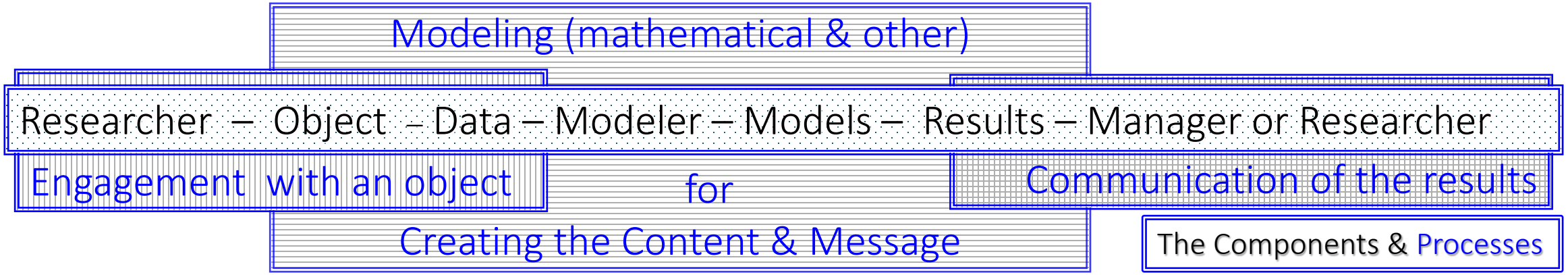


Systemic Analysis of Natural Objects: an Initial Schema with a Researcher (Processes)



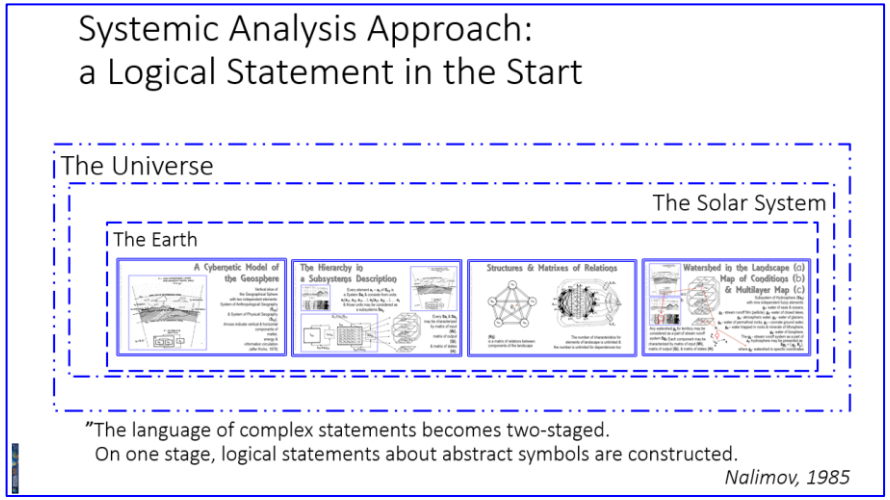
Researcher Is the Principal Component in the System

Researcher – Object – Data – Modeler – Models – Results – Manager or Researcher



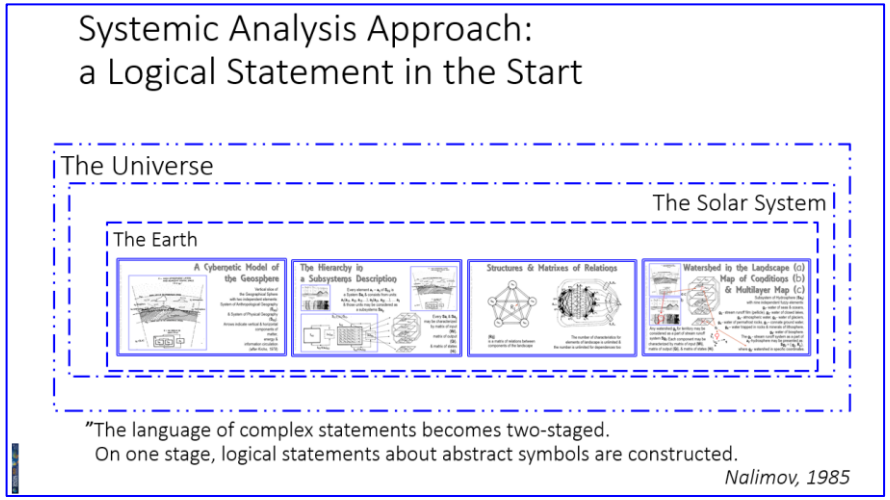
New concepts derived from statistical learning:

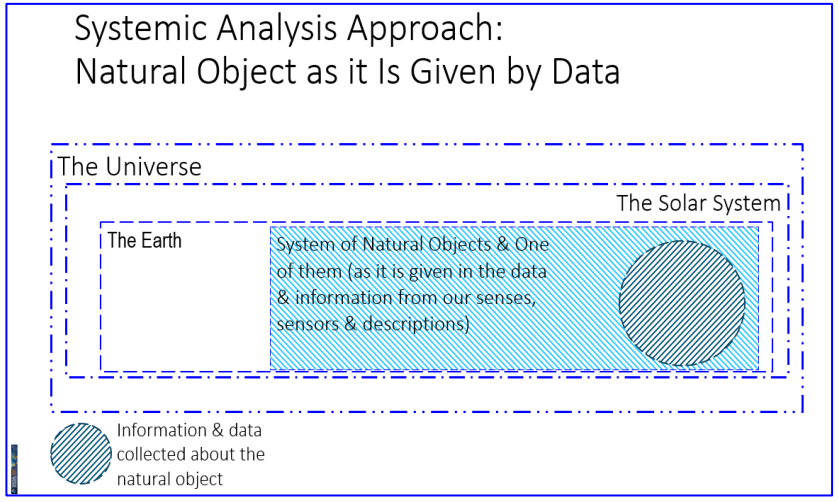
- a reflection of system complexity

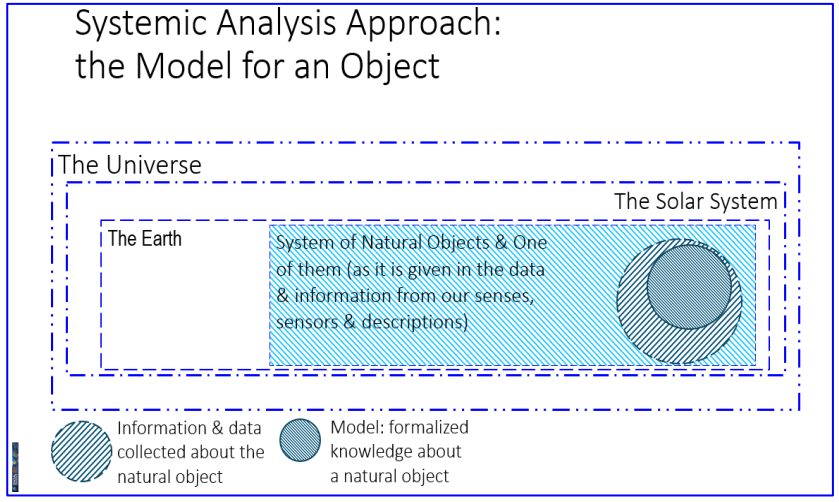


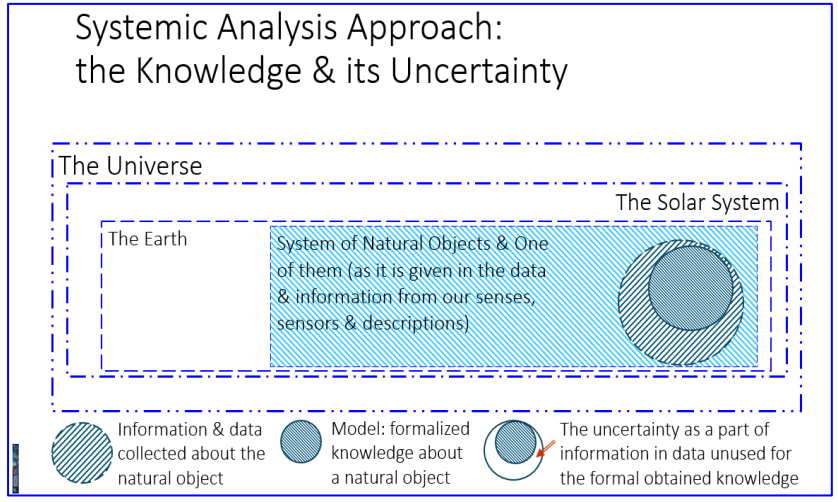
New concepts derived from statistical learning:

- a reflection of system complexity
- a measurable property





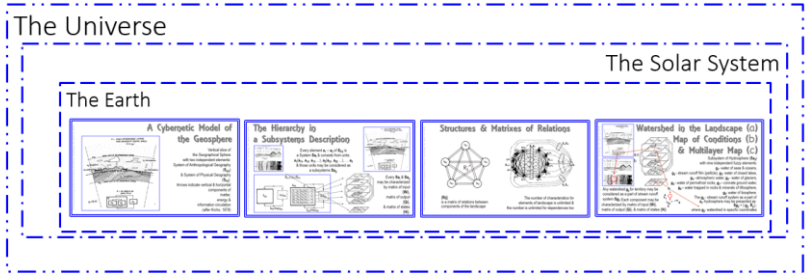




New concepts derived from statistical learning:

- a reflection of system complexity
- a measurable property
- a constraint on model expressiveness

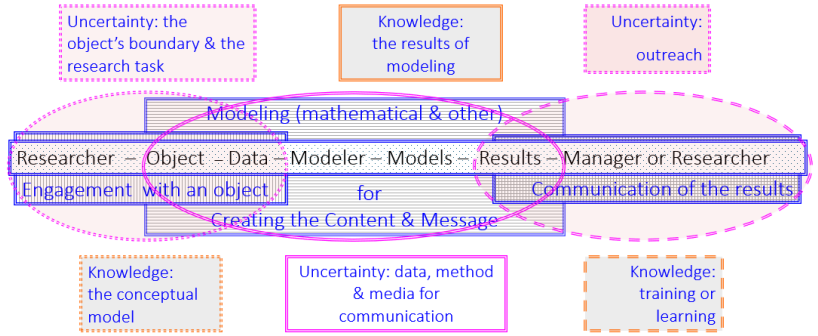
Systemic Analysis Approach: a Logical Statement in the Start



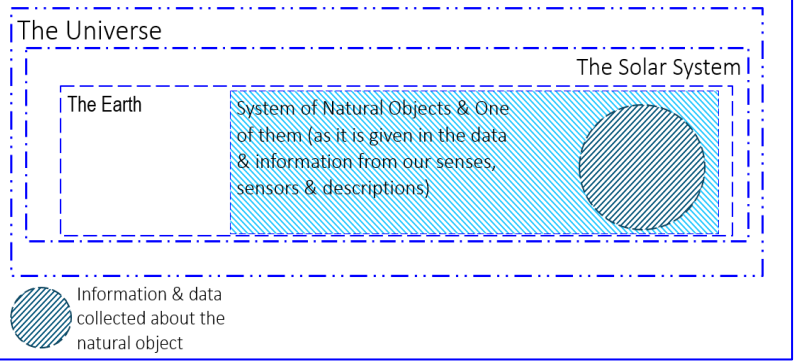
"The language of complex statements becomes two-staged.
On one stage, logical statements about abstract symbols are constructed.

Nalimov, 1985

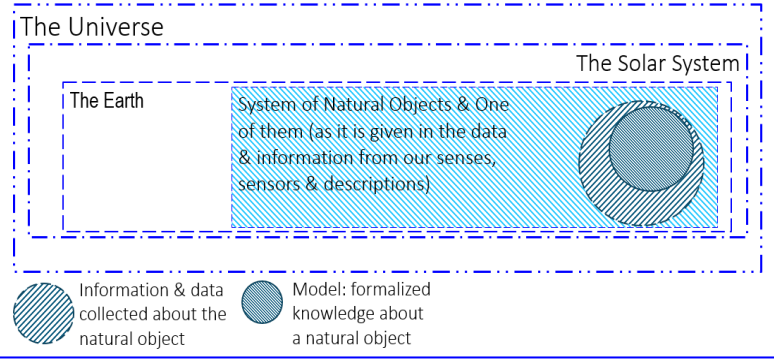
The Knowledge & Uncertainty for Components in the Systemic Analysis



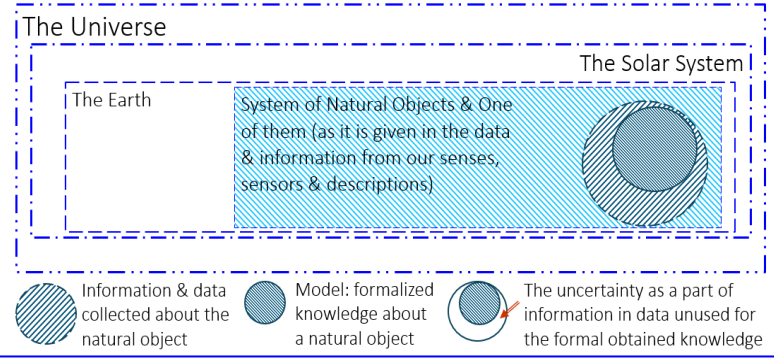
Systemic Analysis Approach: Natural Object as it Is Given by Data



Systemic Analysis Approach: the Model for an Object



Systemic Analysis Approach: the Knowledge & its Uncertainty

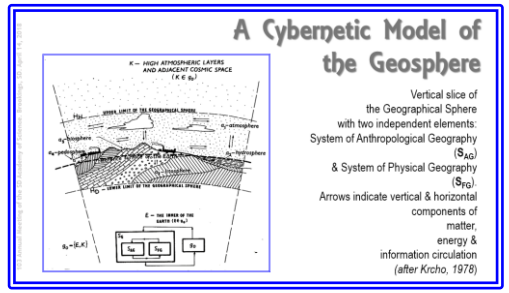


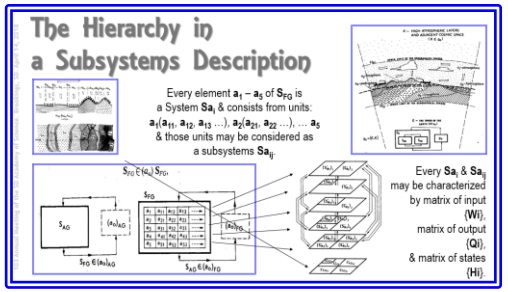
Systemic Analysis Approach: a Logical Statement in the Start

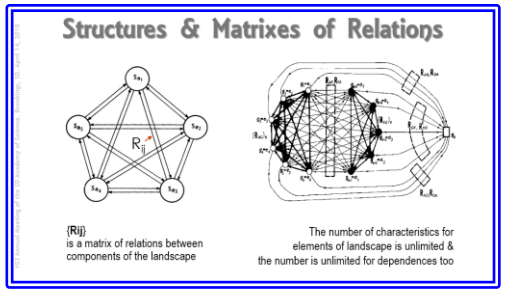
The Universe

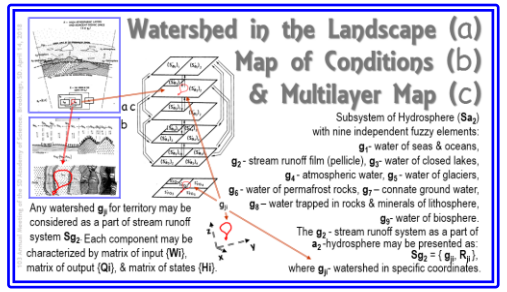
The Solar System

The Earth









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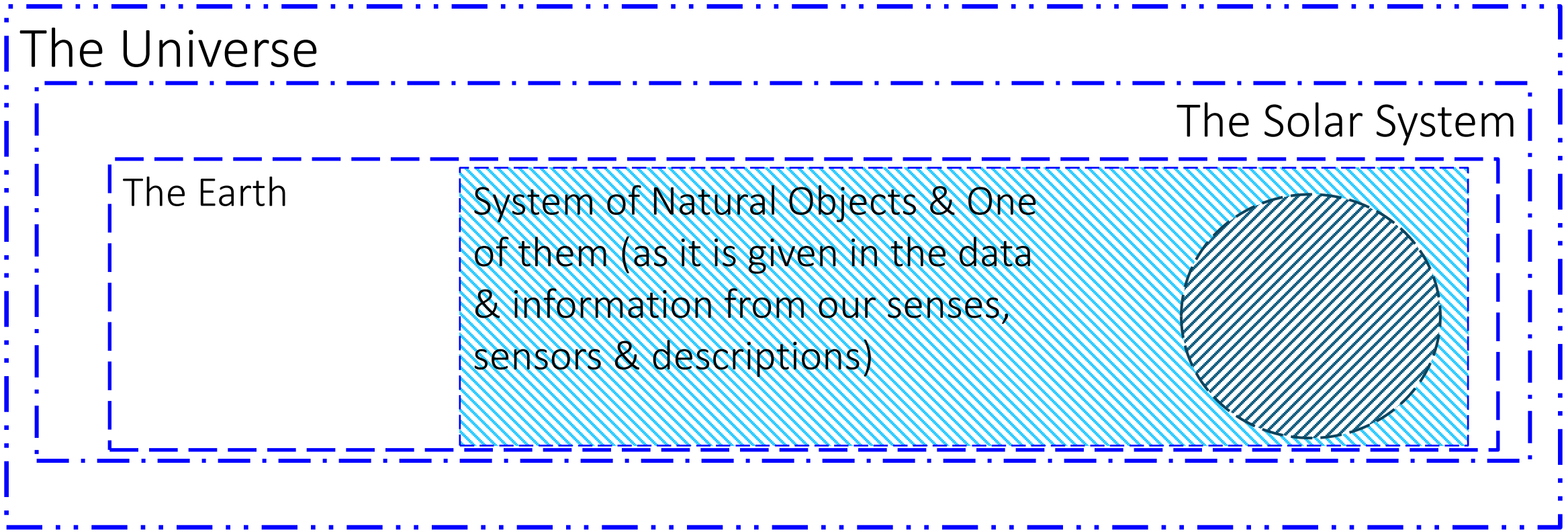
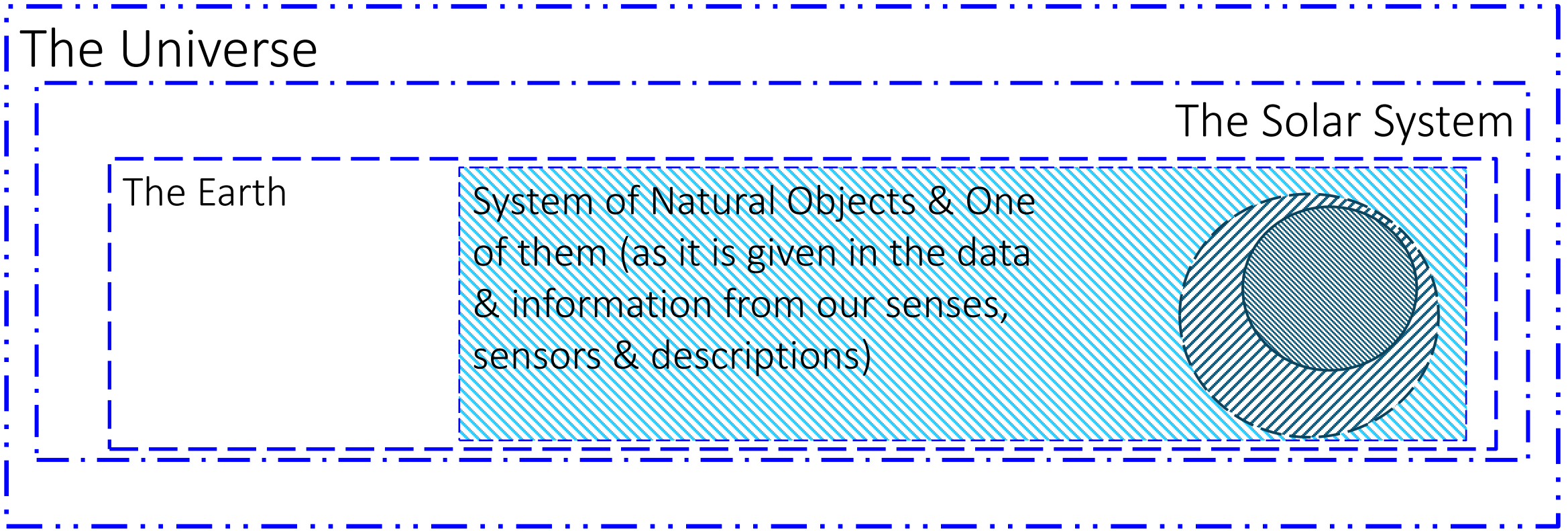
Nalimov, 1985

Systemic Analysis Approach: Natural Object as it Is Given by Data



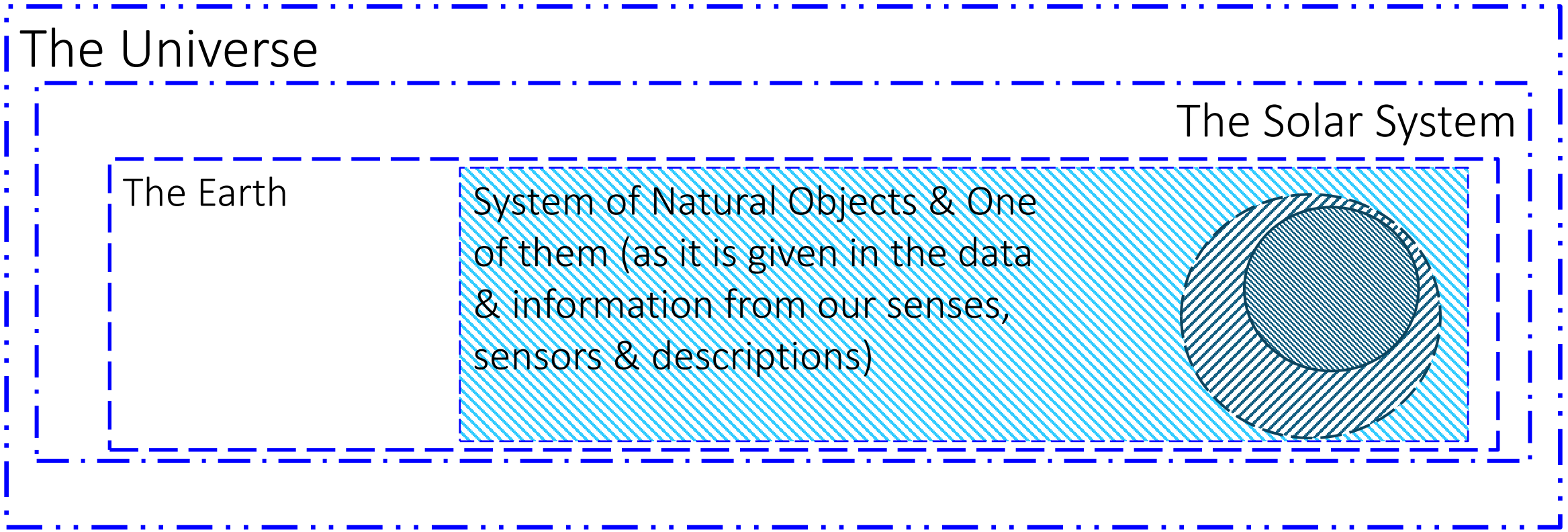
Systemic Analysis Approach: the Model for an Object



Information & data collected about the natural object

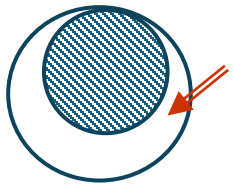
Model: formalized knowledge about a natural object

Systemic Analysis Approach: the Knowledge & its Uncertainty



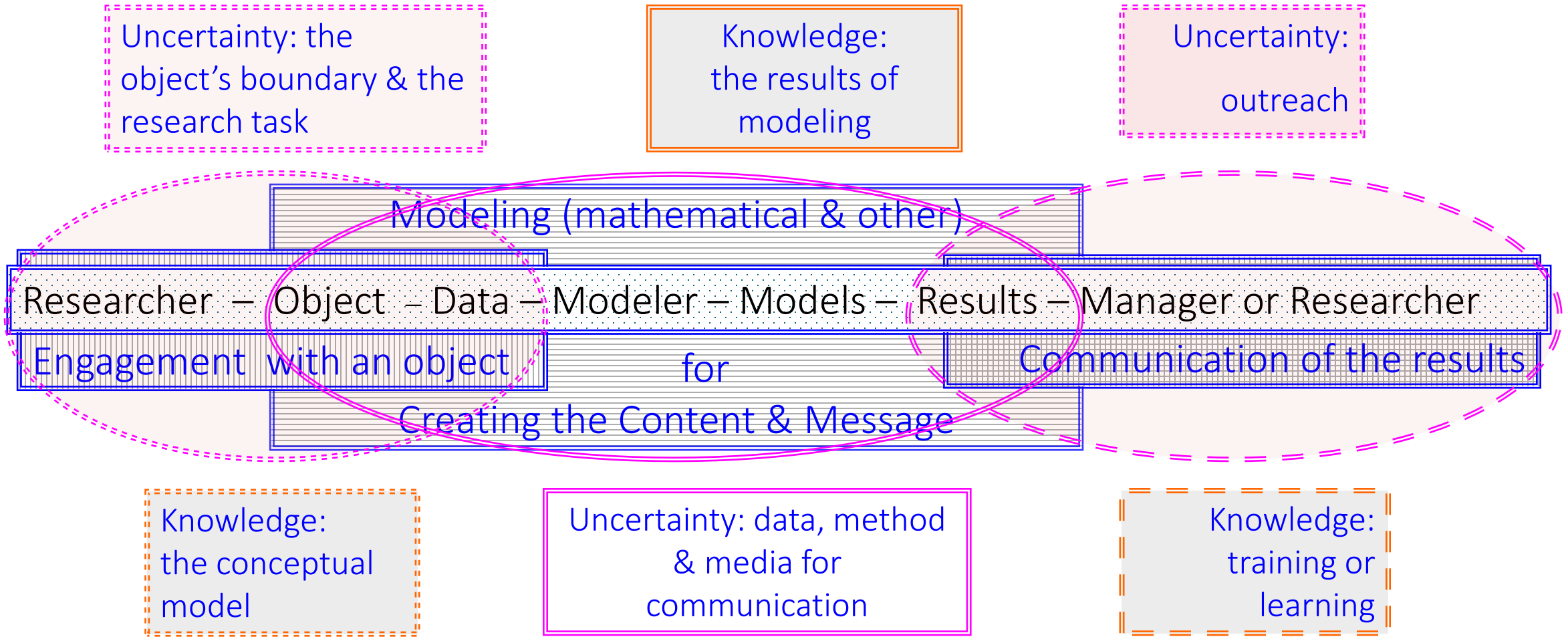
Information & data collected about the natural object

Model: formalized knowledge about a natural object



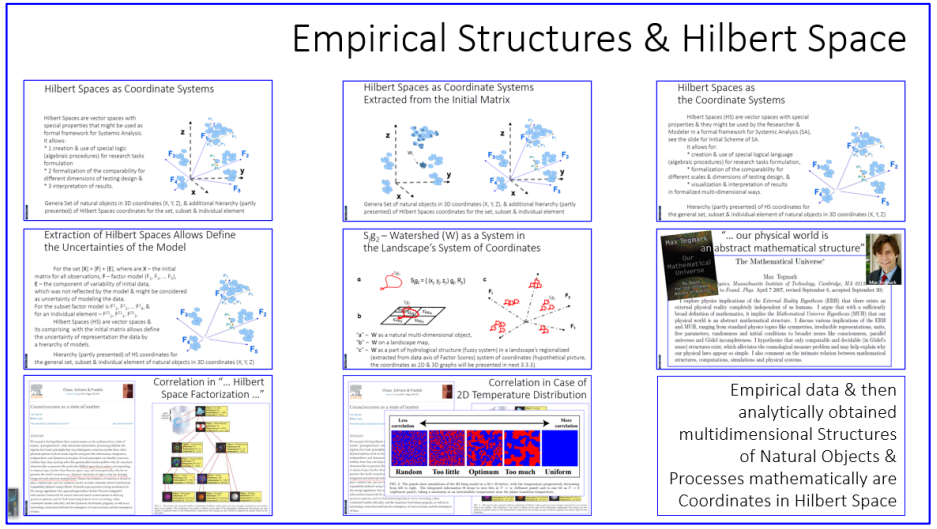
The uncertainty as a part of information in data unused for the formal obtained knowledge

The Knowledge & Uncertainty for Components in the Systemic Analysis



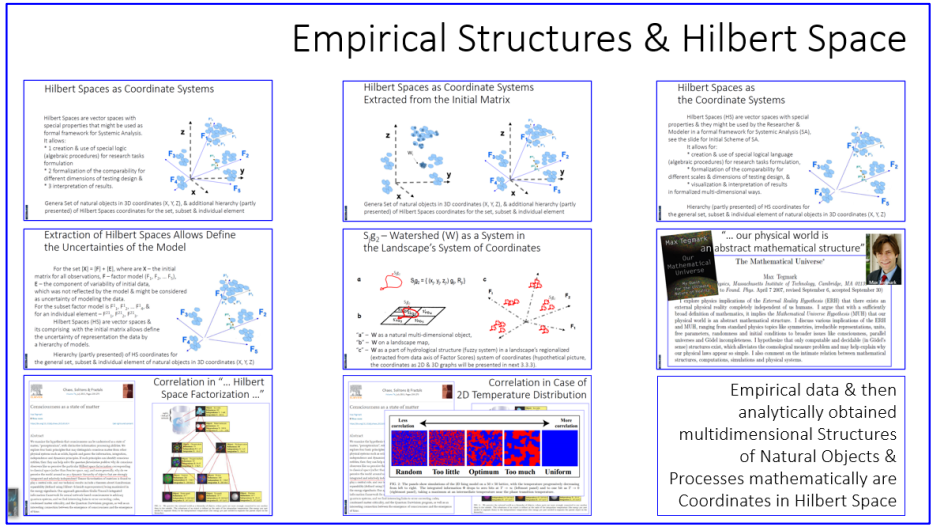
6 – Statistical Learning as a Systemic Tool

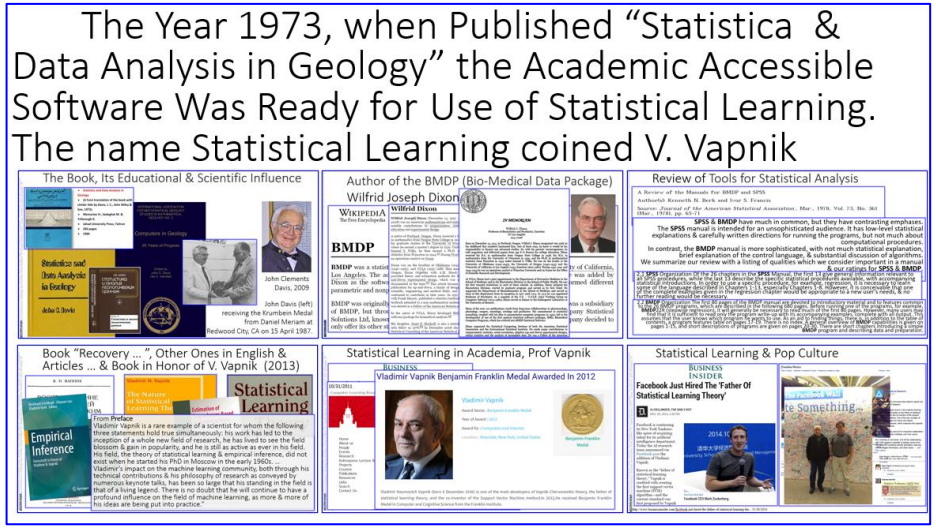
- Extracts Hilbert spaces from empirical data



6 – Statistical Learning as a Systemic Tool

- Extracts Hilbert spaces from empirical data
- Dimensionality & variability emerge from the data



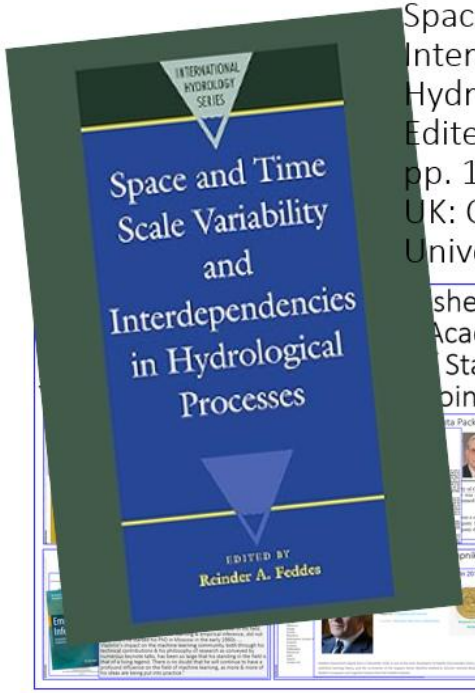


6 – Statistical Learning as a Systemic Tool

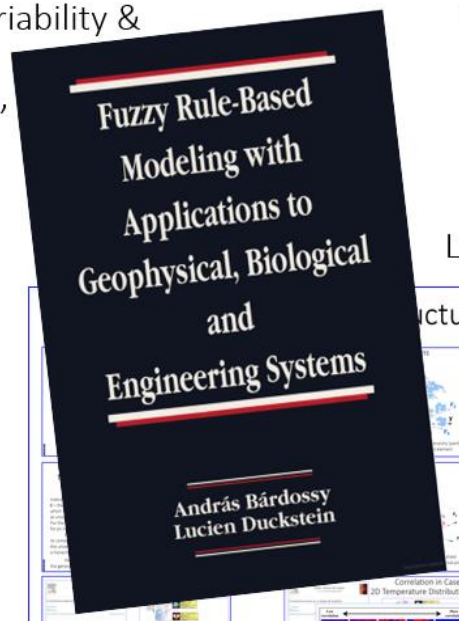
- Extracts Hilbert spaces from empirical data
- Dimensionality & variability emerge from the data
- Defines geosystems as fuzzy time-spatial structures

(for Statistical Learning after Zadeh)

The existing "declarations" about Space & Time & Fuzziness in Hydrology in use now

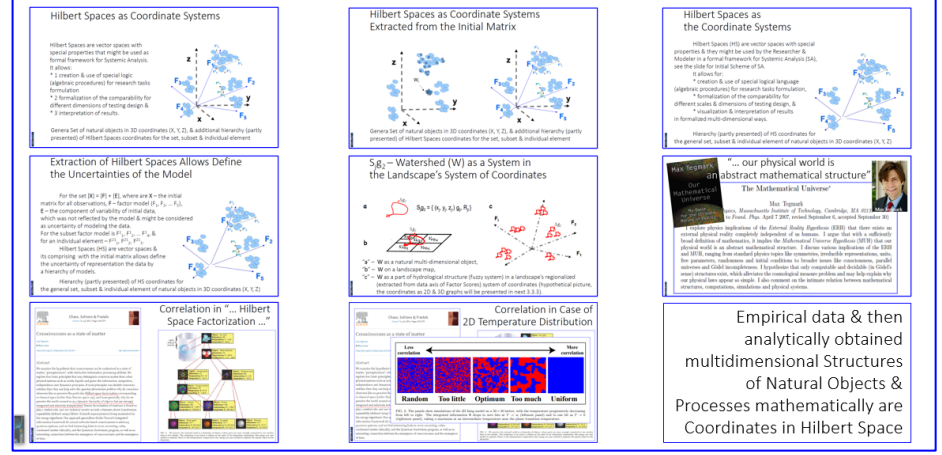


Space & Time Scale Variability & Interdependencies in Hydrological Processes, Edited by R. A. Feddes, pp. 193. Cambridge, UK: Cambridge University Press, 1995.



Fuzzy Rule-Based Modeling with Applications to Geophysical, Biological & Engineering Systems. A. Bárdossy, L. Duckstein, 1995

Empirical Structures & Hilbert Space

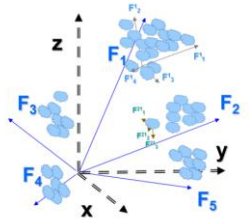


The Year 1973, when Published "Statistica & Data Analysis in Geology" the Academic Accessible Software Was Ready for Use of Statistical Learning. The name Statistical Learning coined V. Vapnik

Empirical Structures & Hilbert Space

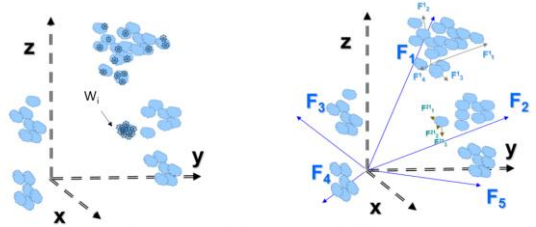
Hilbert Spaces as Coordinate Systems

Hilbert Spaces are vector spaces with special properties that might be used as formal framework for Systemic Analysis.
It allows:
* 1 creation & use of special logic (algebraic procedures) for research tasks formulation
* 2 formalization of the comparability for different dimensions of testing design &
* 3 interpretation of results.



Genera Set of natural objects in 3D coordinates (X, Y, Z), & additional hierarchy (partly presented) of Hilbert Spaces coordinates for the set, subset & individual element

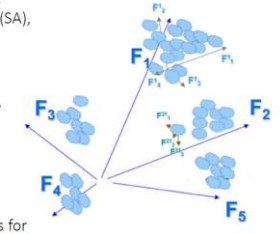
Hilbert Spaces as Coordinate Systems Extracted from the Initial Matrix



Genera Set of natural objects in 3D coordinates (X, Y, Z), & additional hierarchy (partly presented) of Hilbert Spaces coordinates for the set, subset & individual element

Hilbert Spaces as the Coordinate Systems

Hilbert Spaces (HS) are vector spaces with special properties & they might be used by the Researcher & Modeler in a formal framework for Systemic Analysis (SA), see the slide for Initial Scheme of SA.
It allows for:
* creation & use of special logical language (algebraic procedures) for research tasks formulation,
* formalization of the comparability for different scales & dimensions of testing design, &
* visualization & interpretation of results in formalized multi-dimensional ways.

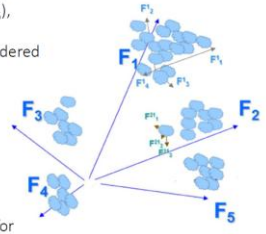


Hierarchy (partly presented) of HS coordinates for the general set, subset & individual element of natural objects in 3D coordinates (X, Y, Z)

Extraction of Hilbert Spaces Allows Define the Uncertainties of the Model

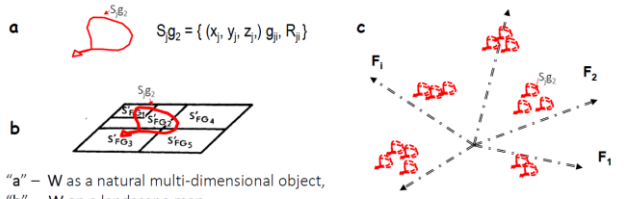
For the set [X] = [F] + [E], where are X – the initial matrix for all observations, F – factor model (F_1, F_2, ... F_5), E – the component of variability of initial data, which was not reflected by the model & might be considered as uncertainty of modeling the data.
For the subset factor model is F^1_1, F^1_2, ... F^1_4, & for an individual element – F^{21}_1, F^{21}_2, F^{21}_3.

Hilbert Spaces (HS) are vector spaces & its comprising with the initial matrix allows define the uncertainty of representation the data by a hierarchy of models.



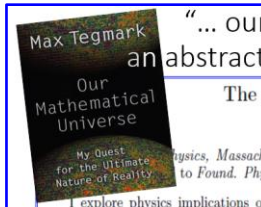
Hierarchy (partly presented) of HS coordinates for the general set, subset & individual element of natural objects in 3D coordinates (X, Y, Z)

S_jg_2 – Watershed (W) as a System in the Landscape's System of Coordinates



"a" – W as a natural multi-dimensional object,
"b" – W on a landscape map,
"c" – W as a part of hydrological structure (fuzzy system) in a landscape's regionalized (extracted from data axis of Factor Scores) system of coordinates (hypothetical picture, the coordinates as 2D & 3D graphs will be presented in next 3.3.3).

"... our physical world is an abstract mathematical structure"



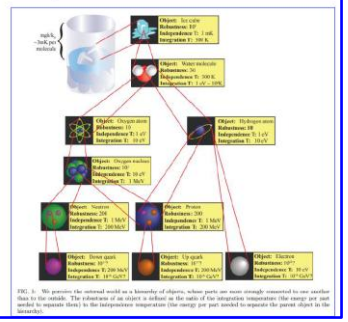
The Mathematical Universe*

Max Tegmark

ysics, Massachusetts Institute of Technology, Cambridge, MA 0213 to Found. Phys. April 7 2007, revised September 6, accepted September 30)

I explore physics implications of the *External Reality Hypothesis* (ERH) that there exists an external physical reality completely independent of us humans. I argue that with a sufficiently broad definition of mathematics, it implies the *Mathematical Universe Hypothesis* (MUH) that our physical world is an abstract mathematical structure. I discuss various implications of the ERH and MUH, ranging from standard physics topics like symmetries, irreducible representations, units, free parameters, randomness and initial conditions to broader issues like consciousness, parallel universes and Gödel incompleteness. I hypothesize that only computable and decidable (in Gödel's sense) structures exist, which alleviates the cosmological measure problem and may help explain why our physical laws appear so simple. I also comment on the intimate relation between mathematical structures, computations, simulations and physical systems.

Correlation in "... Hilbert Space Factorization ..."



Correlation in Case of 2D Temperature Distribution

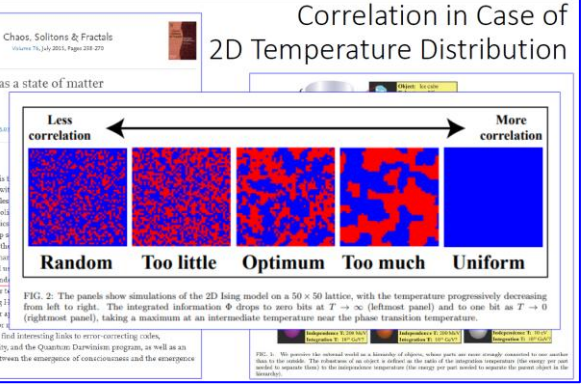
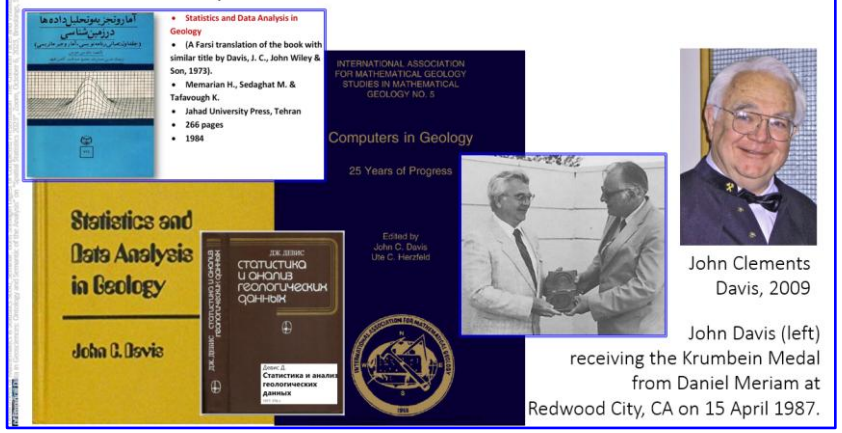


FIG. 2: The panels show simulations of the 2D Ising model on a 50 × 50 lattice, with the temperature progressively decreasing from left to right. The integrated information Φ drops to zero bits at $T \to \infty$ (leftmost panel) and to one bit as $T \to 0$ (rightmost panel), taking a maximum at an intermediate temperature near the phase transition temperature.

Empirical data & then analytically obtained multidimensional Structures of Natural Objects & Processes mathematically are Coordinates in Hilbert Space

The Year 1973, when Published “Statistica & Data Analysis in Geology” the Academic Accessible Software Was Ready for Use of Statistical Learning. The name Statistical Learning coined V. Vapnik

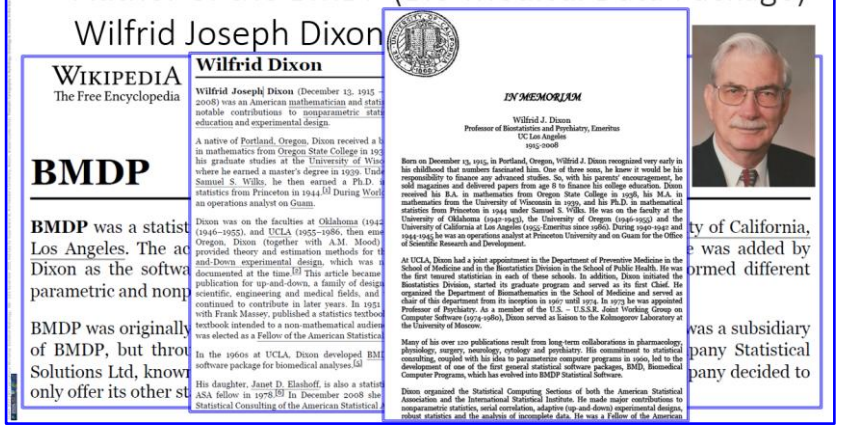
The Book, Its Educational & Scientific Influence



John Clements Davis, 2009

John Davis (left) receiving the Krumbein Medal from Daniel Meriam at Redwood City, CA on 15 April 1987.

Author of the BMDP (Bio-Medical Data Package) Wilfrid Joseph Dixon



Review of Tools for Statistical Analysis

A Review of the Manuals for BMDP and SPSS

Author(s): Kenneth N. Berk and Ivor S. Francis

Source: *Journal of the American Statistical Association*, Mar., 1978, Vol. 73, No. 361 (Mar., 1978), pp. 65-71

SPSS & **BMDP** have much in common, but they have contrasting emphases. The **SPSS** manual is intended for an unsophisticated audience. It has low-level statistical explanations & carefully written directions for running the programs, but not much about computational procedures.

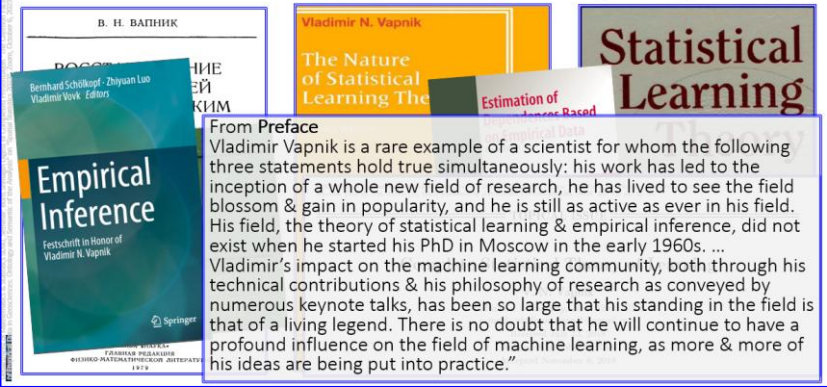
In contrast, the **BMDP** manual is more sophisticated, with not much statistical explanation, brief explanation of the control language, & substantial discussion of algorithms.

We summarize our review with a listing of qualities which we consider important in a manual & our ratings for **SPSS** & **BMDP**.

2.1 **SPSS** Organization Of the 26 chapters in the **SPSS** Manual, the first 13 give general information relevant to all SPSS procedures, while the last 13 describe the specific statistical procedures available, with accompanying statistical introductions. In order to use a specific procedure, for example, regression, it is necessary to learn some of the language described in Chapters 1-13, especially Chapters 1-8. However, it is conceivable that one of the complete examples given in the regression chapter would be appropriate to a new user's needs, & no further reading would be necessary.

2.2 **BMDP** Organization The first 80 pages of the BMDP manual are devoted to introductory material and to features common to the 26 BMDP programs, which are described in the following 680 pages. Before running one of the programs, for example, **BMDP**2R (stepwise regression), it will generally be necessary to read much of the first 80 pages. However, many users may find that it is sufficient to read only the program write-up with its accompanying examples, complete with all output. This assumes that the user knows which program he wants to use. As an aid to finding things, there is, in addition to the table of contents, a program features table on pages 17-19. There is no index. A general overview of **BMDP** capabilities is given on pages 1-15, and short descriptions of programs are given on pages 20-30. There are short chapters introducing a simple **BMDP** program and describing data and preparation.

Book “Recovery … ”, Other Ones in English & Articles … & Book in Honor of V. Vapnik (2013)



From Preface

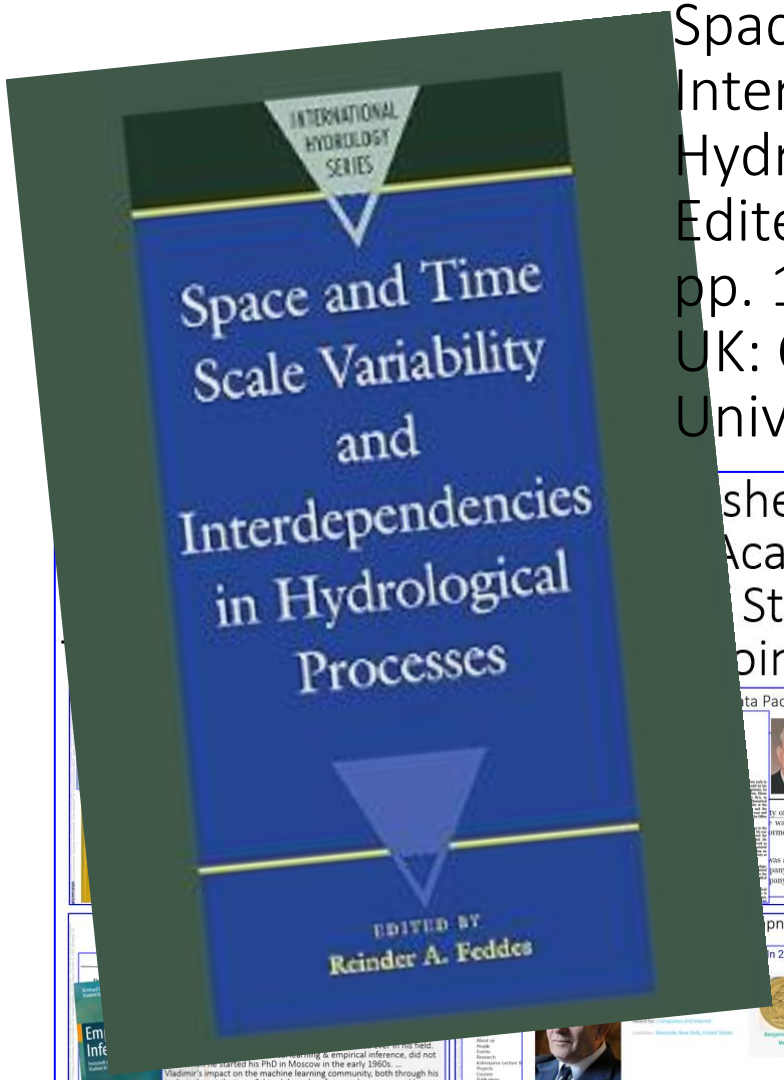
Vladimir Vapnik is a rare example of a scientist for whom the following three statements hold true simultaneously: his work has led to the inception of a whole new field of research, he has lived to see the field blossom & gain in popularity, and he is still as active as ever in his field. His field, the theory of statistical learning & empirical inference, did not exist when he started his PhD in Moscow in the early 1960s. …

Vladimir’s impact on the machine learning community, both through his technical contributions & his philosophy of research as conveyed by numerous keynote talks, has been so large that his standing in the field is that of a living legend. There is no doubt that he will continue to have a profound influence on the field of machine learning, as more & more of his ideas are being put into practice.”

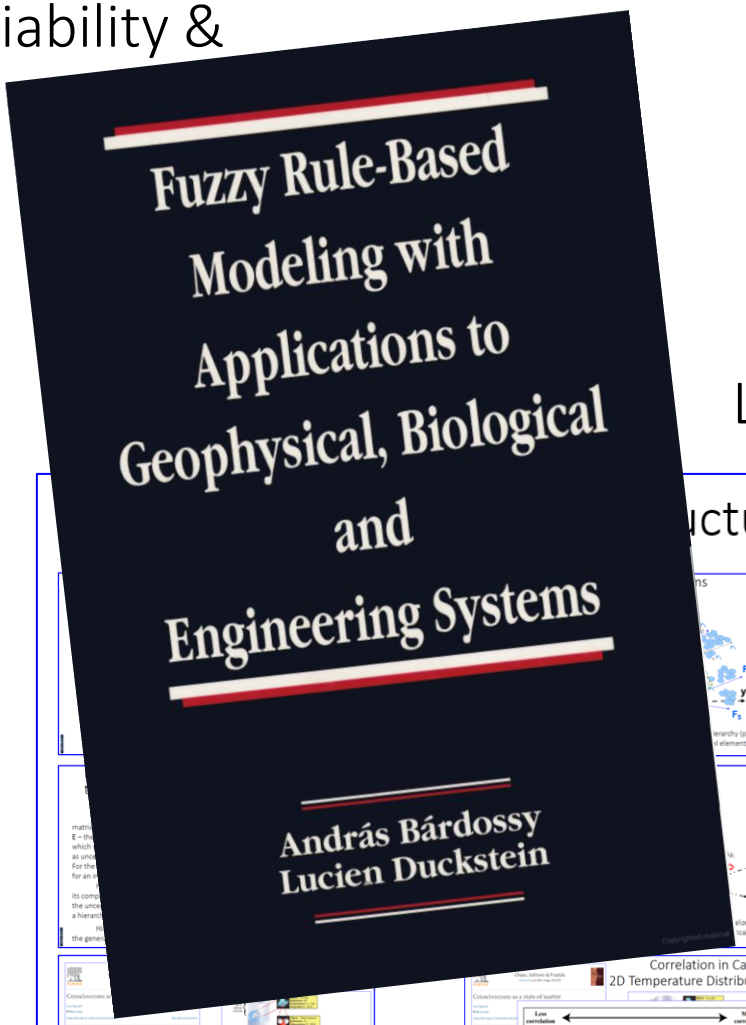
Statistical Learning in Academia, Prof Vapnik

Statistical Learning & Pop Culture

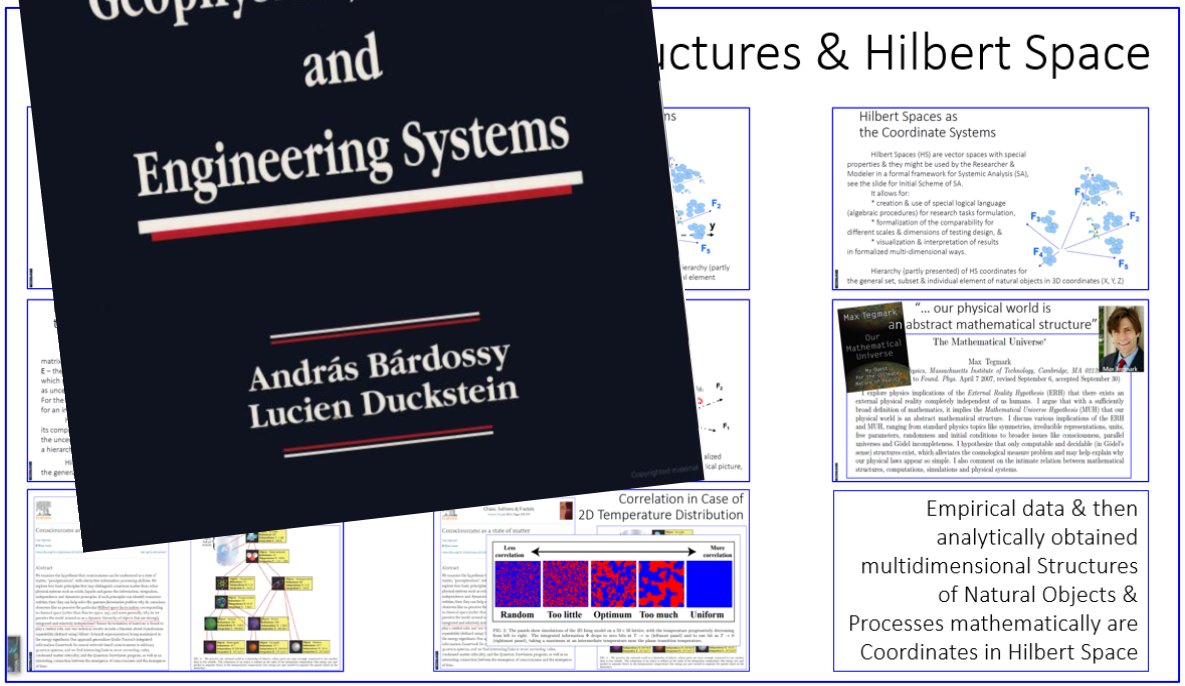
The existing “declarations” about Space & Time & Fuzziness in Hydrology in use now



Space & Time Scale Variability & Interdependencies in Hydrological Processes, Edited by R. A. Feddes, pp. 193. Cambridge, UK: Cambridge University Press, 1995.



Fuzzy Rule-Based Modeling with Applications to Geophysical, Biological & Engineering Systems. A. Bárdossy, L. Duckstein, 1995



7 – Case Studies

Two mesoscale watersheds:

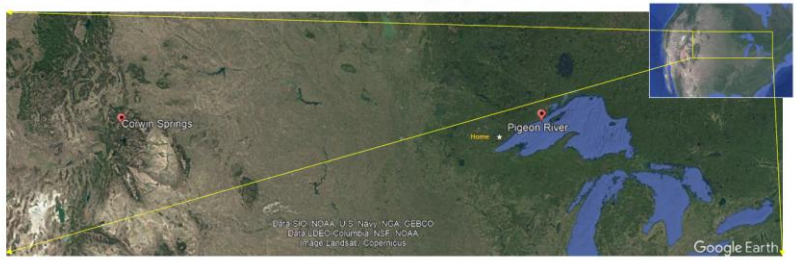
- USGS 04010500 — Pigeon River at Middle Falls, MN.
- USGS 06191500 — Yellowstone River at Corwin Springs, MT.

We present:

- Six models [101x12], [101x12D], [12Dx101] & [114x12], [114x12D], [12Dx114].
- Based on three types of fuzzy structures.
- Derived from statistical learning on empirical, recalculated & transposed hydrological data/

Visualization results will be presented following the above concepts with interpretation on the example of two watersheds

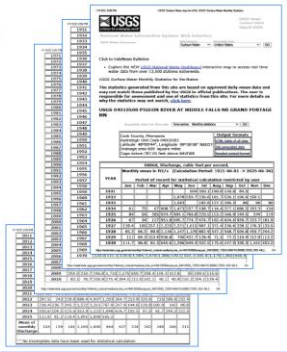
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN
USGS 06191500 Yellowstone River at Corwin Springs MT



USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Initial information

Cook County, Minnesota
Hydrologic Unit Code 04010101
Latitude 48°00'44", Longitude 89°36'58" NAD27
Drainage area 609 square miles
Gage datum 787.05 feet above NAVD88

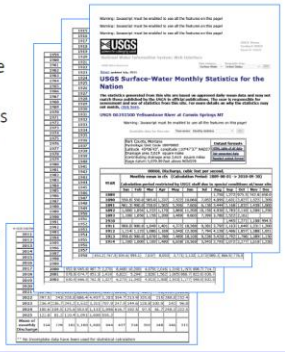
The Matrix Q_{101x13} used for analysis had twelve monthly & annual mean for 101 years (1924-2024) of discharge observations



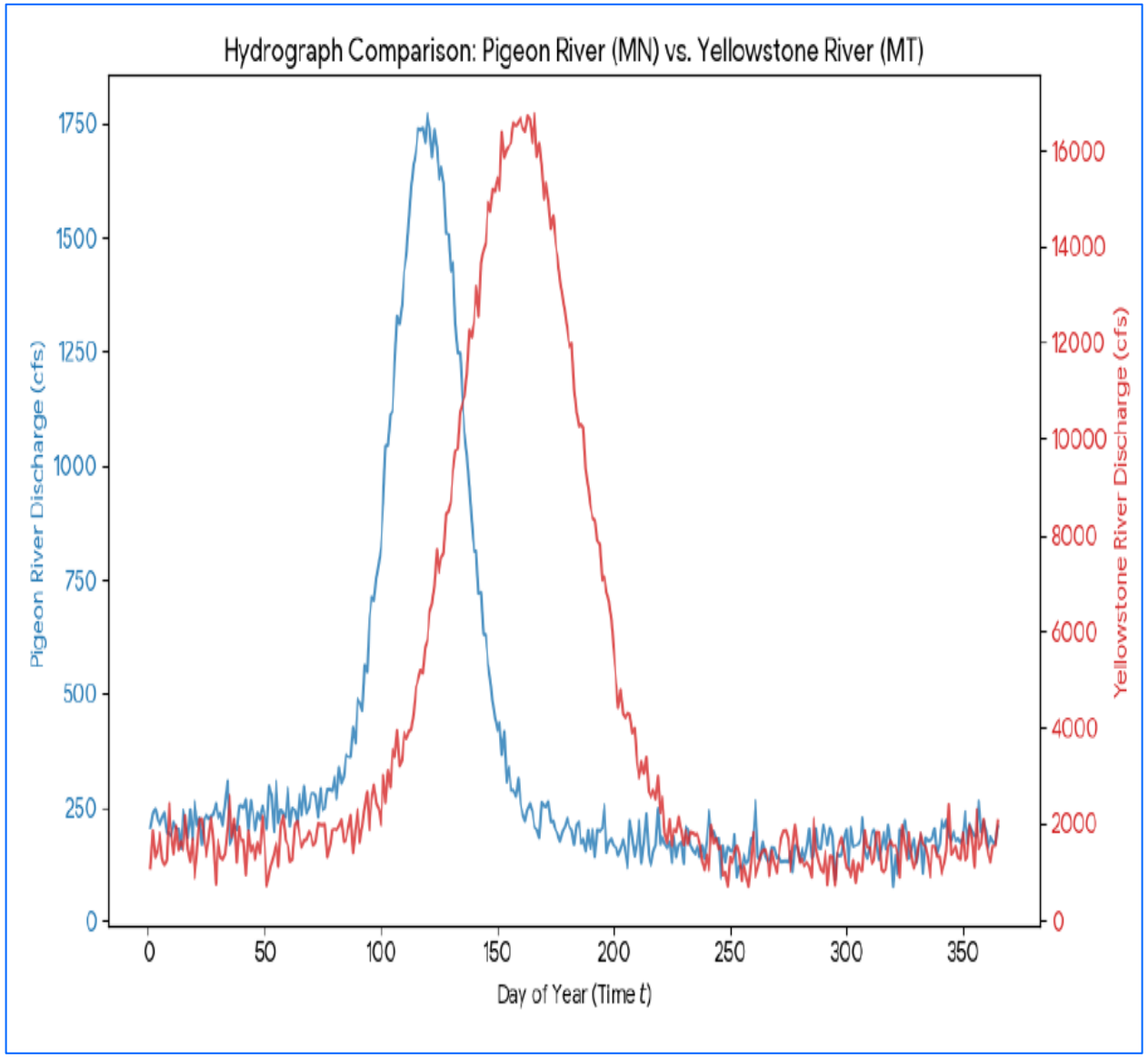
USGS 06191500 Yellowstone River at Corwin Springs MT. Initial information

Park County, Montana
Hydrologic Unit Code 10070002
Latitude 45°06'43.63", Longitude 110°47'37.20" NAD83
Drainage area 2,615 square miles
Gage datum 5,079.09 feet above NGVD29

The Matrix Q_{101x13} used for analysis had twelve monthly & annual mean for 101 years (1924-2024) of discharge observations



Characteristics of the two watersheds by Gemini



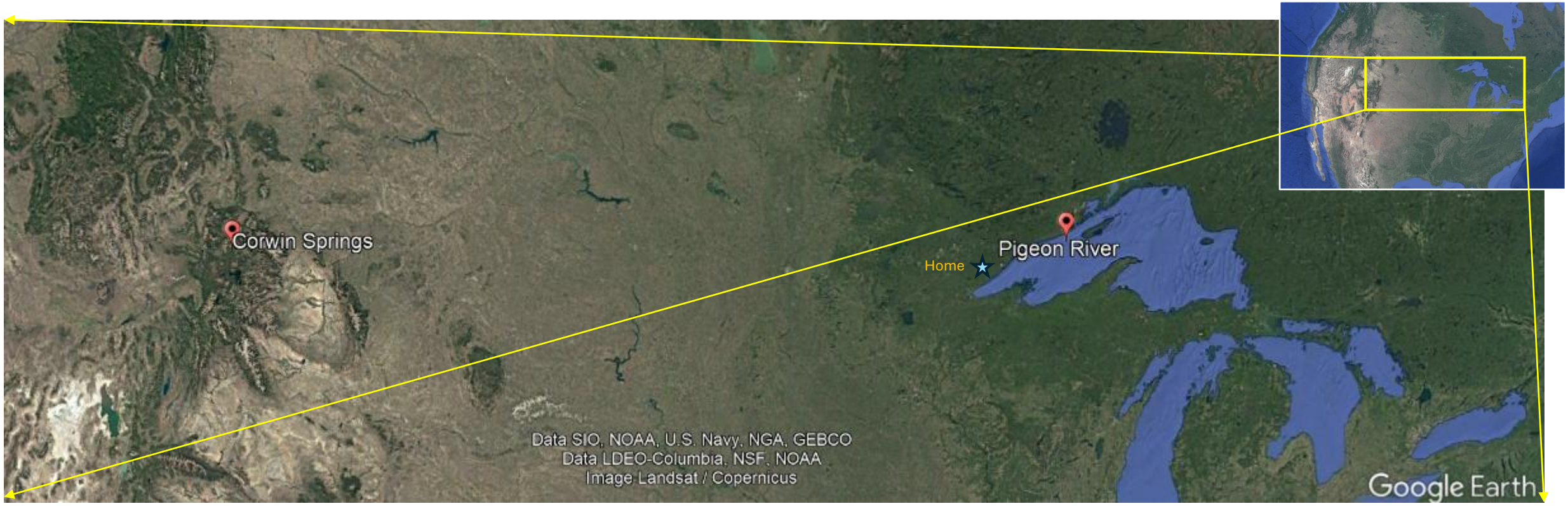
The following data points reflect the empirical representation of these two distinct subsystems:

Pigeon River at Middle Falls, MN (USGS 04010500): Drainage Area: 609 square miles. Hydrologic Unit Code (HUC): 04010101. Context: Located in the Northern Lakes and Forests ecoregion, characterized by humid continental climates and significant snowmelt runoff in early spring.

Yellowstone River at Corwin Springs, MT (USGS 06191500): Drainage Area: 2,615 square miles. Hydrologic Unit Code (HUC): 10070002. Annual Runoff Ratio: Approximately 54.6%. Dynamics: Exhibits a high-volume snow-driven hydrograph with peaks typically occurring in June. Recent extreme events, such as the 2022 flood (peaking at 43,300 cfs), illustrate the "fuzzy" nature of these systems where record-breaking variability challenges traditional static models.

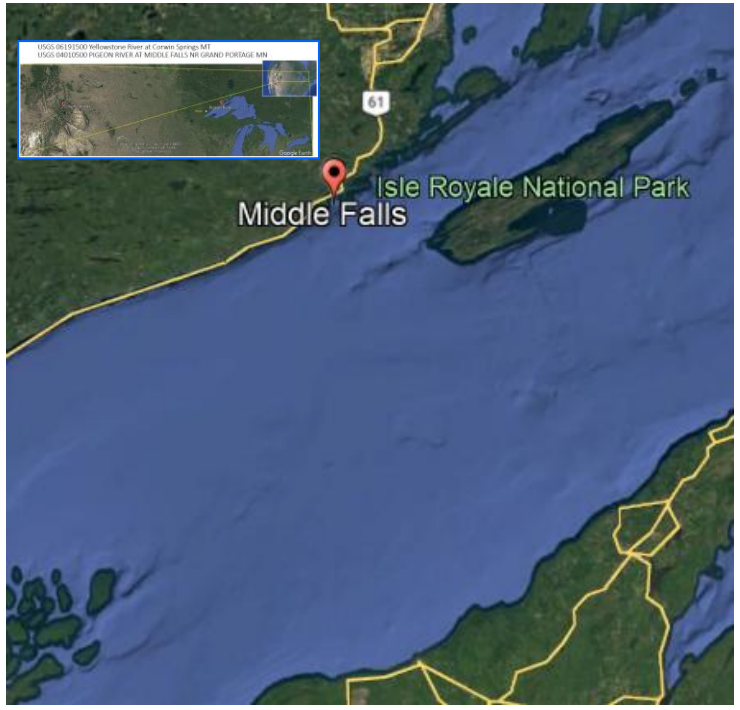
Visualization results will be presented following the above concepts with interpretation on the example of two watersheds

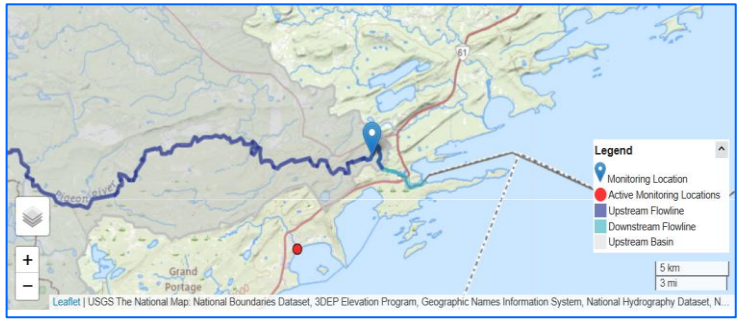
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN
USGS 06191500 Yellowstone River at Corwin Springs MT



©B. Shmagin, 2026

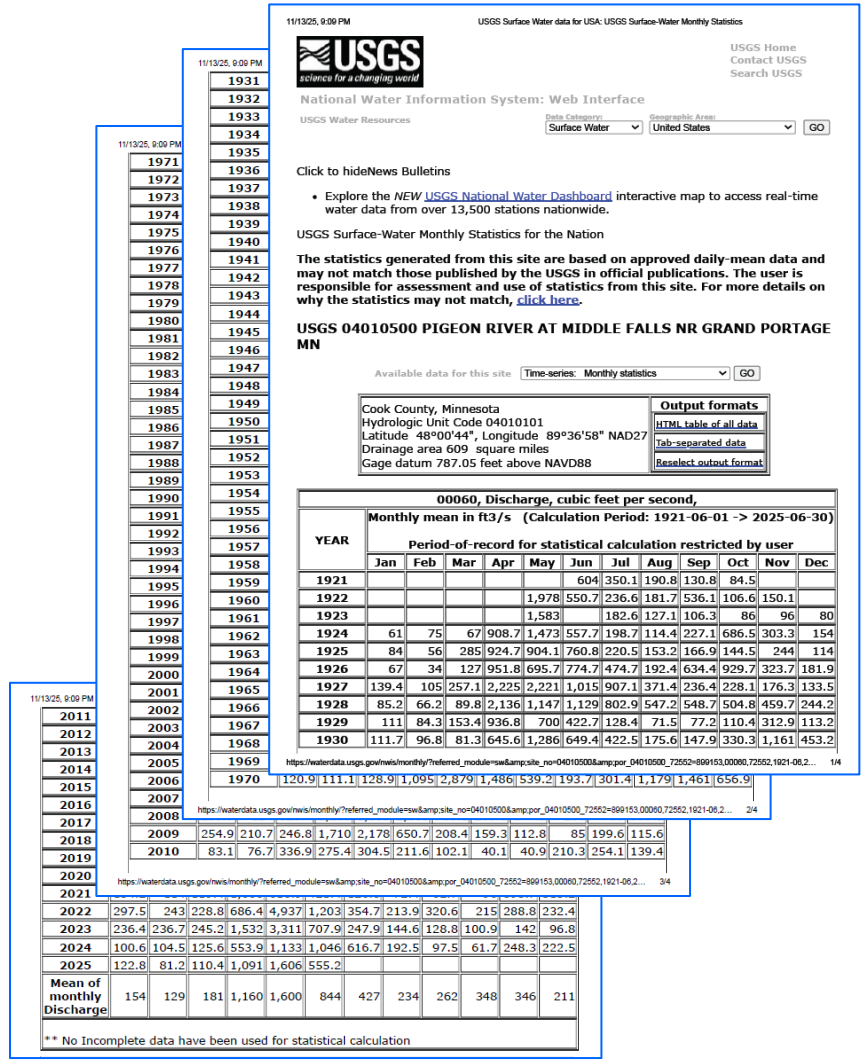
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Initial information





Cook County, Minnesota
Hydrologic Unit Code 04010101
Latitude 48°00'44", Longitude 89°36'58" NAD27
Drainage area 609 square miles
Gage datum 787.05 feet above NAVD88

The Matrix Q_{101x13} used for analysis had twelve monthly & annual mean for 101 years (1924-2024) of discharge observations



11/13/25, 9:09 PM — USGS Surface Water data for USA: USGS Surface-Water Monthly Statistics

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USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN

Available data for this site: Time-series: Monthly statistics | GO

Cook County, Minnesota
Hydrologic Unit Code 04010101
Latitude 48°00'44", Longitude 89°36'58" NAD27
Drainage area 609 square miles
Gage datum 787.05 feet above NAVD88

Output formats: HTML table of all data | Tab-separated data | Reselect output format

00060, Discharge, cubic feet per second,
Monthly mean in ft3/s (Calculation Period: 1921-06-01 -> 2025-06-30)
Period-of-record for statistical calculation restricted by user

YEAR	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1921						604	350.1	190.8	130.8	84.5		
1922					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1923					1,583		182.6	127.1	106.3	86	96	80
1924	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1925	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1926	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1927	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1928	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1929	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1930	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1931												
1932												
1933												
1934												
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1962												
1963												
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1965												
1966												
1967												
1968												
1969												
1970	120.9	111.1	128.9	1,095	2,879	1,486	539.2	193.7	301.4	1,179	1,461	656.9
1971												
1972												
1973												
1974												
1975												
1976												
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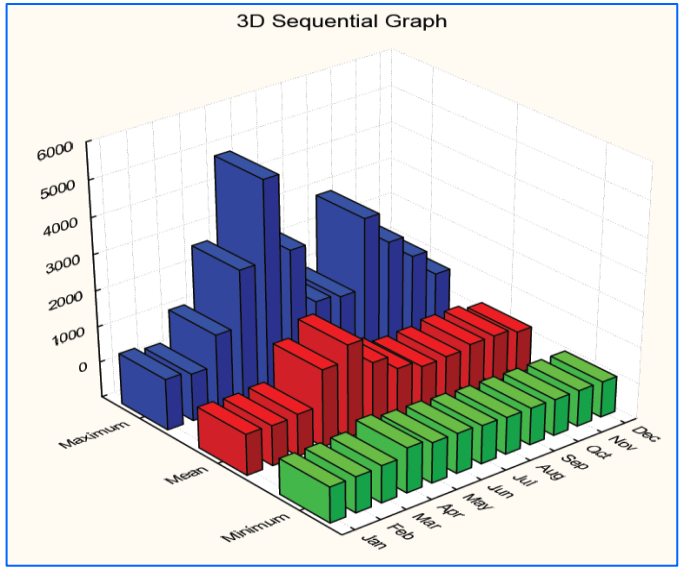
2003												
2004												
2005												
2006												
2007												
2008												
2009	254.9	210.7	246.8	1,710	2,178	650.7	208.4	159.3	112.8	85	199.6	115.6
2010	83.1	76.7	336.9	275.4	304.5	211.6	102.1	40.1	40.9	210.3	254.1	139.4
2011												
2012												
2013												
2014												
2015												
2016												
2017												
2018												
2019												
2020												
2021												
2022	297.5	243	228.8	686.4	4,937	1,203	354.7	213.9	320.6	215	288.8	232.4
2023	236.4	236.7	245.2	1,532	3,311	707.9	247.9	144.6	128.8	100.9	142	96.8
2024	100.6	104.5	125.6	553.9	1,133	1,046	616.7	192.5	97.5	61.7	248.3	222.5
2025	122.8	81.2	110.4	1,091	1,606	555.2						
Mean of monthly Discharge	154	129	181	1,160	1,600	844	427	234	262	348	346	211

** No Incomplete data have been used for statistical calculation

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Descriptive statistics

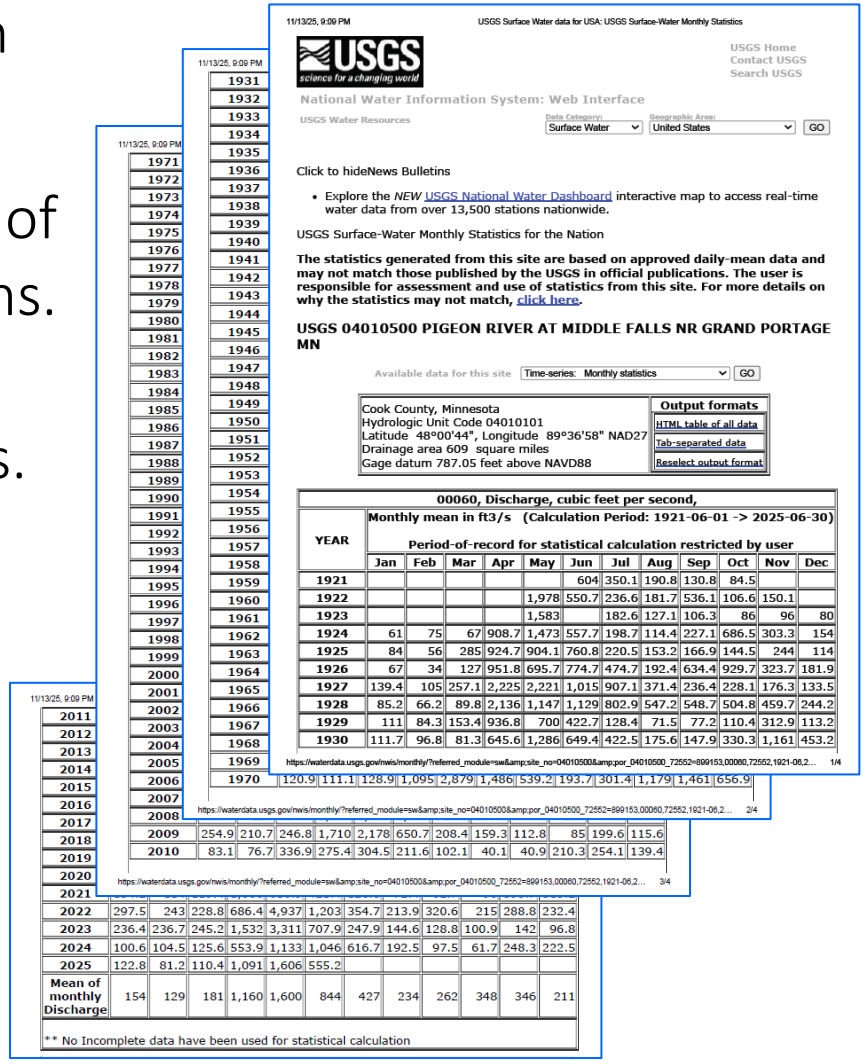


Monthly Hydrograph, visualization of monthly discharge: Minimum, Mean & Maximum for the 101 years.

The Matrix Q_{101x12} (1924-2024) of monthly discharge observations.

Descriptive statistical characteristics.

Variable	Descriptive Statistics								
	Valid N	Mean	Median	Minimum	Maximum	Lower Quartile	Upper Quartile	Variance	Std.Dev.
Jan	101	154.4	126.2	2.2	431.1	92.5	197.9	8042.8	89.7
Feb	101	129.8	111.1	8.0	324.2	87.1	177.1	4264.8	65.3
Mar	101	181.7	140.6	41.5	1169.0	111.3	203.9	19813.8	140.8
Apr	101	1164.5	1095.0	262.7	2724.0	745.2	1525.0	320374.8	566.0
May	101	1596.2	1375.0	137.8	4937.0	1031.0	2162.0	747480.7	864.6
Jun	101	851.9	748.9	124.9	2801.0	509.8	1135.0	203349.9	450.9
Jul	101	432.0	358.5	78.0	1127.0	258.7	531.5	59350.2	243.6
Aug	101	235.9	194.2	40.1	1029.0	129.5	278.3	25590.2	160.0
Sep	101	261.7	168.5	36.6	2985.0	109.8	305.9	111085.0	333.3
Oct	101	356.0	220.4	17.4	2095.0	126.4	471.0	130313.9	361.0
Nov	101	350.6	288.1	11.4	1461.0	179.5	413.0	71404.0	267.2
Dec	101	211.8	172.1	2.9	720.3	112.9	250.5	22099.8	148.7



11/13/25, 9:09 PM — USGS Surface Water data for USA: USGS Surface-Water Monthly Statistics

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USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN

Cook County, Minnesota
Hydrologic Unit Code 04010101
Latitude 48°00'44", Longitude 89°36'58" NAD27
Drainage area 609 square miles
Gage datum 787.05 feet above NAVD88

Output formats: HTML table of all data | Tab-separated data | Reselect output format

00060, Discharge, cubic feet per second,
Monthly mean in ft3/s (Calculation Period: 1921-06-01 -> 2025-06-30)
Period-of-record for statistical calculation restricted by user

YEAR	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1921						604	350.1	190.8	130.8	84.5		
1922					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1923					1,583		182.6	127.1	106.3	86	96	80
1924	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1925	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1926	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1927	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1928	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1929	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1930	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1931						604	350.1	190.8	130.8	84.5		
1932					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1933					1,583		182.6	127.1	106.3	86	96	80
1934	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1935	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1936	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1937	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1938	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1939	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1940	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1941						604	350.1	190.8	130.8	84.5		
1942					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1943					1,583		182.6	127.1	106.3	86	96	80
1944	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1945	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1946	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1947	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1948	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1949	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1950	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1951						604	350.1	190.8	130.8	84.5		
1952					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1953					1,583		182.6	127.1	106.3	86	96	80
1954	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1955	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1956	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1957	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1958	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1959	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1960	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1961						604	350.1	190.8	130.8	84.5		
1962					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1963					1,583		182.6	127.1	106.3	86	96	80
1964	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1965	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1966	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1967	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1968	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1969	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1970	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1971						604	350.1	190.8	130.8	84.5		
1972					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1973					1,583		182.6	127.1	106.3	86	96	80
1974	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1975	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1976	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1977	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1978	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1979	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1980	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1981						604	350.1	190.8	130.8	84.5		
1982					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1983					1,583		182.6	127.1	106.3	86	96	80
1984	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1985	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1986	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1987	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1988	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1989	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1990	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1991						604	350.1	190.8	130.8	84.5		
1992					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1993					1,583		182.6	127.1	106.3	86	96	80
1994	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1995	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1996	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1997	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1998	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1999	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
2000	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
2001						604	350.1	190.8	130.8	84.5		
2002					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
2003					1,583		182.6	127.1	106.3	86	96	80
2004	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
2005	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
2006	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
2007	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
2008	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
2009	254.9	210.7	246.8	1,710	2,178	650.7	208.4	159.3	112.8	85	199.6	115.6
2010	83.1	76.7	336.9	275.4	304.5	211.6	102.1	40.1	40.9	210.3	254.1	139.4
2011						604	350.1	190.8	130.8	84.5		
2012					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
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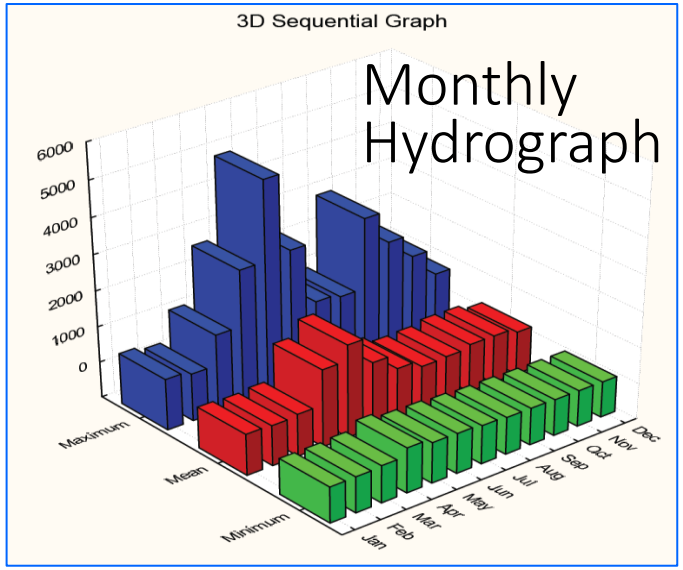
2014	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
2015	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
2016	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
2017	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
2018	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
2019	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
2020	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
2021						604	350.1	190.8	130.8	84.5		
2022	297.5	243	228.8	686.4	4,937	1,203	354.7	213.9	320.6	215	288.8	232.4
2023	236.4	236.7	245.2	1,532	3,311	707.9	247.9	144.6	128.8	100.9	142	96.8
2024	100.6	104.5	125.6	553.9	1,133	1,046	616.7	192.5	97.5	61.7	248.3	222.5
2025	122.8	81.2	110.4	1,091	1,606	555.2						
Mean of monthly Discharge	154	129	181	1,160	1,600	844	427	234	262	348	346	211

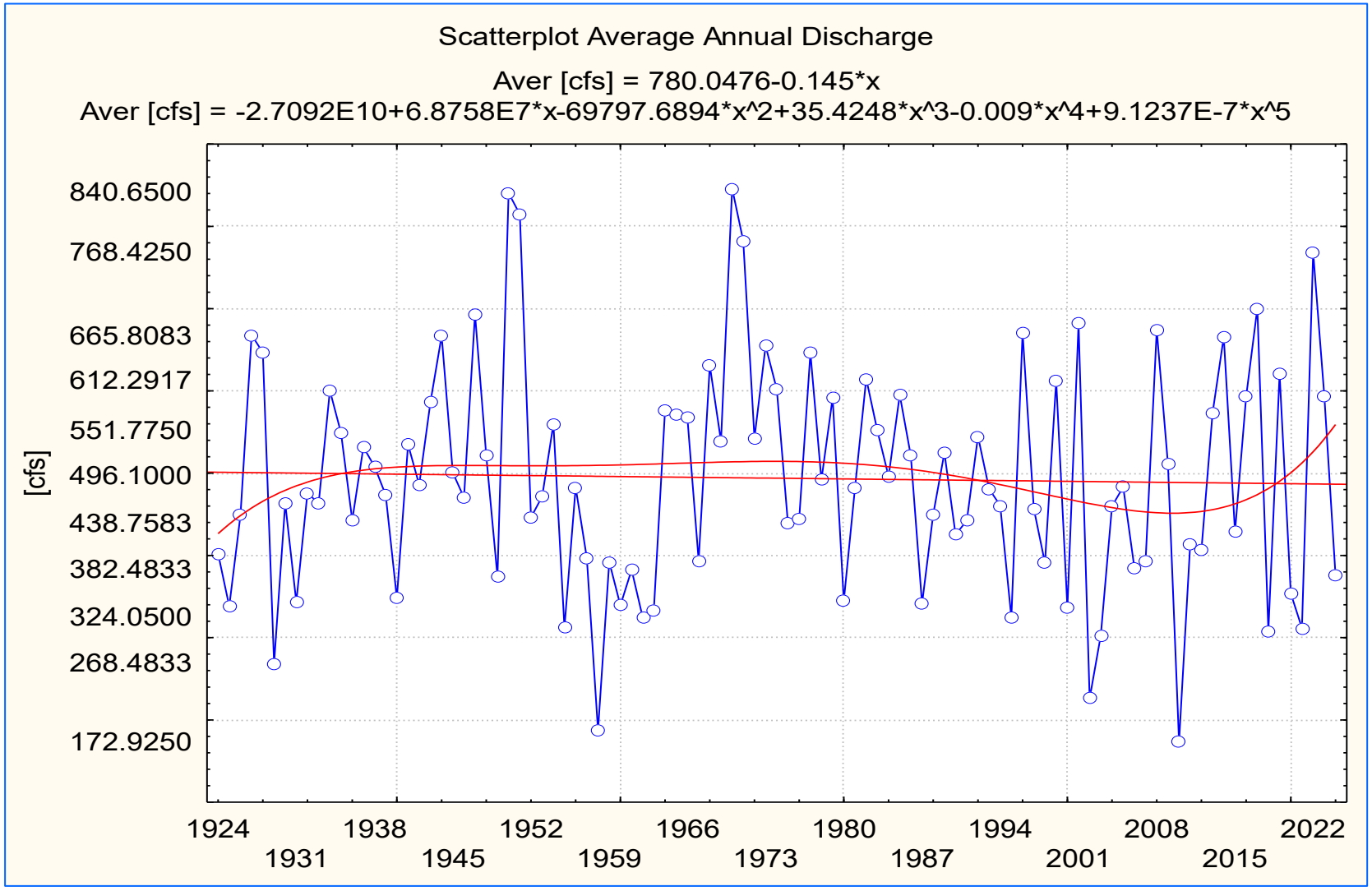
** No Incomplete data have been used for statistical calculation

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Traditional visualization

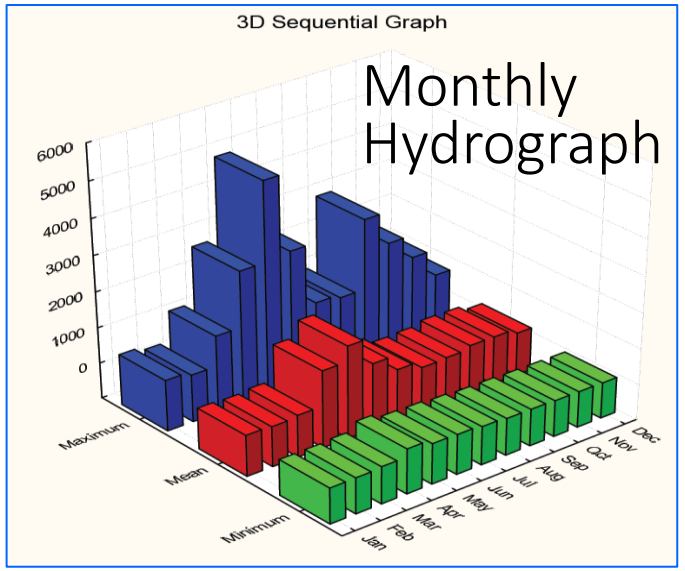


Variable	Descriptive Statistics					
	Valid N	Mean	Minimum	Maximum	Variance	Std.Dev.
Jan	101	154.4	2.2	431.1	8042.8	89.7
Feb	101	129.8	8.0	324.2	4264.8	65.3
Mar	101	181.7	41.5	1169.0	19813.8	140.8
Apr	101	1164.5	262.7	2724.0	320374.8	566.0
May	101	1596.2	137.8	4937.0	747480.7	864.6
Jun	101	851.9	124.9	2801.0	203349.9	450.9
Jul	101	432.0	78.0	1127.0	59350.2	243.6
Aug	101	235.9	40.1	1029.0	25590.2	160.0
Sep	101	261.7	36.6	2985.0	111085.0	333.3
Oct	101	356.0	17.4	2095.0	130313.9	361.0
Nov	101	350.6	11.4	1461.0	71404.0	267.2
Dec	101	211.8	2.9	720.3	22099.8	148.7

Graph of annual discharge with trends: linear & polynomic X^5



USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Correlation Analysis

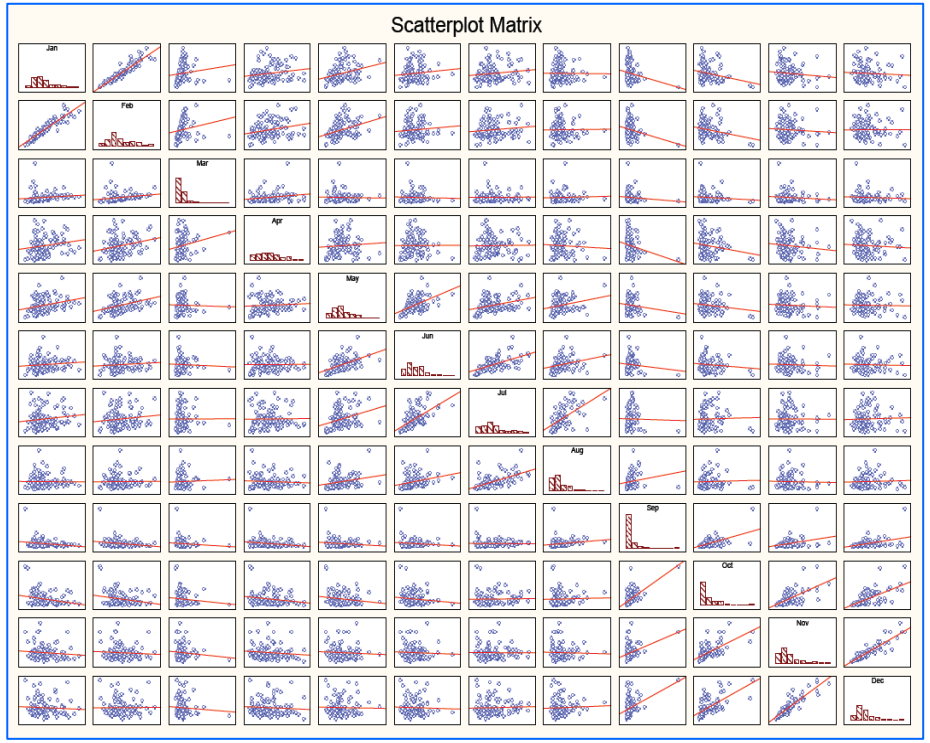


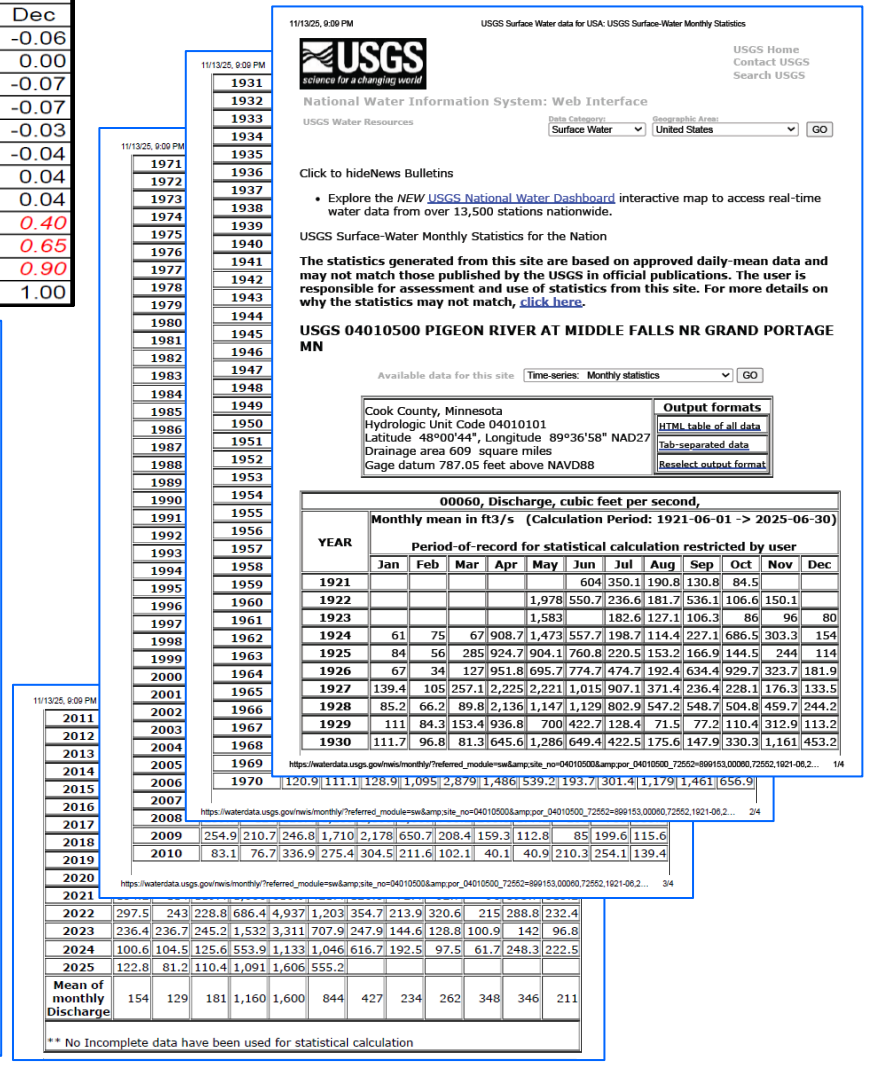
The Matrix Q_{101x12} (1924-2024) of monthly discharge observations.

Correlations statistical characteristics.

Correlations
Marked correlations are significant at p < .05000
N=101 (Casewise deletion of missing data)

Variable	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	1.00	0.93	0.13	0.18	0.30	0.11	0.13	-0.01	-0.21	-0.24	-0.11	-0.06
Feb	0.93	1.00	0.20	0.25	0.36	0.10	0.13	0.01	-0.21	-0.23	-0.07	0.00
Mar	0.13	0.20	1.00	0.21	-0.03	-0.04	-0.00	0.04	-0.10	-0.10	-0.10	-0.07
Apr	0.18	0.25	0.21	1.00	0.07	0.00	0.02	-0.06	-0.22	-0.19	-0.11	-0.07
May	0.30	0.36	-0.03	0.07	1.00	0.53	0.32	0.24	-0.13	-0.16	-0.06	-0.03
Jun	0.11	0.10	-0.04	0.00	0.53	1.00	0.58	0.27	-0.11	-0.12	-0.05	-0.04
Jul	0.13	0.13	-0.00	0.02	0.32	0.58	1.00	0.59	-0.02	0.03	-0.01	0.04
Aug	-0.01	0.01	0.04	-0.06	0.24	0.27	0.59	1.00	0.18	0.04	-0.03	0.04
Sep	-0.21	-0.21	-0.10	-0.22	-0.13	-0.11	-0.02	0.18	1.00	0.62	0.35	0.40
Oct	-0.24	-0.23	-0.10	-0.19	-0.16	-0.12	0.03	0.04	0.62	1.00	0.65	0.65
Nov	-0.11	-0.07	-0.10	-0.11	-0.06	-0.05	-0.01	-0.03	0.35	0.65	1.00	0.90
Dec	-0.06	0.00	-0.07	-0.07	-0.03	-0.04	0.04	0.04	0.40	0.65	0.90	1.00





11/13/25, 9:09 PM — USGS Surface Water data for USA: USGS Surface-Water Monthly Statistics

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USGS Water Resources

Data Category: Surface Water | Geographic Area: United States | GO

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USGS Surface-Water Monthly Statistics for the Nation

The statistics generated from this site are based on approved daily-mean data and may not match those published by the USGS in official publications. The user is responsible for assessment and use of statistics from this site. For more details on why the statistics may not match, click here.

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN

Available data for this site: Time-series: Monthly statistics | GO

Cook County, Minnesota
Hydrologic Unit Code 04010101
Latitude 48°00'44", Longitude 89°36'58" NAD27
Drainage area 609 square miles
Gage datum 787.05 feet above NAVD88

Output formats: HTML table of all data | Tab-separated data | Reselect output format

00060, Discharge, cubic feet per second,
Monthly mean in ft3/s (Calculation Period: 1921-06-01 -> 2025-06-30)
Period-of-record for statistical calculation restricted by user

YEAR	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1921						604	350.1	190.8	130.8	84.5		
1922					1,978	550.7	236.6	181.7	536.1	106.6	150.1	
1923					1,583		182.6	127.1	106.3	86	96	80
1924	61	75	67	908.7	1,473	557.7	198.7	114.4	227.1	686.5	303.3	154
1925	84	56	285	924.7	904.1	760.8	220.5	153.2	166.9	144.5	244	114
1926	67	34	127	951.8	695.7	774.7	474.7	192.4	634.4	929.7	323.7	181.9
1927	139.4	105	257.1	2,225	2,221	1,015	907.1	371.4	236.4	228.1	176.3	133.5
1928	85.2	66.2	89.8	2,136	1,147	1,129	802.9	547.2	548.7	504.8	459.7	244.2
1929	111	84.3	153.4	936.8	700	422.7	128.4	71.5	77.2	110.4	312.9	113.2
1930	111.7	96.8	81.3	645.6	1,286	649.4	422.5	175.6	147.9	330.3	1,161	453.2
1931												
1932												
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1970	120.9	111.1	128.9	1,095	2,879	1,486	539.2	193.7	301.4	1,179	1,461	656.9
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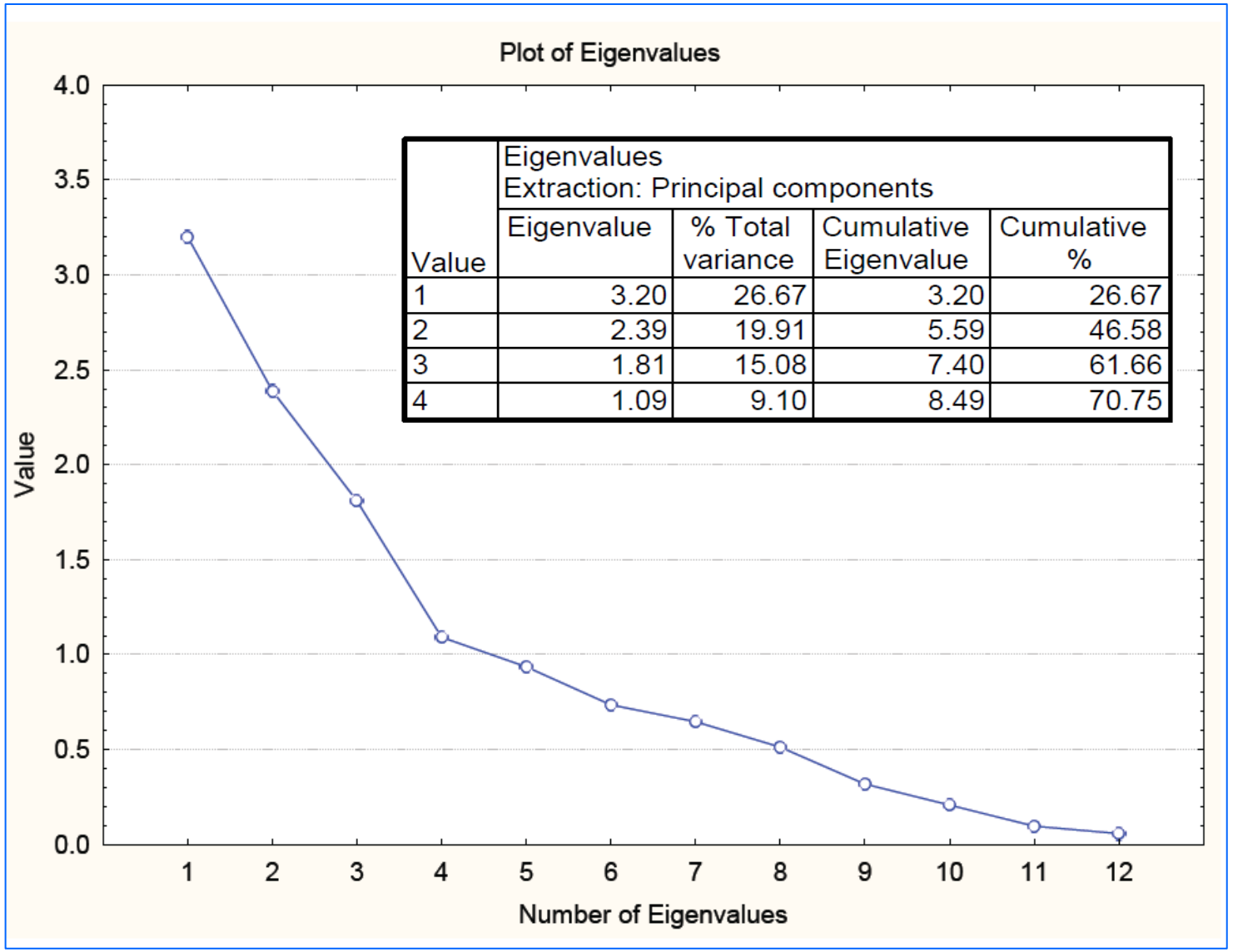
2009	254.9	210.7	246.8	1,710	2,178	650.7	208.4	159.3	112.8	85	199.6	115.6
2010	83.1	76.7	336.9	275.4	304.5	211.6	102.1	40.1	40.9	210.3	254.1	139.4
2011												
2012												
2013												
2014												
2015												
2016												
2017												
2018												
2019												
2020												
2021												
2022	297.5	243	228.8	686.4	4,937	1,203	354.7	213.9	320.6	215	288.8	232.4
2023	236.4	236.7	245.2	1,532	3,311	707.9	247.9	144.6	128.8	100.9	142	96.8
2024	100.6	104.5	125.6	553.9	1,133	1,046	616.7	192.5	97.5	61.7	248.3	222.5
2025	122.8	81.2	110.4	1,091	1,606	555.2						
Mean of monthly Discharge	154	129	181	1,160	1,600	844	427	234	262	348	346	211

** No Incomplete data have been used for statistical calculation

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, first step



Value	Eigenvalues Extraction: Principal components			
	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

Factor Analysis by four Principal components (with Eigenvalue > 1.0) reflects 70.75% of variance contained by data in Matrix Q_{101x12} (1924-2024) of monthly discharge observations.

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

Factor Analysis by four Principal components (with Eigenvalue > 1.0) reflects 70.75% of variance contained by data in Matrix Q_{101x12} (1924-2024) of monthly discharge observations. Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year .

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000)			
	Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
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May	-0.10	0.56	0.48	-0.22
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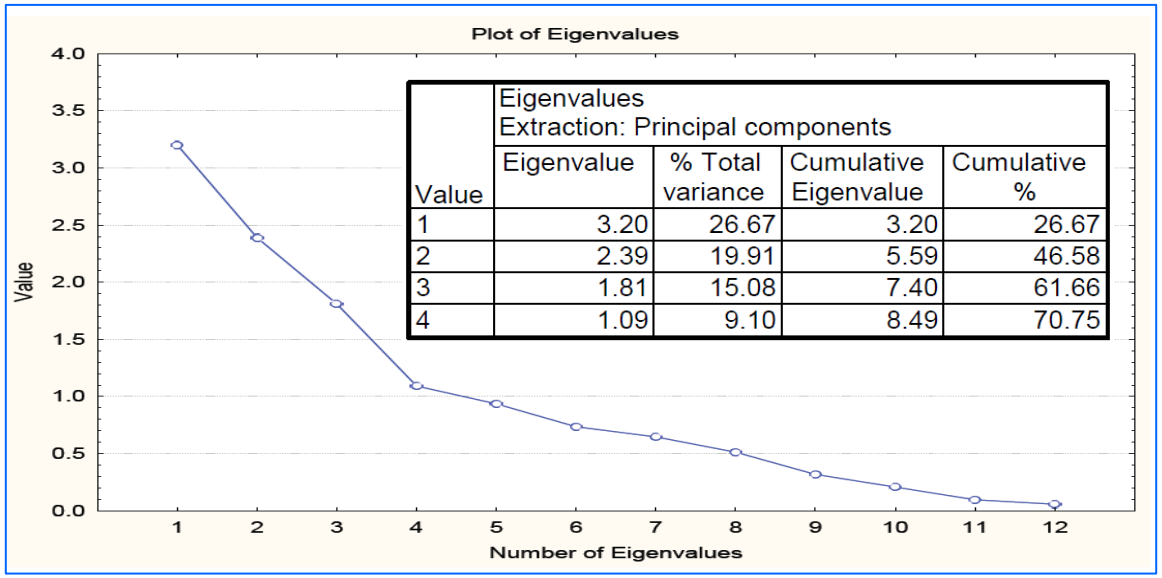
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10



Value	Eigenvalues Extraction: Principal components			
	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

Factor Analysis by four Principal components (with Eigenvalue > 1.0) reflects 70.75% of variance contained by data in Matrix Q_{101x12} (1924-2024) of monthly discharge observations. Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year .

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
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Feb	-0.05	0.06	*0.92*	0.22
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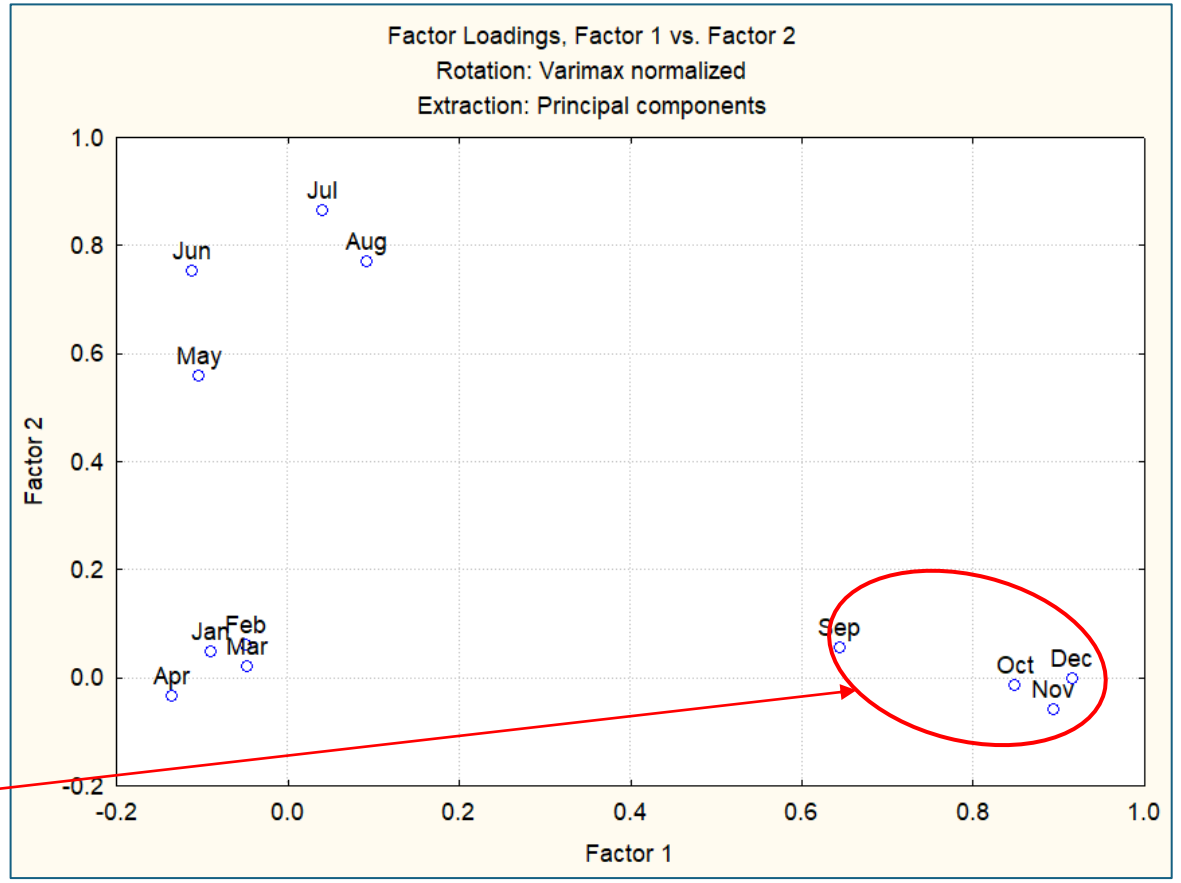
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10



Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
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Nov	*0.89*	-0.06	0.08	-0.12
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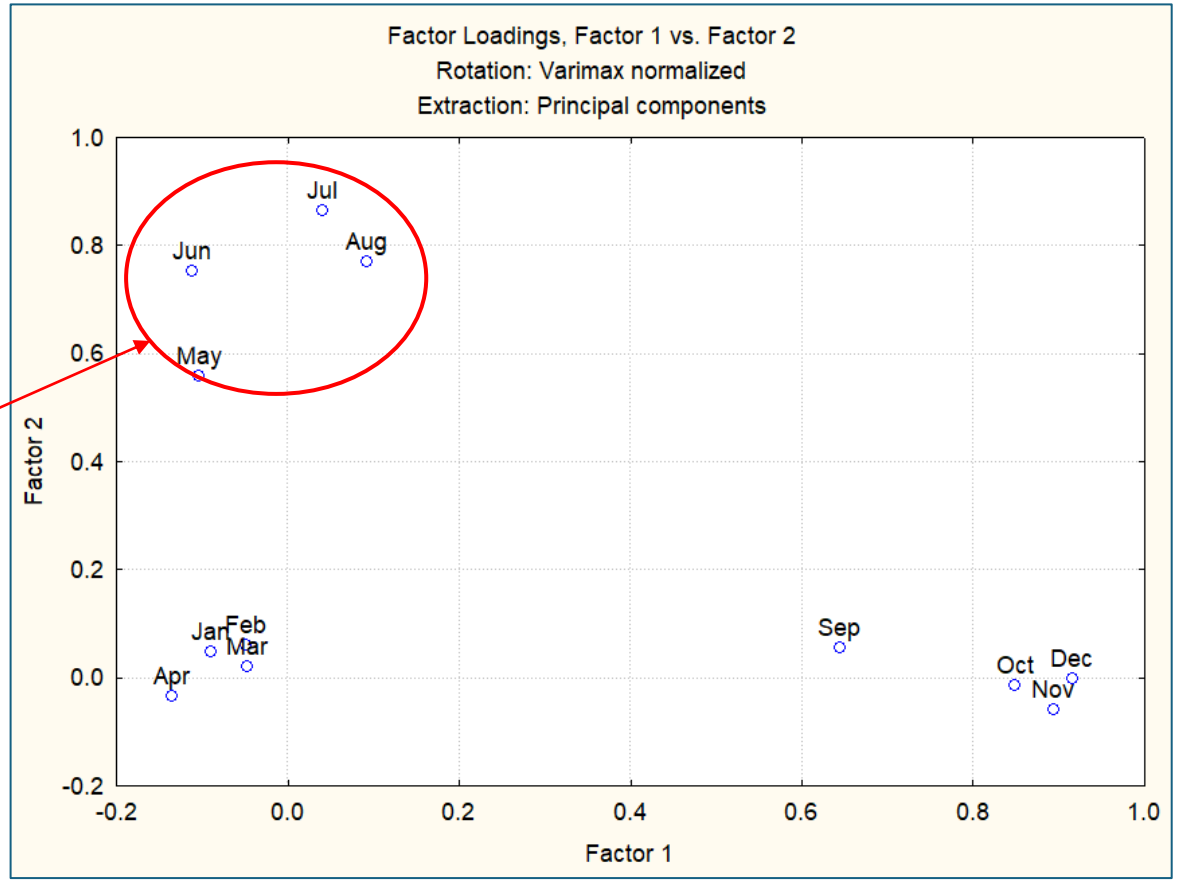
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10



Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
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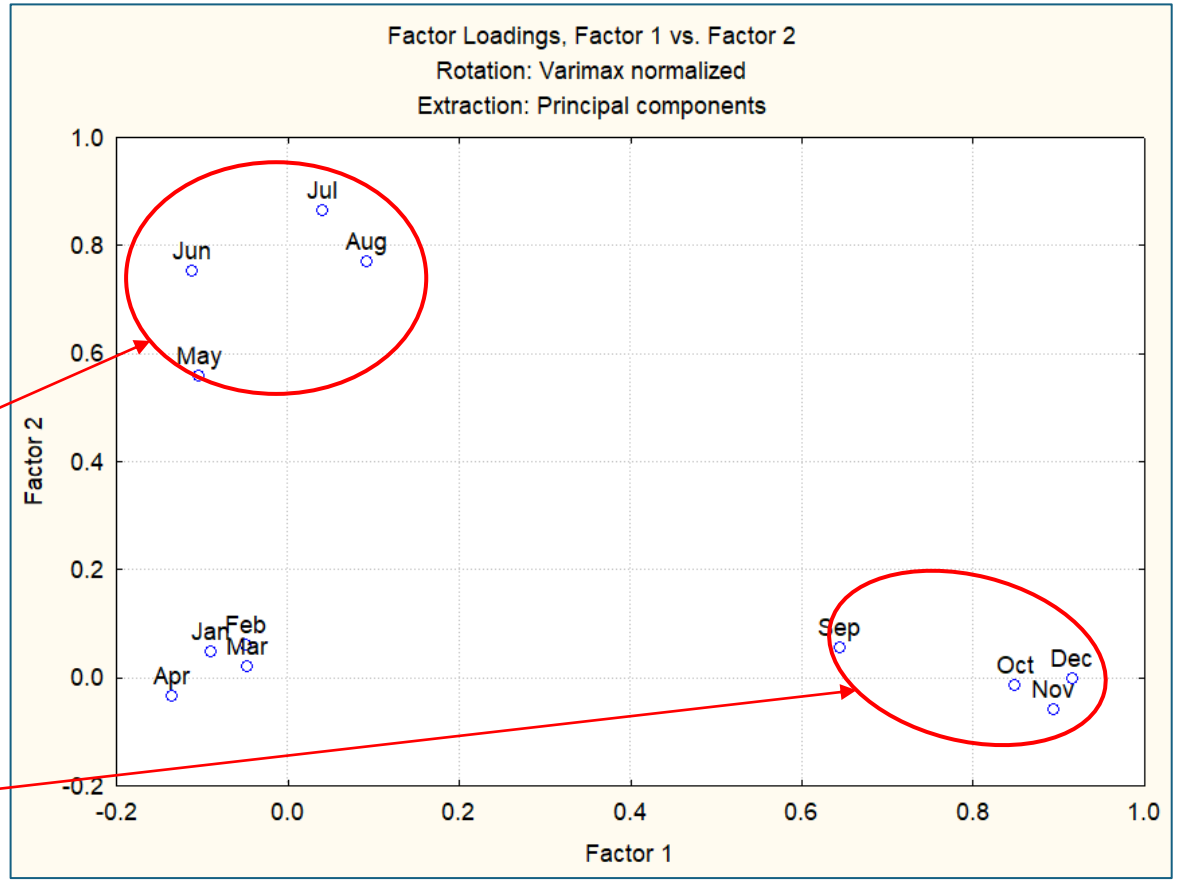

Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
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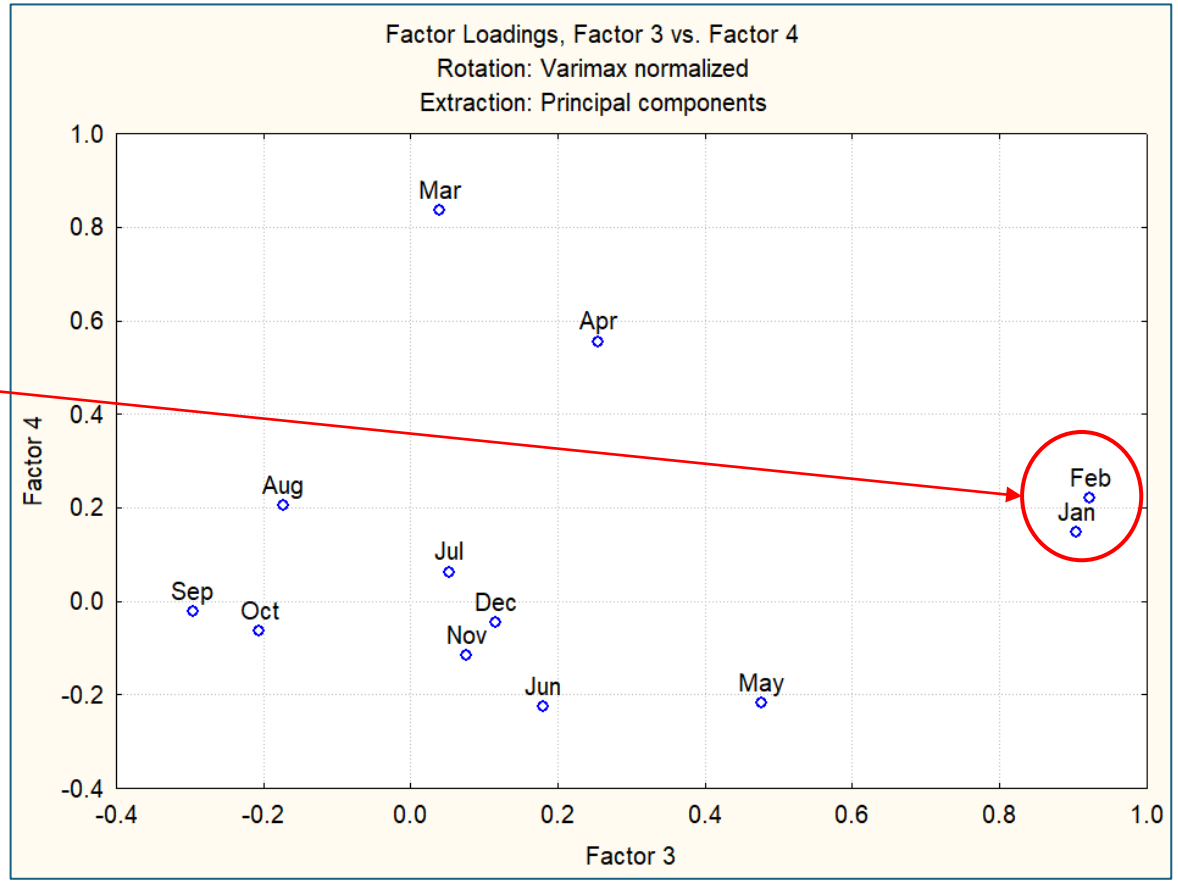

Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Analysis, structure

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
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Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
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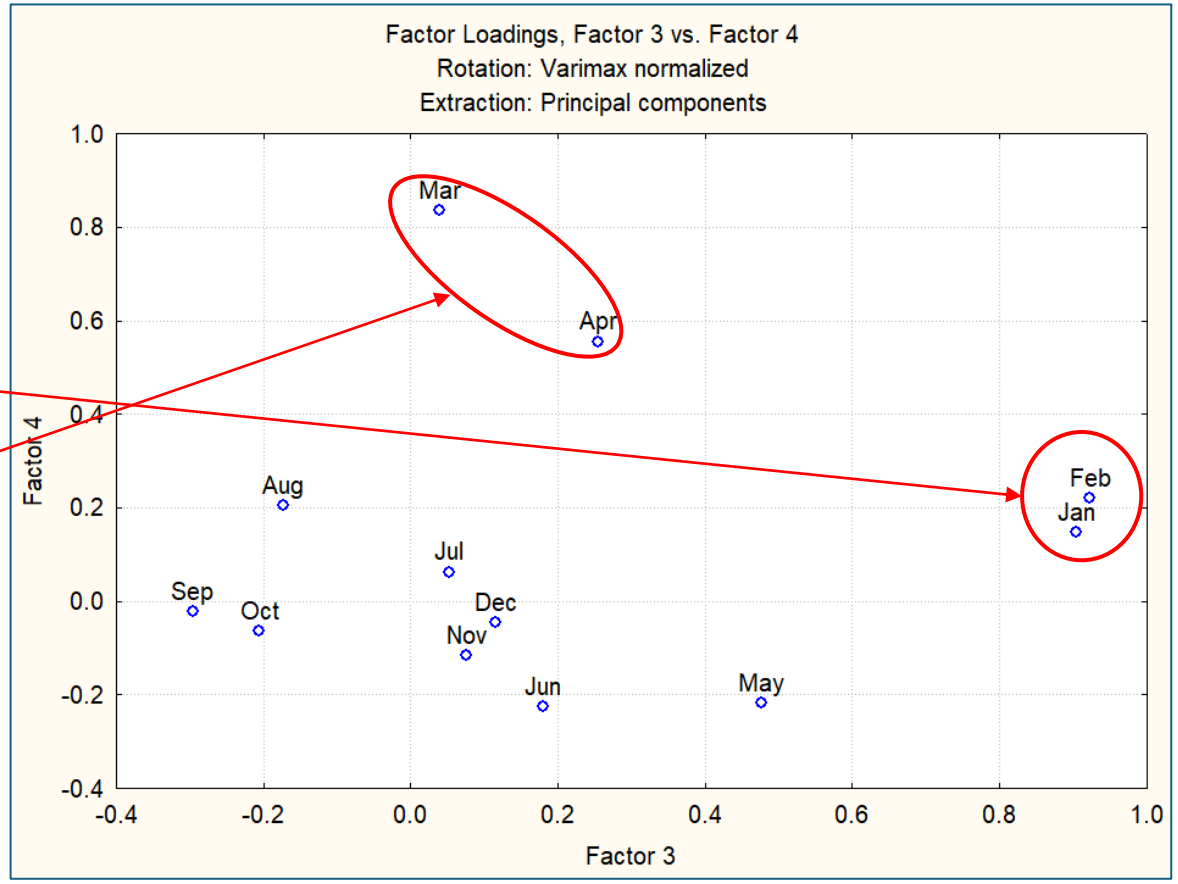
Prp.Totl	0.24	0.19	0.18	0.10



Model after varimax normalization with four Factor represent a structure variance groups of month as seasons for a year

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor 1 Scores

Factor scores for 4

1 Year Pig	2 FACTOR1	3 FACTOR2	4 FACTOR3	5 FACTOR4
1924	-0.08	-0.84	-0.72	-0.92
1925	-0.67	-0.65	-1.03	0.26
1926				
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1976				

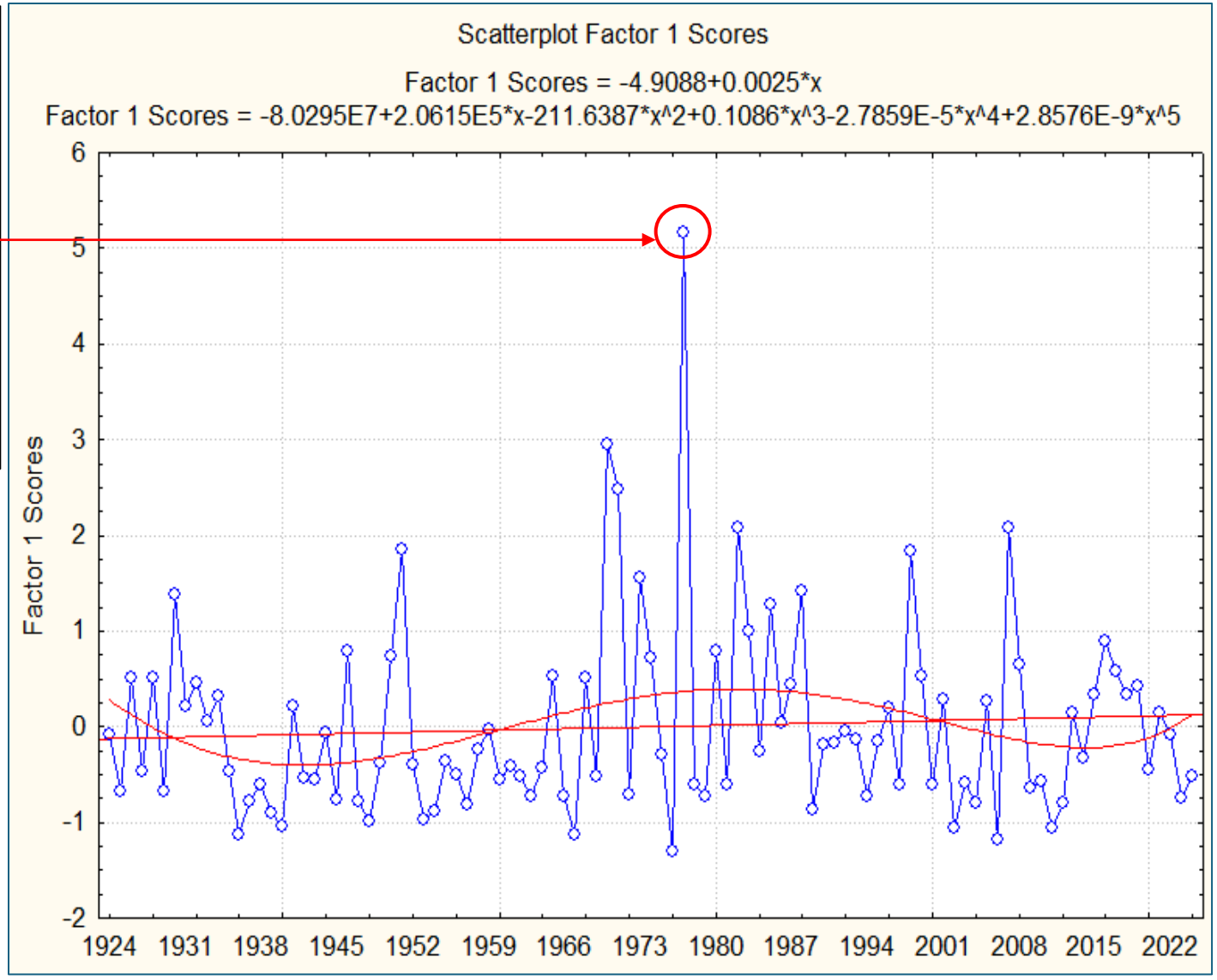
Factor scores for 4

1 Year Pig	2 FACTOR1	3 FACTOR2	4 FACTOR3	5 FACTOR4
1977	5.17	-1.27	-2.10	-0.49
1978	-0.61	0.12	1.76	-0.34
1979	-0.12	0.26	-0.04	-0.89
1980	0.79	-1.43	-0.95	-0.29
1981	-0.60	-0.17	0.04	0.70
1982	2.08	-0.31	0.16	-0.24
1983	1.00	0.13	1.60	0.44
1984	-0.25	0.09	1.00	-0.05
1985	1.27	0.99	-0.83	0.34
1986	0.04	-0.53	0.86	0.90
1987	0.44	0.08	-1.26	0.78
1988	1.42	-0.46	-0.33	0.10
1989	-0.86	0.05	1.69	-0.18
1990	-0.19	0.68	-1.14	-0.48
1991	-0.16	-0.78	0.36	0.23
1992	-0.05	0.27	0.41	-0.67
1993	-0.13	1.46	-0.64	0.60
1994	-0.72	0.26	-0.25	-0.25
1995	-0.14	-0.94	-0.51	-0.30
1996	0.20	1.65	0.58	-0.32
1997	-0.61	-0.13	1.14	0.08
1998	1.83	-1.73	-0.27	0.13
1999	0.52	0.95	0.87	1.62
2000	-0.61	-0.50	-0.05	0.62
2001	0.29	0.31	-0.14	0.71
2002	-1.06	-1.28	-0.19	-0.44
2003	-0.59	-0.47	-1.44	-0.55
2004	-0.79	-0.21	0.19	0.18
2005	0.27	-0.40	0.23	-0.34
2006	-1.18	-1.12	0.85	0.12
2007	2.07	-1.35	-1.40	-1.20
2008	0.65	1.36	0.85	-0.63
2009	-0.63	-0.66	1.22	0.55
2010	-0.57	-1.65	-0.98	0.20
2011	-1.05	0.37	-0.88	-0.07
2012	-0.80	0.69	-1.62	1.17
2013	0.14	1.80	-0.92	-0.99
2014	-0.32	2.47	0.29	-0.54
2015	0.34	-0.16	-0.01	-0.30
2016	0.90	0.75	0.81	1.61
2017	0.57	1.21	1.73	2.11
2018	0.34	-1.31	0.12	-0.73
2019	0.43	0.20	0.64	0.52
2020	-0.45	-0.91	0.64	0.20
2021	0.15	-1.51	0.08	-0.42
2022	-0.07	0.75	2.25	-1.08
2023	-0.74	-0.31	1.53	0.11
2024	-0.51	0.25	-0.48	-0.77

Factor Loadings (Varimax normalized)
Extraction: Principal components
(Marked loadings are >.700000)

Variable	Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

Factor Loadings represent a quantitively structure months of as four seasons in a year & with Scores – variability for seasons; fall-winter - 1977



USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor 2 Scores

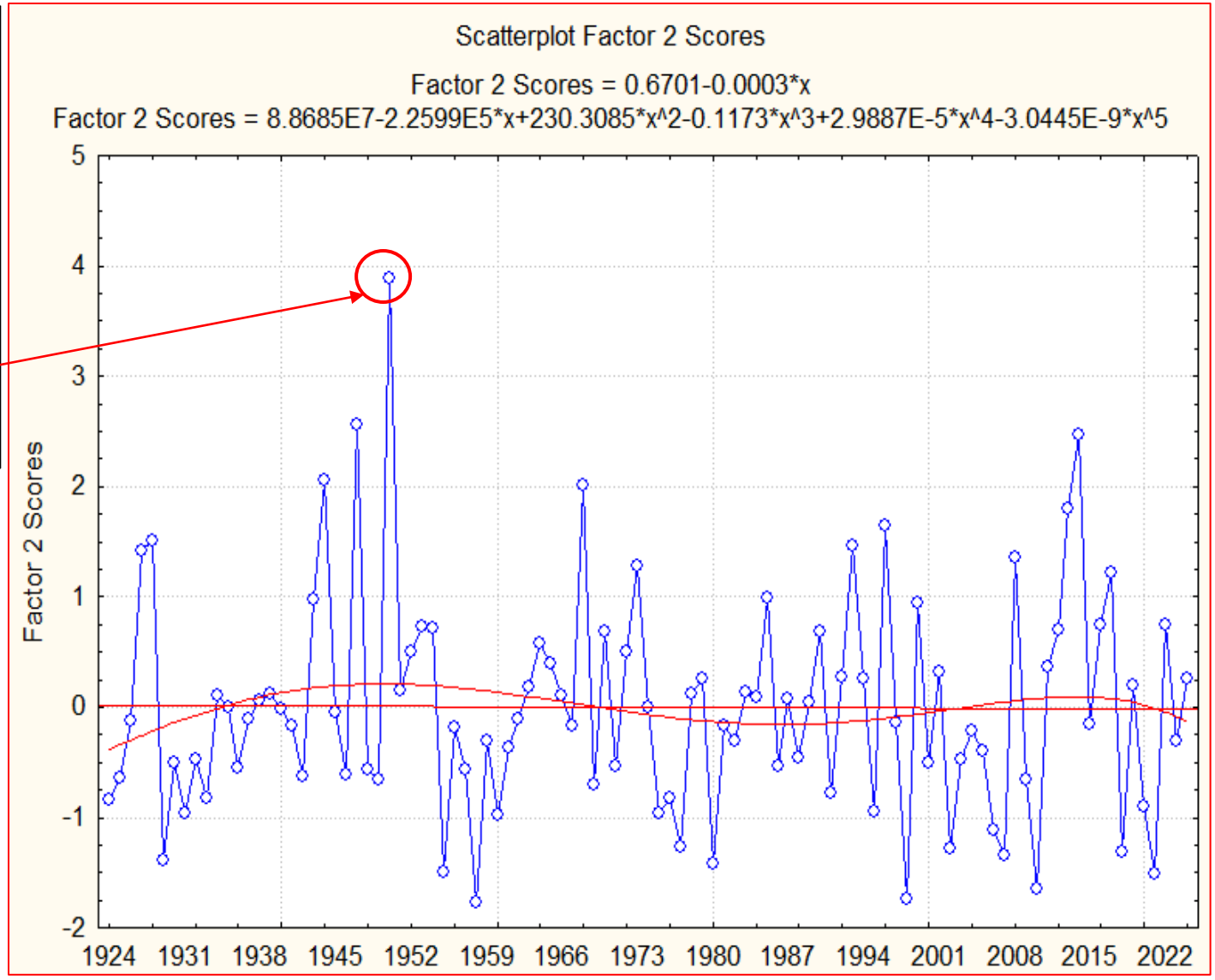
Factor scores for 4

1 Year Pig	2 FACTOR1	3 FACTOR2	4 FACTOR3	5 FACTOR4
1924	-0.08	-0.84	-0.72	-0.92
1925	-0.67	-0.65	-1.03	0.26
1926	0.50	-0.12	-1.46	-0.20
1927	-0.46	1.42	-0.39	1.32
1928	0.51	1.51	-1.15	0.86
1929	-0.67	-1.38	-0.53	-0.36
1930	1.39	-0.50	0.16	-1.04
1931	0.21	-0.96	0.37	-0.96
1932	0.46	-0.48	0.22	-0.19
1933	0.06	-0.82	0.61	0.01
1934	0.33	0.10	0.77	-1.35
1935	-0.46	0.00	0.89	1.10
1936	-1.12	-0.55	-0.15	-1.15
1937	-0.77	-0.11	-0.25	-0.24
1938	-0.60	0.05	-0.80	2.10
1939	-0.89	0.12	-0.03	-0.64
1940	-1.05	-0.01	-0.88	-1.50
1941	0.21	-0.17	-1.00	0.20
1942	-0.54	-0.62	-0.09	-0.37
1943	-0.55	0.97	-0.25	-0.88
1944	-0.06	2.06	-0.49	-1.05
1945	-0.76	-0.05	-1.56	5.79
1946	0.78	-0.62	-1.24	2.13
1947	-0.78	2.57	0.10	-1.86
1948	-0.98	-0.57	-0.24	0.36
1949	-0.38	0.66	-0.76	-0.05
1950	0.73	3.89	-0.61	-0.28
1951	1.85	0.15	1.71	0.19
1952	-0.40	0.50	0.76	1.19
1953	-0.97	0.73	-0.56	-0.77
1954	-0.88	0.71	-0.69	-0.89
1955	-0.36	-1.49	-0.50	0.12
1956	-0.50	-0.18	0.27	-0.48
1957	-0.81	-0.56	-0.36	-0.02
1958	-0.23	-1.77	-0.77	-0.78
1959	-0.03	-0.31	-0.77	-0.79
1960	-0.56	-0.97	-0.22	-0.41
1961	-0.41	-0.37	-0.71	-0.58
1962	-0.52	-0.11	-0.76	-0.31
1963	-0.73	0.18	-1.01	-0.65
1964	-0.43	0.58	-0.17	-0.76
1965	0.54	0.40	0.08	-0.31
1966	-0.73	0.10	1.27	-0.12
1967	-1.12	-0.17	-0.86	-0.22
1968	0.51	2.01	-1.48	0.22
1969	-0.51	-0.70	1.98	1.36
1970	2.95	0.69	0.89	-1.10
1971	2.49	-0.54	3.02	0.84
1972	-0.71	0.51	1.31	-0.67
1973	1.56	1.28	-1.12	1.58
1974	0.73	-0.01	1.11	-0.41
1975	-0.29	-0.96	2.46	0.03
1976	-1.30	-0.82	-0.02	1.31

Factor Loadings (Varimax normalized)
Extraction: Principal components
(Marked loadings are >.700000)

Variable	Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

Factor Loadings represent a quantitively structure months of as four seasons in a year & with Scores – variability for seasons; summer - 1950



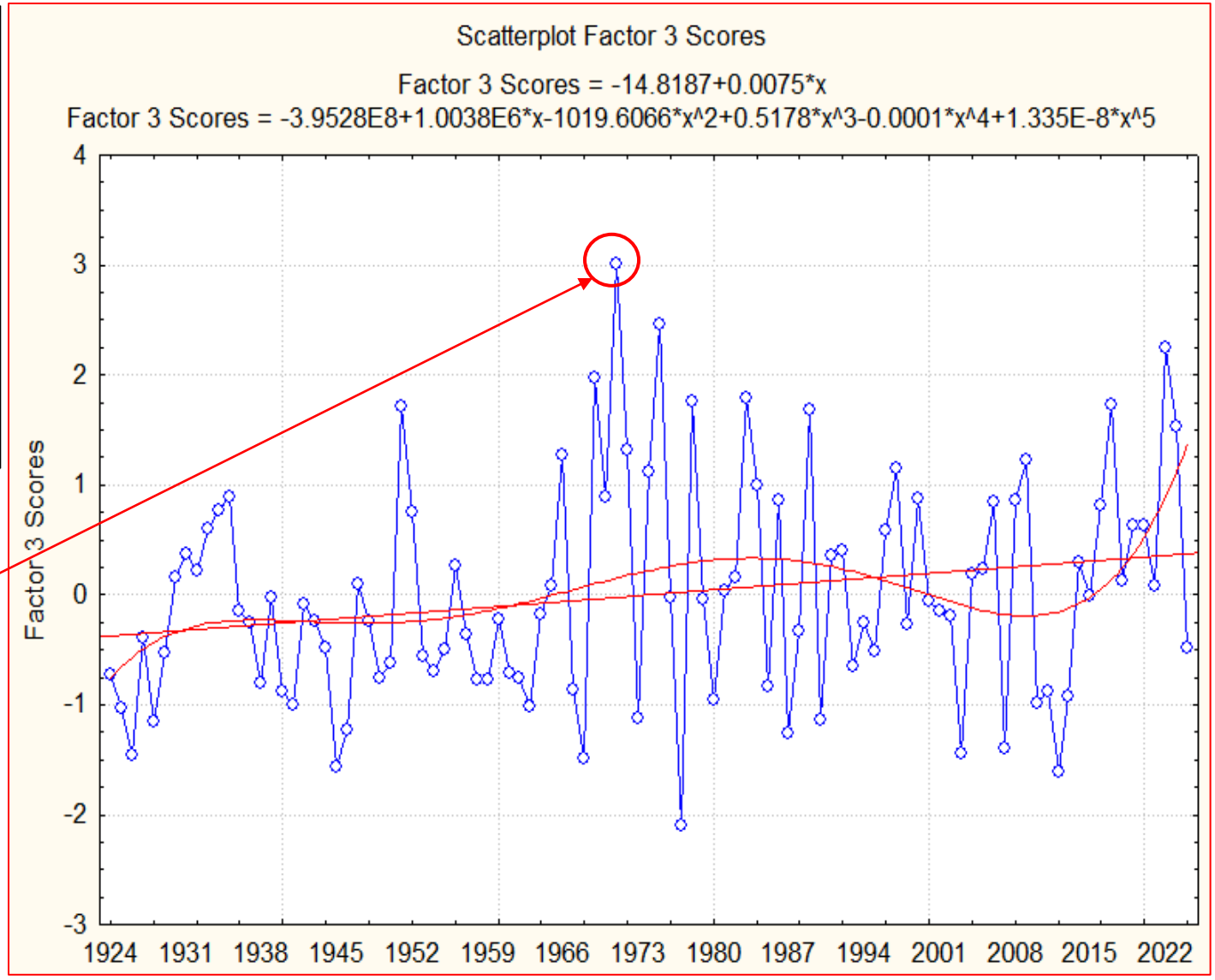
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor 3 Scores

Factor scores for 4

1 Year Pig	2 FACTOR1	3 FACTOR2	4 FACTOR3	5 FACTOR4
1924	-0.08	-0.84	-0.72	-0.92
1925	-0.67	-0.65	-1.03	0.26
1926	0.50	-0.12	-1.46	-0.20
1927	-0.46	1.42	-0.39	1.32
1928	0.51	1.51	-1.15	0.86
1929	-0.67	-1.38	-0.53	-0.36
1930	1.39	-0.50	0.16	-1.04
1931	0.21	-0.96	0.37	-0.96
1932	0.46	-0.48	0.22	-0.19
1933	0.06	-0.82	0.61	0.01
1934	0.33	0.10	0.77	-1.35
1935	-0.46	0.00	0.89	1.10
1936	-1.12	-0.55	-0.15	-1.15
1937	-0.77	-0.11	-0.25	-0.24
1938	-0.60	0.05	-0.80	2.10
1939	-0.89	0.12	-0.03	-0.64
1940	-1.05	-0.01	-0.88	-1.50
1941	0.21	-0.17	-1.00	0.20
1942	-0.54	-0.62	-0.09	-0.37
1943	-0.55	0.97	-0.25	-0.88
1944	-0.06	2.06	-0.49	-1.05
1945	-0.76	-0.05	-1.56	5.79
1946	0.78	-0.62	-1.24	2.13
1947	-0.78	2.57	0.10	-1.99
1948	-0.98	-0.57	-0.24	0.36
1949	-0.38	-0.66	-0.76	-0.05
1950	0.73	3.89	-0.61	-0.28
1951	1.85	0.15	1.71	0.19
1952	-0.40	0.50	0.76	1.19
1953	-0.97	0.73	-0.56	-0.77
1954	-0.88	0.71	-0.69	-0.89
1955	-0.36	-1.49	-0.50	0.12
1956	-0.50	-0.18	0.27	-0.48
1957	-0.81	-0.56	-0.36	-0.02
1958	-0.23	-1.77	-0.77	-0.78
1959	-0.03	-0.31	-0.77	-0.79
1960	-0.56	-0.97	-0.22	-0.41
1961	-0.41	-0.37	-0.71	-0.58
1962	-0.52	-0.11	-0.76	-0.31
1963	-0.73	0.18	-1.01	-0.65
1964	-0.43	0.58	-0.17	-0.76
1965	0.54	0.40	0.08	-0.31
1966	-0.73	0.10	1.27	-0.12
1967	-1.12	-0.17	-0.86	-0.22
1968	0.51	2.01	-1.48	0.22
1969	-0.51	-0.70	1.98	1.36
1970	2.95	0.69	0.89	-1.10
1971	2.49	-0.54	3.02	0.84
1972	-0.71	0.51	1.37	-0.67
1973	1.56	1.28	-1.12	1.58
1974	0.73	-0.01	1.11	-0.41
1975	-0.29	-0.96	2.46	0.03
1976	-1.30	-0.82	-0.02	1.31

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.64	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

Factor Loadings represent a quantitively structure months of as four seasons in a year & with Scores – variability for seasons; wint. (Jan-Feb) - 1971



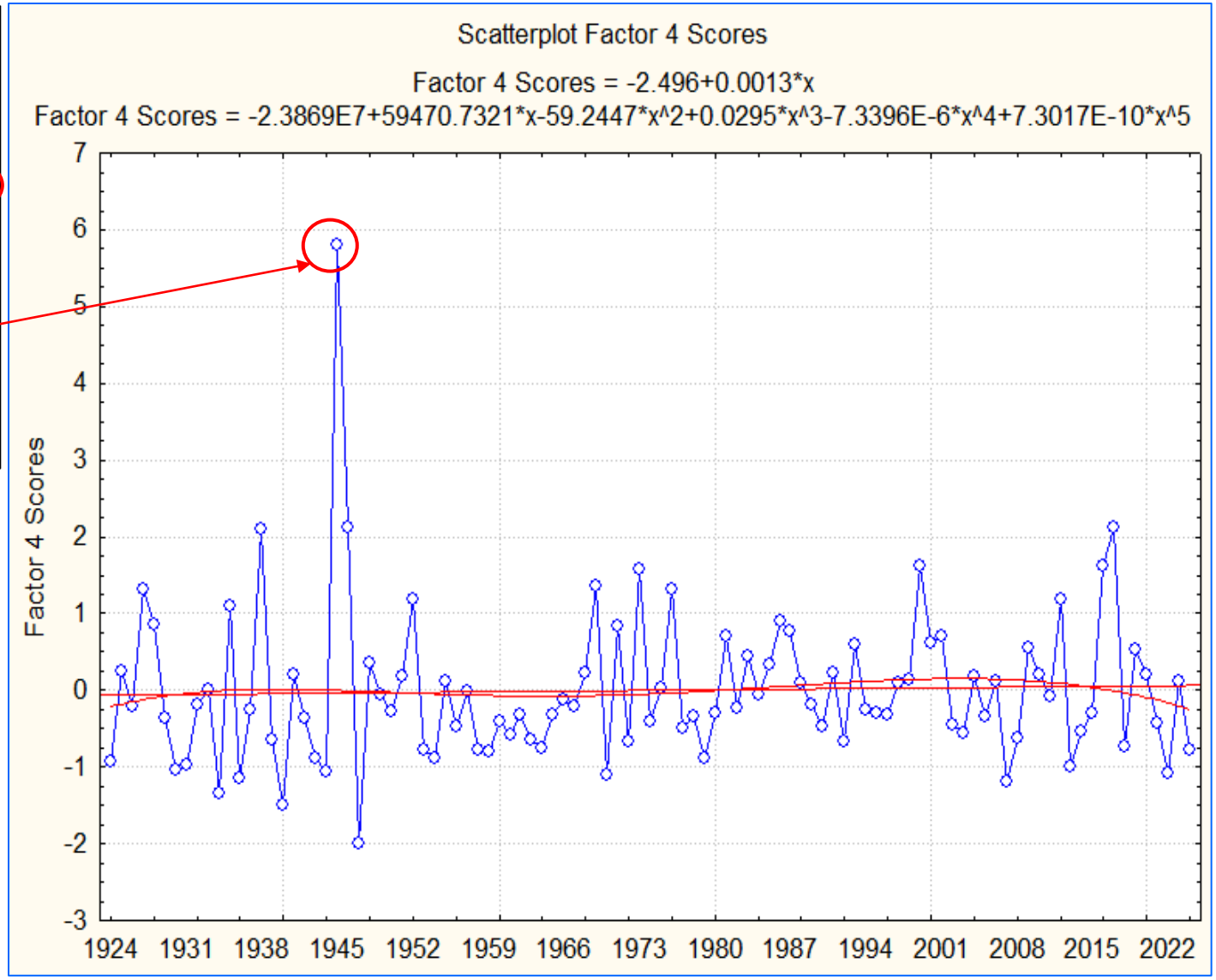
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor 4 Scores

Factor scores for 4

1 Year Pig	2 FACTOR1	3 FACTOR2	4 FACTOR3	5 FACTOR4
1924	-0.08	-0.84	-0.72	-0.92
1925	-0.67	-0.65	-1.03	0.26
1926	0.50	-0.12	-1.46	-0.20
1927	-0.46	1.42	-0.39	1.32
1928	0.51	1.51	-1.15	0.86
1929	-0.67	-1.38	-0.53	-0.36
1930	1.39	-0.50	0.16	-1.04
1931	0.21	-0.96	0.37	-0.96
1932	0.46	-0.48	0.22	-0.19
1933	0.06	-0.82	0.61	0.01
1934	0.33	0.10	0.77	-1.35
1935	-0.46	0.00	0.89	1.10
1936	-1.12	-0.55	-0.15	-1.15
1937	-0.77	-0.11	-0.25	-0.24
1938	-0.60	0.05	-0.80	2.10
1939	-0.89	0.12	-0.03	-0.64
1940	-1.05	-0.01	-0.88	-1.50
1941	0.21	-0.17	-1.00	0.20
1942	-0.54	-0.62	-0.09	-0.37
1943	-0.55	0.97	-0.25	0.88
1944	-0.06	2.06	-0.49	-1.05
1945	-0.76	-0.05	-1.56	5.79
1946	0.78	-0.62	-1.24	2.13
1947	-0.78	2.57	0.10	-1.99
1948	-0.98	-0.57	-0.24	0.36
1949	-0.38	-0.66	-0.76	-0.05
1950	0.73	3.89	-0.61	-0.28
1951	1.85	0.15	1.71	0.19
1952	-0.40	0.50	0.76	1.19
1953	-0.97	0.73	-0.56	-0.77
1954	-0.88	0.71	-0.69	-0.89
1955	-0.36	-1.49	-0.50	0.12
1956	-0.50	-0.18	0.27	-0.48
1957	-0.81	-0.56	-0.36	-0.02
1958	-0.23	-1.77	-0.77	-0.78
1959	-0.03	-0.31	-0.77	-0.79
1960	-0.56	-0.97	-0.22	-0.41
1961	-0.41	-0.37	-0.71	-0.58
1962	-0.52	-0.11	-0.76	-0.31
1963	-0.73	0.18	-1.01	-0.65
1964	-0.43	0.58	-0.17	-0.76
1965	0.54	0.40	0.08	-0.31
1966	-0.73	0.10	1.27	-0.12
1967	-1.12	-0.17	-0.86	-0.22
1968	0.51	2.01	-1.48	0.22
1969	-0.51	-0.70	1.98	1.36
1970	2.95	0.69	0.89	-1.10
1971	2.49	-0.54	3.02	0.84
1972	-0.71	0.51	1.31	-0.67
1973	1.56	1.28	-1.12	1.58
1974	0.73	-0.01	1.11	-0.41
1975	-0.29	-0.96	2.46	0.03
1976	-1.30	-0.82	-0.02	1.31

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

Factor Loadings represent a quantitively structure months of as four seasons in a year & with Scores – variability for seasons; spring (March) - 1945

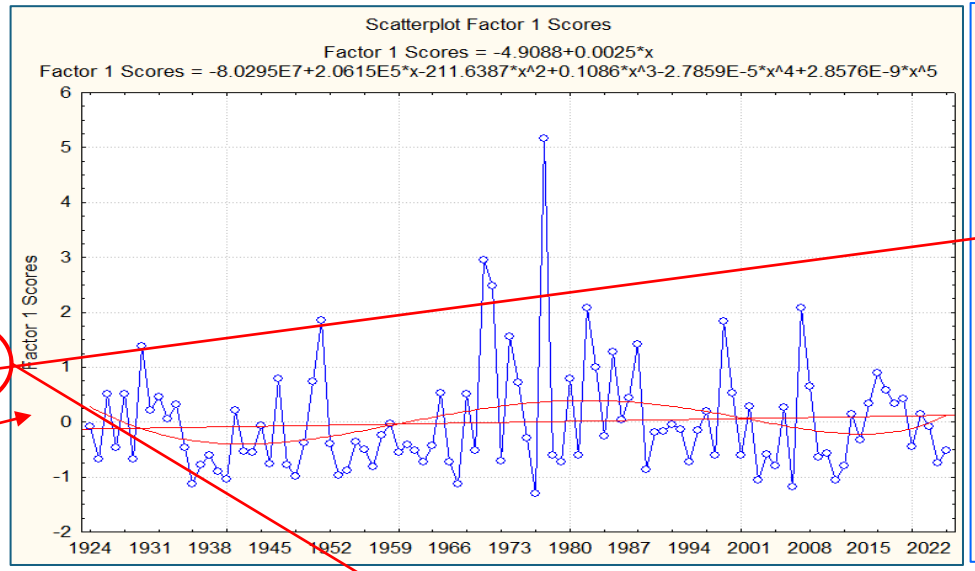


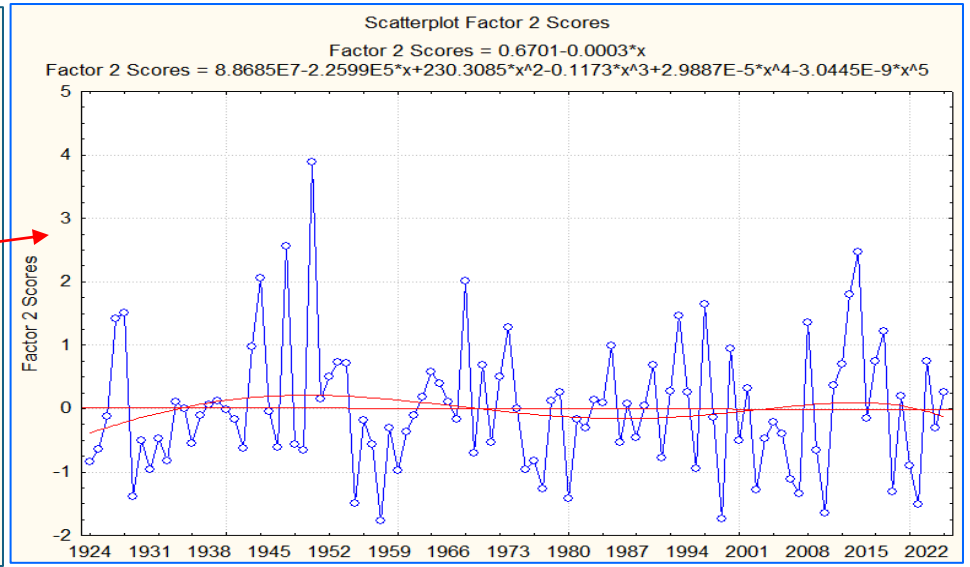
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor structure of seasonal discharge

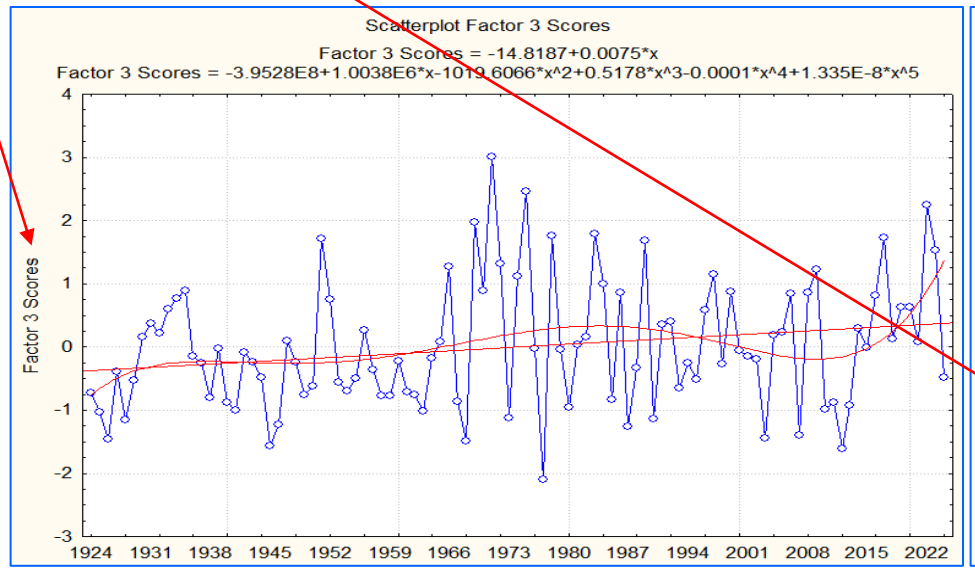
Value	Eigenvalues Extraction: Principal components			
	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

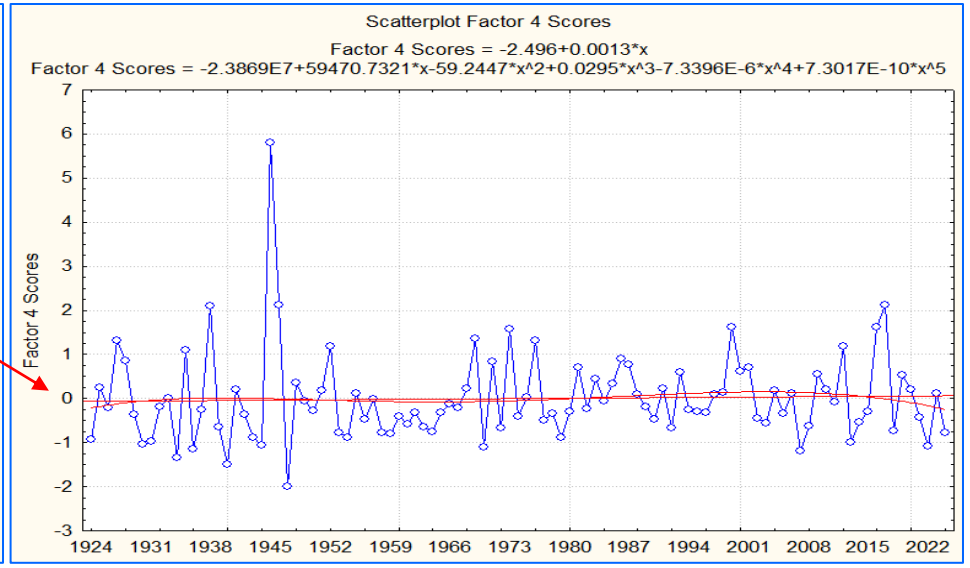
Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.64	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

Factor Loadings as structure of seasonality & Factor Scores – seasons variability through the years







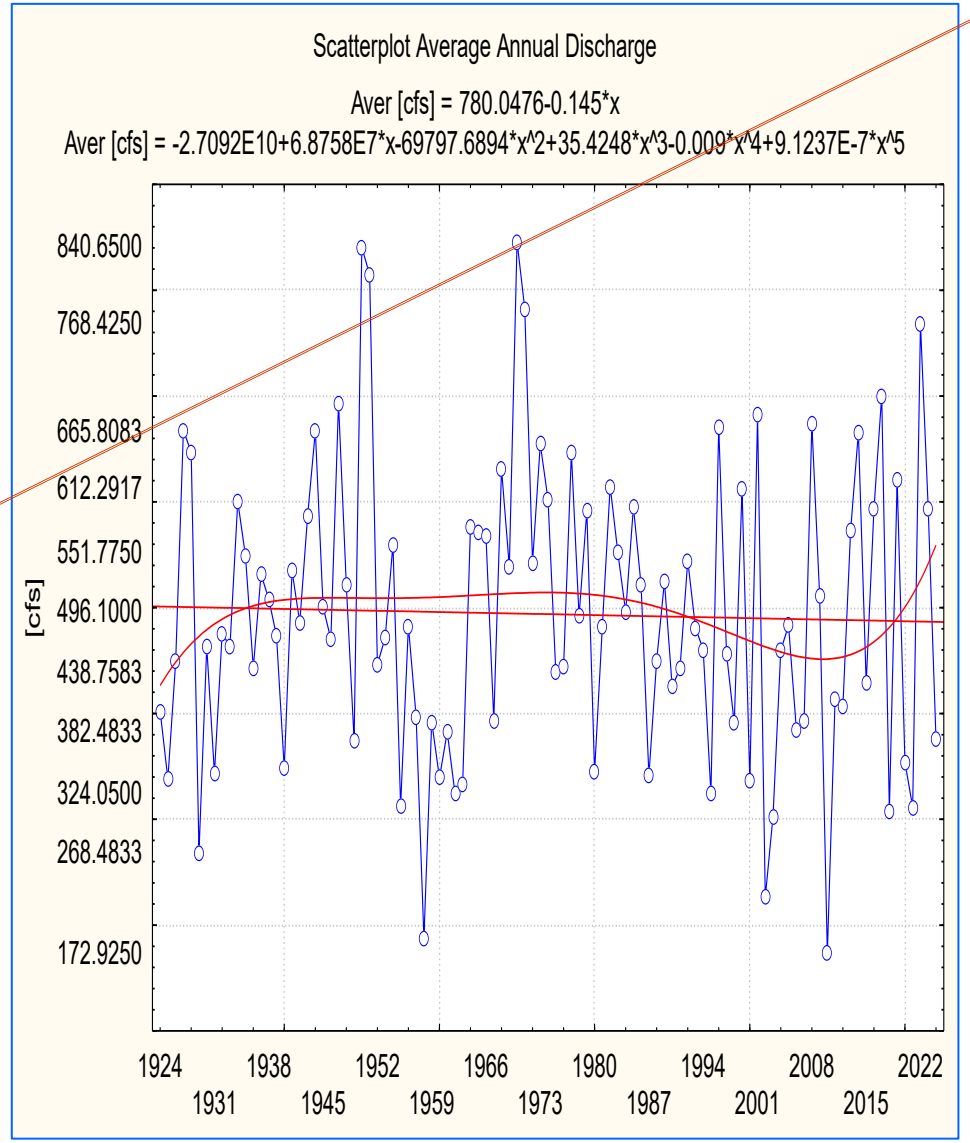


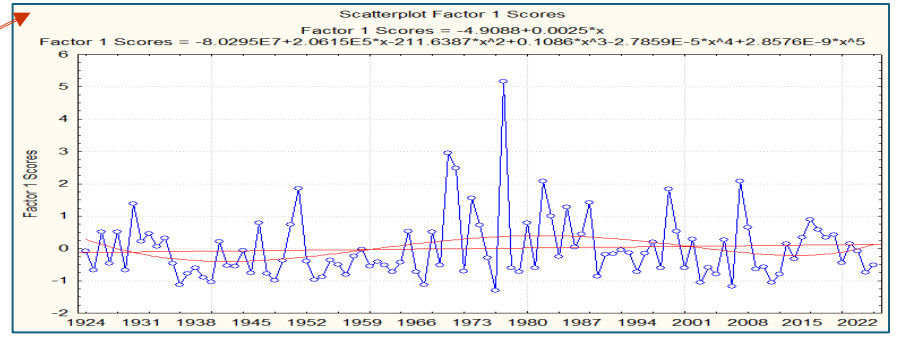
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Annual & Factor Scores as seasonal discharge

Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

The graph of annual discharge reflects all variability from empirical data, Factor Scores only parts, for Factor 1- 0.24



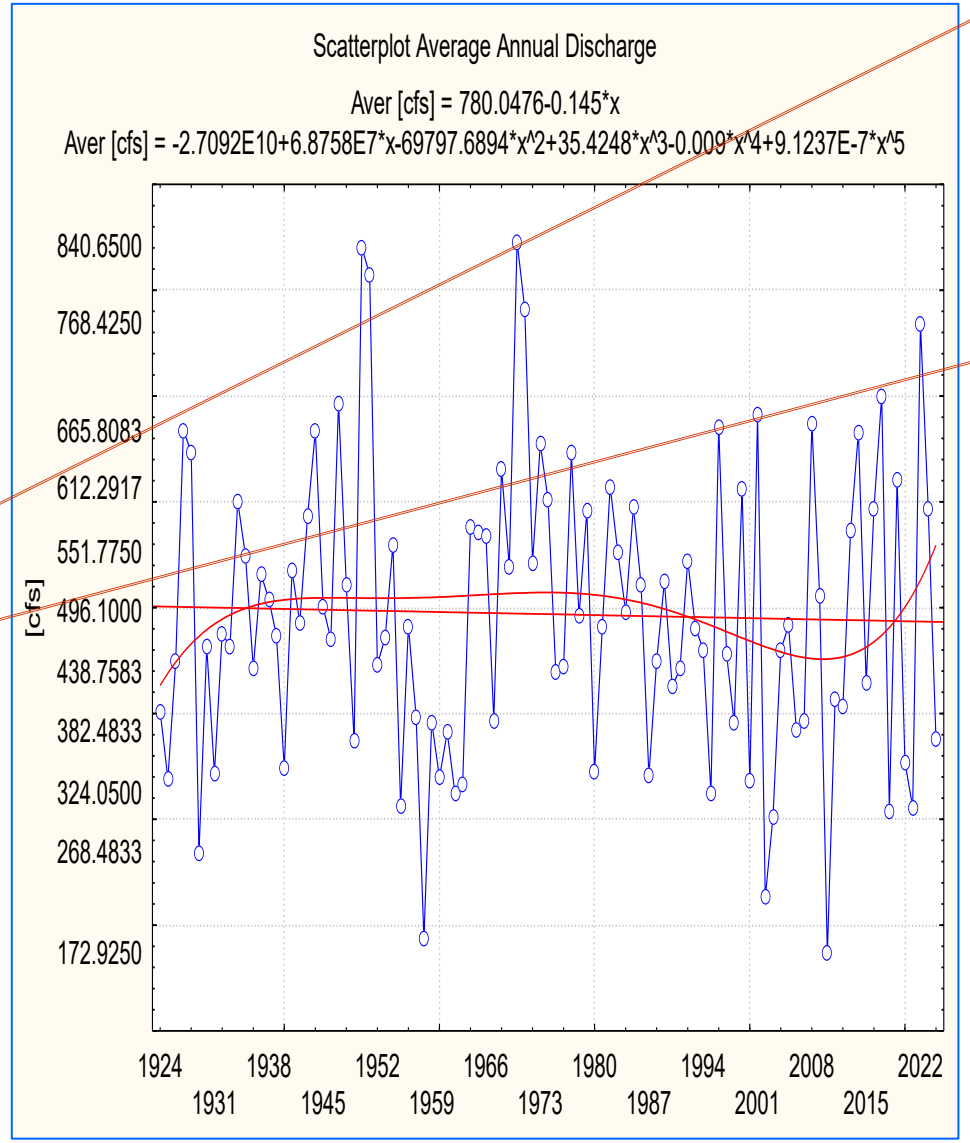


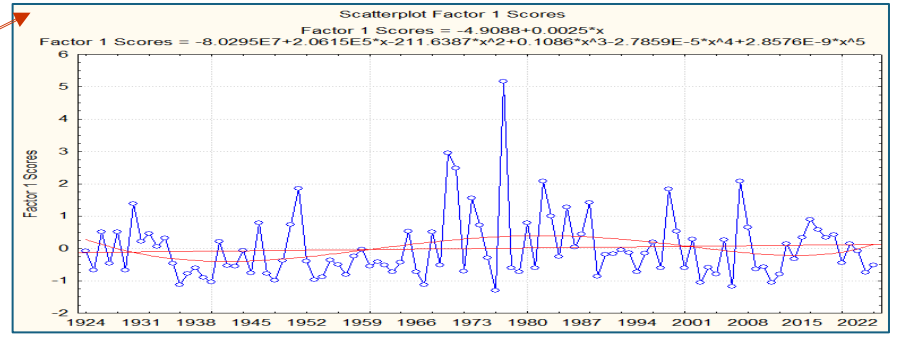
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Annual & Factor Scores as seasonal discharge

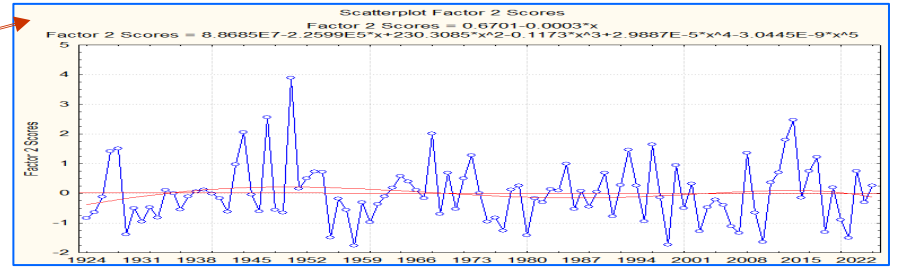
Value	Eigenvalues Extraction: Principal components			
	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

The graph of annual discharge reflects all variability from empirical data, Factor Scores only parts, for Factor 2- 0.19





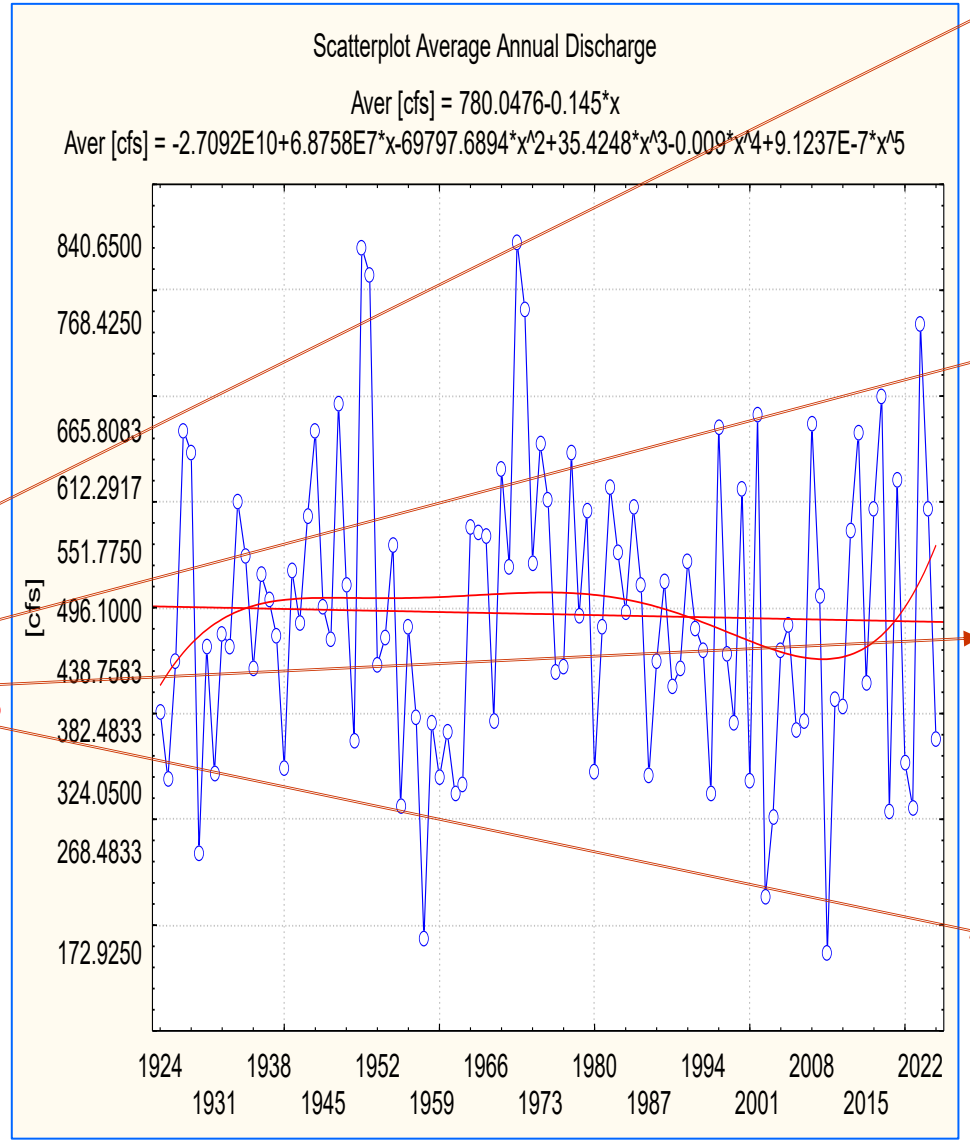


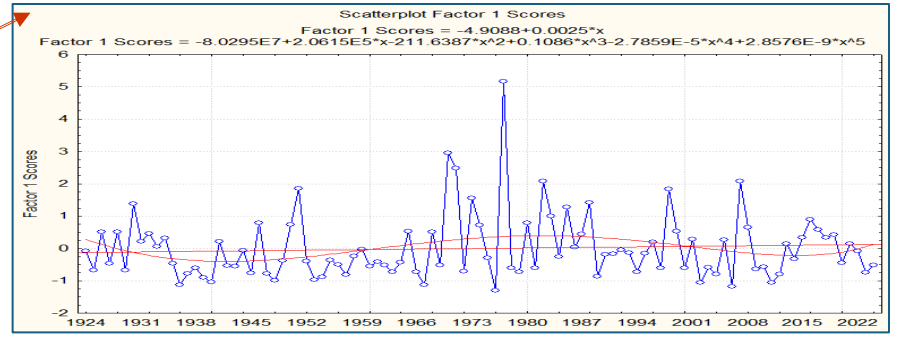
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Annual & Factor Scores as seasonal discharge

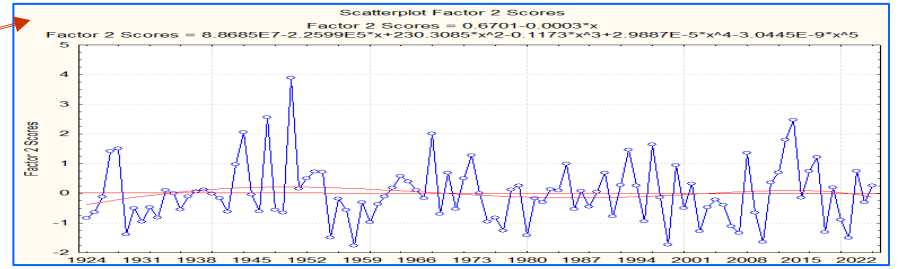
Value	Eigenvalues Extraction: Principal components			
	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

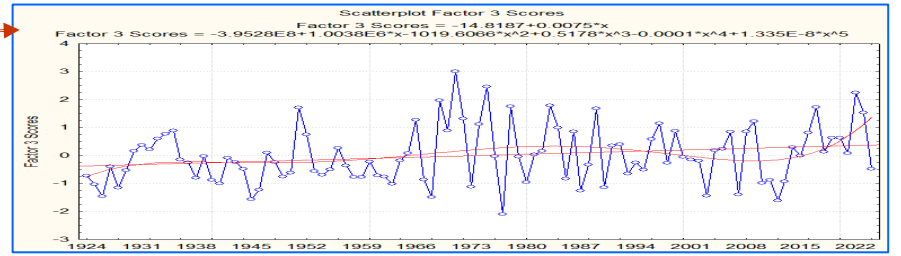
Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	-0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

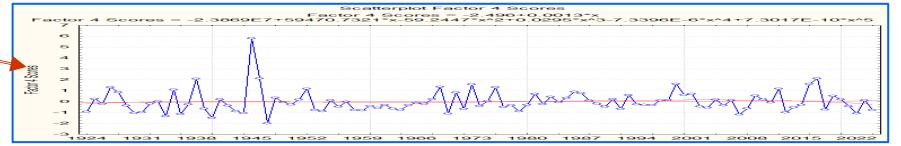
The graph of annual discharge reflects all variability from empirical data, Factor Scores only parts, for Factors 1-4 0.71











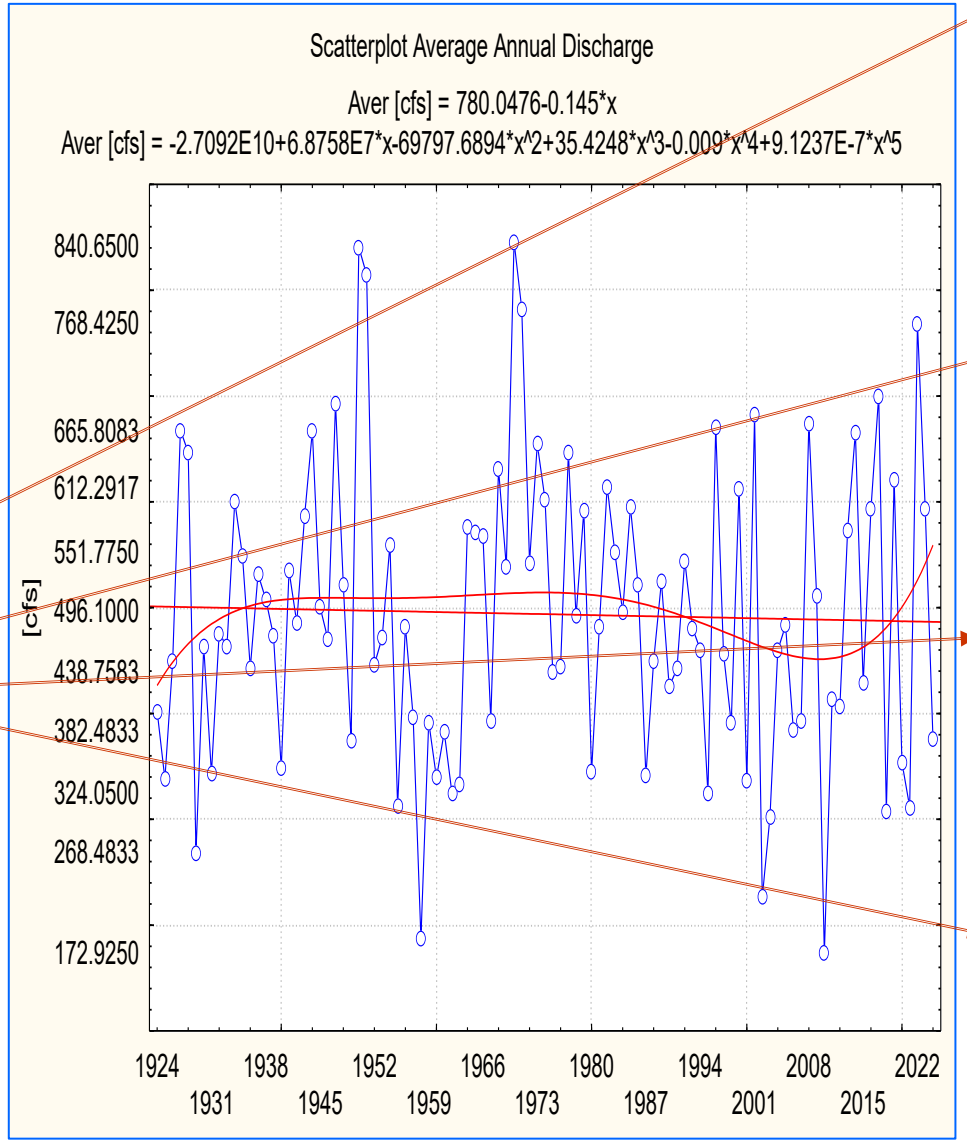
an attempt to reflect parts by size of graph

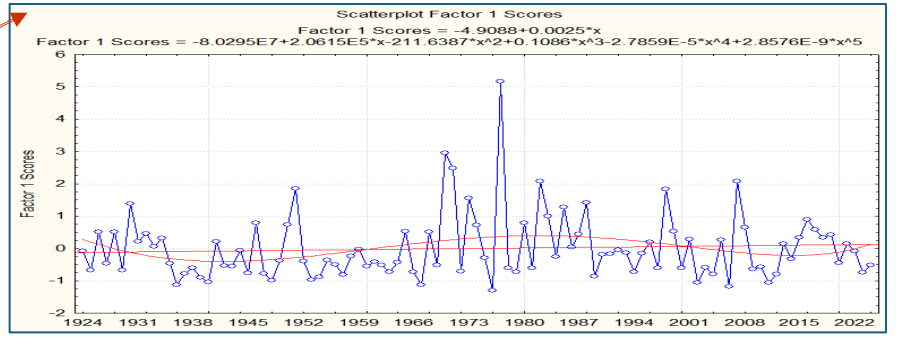
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Annual, seasonal discharge & uncertainty

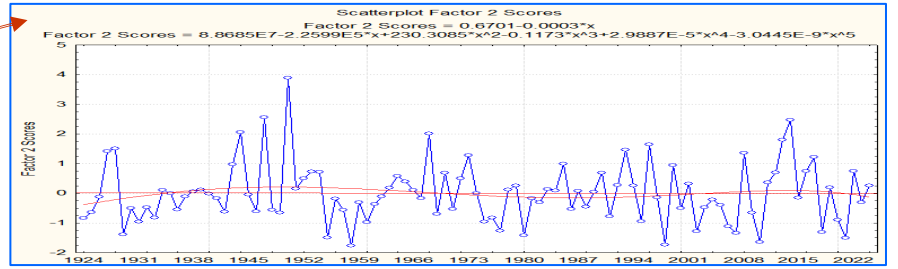
Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.20	26.67	3.20	26.67
2	2.39	19.91	5.59	46.58
3	1.81	15.08	7.40	61.66
4	1.09	9.10	8.49	70.75

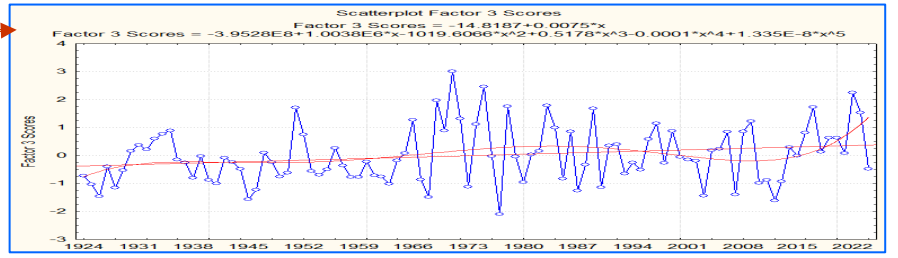
Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
Jan	-0.09	0.05	*0.90*	0.15
Feb	-0.05	0.06	*0.92*	0.22
Mar	-0.05	0.02	0.04	*0.83*
Apr	-0.13	-0.03	0.26	0.56
May	-0.10	0.56	0.48	-0.22
Jun	-0.11	*0.75*	0.18	-0.23
Jul	0.04	*0.87*	0.05	0.06
Aug	0.09	*0.77*	-0.17	0.20
Sep	0.64	0.06	-0.30	-0.02
Oct	*0.85*	-0.01	-0.21	-0.06
Nov	*0.89*	-0.06	0.08	-0.12
Dec	*0.92*	0.00	0.12	-0.04
Expl.Var	2.84	2.24	2.18	1.24
Prp.Totl	0.24	0.19	0.18	0.10

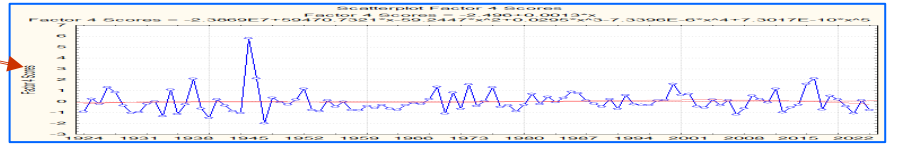
Uncertainty is 29.25%, model does not reflect this part of the variability of the data





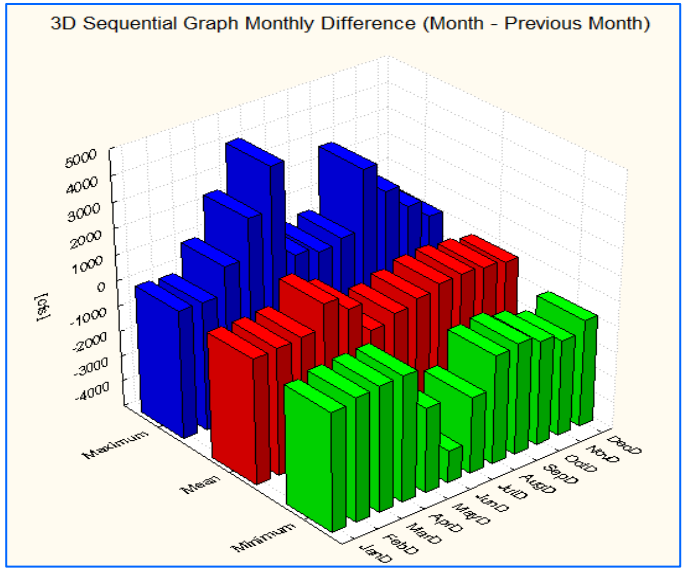






Hydrograph of annual discharge is one- & Factor Scores as year's seasons are multi - dimensional characteristics

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Data of monthly change



Monthly Hydrograph, visualization of monthly changes of discharge: Minimum, Mean & Maximum for the 101 years.

Calculated Matrix $Q_{101x12D}$ (1924-2024) of monthly discharge changes.

Descriptive statistical characteristics.

Variable	Descriptive Statistics								
	Valid N	Mean	Median	Minimum	Maximum	Lower Quartile	Upper Quartile	Variance	Std.Dev.
JanD	101	-56.0	-32.6	-345.8	73.7	-74.8	-8.8	6591.2	81.2
FebD	101	-24.6	-16.7	-161.6	35.8	-33.0	-0.7	1450.1	38.1
MarD	101	52.0	12.0	-78.5	1091.0	-4.4	49.9	20465.0	143.1
AprD	101	982.7	945.4	-61.5	2610.5	564.3	1268.5	307197.9	554.3
MayD	101	431.8	205.0	-1674.0	4250.6	-236.8	838.3	995857.2	997.9
JunD	101	-744.4	-632.2	-3734.0	558.9	-1153.0	-230.0	537798.0	733.3
JulD	101	-419.9	-342.6	-1997.7	357.1	-619.3	-155.0	135104.6	367.6
AugD	101	-196.1	-179.3	-729.2	570.5	-266.8	-89.1	38885.8	197.2
SepD	101	25.8	-31.1	-621.6	2897.3	-82.4	22.4	117536.3	342.8
OctD	101	94.3	14.7	-890.0	1740.7	-35.2	173.5	93002.9	305.0
NovD	101	-5.4	5.7	-1237.4	830.7	-67.0	123.6	76280.6	276.2
DecD	101	-138.7	-103.6	-804.1	112.5	-187.1	-36.8	22088.3	148.6

	1 Year Pig D	2 JanD	3 FebD	4 MarD	5 AprD	6 MayD	7 JunD	8 JulD	12 NovD	13 DecD	14 AverD [qfs]
1	1924	-19	14	-8	841.7	564.3	-915.3	-359	-383.2	-149.3	6.2
2	1925	-70	-28	229	639.7	-20.6	-143.3	-540.3	99.5	-130	-3.3
3	1926	-47	-33	93	824.8	-256.1	79	-300	-606	-141.8	5.7
4	1927	-42.5	-34.4	152.1	1967.9	-4	-1206	-107.9	-51.8	-42.8	-4.0
5	1928	-48.3	-19	23.6	2046.2	-989	-18	-326.1	-45.1	-215.5	9.2
6	1929	-133.2	-26.7	69.1	783.4	-236.8	-277.3	-294.3	202.5	-199.7	-10.9
7	1930	-1.5	-14.9	-15.5	564.3	640.4	-636.6	-226.9	830.7	-707.8	28.3
8	1931	-262.6	-53.3	-25	337.7	444	-280.2	-271.9	314.6	-450.7	-21.2
9	1932	-65.4	-17.6	17.4	1236.1	68	-733.9	-374.2	608	-326.5	14.7
10	1933	-170.5	-39.6	-10.7	1289.5	-184	-534.1	-471	-20.1	-187.1	-14.5
11	1934	-43.8	-6.4	-11	494.1	2278.7	-1638	-972.9	-28.3	-216.2	6.6
12	1935	-52.9	-12.1	86.9	1639.8	-578	-96	-879.7	88.8	-92.2	-11.7
13	1936	-31.6	-29.3	11.1	918.8	1827	-2283.3	-317.7	14.5	-0.9	-4.9
14	1937	23.6	-16.8	7.5	1950.9	-442	-265	-1026.7	21.9	-113.8	2.0
15	1938	-22.8	17.4	404.2	1262.7	-108	-910.1	-418.1	126.4	-143	-0.5
16	1939	51.1	-32.5	12	945.4	908	-844	-766.7	-67	-16.6	-1.2
17	1940	-29.3	5.6	4.4	367.9	999.3	-230	-845.5	139.9	-105.5	-0.1
18	1941	-11.4	-14.3	1.2	1989	-1033	-320.8	-113.7	-469.1	-81.9	7.0
19	1942	-51.9	-26.1	49.9	1384.4	586	-1391.7	-480.2	-18.5	-131.9	-2.5
20	1943	-32.6	-5.9	11.7	1096.7	865	-265	-1262.9	5.7	-123.1	-0.1
21	1944	-17.7	-19.4	-11.4	656.3	1575.8	-285	-1301.4	49.4	-182.2	1.7
22	1945	-77.7	-2	1091	805	-629	-861.3	-216.1	24.6	-22.4	-7.9
23	1946	6.6	2	560.9	308.5	-146.3	-150	-315.3	-461.5	-213.3	13.2
24	1947	-74.8	-41.8	-4.4	280.7	2609.7	-190	-1997.7	171.9	-155.5	-8.1
25	1948	-24.5	-3.2	-4.6	2393.4	-323	-1655	-293.5	24.3	-26.9	-3.6
26	1949	8.9	-3.1	1.6	1291.3	-415.5	-641.1	80.7	-319.5	-101.5	2.6
27	1950	-1.3	14.1	17.5	238.8	3634	-2512	-529.7	-339.9	-58.9	16.6
28	1951	-67.8	-24.5	7.7	1452.3	1341	-1911	-772	-127.8	-232.1	15.8
29	1952	-187.4	-74.7	-48.7	1134.2	-265	-622.4	154.3	-25.2	-21	-33.3
30	1953	-3.5	1.6	24.5	783	727.9	-84	-1045.7	-30.2	-13	-1.6
31	1954	-24.9	4.1	12	1213.6	1456	-1590	-689.4	-9.7	-39.5	1.5
32	1955	-9	5.8	23.1	1405.6	-939.5	-369.5	-79	116	-224.3	3.8
33	1956	1.4	5.1	-13.6	884.5	1296	-1623.2	-378.6	43.5	-36.8	0.6
34	1957	-35.3	-12.5	26.8	1392.2	-403	-335.5	-494.1	54	-60.2	-4.2
35	1958	-14	-2.3	4.5	329.1	-200	-91.9	-52.3	145.2	-179.2	5.6
36	1959	-58.3	-34.3	13.7	416.5	838.3	-632.2	-555.8	-269.4	-55	1.2
37	1960	-40.2	-52.2	-1.9	1087.7	-67	-684.8	-189.9	153.4	-69.4	-0.5
38	1961	-83.5	-15.9	12.9	855.7	169.3	-371.9	-402.2	52.7	-153.9	-3.9
39	1962	-16.2	-4.9	15.8	358.6	563.5	-541.2	-160.5	-31.5	-17.5	0.1
40	1963	-37.9	-16.7	20.8	512.9	-38.3	558.9	-674.3	-22.2	-29.5	-1.8
41	1964	-6.2	-9.4	-3.3	1100.1	1596	-1808.4	-491.2	5.3	-88.4	4.2
42	1965	-17.7	-10.2	21.8	753.9	1293.4	-1378.2	-303.7	-340.5	-23.9	12.4
43	1966	-29.4	-93	38.5	1037.3	1451	-1620	-744.5	-42.9	17.5	-15.9
44	1967	-38.7	-10.5	0.8	1475.4	-302	-428.3	-499.6	-2	-20	-5.2
45	1968	-8.8	-19.2	104.5	1257.8	-238	349	-378	-401.4	-93	18.4
46	1969	37.7	-18	-51.9	2082.6	-700	-990.2	-335.9	-2.3	-33.3	-13.4
47	1970	1.5									
48	1971	-317									
49	1972	-345.8									
50	1973	11.8									
51	1974	-171.2									
52	1975	54.8									
53	1976	-40									
54	1977	-0.67									
55	1978	-326.4									
56	1979	-3									
57	1980	-14.3									
58	1981	-56									
59	1982	-16.6									
60	1983	-156.6									
61	1984	-95.5									
62	1985	-88.8									
63	1986	-96.2									
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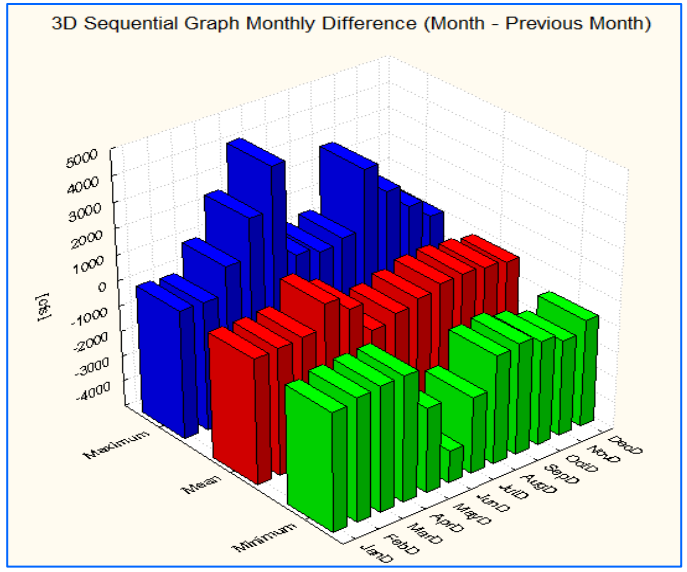
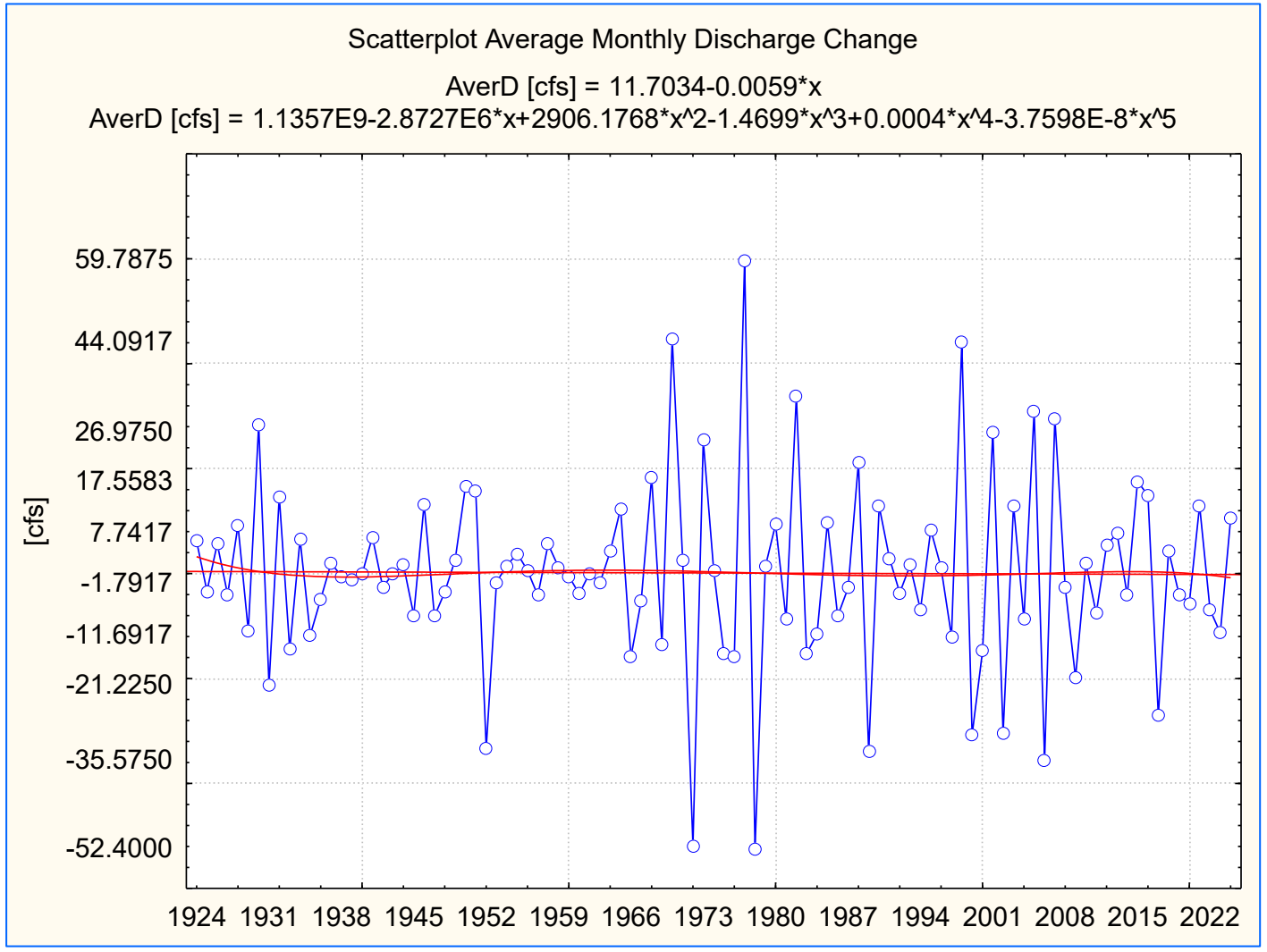
66	1989	-93.5									
67	1990	5.5									
68	1991	-41.4									
69	1992	-40									
70	1993	-9.2									
71	1994	-75.9									
72	1995	-24.7									
73	1996	-43.6									
74	1997	73.7									
75	1998	-19.7									
76	1999	-321.5									
77	2000	-76									
78	2001	4.7									
79	2002	-217									
80	2003	-5.3									
81	2004	-17									
82	2005	15.6									
83	2006	-200.7									
84	2007	-6.4									
85	2008	-125.1									
86	2009	-98.3									
87	2010	-32.5									
88	2011	-37.4									
89	2012	-0.6									
90	2013	-37.2	0.4	9.5	178.1	2343.3	-1656.5	-19.2	-453.5	-90.6	40.1	-28.1	-193.4	7.7
91	2014	-18.1	-8.3	-13.9	396.5	2228	-1153	-527	-657	-128.2	-81.7	-50.3	-36.4	-4.1
92	2015	-19.6	-18.2	45.4	670.7	381.3	-454.1	-268.9	-281.1	-26	-68.7	430.7	-180.8	17.6
93	2016	-236.7	15.6	144.5	1095.3	302	-730	-155	-49.8	-51.4	1.5	28.8	-20.9	-5.6
94	2017	-63.1	-16	-6.3	234.4	476.3	-511.3	-141	-54.4	-9.7	33.3	497.7	-278.5	13.0
95	2018	-70.2	-19.8	72.8	1268.5	287	-635	-728.2	-140.8	106.7	-105.6	73.8	-56.4	-6.9
96	2019	-3.4	-24.1	-12.4	838.1	116	-765.1	-124.8	-103.3	-15.8	-27.9	41.1	-45.2	-11.3
97	2020	-25.1	-20.2	5.4	946.6	-449.1	-195.5	-294.6	-424.2	-95	-35.8	186.6	-25.8	10.5
98	2021	-17.7	-54.5	-14.2	457.6	4250.6	-3734	-848.3						
99	2022	4	0.3	8.5	1286.8	1779	-2603.1	-460						
100	2023	3.8	3.9	21.1	428.3	579.1	-87	-429.3						
101	2024													

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Data of monthly change

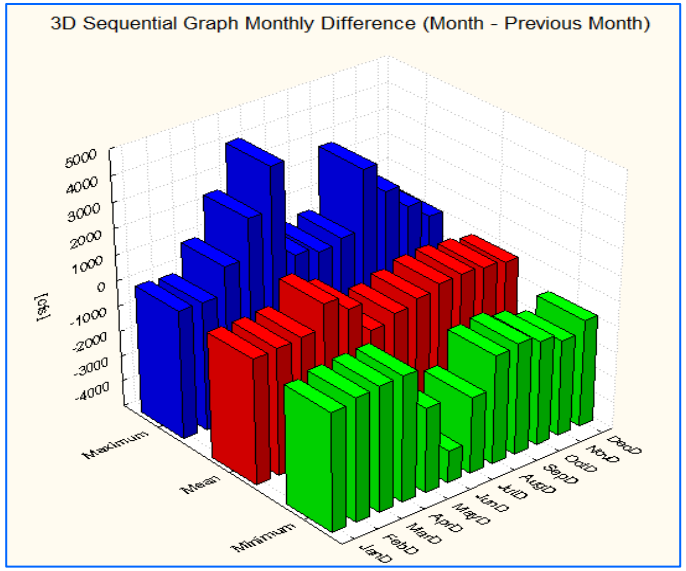


Graph of annual discharge change with trends: linear & polynomic X^5



Variable	Descriptive Statistics						
	MonthD	Valid N	Mean	Minimum	Maximum	Variance	Std.Dev.
JanD	JanD	101	-56.0	-345.8	73.7	6591.2	81.2
FebD	FebD	101	-24.6	-161.6	35.8	1450.1	38.1
MarD	MarD	101	52.0	-78.5	1091.0	20465.0	143.1
AprD	AprD	101	982.7	-61.5	2610.5	307197.9	554.3
MayD	MayD	101	431.8	-1674.0	4250.6	995857.2	997.9
JunD	JunD	101	-744.4	-3734.0	558.9	537798.0	733.3
JulD	JulD	101	-419.9	-1997.7	357.1	135104.6	367.6
AugD	AugD	101	-196.1	-729.2	570.5	38885.8	197.2
SepD	SepD	101	25.8	-621.6	2897.3	117536.3	342.8
OctD	OctD	101	94.3	-890.0	1740.7	93002.9	305.0
NovD	NovD	101	-5.4	-1237.4	830.7	76280.6	276.2
DecD	DecD	101	-138.7	-804.1	112.5	22088.3	148.6

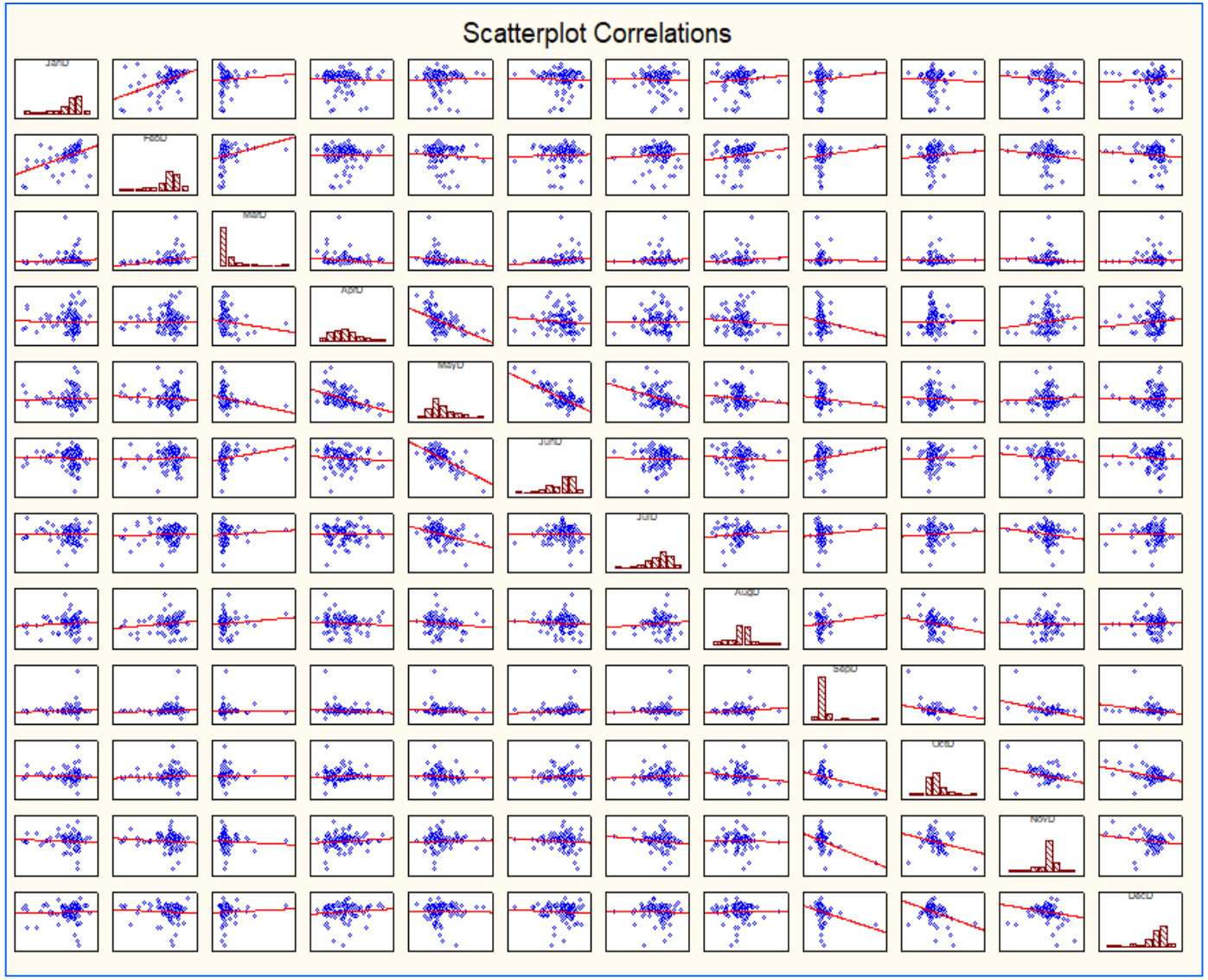
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Correlation Analysis



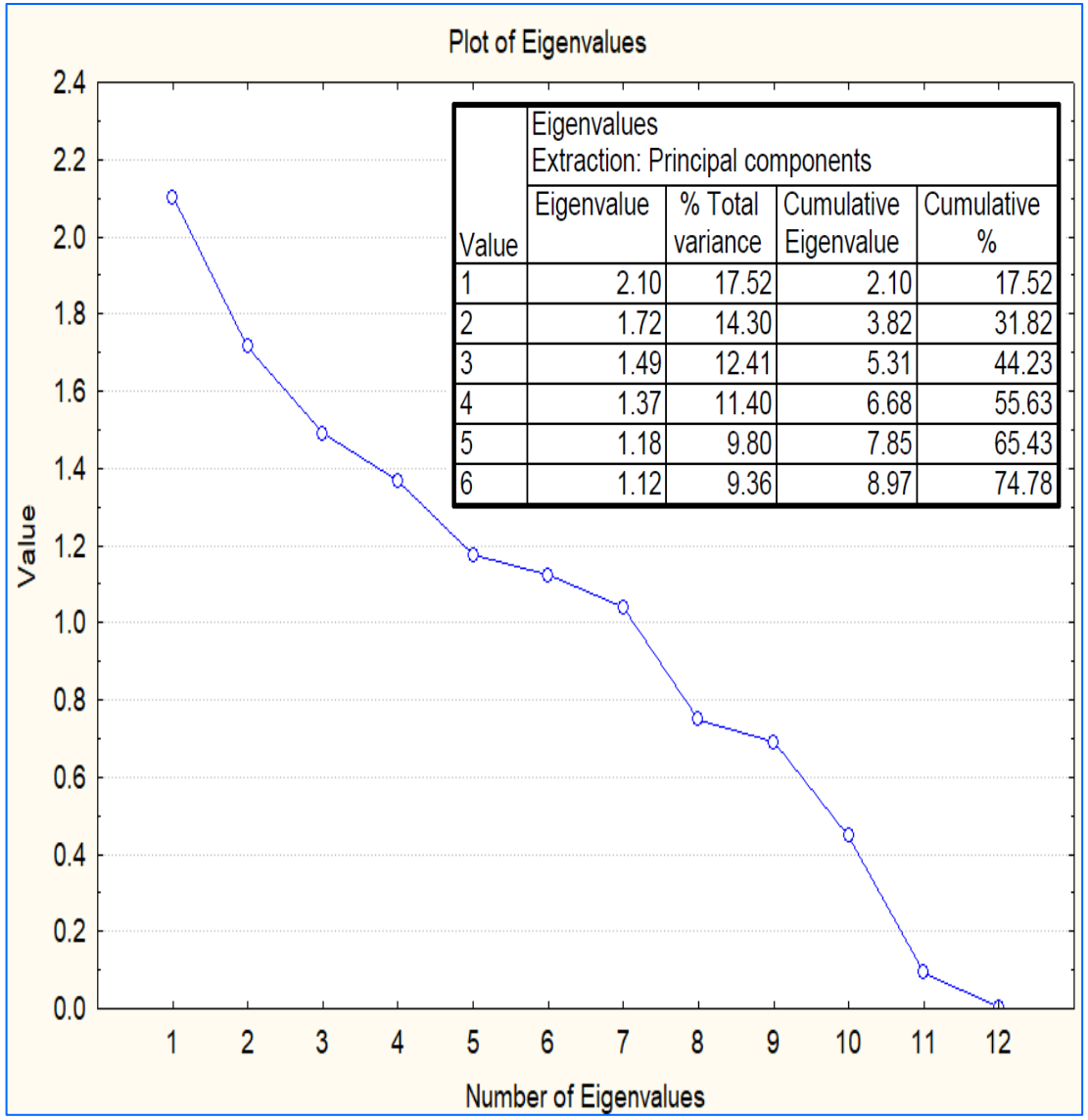
Correlation table & scatterplot

Correlations
Marked correlations are significant at p < .05000
N=101 (Casewise deletion of missing data)

Variable	JanD	FebD	MarD	AprD	MayD	JunD	JulD	AugD	SepD	OctD	NovD	DecD
JanD	1.00	*0.49*	0.07	-0.02	0.04	-0.03	-0.02	0.10	0.08	-0.04	-0.08	0.05
FebD	*0.49*	1.00	*0.24*	-0.00	-0.08	0.04	0.05	0.16	0.10	0.08	-0.11	-0.08
MarD	0.07	*0.24*	1.00	-0.14	*-0.22*	0.18	0.07	0.10	-0.02	0.00	-0.07	0.05
AprD	-0.02	-0.00	-0.14	1.00	*-0.48*	-0.09	-0.00	-0.08	-0.16	0.02	0.13	0.11
MayD	0.04	-0.08	*-0.22*	*-0.48*	1.00	*-0.69*	*-0.39*	-0.13	-0.11	-0.04	0.05	-0.01
JunD	-0.03	0.04	0.18	-0.09	*-0.69*	1.00	-0.00	-0.07	0.14	0.03	-0.10	-0.07
JulD	-0.02	0.05	0.07	-0.00	*-0.39*	-0.00	1.00	0.11	0.09	0.06	-0.17	-0.03
AugD	0.10	0.16	0.10	-0.08	-0.13	-0.07	0.11	1.00	0.13	*-0.20*	-0.01	0.02
SepD	0.08	0.10	-0.02	-0.16	-0.11	0.14	0.09	0.13	1.00	*-0.28*	*-0.41*	*-0.27*
OctD	-0.04	0.08	0.00	0.02	-0.04	0.03	0.06	*-0.20*	*-0.28*	1.00	*-0.30*	*-0.36*
NovD	-0.08	-0.11	-0.07	0.13	0.05	-0.10	-0.17	-0.01	*-0.41*	*-0.30*	1.00	-0.19
DecD	0.05	-0.08	0.05	0.11	-0.01	-0.07	-0.03	0.02	*-0.27*	*-0.36*	-0.19	1.00



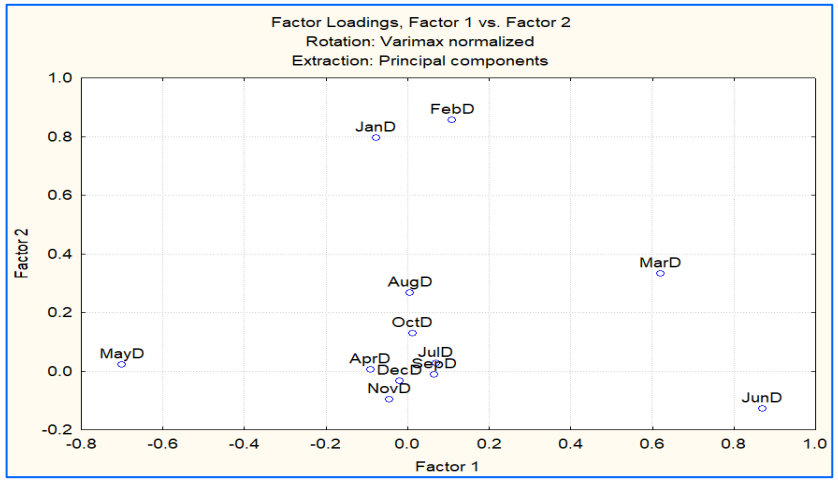
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Analysis: Eigenvalues & Factor Loading

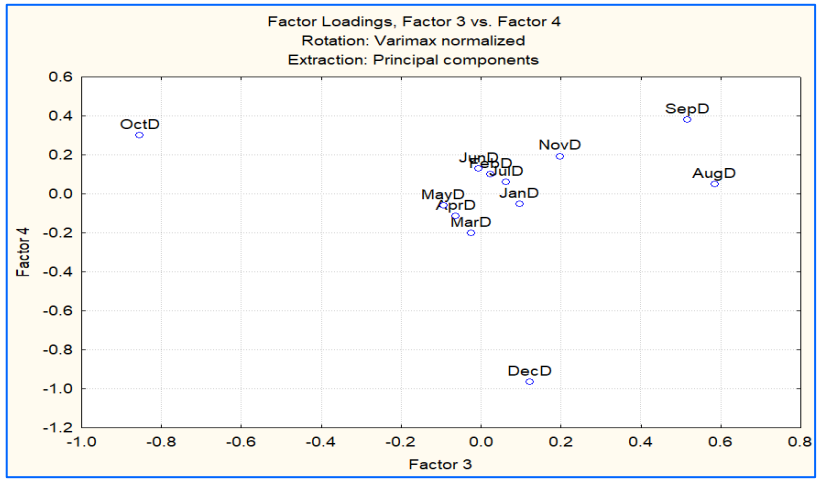


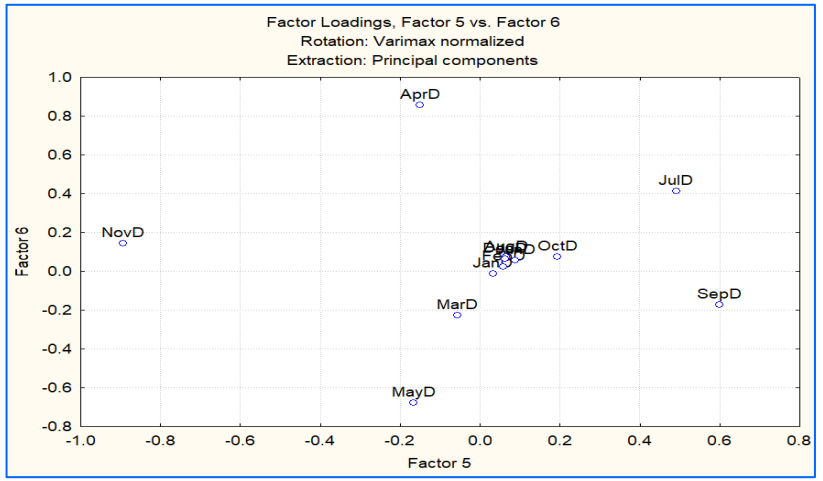
Value	Eigenvalues (Extraction: Principal components) Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	2.10	17.52	2.10	17.52
2	1.72	14.30	3.82	31.82
3	1.49	12.41	5.31	44.23
4	1.37	11.40	6.68	55.63
5	1.18	9.80	7.85	65.43
6	1.12	9.36	8.97	74.78

Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
JanD	-0.08	*0.80*	0.10	-0.05	0.03	-0.01
FebD	0.11	*0.86*	0.02	0.10	0.06	0.02
MarD	0.62	0.33	-0.02	-0.20	-0.06	-0.23
AprD	-0.09	0.01	-0.06	-0.11	-0.15	*0.86*
MayD	-0.70	0.02	-0.09	-0.06	-0.17	-0.68
JunD	*0.87*	-0.13	-0.01	0.13	0.09	0.06
JulD	0.07	0.03	0.06	0.06	0.49	0.41
AugD	0.01	0.27	0.59	0.05	0.07	0.08
SepD	0.06	-0.01	0.52	0.38	0.60	-0.17
OctD	0.01	0.13	*-0.85*	0.30	0.19	0.07
NovD	-0.04	-0.10	0.20	0.19	*-0.89*	0.15
DecD	-0.02	-0.03	0.12	*-0.96*	0.06	0.06
Expl.Var	1.67	1.59	1.42	1.29	1.51	1.49
Prp.Totl	0.14	0.13	0.12	0.11	0.13	0.12

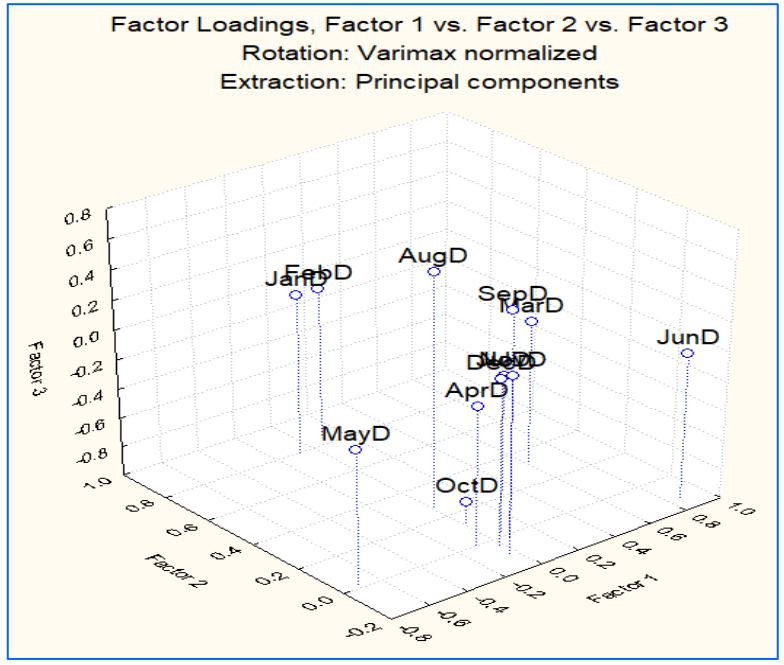
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Analysis: Factor Loading

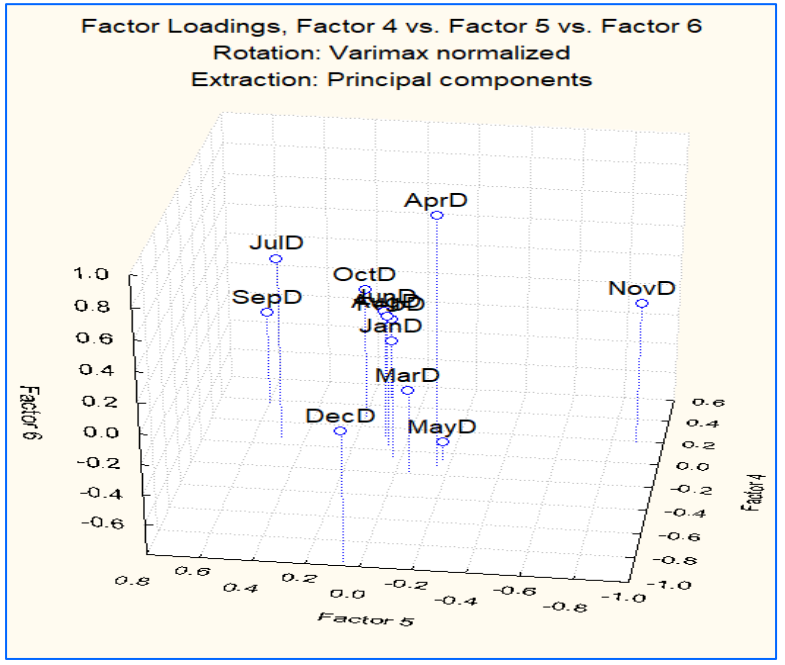




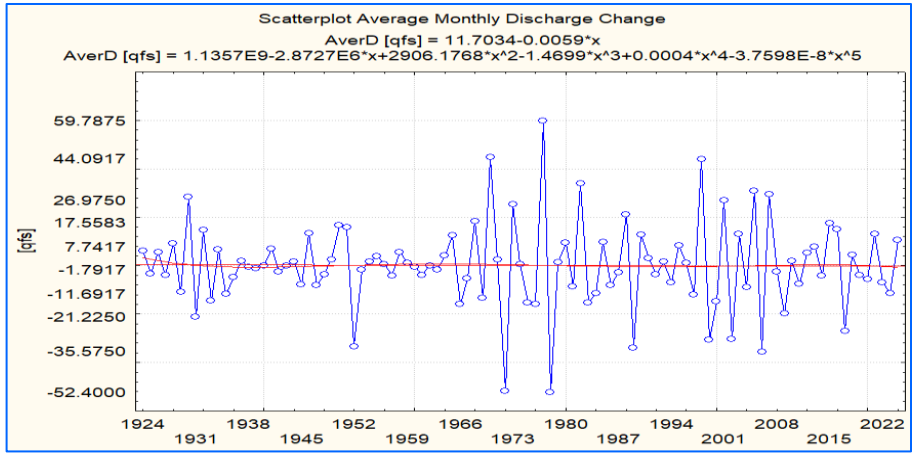


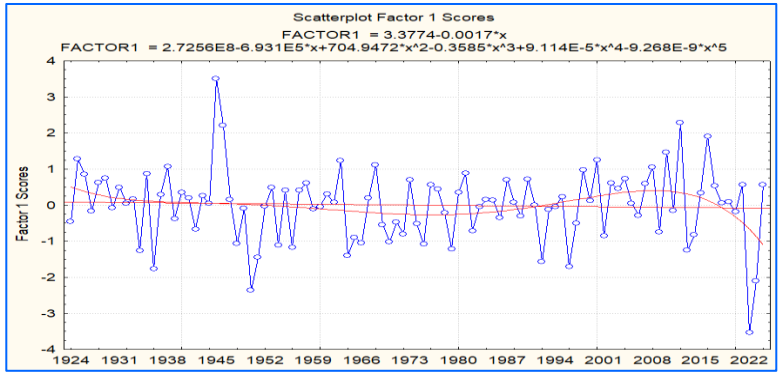
Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000)					
JanD	-0.08	*0.80*	0.10	-0.05	0.03	-0.01
FebD	0.11	*0.86*	0.02	0.10	0.06	0.02
MarD	0.62	0.33	-0.02	-0.20	-0.06	-0.23
AprD	-0.09	0.01	-0.06	-0.11	-0.15	*0.86*
MayD	-0.70	0.02	-0.09	-0.06	-0.17	-0.68
JunD	*0.87*	-0.13	-0.01	0.13	0.09	0.06
JulD	0.07	0.03	0.06	0.06	0.49	0.41
AugD	0.01	0.27	0.59	0.05	0.07	0.08
SepD	0.06	-0.01	0.52	0.38	0.60	-0.17
OctD	0.01	0.13	*-0.85*	0.30	0.19	0.07
NovD	-0.04	-0.10	0.20	0.19	*-0.89*	0.15
DecD	-0.02	-0.03	0.12	*-0.96*	0.06	0.06
Expl.Var	1.67	1.59	1.42	1.29	1.51	1.49
Prp.Totl	0.14	0.13	0.12	0.11	0.13	0.12

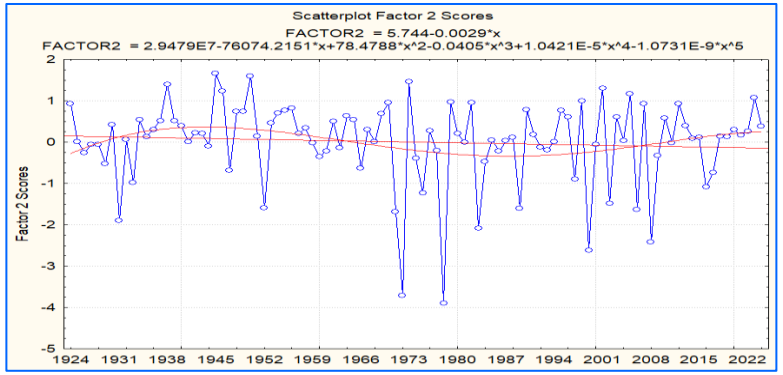




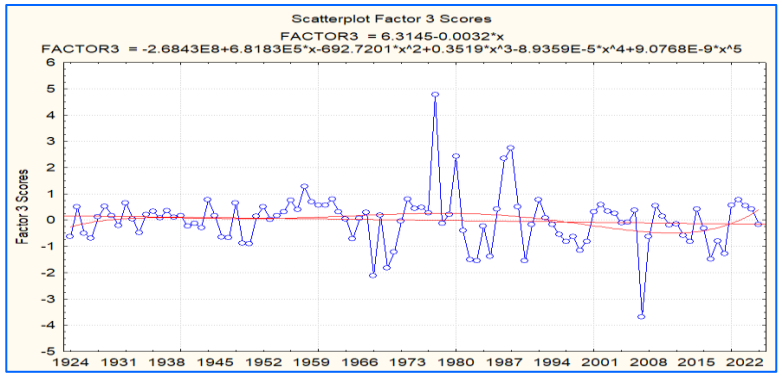
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Annual average monthly change & Factor Scores

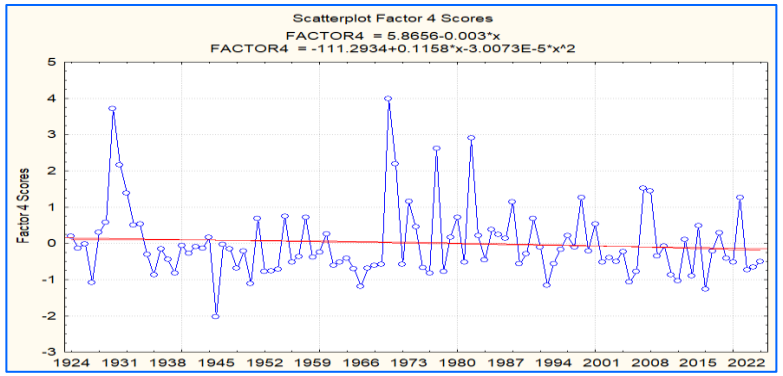


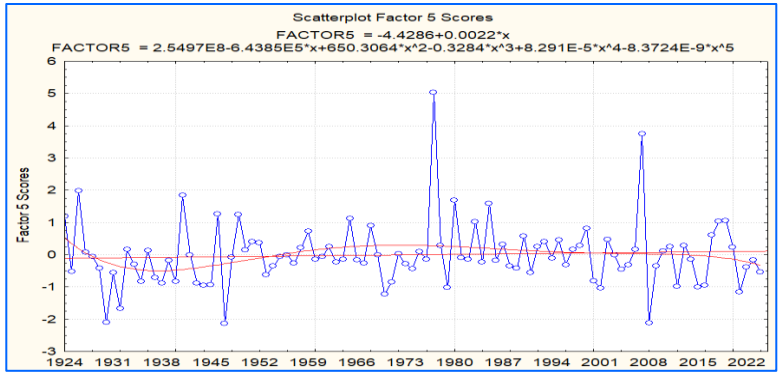


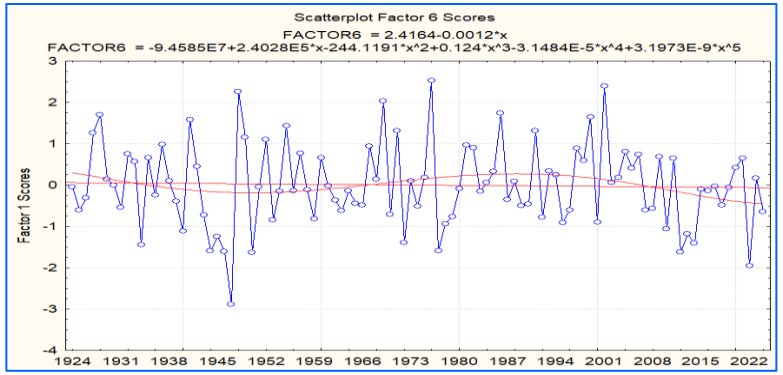


Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
JanD	-0.08	*0.80*	0.10	-0.05	0.03	-0.01
FebD	0.11	*0.86*	0.02	0.10	0.06	0.02
MarD	0.62	0.33	-0.02	-0.20	-0.06	-0.23
AprD	-0.09	0.01	-0.06	-0.11	-0.15	*0.86*
MayD	-0.70	0.02	-0.09	-0.06	-0.17	-0.68
JunD	*0.87*	-0.13	-0.01	0.13	0.09	0.06
JulD	0.07	0.03	0.06	0.06	0.49	0.41
AugD	0.01	0.27	0.59	0.05	0.07	0.08
SepD	0.06	-0.01	0.52	0.38	0.60	-0.17
OctD	0.01	0.13	*-0.85*	0.30	0.19	0.07
NovD	-0.04	-0.10	0.20	0.19	*-0.89*	0.15
DecD	-0.02	-0.03	0.12	*-0.96*	0.06	0.06
Expl.Var	1.67	1.59	1.42	1.29	1.51	1.49
Prp.Totl	0.14	0.13	0.12	0.11	0.13	0.12

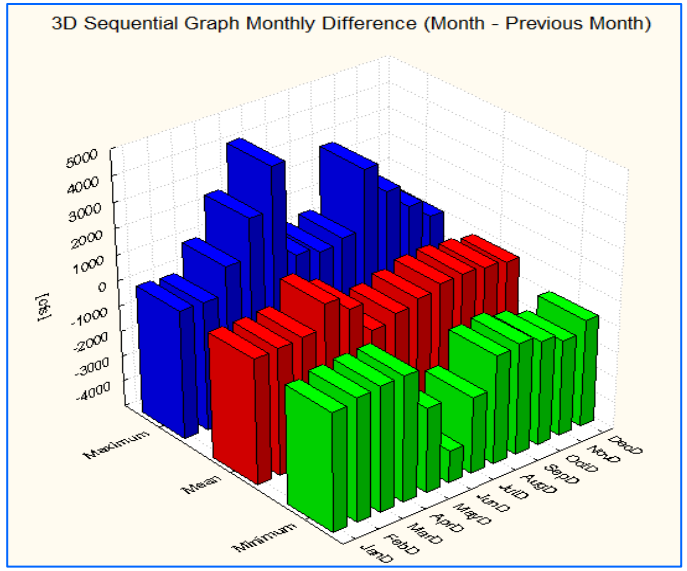








USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Transposed Matrix(12Dx101)



Monthly Hydrograph, visualization of average monthly changes of discharge through 12 months for the 101 years.

The Matrix $Q_{101x12D}$ (1924-2024) of monthly discharge changes is transposed (12Dx101); Parts.

	1 Year	2 Y1924	3 Y1925	4 Y1926	5 Y1927	6 Y1928	7 Y1929	8 Y1930	9 Y1931
1	JanD	-19	-70	-47	-42.5	-48.3	-133.2	-1.5	-262.6
2	FebD	14	-28	-33	-34.4	-19	-26.7	-14.9	-53.3
3	MarD	-8	229	93	152.1	23.6	69.1	-15.5	-25
4	AprD	841.7	639.7	824.8	1967.9	2046.2	783.4	564.3	337.7
5	MayD	564.3	-20.6	-256.1	-4	-989	-236.8	640.4	444
6	JunD	-915.3	-143.3	79	-1206	-18	-277.3	-636.6	-280.2
7	JulD	-359	-540.3	-300	-107.9	-326.1	-294.3	-226.9	-271.
8	AugD	-84.3	-67.3	-282.3	-535.7	-255.7	-56.9	-246.9	-266.
9	SepD	112.7	13.7	442	-135	1.5	5.7	-27.7	4
10	OctD	459.4	-22.4	295.3	-8.3	-43.9	33.2	182.4	212.
11	NovD	-383.2	99.5	-606	-51.8	-45.1	202.5	830.7	314.
12	DecD	-149.3	-130	-141.8	-42.8	-215.5	-199.7	-707.8	-450.
13	AverD [q	5.166667	-3.33333	5.658333	-4.03333	9.225	-10.9167	28.33333	-21.22

	10 Y1932	11 Y1933	12 Y1934	13 Y1935	14 Y1936	15 Y1937	16 Y1938	17 Y1939	18 Y1940
1	-65.4	-170.5	-43.8	-52.9	-31.6	23.6	-22.8	51.1	-29.3
2	-17.6	-39.6	-6.4	-12.1	-29.3	-16.8	17.4	-32.5	5.6
3	17.4	-10.7	-11	86.9	11.1	7.5	404.2	12	4.4
4	1236.1	1289.5	494.1	1639.8	918.8	1950.9	1262.7	945.4	367.9
5	68	-184	2278.7	-578	1827	-442	-108	908	999.3
6	-733.9	-534.1	-1638	-96	-2283.3	-265	-910.1	-844	-230

	19 Y1941	20 Y1942	21 Y1943	22 Y1944	23 Y1945	24 Y1946	25 Y1947	26 Y1948	27 Y1949
1	-11.4	-51.9	-32.6	-17.7	-77.7	6.6	-74.8	-24.5	8.9
2	-14.3	-26.1	-5.9	-19.4	-2	2	-41.8	-3.2	-3.1
3	1.2	49.9	11.7	-11.4	1091	560.9	-4.4	-4.6	1.6
4	1989	1384.4	1096.7	656.3	805	308.5	280.7	2393.4	1291.3
5	-1033	586	865	1575.8	-629	-146.3	2609.7	-323	-415.5
6	-320.8	-1391.7	-265	-285	-861.3	-150	-190	-1655	-641.1
7	-113.7	-480.2	-1262.9	-1301.4	-216.1	-315.3	-1997.7	-293.5	80.7
8	-465.5	-149.6	-328.6	-309.5	38	-256.6	-473.5	-23.1	-221.3
9	513.8	11.2	0.6	221.1	-209.8	463.2	-178.3	-54.1	-73
10	89.9	187.8	37.7	-355.7	-35.2	360.6	-44	-52.4	424.1
11	-469.1	-18.5	5.7	49.4	24.6	-461.5	171.9	24.3	-319.5
12	-81.9	-131.9	-123.1	-182.2	-22.4	-213.3	-155.5	-26.9	-101.5
13	7.016667	-2.55	-0.05833	1.691667	-7.90833	13.23333	-8.14167	-3.55	2.633333

	100 Y2022	101 Y2023	102 Y2024
1	-17.7	4	3.8
2	-54.5	0.3	3.9
3	-14.2	8.5	21.1
4	457.6	1286.8	428.3
5	4250.6	1779	579.1
6	-3734	-2603.1	-87
7	-848.3	-460	-429.3
8	-140.8	-103.3	-424.2
9	106.7	-15.8	-95
10	-105.6	-27.9	-35.8
11	73.8	41.1	186.6
12	-56.4	-45.2	-25.8
13	-6.9	-11.3	10.475

Descriptive statistical characteristics (a part

Descriptive Statistics
Exclude cases: 13

Variable	Valid N	Mean	Minimum	Maximum	Variance	Std.Dev.
Y1924	12	6.1667	-915.30	841.700	217609	466.486
Y1925	12	-3.3333	-540.30	639.700	73828	271.713
Y1926	12	5.6583	-606.00	824.800	145754	381.777

Descriptive Statistics
Exclude cases: 13

Variable	Valid N	Mean	Minimum	Maximum	Variance	Std.Dev.
Y2018	12	4.4000	-511.30	544.500	85728	292.793
Y2019	12	-3.9083	-728.20	1268.500	293909	542.134
Y2020	12	-5.6250	-765.10	838.100	120325	346.879
Y2021	12	12.9917	-449.10	946.600	141020	375.527
Y2022	12	-6.9000	-3734.00	4250.600	2999388	1731.874
Y2023	12	-11.3000	-2603.10	1779.000	1074765	1036.709
Y2024	12	10.4750	-429.30	579.100	85051	291.635

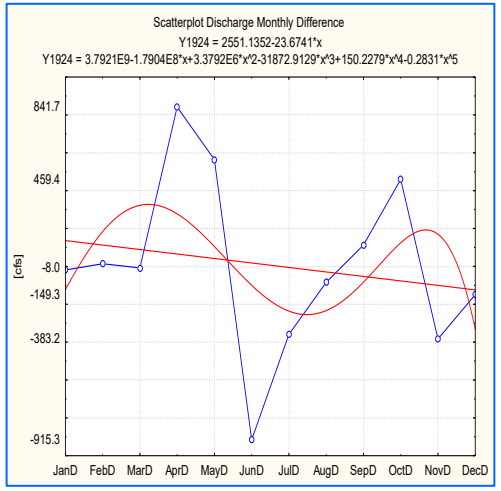
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Matrix & Scatterplots

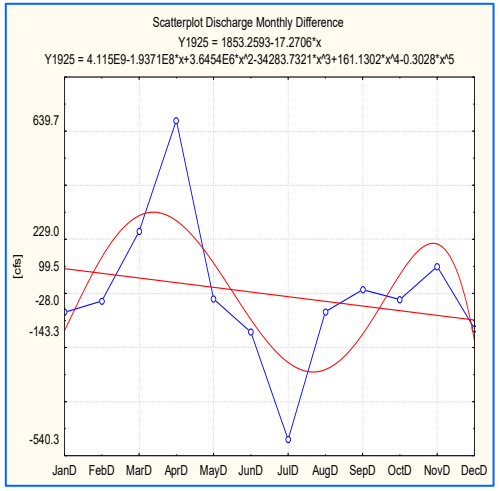
Calculated Matrix $Q_{101x12D}$ (1924-2024) of monthly discharge changes is transposed (12Dx101)

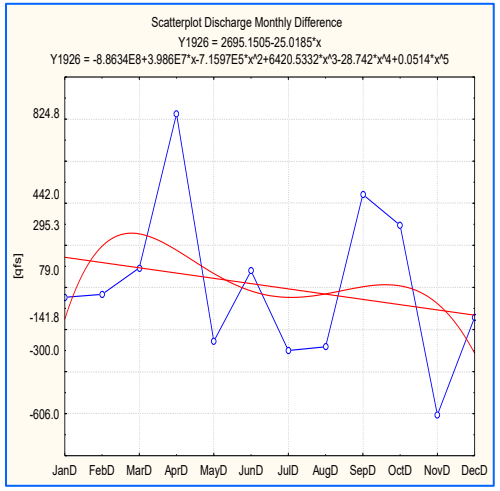
Fragments.

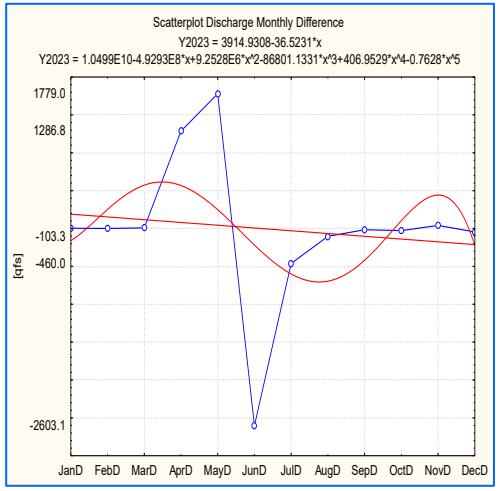
	1 Year	2 Y1924	3 Y1925	4 Y1926	5 Y1927	6 Y1928	7 Y1929	8 Y1930	9 Y1931	10 Y1932	11 Y1933	12 Y1934	13 Y1935	14 Y1936	15 Y1937	16 Y1938	17 Y1939	18 Y1940	19 Y1941	20 Y1942	21 Y1943	22 Y1944	23 Y1945	24 Y1946	25 Y1947	26 Y1948	27 Y1949	100 Y2022	101 Y2023	102 Y2024
1	JanD	-19	-70	-47	-42.5	-48.3	-133.2	-1.5	-262.6	-65.4	-170.5	-43.8	-52.9	-31.6	23.6	-22.8	51.1	-29.3	-11.4	-51.9	-32.6	-17.7	-77.7	6.6	-74.8	-24.5	8.9	-17.7	4	3.8
2	FebD	14	-28	-33	-34.4	-19	-26.7	-14.9	-53.3	-17.6	-39.6	-6.4	-12.1	-29.3	-16.8	17.4	-32.5	5.6	-14.3	-26.1	-5.9	-19.4	-2	2	-41.8	-3.2	-3.1	-54.5	0.3	3.9
3	MarD	-8	229	93	152.1	23.6	69.1	-15.5	-25	17.4	-10.7	-11	86.9	11.1	7.5	404.2	12	4.4	1.2	49.9	11.7	-11.4	1091	560.9	-4.4	-4.6	1.6	-14.2	8.5	21.1
4	AprD	841.7	639.7	824.8	1967.9	2046.2	783.4	564.3	337.7	1236.1	1289.5	494.1	1639.8	918.8	1950.9	1262.7	945.4	367.9	1989	1384.4	1096.7	656.3	805	308.5	280.7	2393.4	1291.3	457.6	1286.8	428.3
5	MayD	564.3	-20.6	-256.1	-4	-989	-236.8	640.4	444	68	-184	2278.7	-578	1827	-442	-108	908	999.3	-1033	586	865	1575.8	-629	-146.3	2609.7	-323	-415.5	4250.6	1779	579.1
6	JunD	-915.3	-143.3	79	-1206	-18	-277.3	-636.6	-280.2	-733.9	-534.1	-1638	-96	-2283.3	-265	-910.1	-844	-230	-320.8	-1391.7	-265	-285	-861.3	-150	-190	-1655	-641.1	-3734	-2603.1	-87
7	JulD	-359	-540.3	-300	-107.9	-326.1	-294.3	-226.9	-271.9	-374.2	-471	-972.9	-879.7	-317.7	-1026.7	-418.1	-766.7	-845.5	-113.7	-480.2	-1262.9	-1301.4	-216.1	-315.3	-1997.7	-293.5	80.7	-848.3	-460	-429.3
8	AugD	-84.3	-67.3	-282.3	-535.7	-255.7	-56.9	-246.9	-266.8	-122.4	-138.3	-172.7	-188.5	-150.2	-193.5	-39.3	-179.3	-199.6	-465.5	-149.6	-328.6	-309.5	38	-256.6	-473.5	-23.1	-221.3	-140.8	-103.3	-424.2
9	SepD	112.7	13.7	442	-135	1.5	5.7	-27.7	47	-92.4	118.5	72.3	-35.4	0.6	23.5	-140.8	-39	-82.4	513.8	11.2	0.6	221.1	-209.8	463.2	-178.3	-54.1	-73	106.7	-15.8	-95
10	OctD	459.4	-22.4	295.3	-8.3	-43.9	33.2	182.4	212.5	-20.3	173.5	323.1	-21	-17.2	54.1	-34.1	14.7	-26.4	89.9	187.8	37.7	-355.7	-35.2	360.6	-44	-52.4	424.1	-105.6	-27.9	-35.8
11	NovD	-383.2	99.5	-606	-51.8	-45.1	202.5	830.7	314.6	608	-20.1	-28.3	88.8	14.5	21.9	126.4	-67	139.9	-469.1	-18.5	5.7	49.4	24.6	-461.5	171.9	24.3	-319.5	73.8	41.1	186.6
12	DecD	-149.3	-130	-141.8	-42.8	-215.5	-199.7	-707.8	-450.7	-326.5	-187.1	-216.2	-92.2	-0.9	-113.8	-143	-16.6	-105.5	-81.9	-131.9	-123.1	-182.2	-22.4	-213.3	-155.5	-26.9	-101.5	-56.4	-45.2	-25.8
13	AverD [q	6.166667	-3.33333	5.658333	-4.03333	9.225	-10.9167	28.33333	-21.225	14.73333	-14.4917	6.575	-11.6917	-4.85	1.975	-0.45833	-1.15833	-0.13333	7.016667	-2.55	-0.05833	1.691667	-7.90833	13.23333	-8.14167	-3.55	2.633333	-6.9	-11.3	10.475

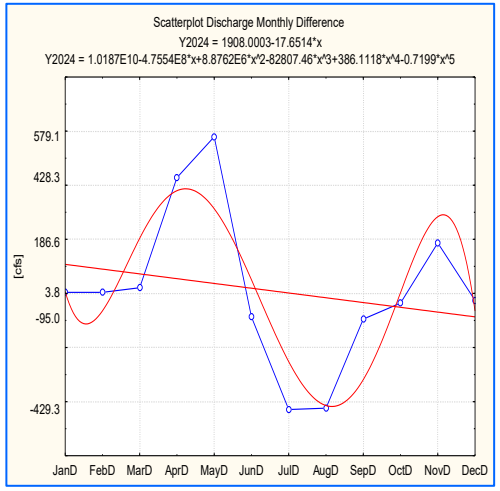
Scatterplots Discharge Monthly Difference (1924, 1925, 1926 … & 2023, 2024)







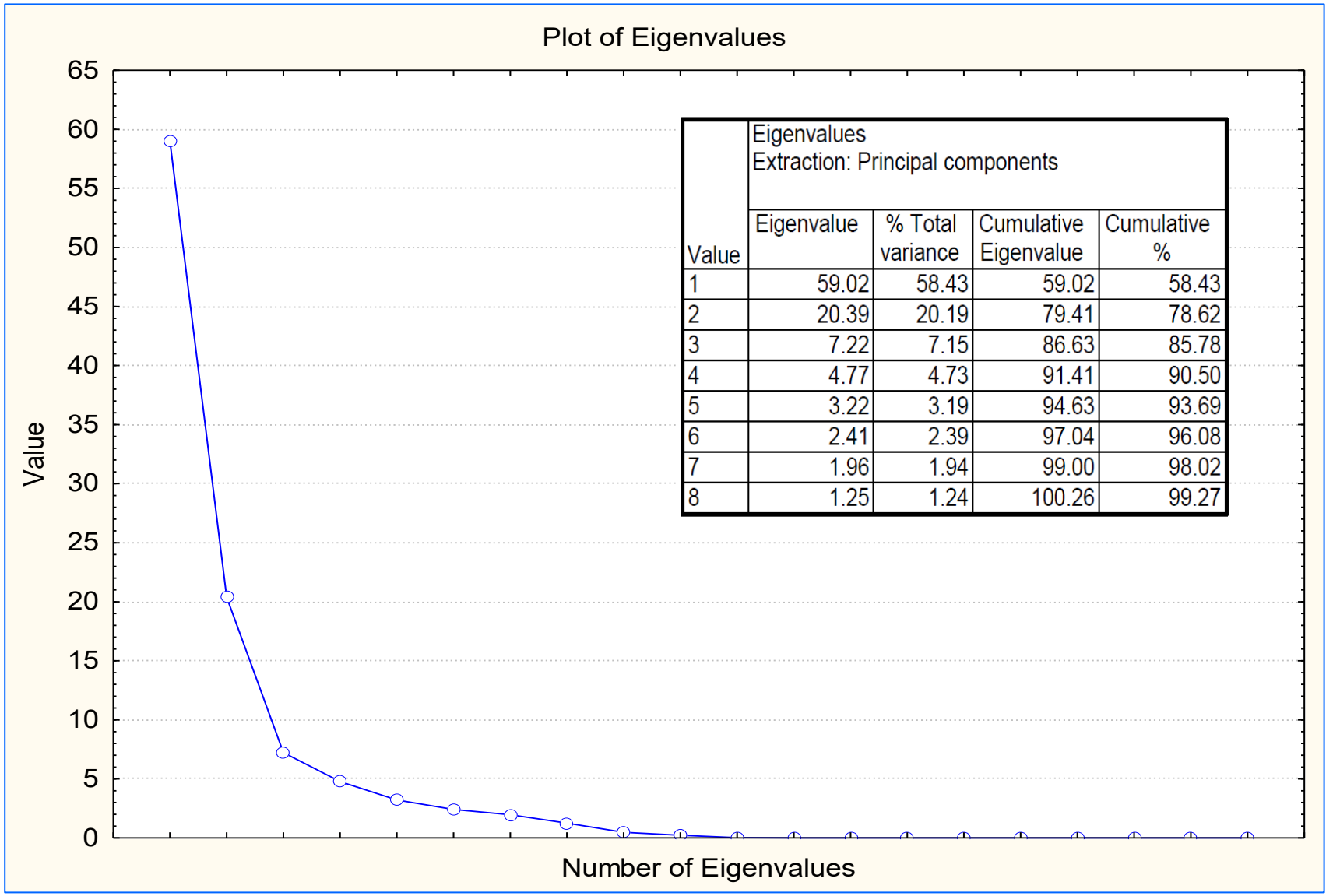




USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Correlation & Eigenvalues

Correlations
Marked correlations are significant at p < .05000
N=12 (Casewise deletion of missing data)
Exclude cases: 13

Variable	Y1924	Y1925	Y1926	Y1927	Y1928	Y1929	Y1930	Y1931	Y1932	Y1933	Y1934	Y1935
Y1924	1.00	0.59	0.53	0.77	0.38	0.57	0.57	0.64	0.66	0.76	0.80	0.47
Y1925	0.59	1.00	0.60	0.73	0.75	0.89	0.53	0.58	0.82	0.87	0.39	0.91
Y1926	0.53	0.60	1.00	0.57	0.73	0.59	0.01	0.22	0.35	0.72	0.05	0.71
Y1927	0.77	0.73	0.57	1.00	0.77	0.85	0.54	0.51	0.87	0.91	0.43	0.76
Y1928	0.38	0.75	0.73	0.77	1.00	0.89	0.21	0.22	0.70	0.86	-0.12	0.94
Y1929	0.57	0.89	0.59	0.85	0.89	1.00	0.57	0.56	0.92	0.95	0.22	0.93
Y1930	0.57	0.53	0.01	0.54	0.21	0.57	1.00	0.92	0.79	0.52	0.68	0.32
Y1931	0.64	0.58	0.22	0.51	0.22	0.56	0.92	1.00	0.71	0.56	0.72	0.36
Y1932	0.66	0.82	0.35	0.87	0.70	0.92	0.79	0.71	1.00	0.88	0.48	0.77
Y1933	0.76	0.87	0.72	0.91	0.86	0.95	0.52	0.56	0.88	1.00	0.36	0.91
Y1934	0.80	0.39	0.05	0.43	-0.12	0.22	0.68	0.72	0.48	0.36	1.00	0.09
Y1935	0.47	0.91	0.71	0.76	0.94	0.93	0.32	0.36	0.77	0.91	0.09	1.00
Variable	Y2008	Y2009	Y2010	Y2011	Y2012	Y2013	Y2014	Y2015	Y2016	Y2017	Y2018	Y2019
Y2014	0.30	0.61	0.23	0.53	0.28	0.96	1.00	0.67	-0.13	0.59	0.71	0.50
Y2015	0.67	0.86	0.24	0.82	0.39	0.61	0.67	1.00	0.46	0.80	0.50	0.66
Y2016	0.79	0.33	0.08	0.45	0.62	-0.32	-0.13	0.46	1.00	0.48	-0.08	0.52
Y2017	0.50	0.85	0.27	0.93	0.27	0.54	0.59	0.80	0.48	1.00	0.69	0.87
Y2018	0.22	0.65	0.46	0.57	0.13	0.74	0.71	0.50	-0.08	0.69	1.00	0.75
Y2019	0.61	0.81	0.27	0.80	0.44	0.41	0.50	0.66	0.52	0.87	0.75	1.00
Y2020	0.38	0.98	0.09	0.97	0.12	0.51	0.51	0.82	0.38	0.86	0.59	0.81
Y2021	0.65	0.61	0.09	0.64	0.30	-0.13	-0.07	0.67	0.79	0.56	0.08	0.55
Y2022	0.11	0.69	0.25	0.53	0.10	0.97	0.94	0.63	-0.29	0.52	0.74	0.46
Y2023	0.22	0.91	0.21	0.79	0.09	0.87	0.84	0.78	-0.03	0.74	0.76	0.67
Y2024	0.78	0.59	0.23	0.59	0.66	0.63	0.80	0.83	0.44	0.67	0.49	0.61



Eigenvalues
Extraction: Principal components

Value	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	59.02	58.43	59.02	58.43
2	20.39	20.19	79.41	78.62
3	7.22	7.15	86.63	85.78
4	4.77	4.73	91.41	90.50
5	3.22	3.19	94.63	93.69
6	2.41	2.39	97.04	96.08
7	1.96	1.94	99.00	98.02
8	1.25	1.24	100.26	99.27

USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Loadings: table & scatterplots

Factor Loadings (Varimax normalized)
Extraction: Principal components
(Marked loadings are >.700000)
Exclude cases: 13

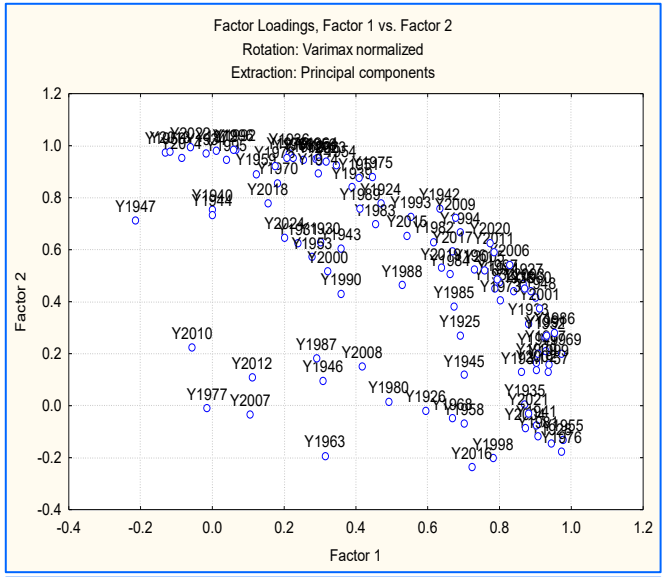
Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Y1924	0.47	*0.78*	0.05	0.40	0.08	0.05	0.02	-0.01
Y1925	0.69	0.27	0.59	0.01	0.10	0.25	0.14	0.08
Y1926	0.59	-0.02	0.29	0.56	0.45	0.03	-0.16	-0.14
Y1927	*0.86*	0.47	0.03	-0.01	0.05	0.05	-0.15	-0.05
Y1928	*0.94*	-0.14	0.23	0.07	0.12	-0.05	-0.04	0.00
Y1929	*0.90*	0.16	0.26	-0.01	0.06	0.13	0.09	0.24
Y1930	0.30	0.63	0.14	-0.16	-0.08	0.13	-0.08	0.65
Y1931	0.24	0.62	0.31	0.08	0.00	0.13	-0.17	0.61
Y1932	*0.79*	0.45	0.19					
Y1933	*0.88*	0.32	0.24					
Y1934	-0.02	*0.97*	0.19					
Y1935	*0.87*	0.01	0.47					
Y1936	0.21	*0.97*	-0.07					
Y1937	*0.86*	0.13	0.46					
Y1938	*0.84*	0.44	0.12					
Y1939	0.39	*0.84*	0.36					
Y1940	0.00	*0.76*	0.64					
Y1941	*0.90*	-0.08	0.07					
Y1942	0.63	*0.76*	0.07					
Y1943	0.36	0.60	0.70					
Y1944	-0.00	*0.73*	0.65					
Y1945	*0.70*	0.12	-0.06					
Y1946	0.31	0.10	0.19					
Y1947	-0.21	*0.71*	0.66					
Y1948	*0.90*	0.42	-0.03					
Y1949	*0.91*	0.20	-0.12					
Y1950	-0.13	*0.98*	0.01					
Y1951	0.41	*0.88*	0.09					
Y1952	*0.93*	0.26	-0.14					
Y1953	0.28	0.57	*0.77*					
Y1954	0.35	*0.92*	0.16	-0.02	0.02	0.01	0.04	-0.00
Y1955	*0.98*	-0.13	-0.00	0.09	0.05	0.05	0.04	0.11
Y1956	0.29	*0.95*	0.01	-0.07	0.06	0.01	0.05	0.01
Y1957	*0.94*	0.13	0.31	0.03	0.09	-0.00	0.05	0.01
Y1958	*0.70*	-0.07	0.07	-0.19	0.56	0.06	0.10	0.38
Y1959	0.12	*0.89*	0.26	0.22	0.11	0.00	0.24	-0.06
Y1960	*0.89*	0.44	0.06	-0.05	0.04	0.01	0.03	0.08
Y1961	*0.73*	0.52	0.37	0.00	0.20	0.00	0.02	0.11
Y1962	0.25	*0.94*	0.09	-0.08	-0.04	-0.02	0.11	-0.06
Y1963	0.31	-0.19	*0.91*	0.09	0.07	-0.09	-0.03	-0.03
Y1964	0.29	*0.95*	0.05	-0.02	0.02	0.02	0.01	0.02
Y1965	0.23	*0.95*	0.00	0.18	0.04	0.04	-0.06	-0.04
Y1966	0.30	*0.94*	0.15	0.01	-0.00	0.04	0.06	-0.01
Y1967	*0.93*	0.21	0.28	0.06	-0.01	-0.04	0.09	-0.04

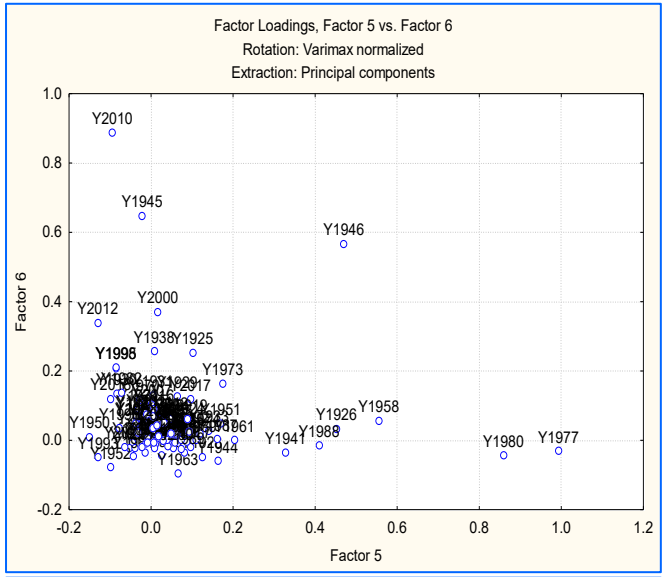
Factor Loadings (Varimax normalized)
Extraction: Principal components
(Marked loadings are >.700000)
Exclude cases: 13

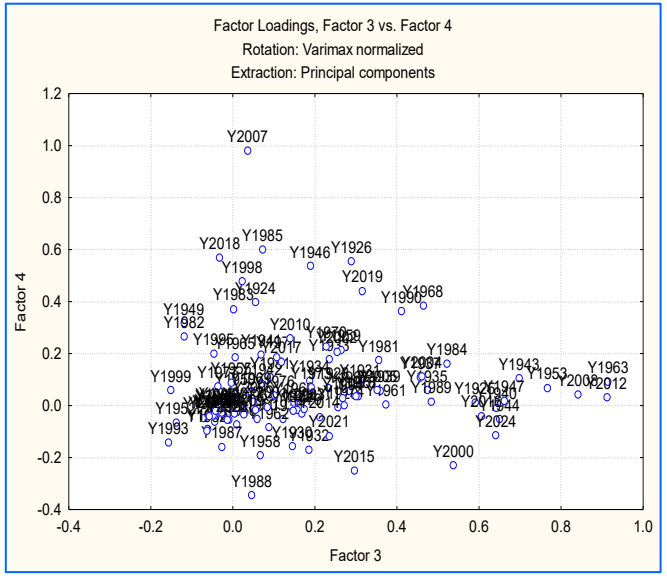
Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Y1968	0.67	-0.05	0.46	0.39	0.06	-0.01	-0.41	-0.09
Y1969	*0.97*	0.20	0.04	0.05	0.04	-0.01	0.05	-0.02
Y1970	0.18	*0.86*	0.23	0.23	-0.03	0.12	-0.03	0.33
Y1971	*0.80*	0.48	0.11	0.19	-0.03	0.09	0.01	0.27
Y1972	0.01	*0.98*	0.15	-0.02	0.01	0.02	-0.01	0.04
Y1973	*0.80*	0.41	0.19	0.07	0.18	0.16	0.20	0.12
Y1974	0.30	*0.89*	0.27	-0.00	0.12	0.02	0.06	0.13
Y1975	0.45	*0.88*	-0.05	-0.06	-0.00	0.00	0.01	0.06
				0.04	0.07	0.00	0.03	-0.05
				0.07	*0.99*	-0.03	0.06	-0.03
				0.04	0.04	-0.02	-0.06	0.02
				-0.03	0.02	0.06	0.07	0.08
				-0.03	*0.86*	-0.04	0.13	-0.02
				0.17	0.05	0.01	0.00	-0.03
				0.27	-0.07	0.14	-0.09	0.32
				0.37	-0.04	0.07	-0.37	0.12
				0.16	-0.04	-0.05	0.02	0.03
				0.60	0.01	0.03	-0.10	-0.05
				-0.01	0.06	0.02	0.03	0.06
				-0.16	0.16	0.00	*0.83*	-0.09
				-0.35	0.41	-0.01	0.39	0.24
				0.01	0.08	-0.04	0.10	0.02
				0.36	-0.04	0.05	-0.61	0.00
				-0.06	0.05	0.06	-0.15	0.17
				-0.10	0.09	0.03	-0.01	0.04
				-0.14	-0.13	-0.05	-0.23	-0.12
				0.03	0.01	-0.01	-0.08	0.00
				0.20	-0.08	0.21	-0.08	0.06
				-0.03	-0.08	0.04	-0.09	0.07
				-0.04	0.04	0.00	-0.33	0.10
1998	*0.78*	-0.20	0.02	0.48	-0.09	0.21	-0.04	0.24
1999	*0.94*	0.16	-0.15	0.06	0.03	-0.00	-0.22	0.00
2000	0.32	0.52	0.54	-0.23	0.02	0.37	-0.11	0.35
2001	*0.91*	0.38	0.08	-0.00	-0.01	0.01	0.13	0.02
2002	*0.90*	0.14	0.25	0.21	0.01	-0.02	0.11	-0.02
2003	*0.87*	0.45	0.17	-0.01	-0.04	0.05	-0.07	0.02
2004	*0.87*	-0.09	0.46	0.11	0.07	-0.02	-0.01	0.02
2005	*0.76*	0.52	0.35	0.06	0.01	-0.01	-0.03	0.01
2006	*0.83*	0.54	0.05	0.01	0.05	0.02	0.08	-0.01
2007	0.11	-0.03	0.04	*0.98*	0.00	0.10	-0.12	-0.02
2008	0.42	0.15	*0.84*	0.04	-0.01	-0.01	-0.09	0.29
2009	0.68	*0.72*	0.03	-0.03	0.03	0.05	0.10	0.04
2010	-0.06	0.22	0.14	0.26	-0.09	*0.89*	-0.05	0.24
2011	*0.78*	0.59	0.00	-0.03	-0.01	-0.01	-0.17	-0.06

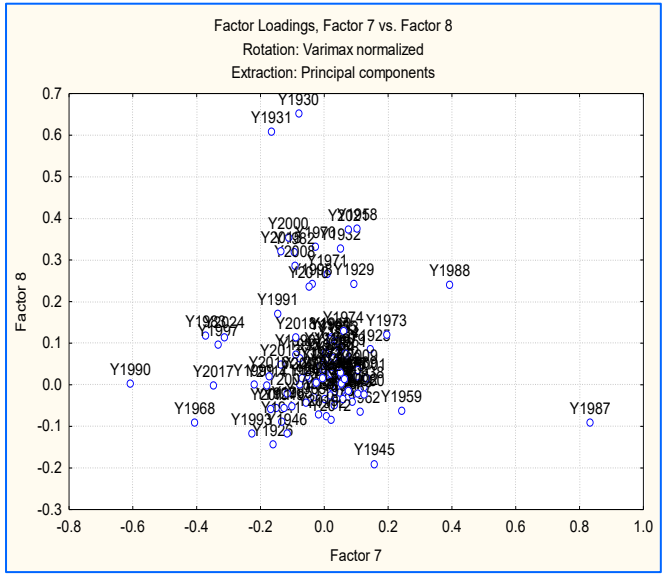
Factor Loadings (Varimax normalized)
Extraction: Principal components
(Marked loadings are >.700000)
Exclude cases: 13

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Y2012	0.11	0.11	*0.91*	0.03	-0.13	0.34	0.02	-0.08
Y2013	-0.12	*0.98*	-0.05	-0.04	-0.03	0.02	-0.14	0.05
Y2014	-0.09	*0.95*	0.21	-0.04	-0.02	-0.02	-0.18	-0.00
Y2015	0.54	0.65	0.30	-0.25	-0.01	0.07	-0.14	0.32
Y2016	*0.72*	-0.24	0.61	-0.04	0.01	0.09	-0.17	0.02
Y2017	0.67	0.59	0.12	0.17	0.10	0.12	-0.35	-0.00
Y2018	0.15	*0.78*	-0.03	0.57	-0.10	0.12	-0.09	0.11
Y2019	0.64	0.53	0.32	0.44	0.09	0.06	-0.02	-0.07
Y2020	*0.77*	0.63	-0.04	-0.02	0.02	0.01	0.05	0.00
Y2021	*0.88*	-0.03	0.23	-0.12	0.00	0.07	0.08	0.37
Y2022	-0.06	*0.99*	-0.01	-0.06	0.00	0.04	0.05	0.03
Y2023	0.32	*0.94*	-0.06	-0.04	0.01	0.04	0.06	0.01
Y2024	0.20	0.65	0.64	-0.12	-0.06	-0.02	-0.31	0.11
Expl.Var	38.24	36.96	9.98	4.49	3.22	2.28	2.68	2.40
Prp.Totl	0.38	0.37	0.10	0.04	0.03	0.02	0.03	0.02





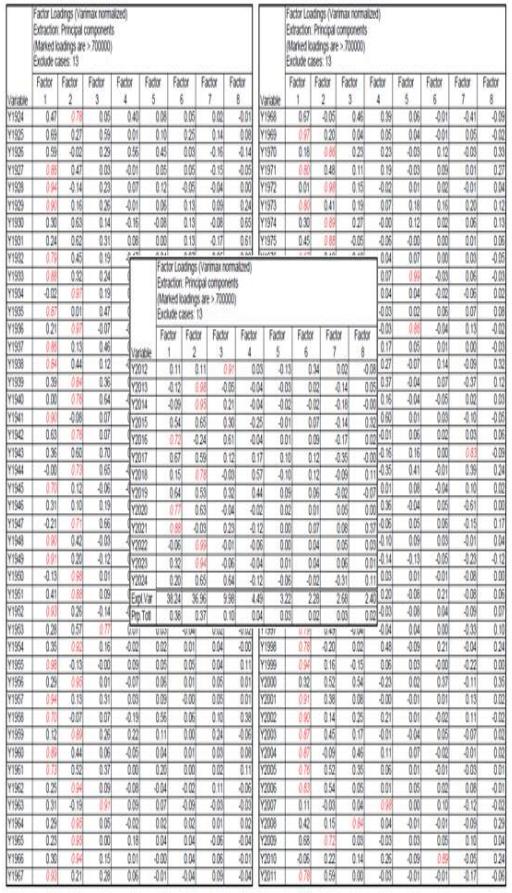


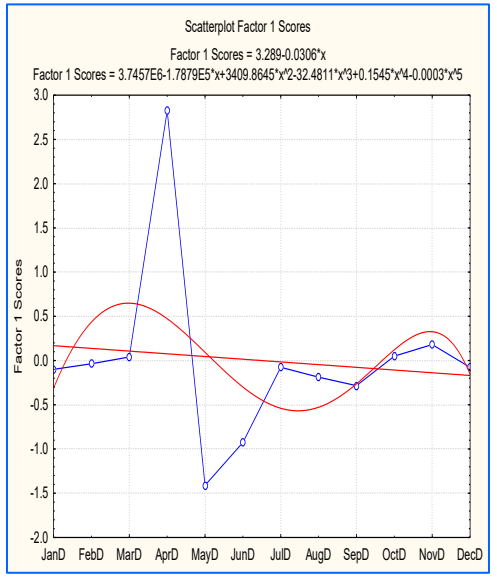


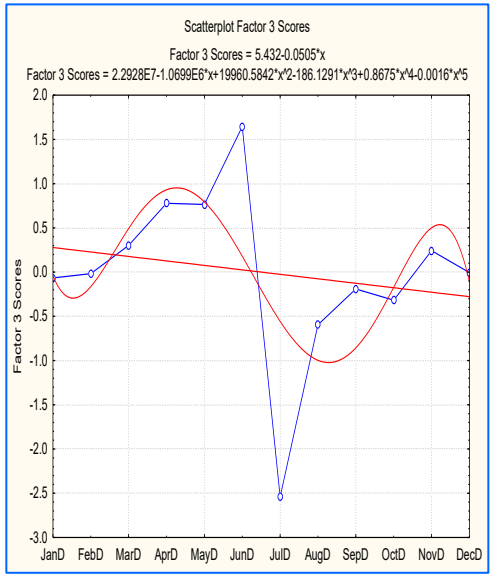
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Factor Scores & Loadings: tables & scatterplots

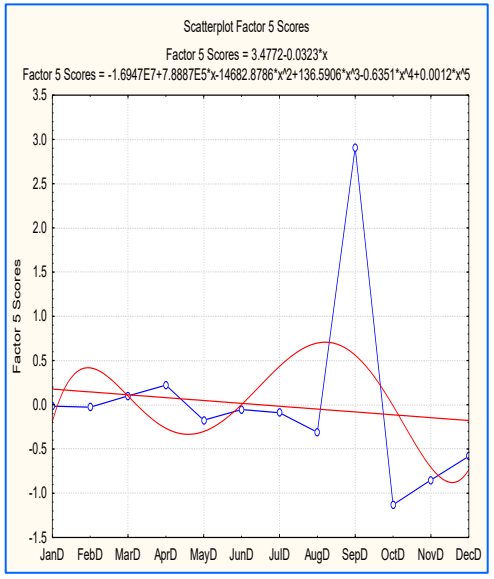
Factor scores for 8

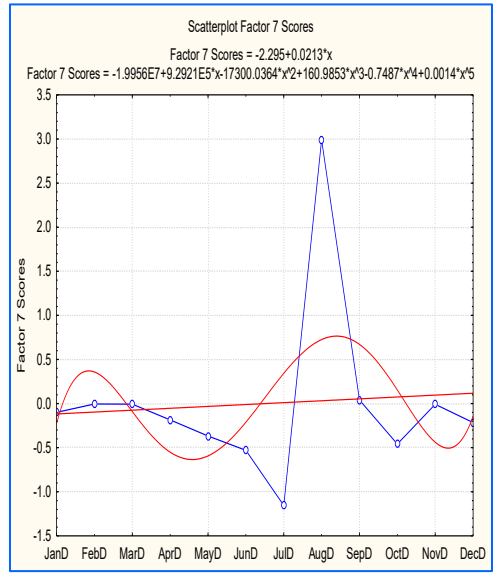
1 Year	2 FACTOR1	3 FACTOR2	4 FACTOR3	5 FACTOR4	6 FACTOR5	7 FACTOR6	8 FACTOR7	9 FACTOR8
JanD	-0.10	-0.04	-0.07	-0.16	-0.02	-0.10	-0.10	-0.36
FebD	-0.04	-0.03	-0.02	-0.04	-0.03	0.02	-0.00	-0.12
MarD	0.04	-0.06	0.30	-0.36	0.10	2.89	-0.01	-0.89
AprD	2.82	0.53	0.78	0.14	0.22	-0.68	-0.19	-0.14
MayD	-1.41	2.47	0.76	-0.06	-0.17	-0.58	-0.37	0.05
JunD	-0.93	-2.09	1.65	0.23	-0.05	-0.80	-0.53	-0.03
JulD	-0.08	-0.42	-2.54	-0.44	-0.09	-0.50	-1.16	-0.08
AugD	-0.19	-0.20	-0.60	0.21	-0.31	-0.41	2.99	-0.06
SepD	-0.28	-0.00	-0.19	0.34	2.91	-0.00	0.04	0.61
OctD	0.05	0.03	-0.32	2.55	-1.13	0.52	-0.46	0.94
NovD	0.18	-0.15	0.24	-1.90	-0.85	0.27	-0.01	2.13
DecD	-0.08	-0.04	-0.01	-0.52	-0.58	-0.64	-0.22	-2.05

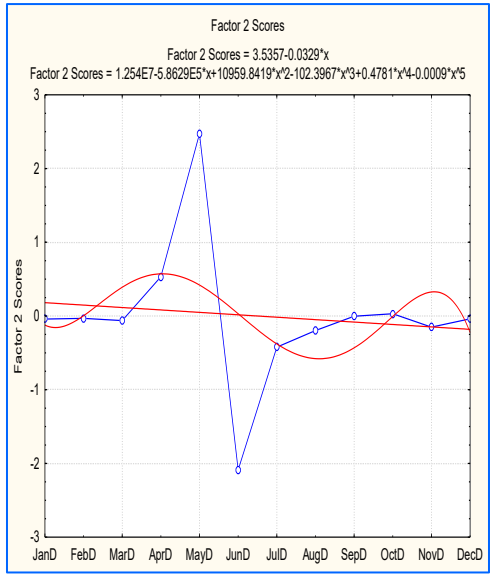


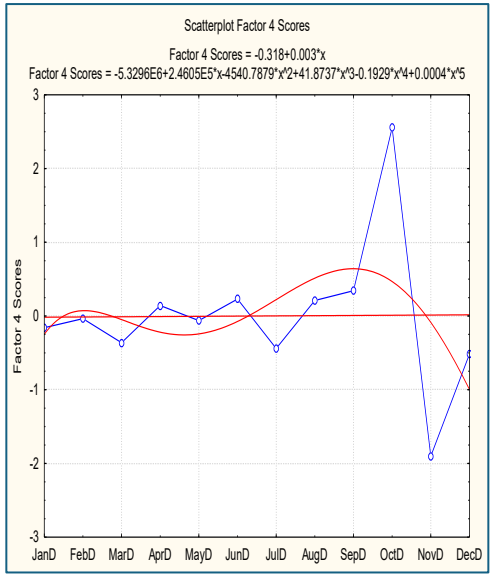


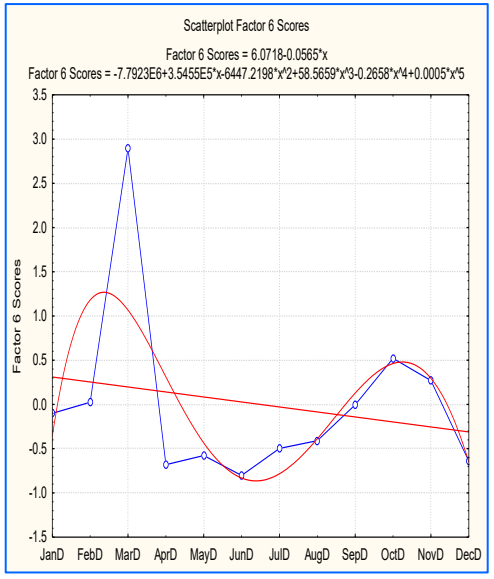


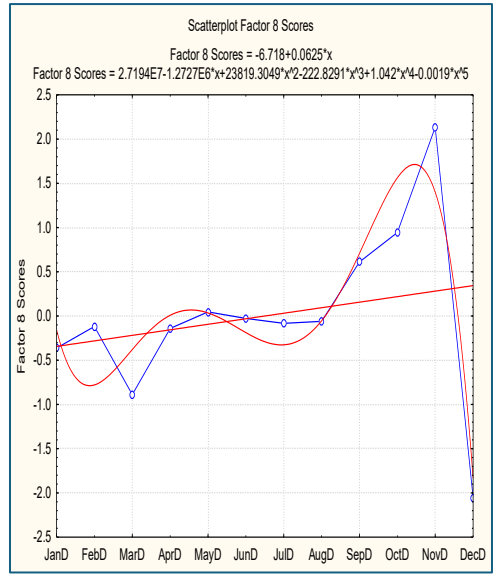




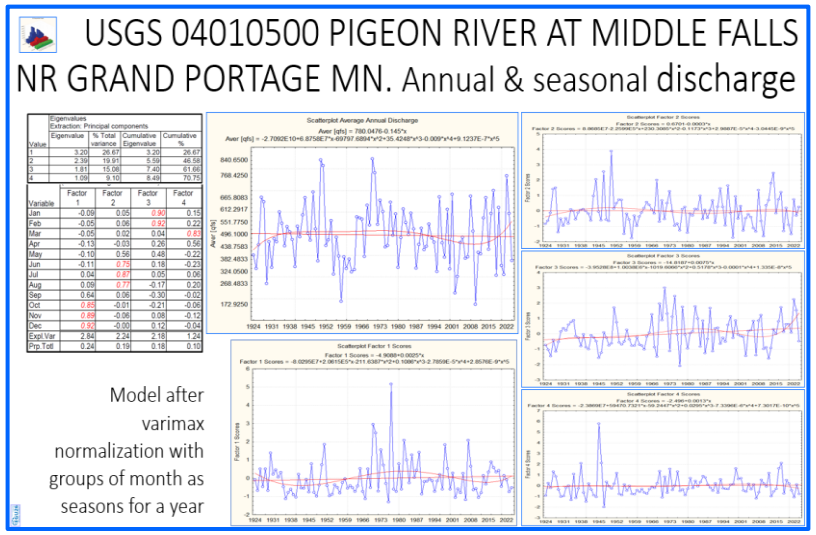




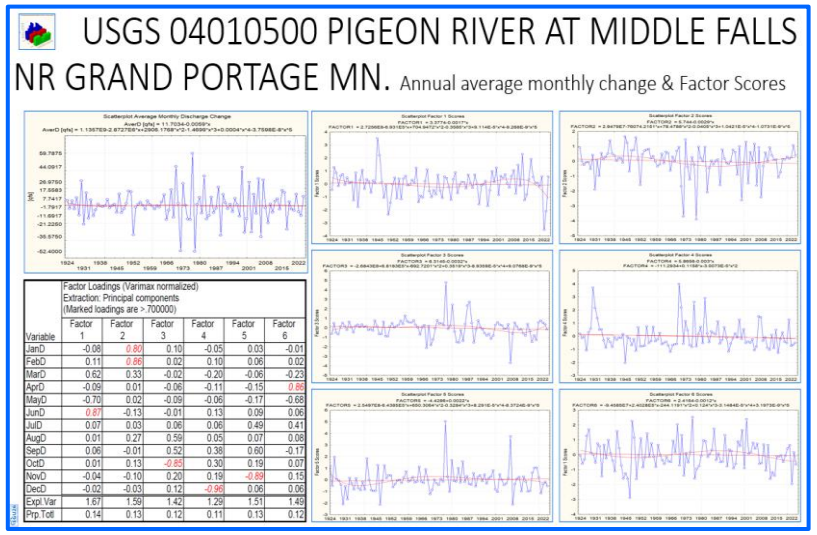




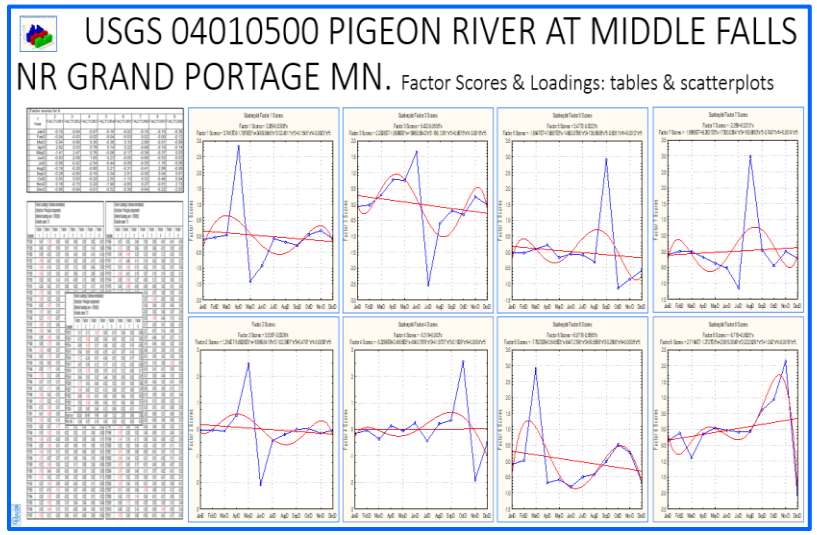
USGS 04010500 PIGEON RIVER AT MIDDLE FALLS NR GRAND PORTAGE MN. Structures of River Discharge from empirical (101x12), calculated (101x12D) & transposed (12Dx101) matrixes



Uncertainty is 29.25%



Uncertainty is 25.22%

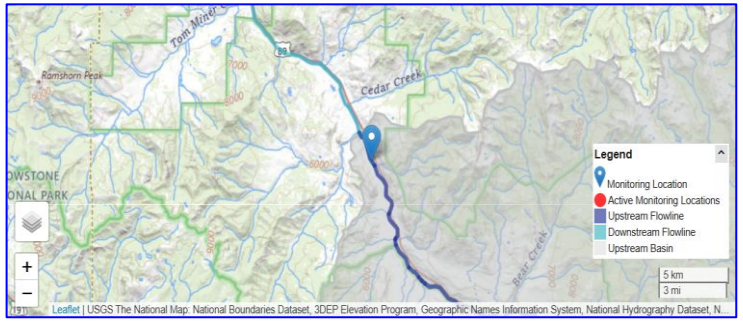


Uncertainty is 0.23%

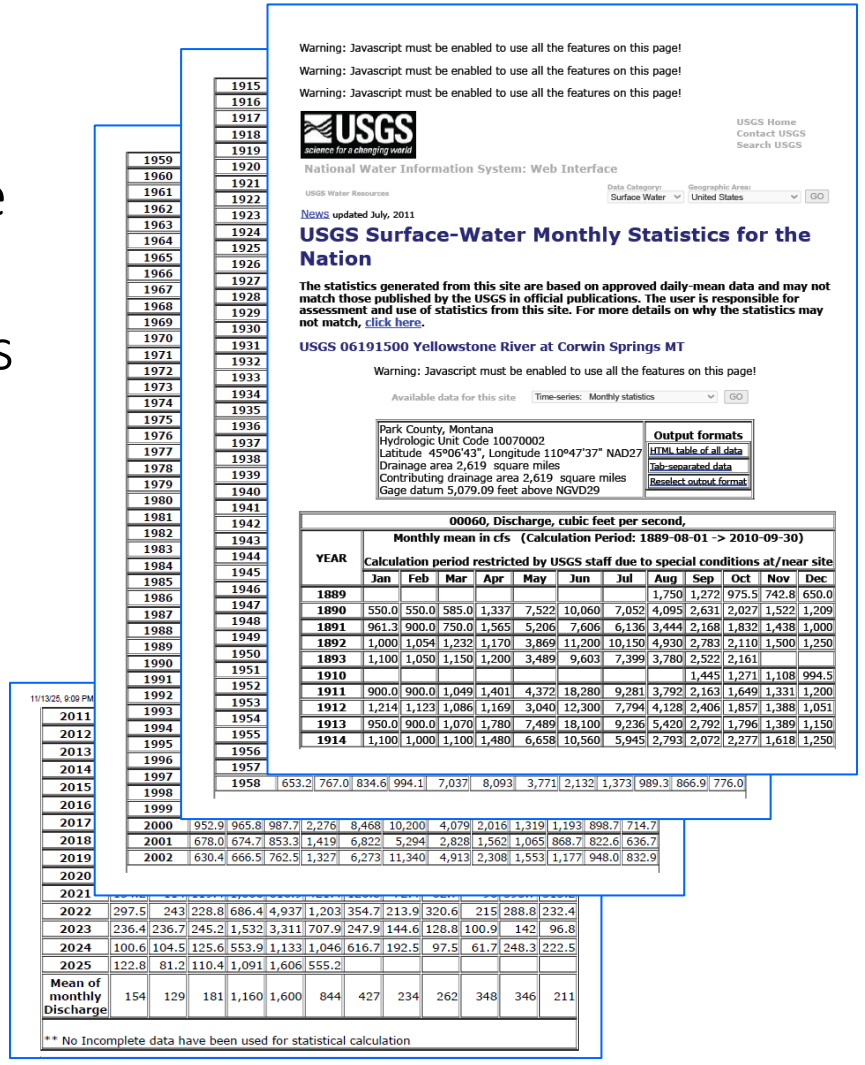
The graphics for models of annual discharge & monthly change of discharge reflects different information from empirical data & calculated data. The monthly discharge actual shows difference of seasons through years, monthly changes – monthly dynamics inside years. The third model shows influence months in distribution of discharge in a year & extract 8 Factors (patterns).

USGS 06191500 Yellowstone River at Corwin Springs MT. Initial information

Park County, Montana
Hydrologic Unit Code 10070002
Latitude 45°06'43.63", Longitude 110°47'37.20" NAD83
Drainage area 2,615 square miles
Gage datum 5,079.09 feet above NGVD29



The Matrix Q_{101x13} used for analysis had twelve monthly & annual mean for 101 years (1924-2024) of discharge observations



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USGS 06191500 Yellowstone River at Corwin Springs MT

Park County, Montana
Hydrologic Unit Code 10070002
Latitude 45°06'43", Longitude 110°47'37" NAD27
Drainage area 2,619 square miles
Contributing drainage area 2,619 square miles
Gage datum 5,079.09 feet above NGVD29

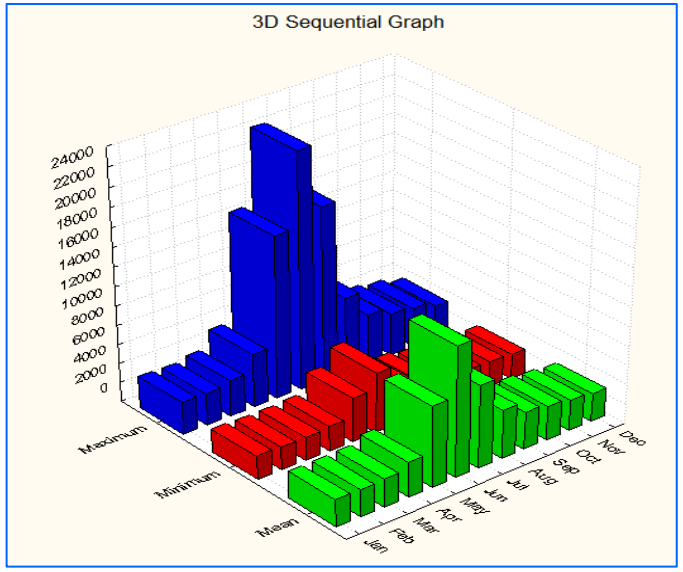
Output formats: HTML table of all data | Tab-separated data | Reselect output format

00060, Discharge, cubic feet per second,
Monthly mean in cfs (Calculation Period: 1889-08-01 -> 2010-09-30)
Calculation period restricted by USGS staff due to special conditions at/near site

YEAR	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1889								1,750	1,272	975.5	742.8	650.0
1890	550.0	550.0	585.0	1,337	7,522	10,060	7,052	4,095	2,631	2,027	1,522	1,209
1891	961.3	900.0	750.0	1,565	5,206	7,606	6,136	3,444	2,168	1,832	1,438	1,000
1892	1,000	1,054	1,232	1,170	3,869	11,200	10,150	4,930	2,783	2,110	1,500	1,250
1893	1,100	1,050	1,150	1,200	3,489	9,603	7,399	3,780	2,522	2,161		
1910									1,445	1,271	1,108	994.5
1911	900.0	900.0	1,049	1,401	4,372	18,280	9,281	3,792	2,163	1,649	1,331	1,200
1912	1,214	1,123	1,086	1,169	3,040	12,300	7,794	4,128	2,406	1,857	1,388	1,051
1913	950.0	900.0	1,070	1,780	7,489	18,100	9,236	5,420	2,792	1,796	1,389	1,150
1914	1,100	1,000	1,100	1,480	6,658	10,560	5,945	2,793	2,072	2,277	1,618	1,250
1958	653.2	767.0	834.6	994.1	7,037	8,093	3,771	2,132	1,373	989.3	866.9	776.0
2000	952.9	965.8	987.7	2,276	8,468	10,200	4,079	2,016	1,319	1,193	898.7	714.7
2001	678.0	674.7	853.3	1,419	6,822	5,294	2,828	1,562	1,065	868.7	822.6	636.7
2002	630.4	666.5	762.5	1,327	6,273	11,340	4,913	2,308	1,553	1,177	948.0	832.9
2022	297.5	243	228.8	686.4	4,937	1,203	354.7	213.9	320.6	215	288.8	232.4
2023	236.4	236.7	245.2	1,532	3,311	707.9	247.9	144.6	128.8	100.9	142	96.8
2024	100.6	104.5	125.6	553.9	1,133	1,046	616.7	192.5	97.5	61.7	248.3	222.5
2025	122.8	81.2	110.4	1,091	1,606	555.2						
Mean of monthly Discharge	154	129	181	1,160	1,600	844	427	234	262	348	346	211

** No Incomplete data have been used for statistical calculation

USGS 06191500 Yellowstone River at Corwin Springs MT. Descriptive statistics

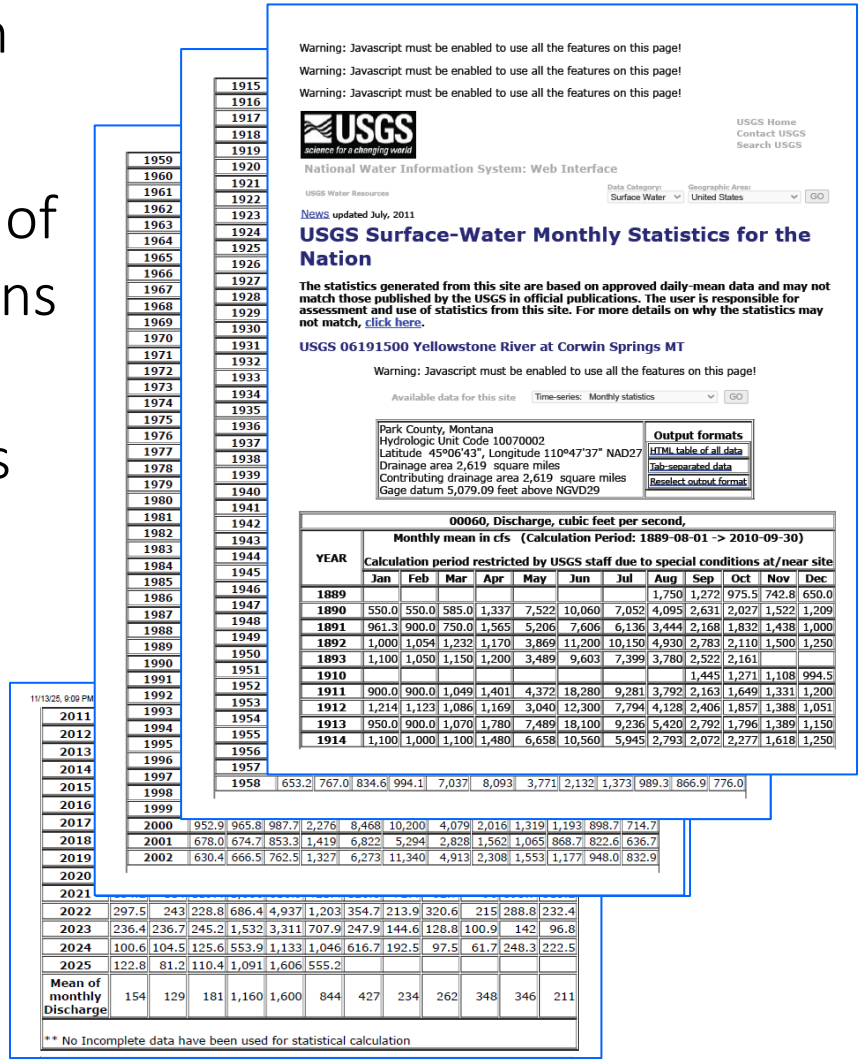


Monthly Hydrograph, visualization of monthly discharge: Minimum, Mean & Maximum for the 114 years

The Matrix Q_{114x12} (1911-2024) of monthly discharge observations

Descriptive statistical characteristics

Variable	Descriptive Statistics (Spreadsheet1)								
	Valid N	Mean	Median	Minimum	Maximum	Lower Quartile	Upper Quartile	Variance	Std.Dev.
Jan	114	839.9	835.9	448.4	1361.0	675.2	980.0	36246	190.4
Feb	114	826.8	812.9	410.6	1340.0	689.9	976.9	33946	184.2
Mar	114	926.0	907.8	411.5	1719.0	815.0	1054.0	36995	192.3
Apr	114	1615.4	1513.5	575.9	3542.0	1161.0	1946.0	323244	568.5
May	114	6421.0	6174.0	2575.0	14890.0	4850.0	7662.0	4741345	2177.5
Jun	114	11538.5	11115.0	4245.0	22540.0	8783.0	13700.0	13980612	3739.1
Jul	114	6620.8	6217.0	2025.0	16170.0	4597.0	8414.0	7610806	2758.8
Aug	114	3083.6	2907.0	1319.0	5810.0	2320.0	3799.0	988236	994.1
Sep	114	1897.4	1837.0	937.5	3207.0	1553.0	2229.0	244739	494.7
Oct	114	1486.3	1417.5	781.5	2429.0	1213.0	1715.0	133076	364.8
Nov	114	1183.4	1154.0	702.0	2058.0	985.7	1349.0	70547	265.6
Dec	114	952.0	939.1	550.8	1424.0	801.4	1083.0	36878	192.0



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USGS Surface-Water Monthly Statistics for the Nation

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Latitude 45°06'43", Longitude 110°47'37" NAD27
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Contributing drainage area 2,619 square miles
Gage datum 5,079.09 feet above NGVD29

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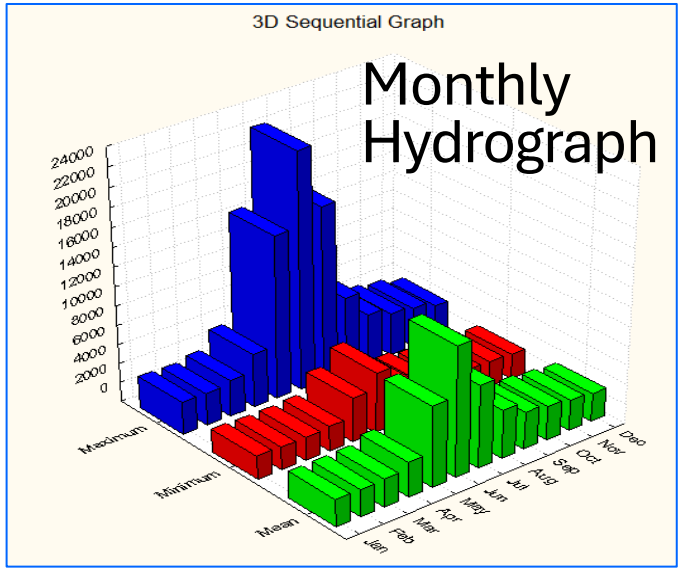
USGS 06191500 Yellowstone River at Corwin Springs MT. Traditional visualization



Variable	Descriptive Statistics (Spreadsheet1)					
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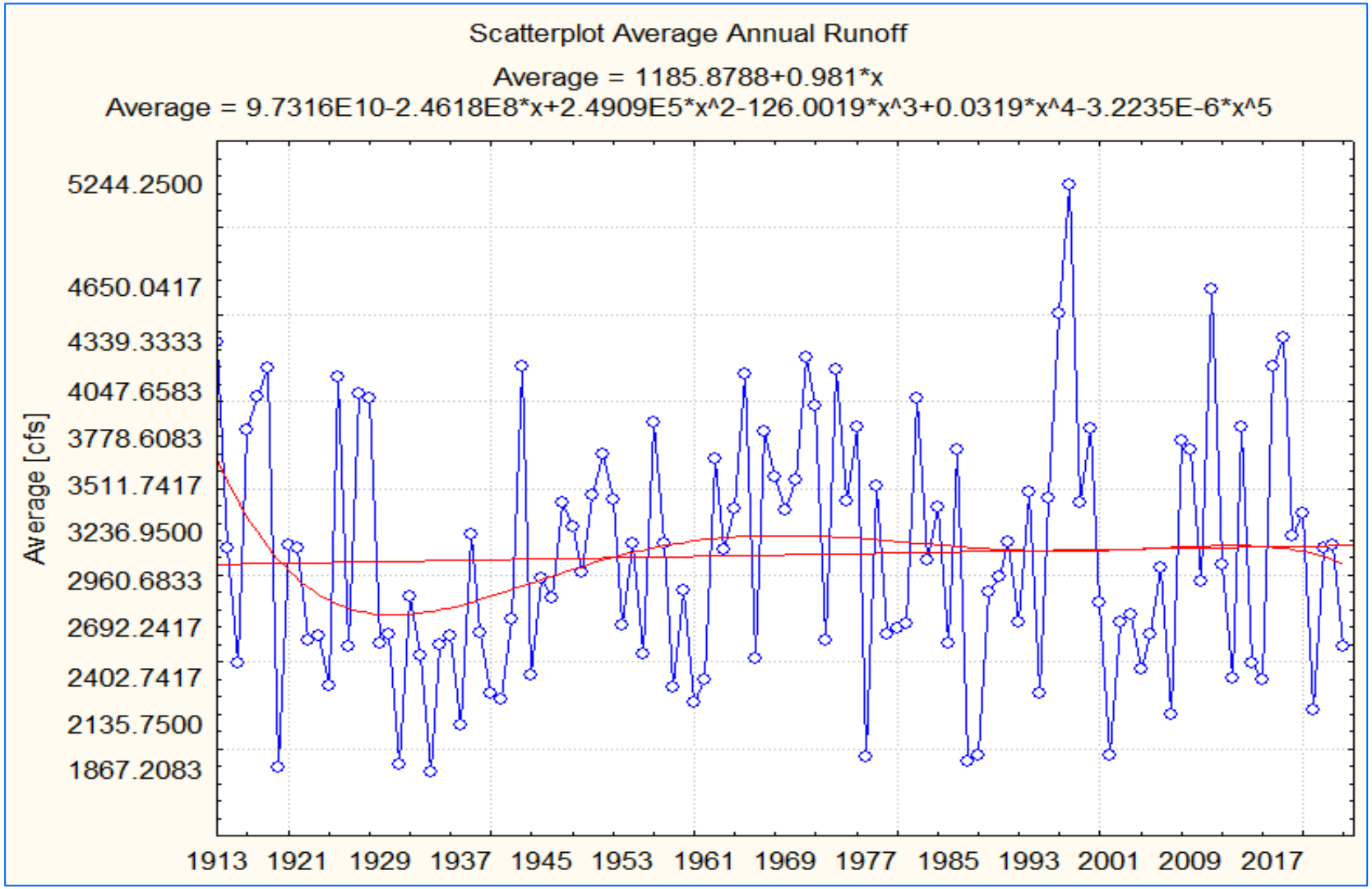
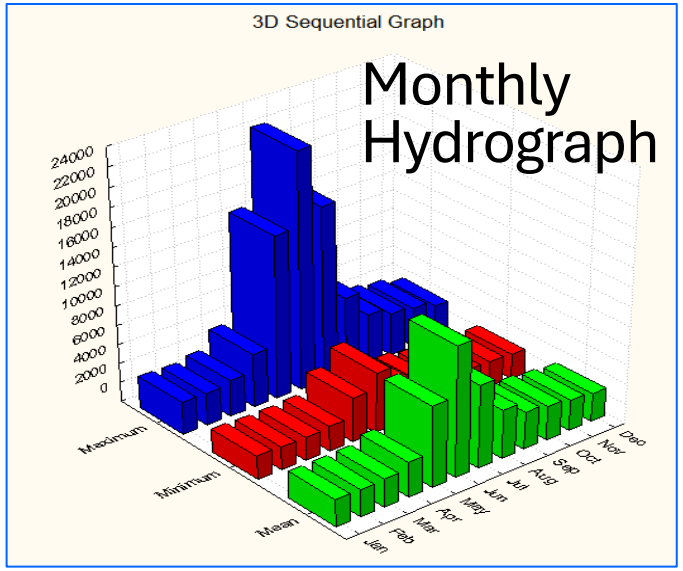
Dec	114	952.0	550.8	1424.0	36878	192.0

Graph of annual discharge with trends: linear & polynomic X^5



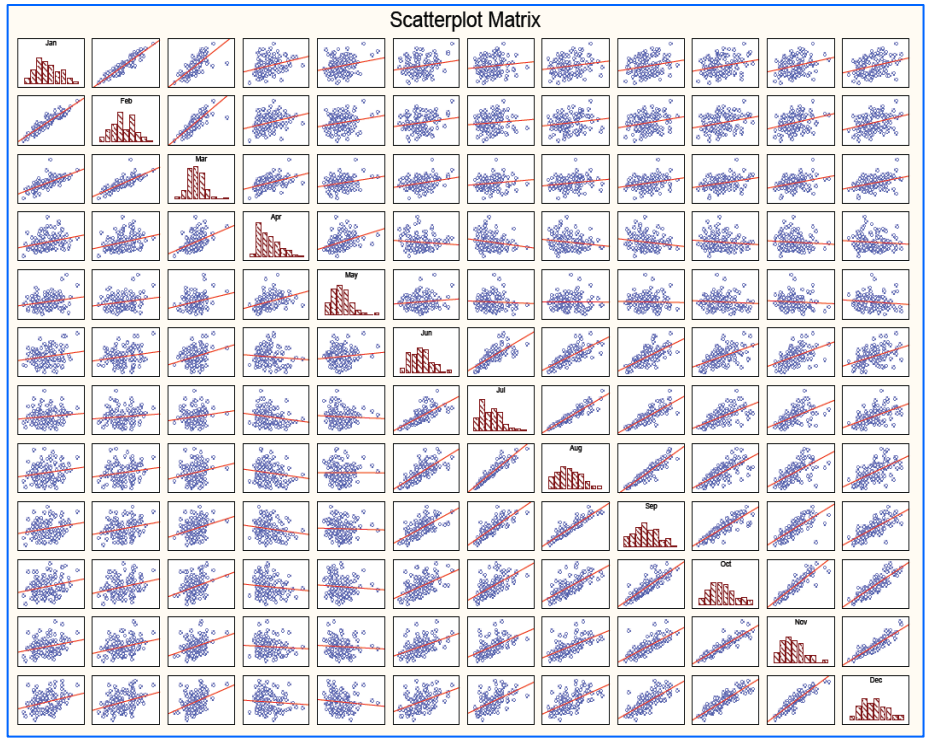
USGS 06191500 Yellowstone River at Corwin Springs MT. Correlation Analysis



The Matrix Q_{114x12} (1911-2024) of monthly discharge observations.

Correlations statistical characteristics.

Variable	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	Correlations Marked correlations are significant at p < .05000 N=114 (Casewise deletion of missing data)											
Jan	1.00	0.93	0.74	0.30	0.22	0.20	0.13	0.18	0.23	0.26	0.28	0.32
Feb	0.93	1.00	0.83	0.31	0.20	0.19	0.11	0.17	0.24	0.27	0.31	0.35
Mar	0.74	0.83	1.00	0.44	0.27	0.30	0.15	0.21	0.29	0.32	0.34	0.40
Apr	0.30	0.31	0.44	1.00	0.39	-0.10	-0.19	-0.17	-0.18	-0.12	-0.08	-0.08
May	0.22	0.20	0.27	0.39	1.00	0.12	-0.06	-0.00	-0.03	-0.08	-0.07	-0.11
Jun	0.20	0.19	0.30	-0.10	0.12	1.00	0.76	0.75	0.70	0.54	0.48	0.46
Jul	0.13	0.11	0.15	-0.19	-0.06	0.76	1.00	0.94	0.84	0.64	0.57	0.56
Aug	0.18	0.17	0.21	-0.17	-0.00	0.75	0.94	1.00	0.93	0.73	0.64	0.62
Sep	0.23	0.24	0.29	-0.18	-0.03	0.70	0.84	0.93	1.00	0.87	0.77	0.76
Oct	0.26	0.27	0.32	-0.12	-0.08	0.54	0.64	0.73	0.87	1.00	0.92	0.88
Nov	0.28	0.31	0.34	-0.08	-0.07	0.48	0.57	0.64	0.77	0.92	1.00	0.92
Dec	0.32	0.35	0.40	-0.08	-0.11	0.46	0.56	0.62	0.76	0.88	0.92	1.00



	1 Year	2 JanD	3 FebD	4 MarD	5 AprD	6 MayD	7 JunD	8 JulD	9 AugD	10 SepD	11 OctD	12 NovD	13 DecD	14 AverageD
1	1911	-94.5	0	149	352	2971	13908	-8999	-5489	-1629	-514	-318	-131	17.1
2	1912	14	-91	-37	83	1871	9260	-4506	-3666	-1722	-549	-469	-337	-12.4
3	1913	-101	-50	170	710	5709	10611	-8864	-3816	-2628	-996	-407	-239	8.3
4	1914	-50	-100	100	380	5178	3902	-4615	-3152	-721	205	-659	-368	8.3
5	1915	-150	-100	100	1511	1655	2039	-1364	-2091	-972	-256	-440	-57	-10.4
6	1916	-212.4	64.3	256.1	506	2391	9330	-1230	-7703	-2084	-585	-492	-166	6.3
7	1917	-100	-134.8	121.8	293	2998	9522	-700	-8445	-1909	-855	-528	-146	9.8
8	1918	-157	-144	180	424	3167	16443	-12694	-4960	-1339	111	-856	-321	-12.2
9	1919	-166	-36.1	72.1	716	4424	-1627	-2529	-706	-298	-97	-39.7	-112.7	-28.3
10	1920	-78.7	52.5	91	57.6	2975	8361	-3876	-4659	-1526	-561	-351	-283	16.9
11	1921	-89.2	29	104.2	137	5052	8133	-9218	-2594	-766	-540	-200	-44	0.3
12	1922	-118	-100	25	323	3037	6635	-5918	-2160	-943	-551	-216	-165	-12.6
13	1923	-52.3	-35.5	-5.8	179.6	3502	4965	-3424	-3023	-1107	-409	-314	-165.3	9.2
14	1924	4.3	-6.1	-18.4	98.5	4130	1546	-2363	-2069	-671	-70	-297	-182	8.5
15	1925	-120	137	46	973	7232	4062	-3581	-5752	-1486	-604	-407	-304	16.3
16	1926	-184	-10	1	1462	4991	-750	-3129	-1591	-554	-174	-216	-88	-20.2
17	1927	-67.1	22.1	70	486	3285	12420	-7950	-5104	-1504	-609	-45	-655	29.1
18	1928	-303	-100	153	331	12106	-2050	-2930	-4713	-1773	-547	-378	-217.7	-35.1
19	1929	-107.8	-73.5	142.6	133.4	2782	6512	-4467	-3342	-929	-347	-279	-41.5	-1.4
20	1930	-183.4	-23.8	80.9	1124.8	3439	2973	-3683	-1580	-1281	131	-753	-190	4.5
21	1931	-118.7	-28.5	-18.3	316.5	3284	2496	-4341	-1058	-455	-151.8	-83.2	-177.5	-28.8
22	1932	-27.9	-5.1	106.9	251.6	4598	5613	-3962	-4302	-1310	-478	-182.3	-256.8	4.6
23	1933	-53.7	-30.6	170.4	385	1872	8798	-6961	-2139	-1270	-450	-50	-100	14.3
24	1934	-95	8.9	30	1756.1	3405	-1760	-2144	-707	-378	-66.6	-80.9	-103.7	-11.3
25	1935	-128.8	-116	-24.4	462.9	3222.5	6999	-4971	-3493	-1079	-517	-254.7	-153.4	-4.4
26	1936	-69.6	5.1	76.5	1200.1	6767	481	-5073	-1811	-901	-432.3	-245	-158.9	-13.4
27	1937	-102.4	-37.8	0.9	164.4	4971.1	2103	-3210	-2311	-785	-236	-262.3	-126.8	14.0
28	1938	-135.5	-33.4	185.7	614.3	3949	8541	-6228	-4381	-1360	-272	-372	-281.7	18.9
29	1939	-62.4	-54	119.2	1083.9	6105	-342	-2904	-2501	-929	-269	-363	-122.9	-20.8
30	1940	-109	-39.9	181.3	421.5	5819	1683	-4973	-1935	-485	-251	-203.2	-127.5	-1.6
31	1941	-8.8	19.8	32.7	206.5	3977.5	1519	-2837	-1311	-5	-191	-503	-284	51.3
32	1942	-228	-112.5	-94.3	1086.8	2339	5756	-3289	-4039	-1180	-442	-149.8	-123.8	-39.7
33	1943	-67.6	-58.9	221.7	1729.4	3451	9398	-2960	-7728	-2253	-901	-365	-292	14.6
34	1944	-139	-5.9	-22.1	161	3401	4002	-2775	-3075	-911	-375	-229	-220.8	-15.7
35	1945	-149.9	-27.5	100.9	123.8	3025.5	6017	-1278	-4745	-1515	-486	-431	-360	22.9
36	1946	-33	-95.6	83.6	1949	3527	3249	-4888	-2561	-735	-195	-269	-176.2	-12.0
37	1947	-238.4	-32.8	188.2	320.2	6848	3612	-3837	-3831	-1735	-502	-454	-220	9.9
38	1948	-129.4	-22.4	36.1	307.7	6092	6585	-7981	-2875	-1219	-474	-304	-256	-20.0
39	1949	-220.7	-19.4	140	1235.1	6301	2683	-5293	-2912	-1029	-353	-280	-281.8	-2.5
40	1950	-253.3	53.1	-16	596	2538	9081	-2400	-6041	-1893	-362	-411	-337	46.2
41	1951	-341.5	-9.5	-7	593	6413	3056	-2104	-3984	-2294	-767	-491	-401.2	-28.1
42	1952	-187	1.2	116.7	1493.3	6929	3508	-6635	-3017	-1413	-585	-365.2	-142.8	-24.7
43	1953	59.9	-17.6	54	266.7	2106	8239	-4500	-3954	-1252	-496	-232	-187.3	7.2
44	1954	-146.5	-25.1	16.8	602.1	6416	2888	-2176	-4883	-1553	-644	-287.6	-161.1	3.9
45	1955	-111.9	11	64	217.6	3089	5845	-4267	-2829	-1225	-416	-251.6	-30.5	8.0
46	1956	79.1	-16.8	89.8	718	6996	7793	-9048	-4087	-1511	-604	-236	-256.8	-7.0
47	1957	-179.9	98.6	198.4	46.7	5110	6859	-5911	-3804	-1257	-552	-394	-237.9	-1.9
48	1958	-170.9	113.8	36	556	9738	10870	-12716	-5000	-1914	-752	-500	-320	21.3
49	1959	-155.7	-75.2	-2	554	6190	2292	-2243	-4616	-1281	-555	-213	-222	-19.0
50	1960	-82.3	-24.8	79	473	5957	7322	-6320	-4656	-1612	-858	-328	-174.5	-10.7
51	1961	-63.3	88.2	21.9	1288.3	6192	1732	-5121	-2063	-697	-126	-294.3	-184	-22.2
52	1962	-151.3	143.6	178.6	565.7	5403	-1528	-2466	-1266	-497	-196.3	-46.1	-185.9	-6.5
53	1963	-346.8	145.3	96	564.5	4946	5067	-5427	-2605	-755	-376	-229	-115.1	16.3
54	1964	-195.3	-108.6	57.1	681.4	5427	3700	-6195	-2411	-835	-339	-213.3	-94.4	-5.0
55	1965	-80.1	-48.3	132.5	1202.4	3010	2209	-3030	-1951	-347	-293	-372	-286	21.3
56	1966	-274.2	-105.4											
57	1967	46.9	106.9											
58	1968	-165.9	126.3											
59	1969	-139	111											
60	1970	-59	75.3											
61	1971	-46	53											
62	1972	-76.6	-65.4											
63	1973	-97	-30											
64	1974	-298.1	19.5											
65	1975	-196.8	-43.9											
66	1976	-52	-88.6											
67	1977	-178.3	19.3											
68	1978	-82	24.1											
69	1979	-211.3	-119.8											
70	1980	-102.2	97.2											
71	1981	-157.6	-21.5											
72	1982	-187.1	-7.7											
73	1983	-83	-74											
74	1984	-193	-96											
75	1985	-74	-23											
76	1986	-11.4	194.9											
77	1987	-63.4	15											
78	1988	-141.7	2.3											
79	1989	-111.7	-67.2											
80	1990	-60.2	-40.7											
81	1991	-164.2	0.3											
82	1992	-108.2	-2											
83	1993	-145.3	-64.1											
84	1994	-86.7	-79.1											
85	1995	-120.7	92.6											
86	1996	-94	-53											
87	1997	278	-21											
88	1998	-93	-39											
89	1999	9	-20											
90	2000	-28.6	12.9											
91	2001	-36.7	-3.3											
92	2002	-6.3	36.1											
93	2003	-54.1	15.7											
94	2004	-25.6	6.4											

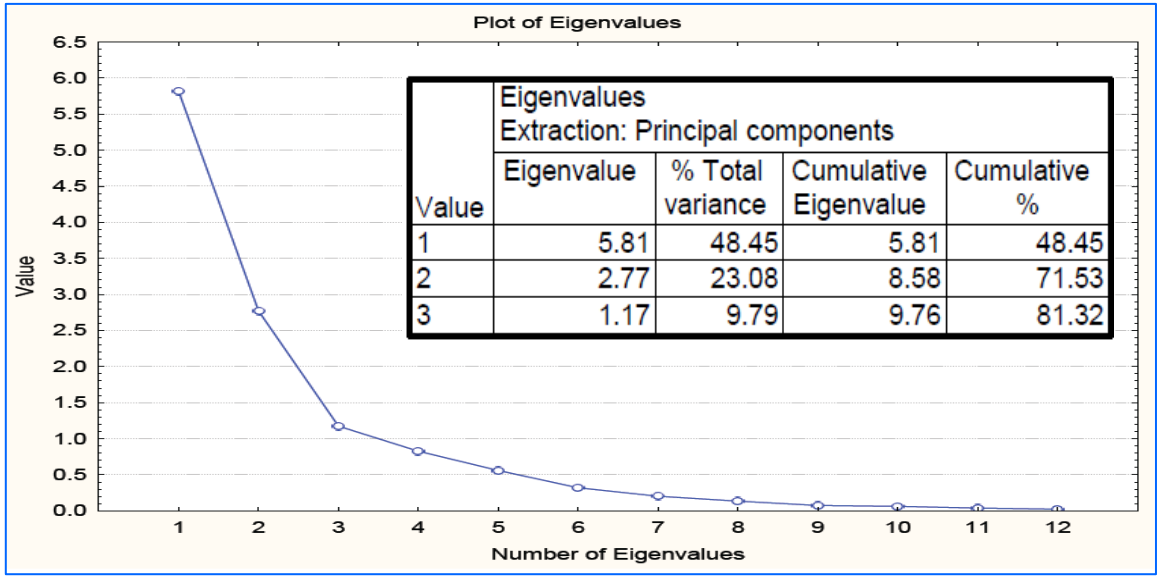
1915			
1916			
1917			

2011														66.9	776.0
2012															
2013															
2014															
2015															
2016															
2017															
2018															
2019															
2020															
2021															
2022	297.5	243	228.8	686.4	4,937	1,203	354.7	213.9	320.6	215	288.8	232.4			
2023	236.4	236.7	245.2	1,532	3,311	707.9	247.9	144.6	128.8	100.9	142	96.8			
2024	100.6	104.5	125.6	553.9	1,133	1,046	616.7	192.5	97.5	61.7	248.3	222.5			
2025	122.8	81.2	110.4	1,091	1,606	555.2									
Mean of monthly Discharge	154	129	181	1,160	1,600	844	427	234	262	348	346	211			

** No Incomplete data have been used for statistical calculation

USGS 06191500 Yellowstone River at Corwin Springs MT. Factor Analysis, structure

Variable	Factor Loadings Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3
Jan	0.12	*0.90*	0.08
Feb	0.12	*0.93*	0.05
Mar	0.20	*0.88*	0.16
Apr	-0.20	0.49	0.50
May	0.04	0.22	*0.79*
Jun	*0.83*	0.04	0.25
Jul	*0.92*	-0.06	0.05
Aug	*0.96*	0.02	0.03
Sep	*0.94*	0.14	-0.13
Oct	*0.82*	0.28	-0.35
Nov	*0.74*	0.34	-0.40
Dec	*0.72*	0.39	-0.43
Expl.Var	5.19	3.09	1.47
Prp.Totl	0.43	0.26	0.12



Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	5.81	48.45	5.81	48.45
2	2.77	23.08	8.58	71.53
3	1.17	9.79	9.76	81.32

Factor Analysis by three Principal components (with Eigenvalue > 1.0) reflects 81.32% of variance contained by data in Matrix Q_{114x12} (1924-2024) of monthly discharge observations. Model after varimax normalization with three Factor represent a structure variance groups of month as seasons for a year .

USGS 06191500 Yellowstone River at Corwin Springs MT. Factor Analysis, structure

Variable	Factor Loadings Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3
Jan	0.12	*0.90*	0.08
Feb	0.12	*0.93*	0.05
Mar	0.20	*0.88*	0.16
Apr	-0.20	0.49	0.50
May	0.04	0.22	*0.79*
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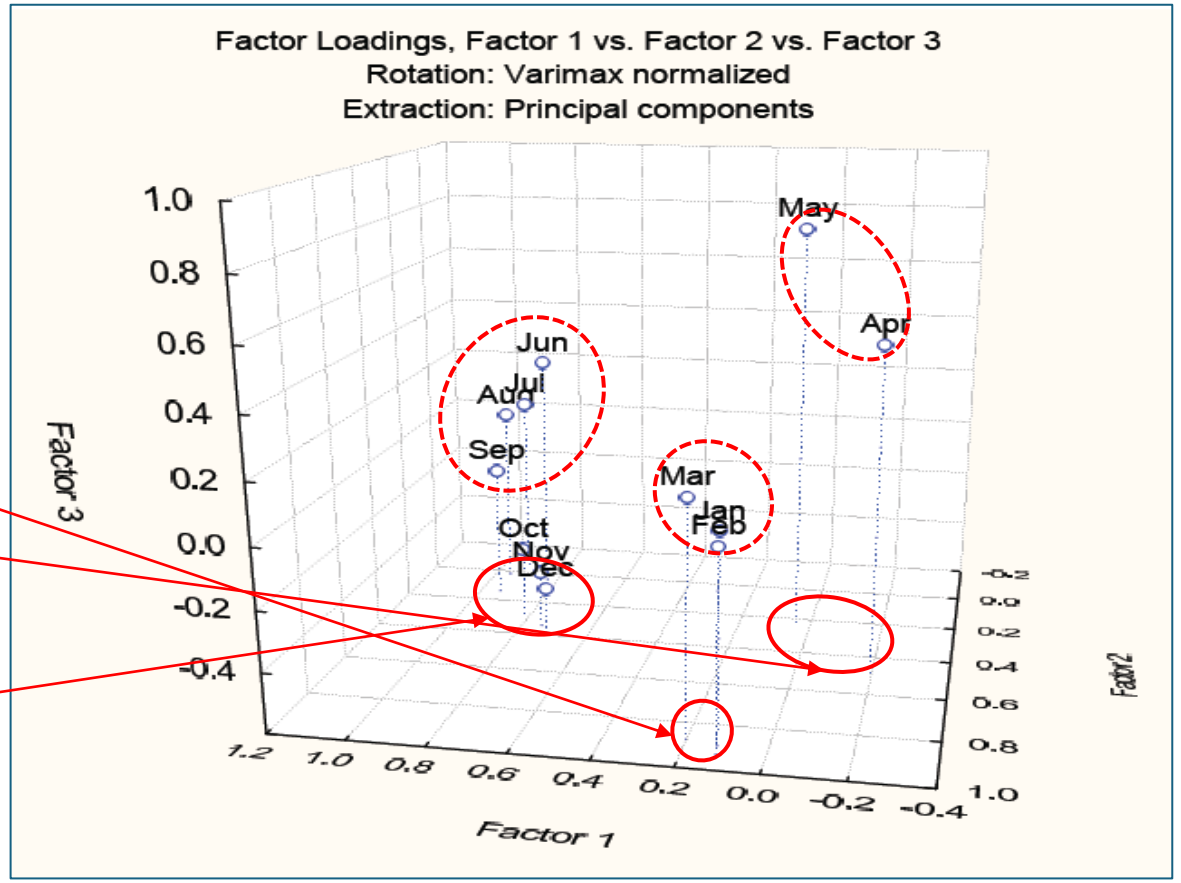
Aug	*0.96*	0.02	0.03
Sep	*0.94*	0.14	-0.13
Oct	*0.82*	0.28	-0.35
Nov	*0.74*	0.34	-0.40
Dec	*0.72*	0.39	-0.43
Expl.Var	5.19	3.09	1.47
Prp.Totl	0.43	0.26	0.12



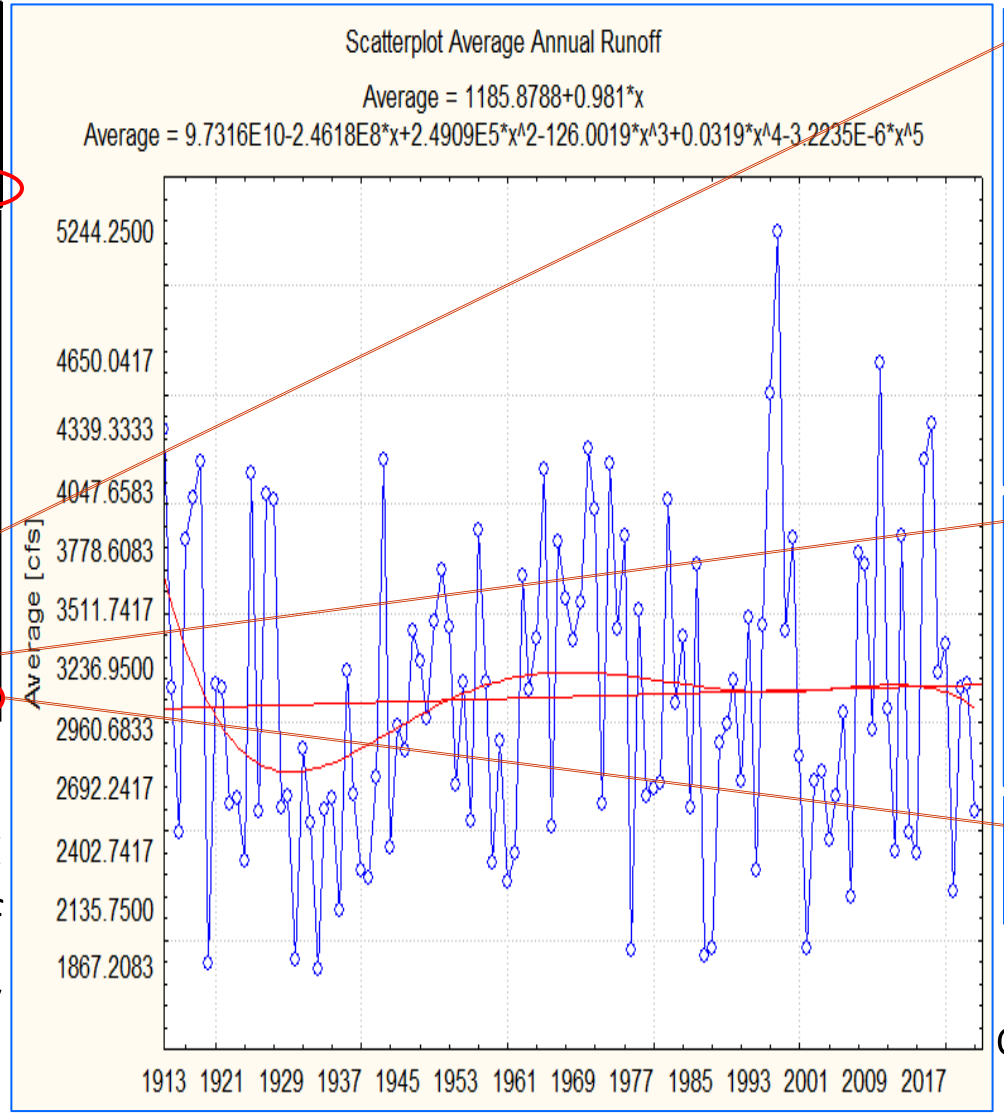
Model after varimax normalization with three Factor represent a structure variance groups of month as seasons for a year

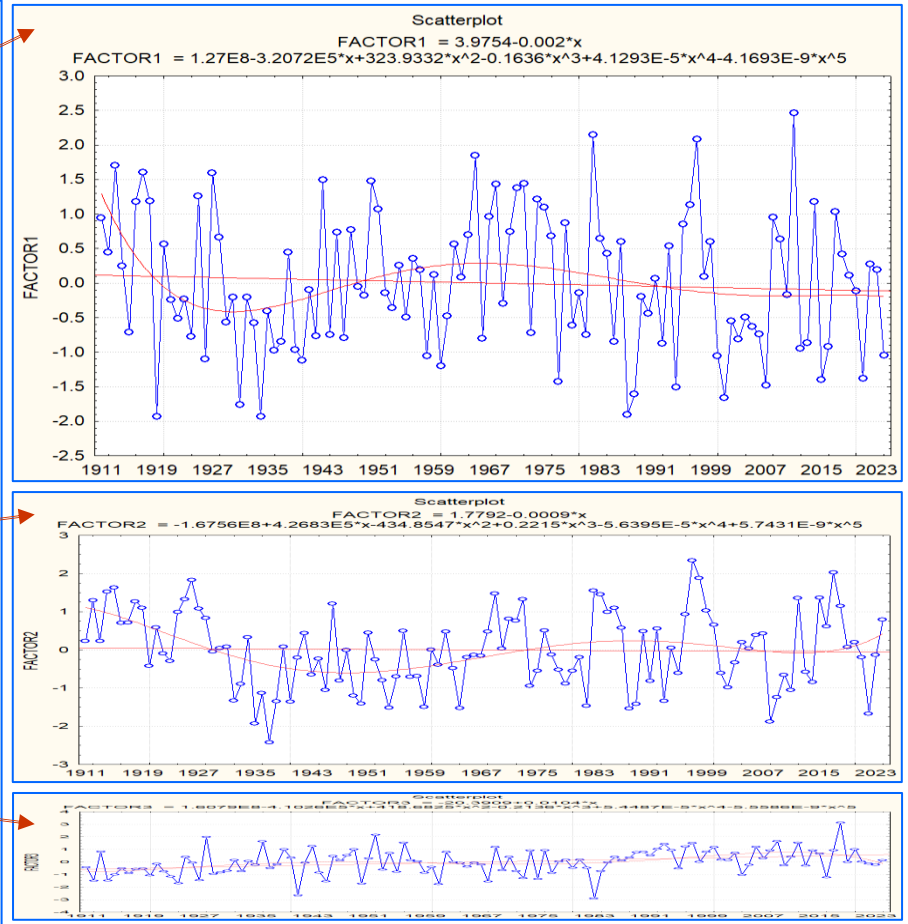
USGS 06191500 Yellowstone River at Corwin Springs MT. Annual, seasonal discharge & uncertainty

Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	5.81	48.45	5.81	48.45
2	2.77	23.08	8.58	71.53
3	1.17	9.79	9.76	81.32

Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3
Jan	0.12	*0.90*	0.08
Feb	0.12	*0.93*	0.05
Mar	0.20	*0.88*	0.16
Apr	-0.20	0.49	0.50
May	0.04	0.22	*0.79*
Jun	*0.83*	0.04	0.25
Jul	*0.92*	-0.06	0.05
Aug	*0.96*	0.02	0.03
Sep	*0.94*	0.14	-0.13
Oct	*0.82*	0.28	-0.35
Nov	*0.74*	0.34	-0.40
Dec	*0.72*	0.39	-0.43
Expl.Var	5.19	3.09	1.47
Prp.Totl	0.43	0.26	0.12

Uncertainty is 18.68%, model does not reflect this part of the variability of the data





Hydrograph of annual discharge is one- & Factor Scores as year's seasons are multi-dimensional characteristics

USGS 06191500 Yellowstone River at Corwin Springs MT. Annual, seasonal discharge & uncertainty

Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	5.81	48.45	5.81	48.45
2	2.77	23.08	8.58	71.53
3	1.17	9.79	9.76	81.32

Variable	(Marked loadings are >.700000) Factor 1	Factor 2	Factor 3
Jan	0.12	*0.90*	0.08
Feb	0.12	*0.93*	0.05
Mar	0.20	*0.88*	0.16
Apr	-0.20	0.49	0.50
May	0.04	0.22	*0.79*
Jun	*0.83*	0.04	0.25
Jul	*0.92*	-0.06	0.05
Aug	*0.96*	0.02	0.03
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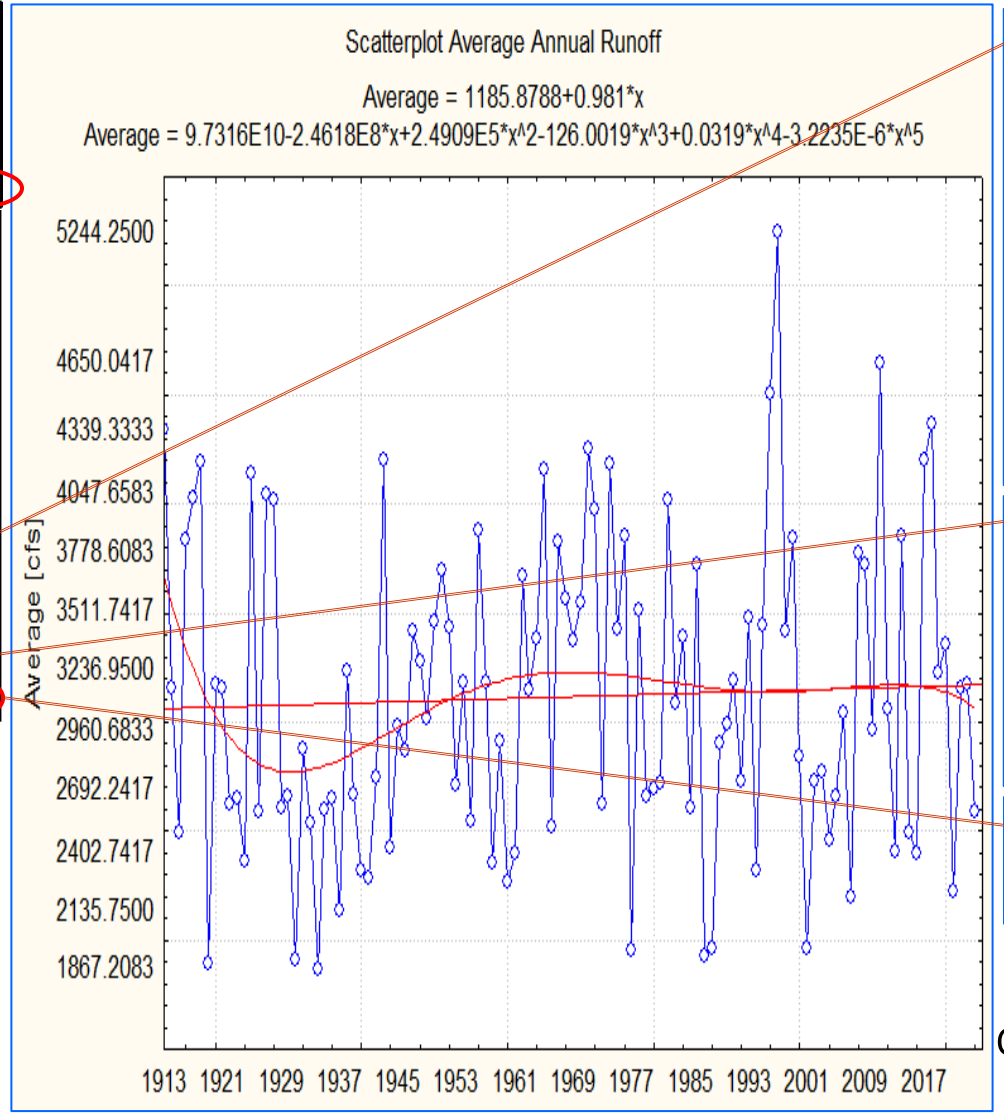
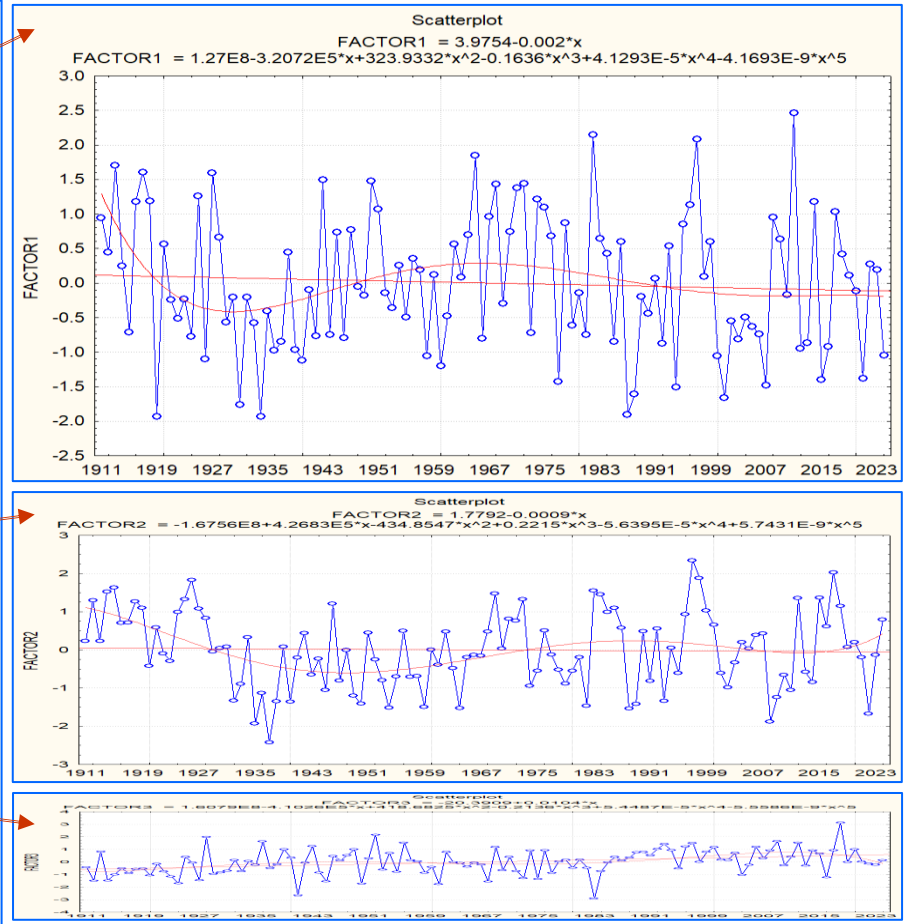
Oct	*0.82*	0.28	-0.35
Nov	*0.74*	0.34	-0.40
Dec	*0.72*	0.39	-0.43
Expl.Var	5.19	3.09	1.47
Prp.Totl	0.43	0.26	0.12

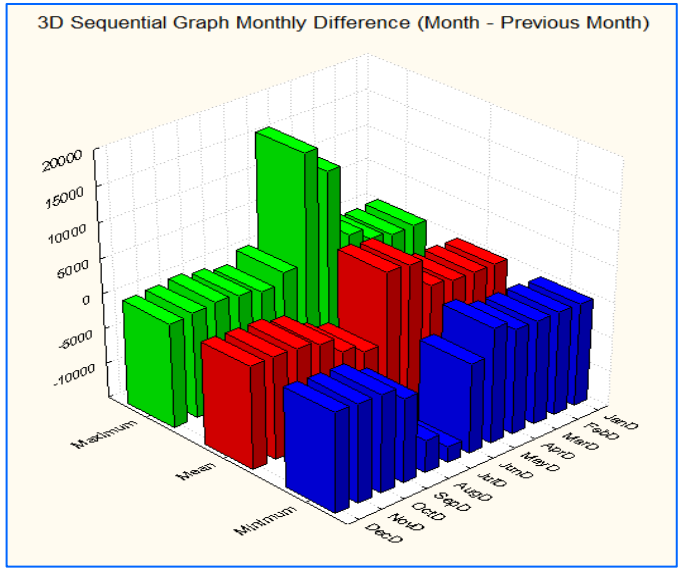
The graph of annual discharge reflects all variability from empirical data, Factor Scores only parts, for Factors 1-4 0.81





Hydrograph of annual discharge is one- & Factor Scores as year's seasons are multi-dimensional characteristics

USGS 06191500 Yellowstone River at Corwin Springs MT. Data of monthly change & Descriptive statistics



Monthly Hydrograph, visualization of changes of monthly discharge: Minimum, Mean & Maximum for the 114 years

Calculated Matrix $Q_{114x12D}$ (1911-2024) of monthly discharge changes

Descriptive statistical characteristics

Variable	Descriptive Statistics (Spreadsheet10)					
	Valid N	Mean	Minimum	Maximum	Variance	Std.Dev.
JanD	114	-114.3	-346.8	278.0	7671	87.6
FebD	114	-13.1	-144.0	194.9	4680	68.4
MarD	114	99.2	-94.3	769.3	12273	110.8
AprD	114	689.3	9.0	2605.5	262989	512.8
MayD	114	4805.7	1531.0	12741.0	4089241	2022.2
JunD	114	5117.5	-2208.0	16443.0	16789728	4097.5
JulD	114	-4917.7	-12716.0	860.0	5971367	2443.6
AugD	114	-3537.3	-10360.0	-706.0	3423648	1850.3
SepD	114	-1186.2	-2945.0	-5.0	319970	565.7
OctD	114	-411.1	-996.0	445.0	65587	256.1
NovD	114	-302.9	-856.0	203.0	25145	158.6
DecD	114	-231.4	-655.0	-30.5	13151	114.7

	1 Year	2 JanD	3 FebD	4 MarD	5 AprD	6 MayD	7 JunD	8 JulD	9 AugD	10 SepD	11 OctD	12 NovD	13 DecD	14 AverageD
1	1911	-94.5	0	149	352	2971	13908	-8999	-5489	-1629	-514	-318	-131	17.1
2	1912	14	-91	-37	83	1871	9260	-4506	-3666	-1722	-549	-469	-337	-12.4
3	1913	-101	-50	170	710	5709	10611	-8864	-3816	-2628	-996	-407	-239	8.3
4	1914	-50	-100	100	380	5178	3902	-4615	-3152	-721	205	-659	-368	8.3
5	1915	-150	-100	100	1511	1655	2039	-1364	-2091	-972	-256	-440	-57	-10.4
6	1916	-212.4	64.3	256.1	506	2391	9330	-1230	-7703	-2084	-585	-492	-166	6.3
7	1917	-100	-134.8	121.8	293	2998	9522	-700	-8445	-1905	-855	-528	-146	9.8
8	1918	-157	-144	180	424	3167	16443	-12694	-4960	-1339	111	-856	-321	-12.2
9	1919	-166	-36.1	72.1	716	4424	-1627	-2529	-706	-238	-97	-39.7	-112.7	-28.3
10	1920	-78.7	52.5	91	57.6	2975	8361	-3876	-4659	-1526	-561	-351	-283	16.9
11	1921	-89.2	29	104.2	137	5052	8133	-9218	-2594	-766	-540	-200	-44	0.3
12	1922	-118	-100	25	323	3037	6635	-5918	-2160	-943	-551	-216	-165	-12.6
13	1923	-52.3	-35.5	-5.8	179.6	3502	4965	-3424	-3023	-1107	-409	-314	-165.3	9.2
14	1924	4.3	-6.1	-18.4	98.5	4130	1546	-2363	-2069	-671	-70	-297	-182	8.5
15	1925	-120	137	46	973	7232	4062	-3581	-5752	-1486	-604	-407	-304	16.3
16	1926	-184	-10	1	1462	4991	-750	-3129	-1591	-554	-174	-216	-88	-20.2
17	1927	-67.1	22.1	70	486	3285	12420	-7950	-5104	-1504	-609	-45	-655	29.1
18	1928	-303	-100	153	331	12106	-2050	-2930	-4713	-1773	-547	-378	-217.7	-35.1
19	1929	-107.8	-73.5	142.6	133.4	2782	6512	-4467	-3342	-929	-347	-279	-41.5	-1.4
20	1930	-183.4	-23.8	80.9	1124.8	3439	2973	-3683	-1580	-1281	131	-753	-190	4.5
21	1931	-118.7	-28.5	-18.3	316.5	3284	2486	-4341	-1058	-455	-151.8	-83.2	-177.5	-28.8
22	1932	-27.9	-5.1	106.9	251.6	4598	5613	-3962	-4302	-1310	-478	-182.3	-256.8	4.6
23	1933	-53.7	-30.6	170.4	385	1872	8798	-6961	-2139	-1270	-450	-50	-100	14.3
24	1934	-95	8.9	30	1756.1	3405	-1760	-2144	-707	-378	-66.6	-80.9	-103.7	-11.3
25	1935	-128.8	-116	-24.4	462.9	3222.5	6999	-4971	-3493	-1079	-517	-254.7	-153.4	-4.4
26	1936	-69.6	5.1	76.5	1200.1	6767	481	-5073	-1811	-901	-432.3	-245	-158.9	-13.4
27	1937	-102.4	-37.8	0.9	164.4	4971.1	2103	-3210	-2311	-785	-236	-262.3	-126.8	14.0
28	1938	-135.5	-33.4	185.7	614.3	3949	8541	-6226	-4381	-1360	-272	-372	-261.7	18.9
29	1939	-62.4	-64	119.2	1083.9	6105	-342	-2904	-2501	-929	-269	-363	-122.9	-20.8
30	1940	-109	-39.9	181.3	421.5	5819	1683	-4973	-1935	-485	-251	-203.2	-127.5	-1.6
31	1941	-8.8	19.8	32.7	206.5	3977.5	1519	-2837	-1311	-5	-191	-503	-284	51.3
32	1942	-226	-112.5	-94.3	1086.8	2339	5756	-3289	-4039	-1180	-442	-149.8	-123.8	-39.7
33	1943	-67.6	-58.9	221.7	1729.4	3451	9398	-2960	-7728	-2253	-901	-365	-292	14.6
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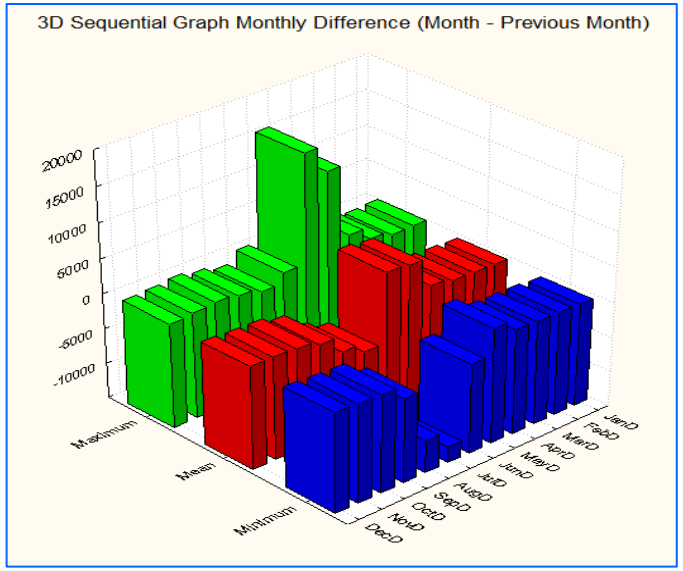
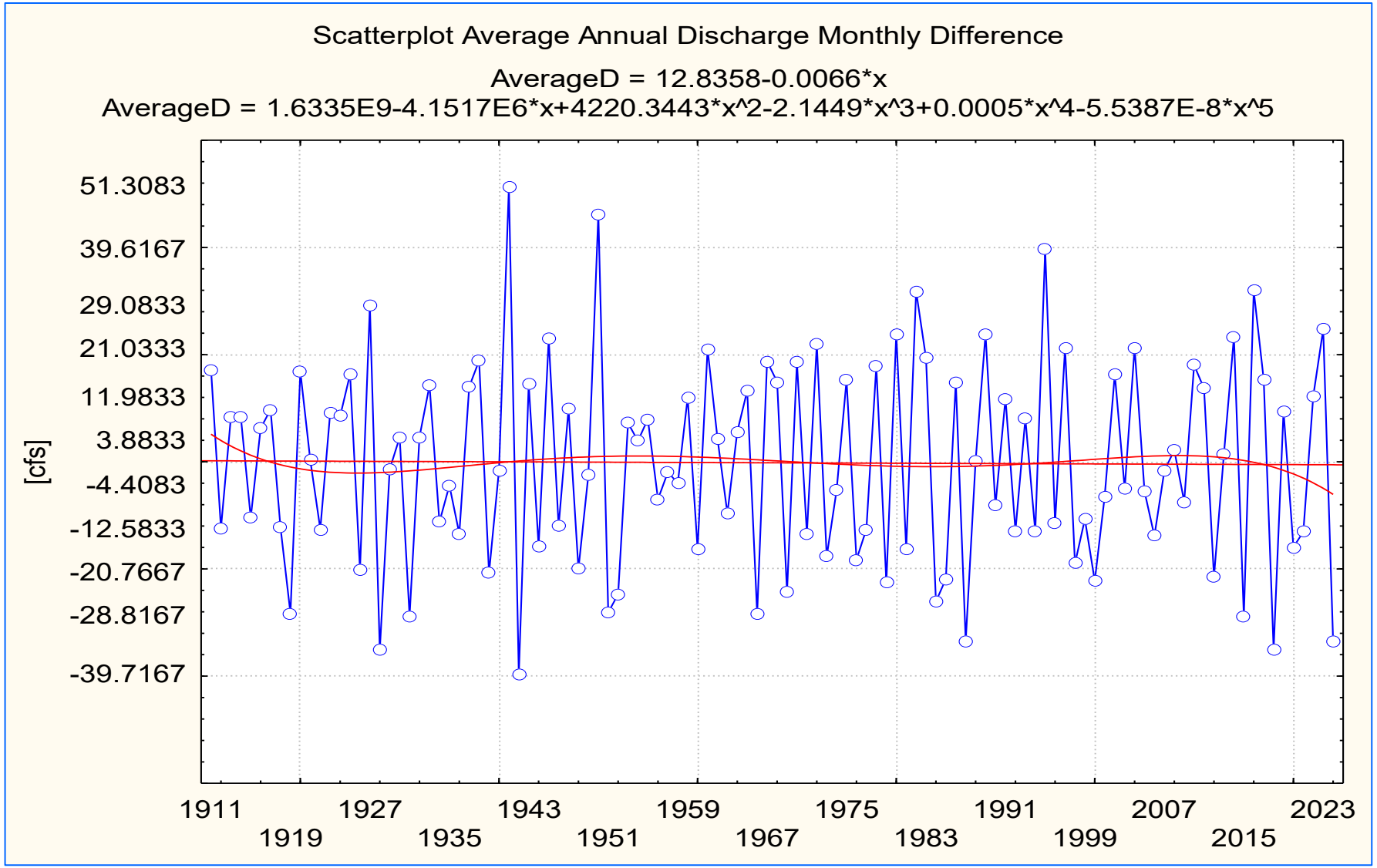
36	1946	-33	-95.6	83.6	1949	3527	3249	-4888	-2561	-735	-195	-269	-176.2	-12.0
37	1947	-238.4	-32.8	188.2	320.2	6848	3612	-3837	-3831	-1735	-502	-454	-220	9.9
38	1948	-129.4	-22.4	36.1	307.7	6092	6585	-7981	-2875	-1219	-474	-304	-256	-20.0
39	1949	-220.7	-19.4	140	1235.1	6301	2683	-5293	-2912	-1029	-353	-280	-261.8	-2.5
40	1950	-253.3	53.1	-16	596	2538	9081	-2400	-6041	-1893	-362	-411	-337	46.2
41	1951	-341.5	-9.5	-7	593	6413	3056	-2104	-3984	-2294	-767	-491	-401.2	-28.1
42	1952	-187	1.2	116.7	1493.3	6929	3508	-6635	-3017	-1413	-585	-365.2	-142.8	-24.7
43	1953	59.9	-17.6	54	266.7	2106	8239	-4500	-3954	-1252	-496	-232	-187.3	7.2
44	1954	-146.5	-25.1	16.8	602.1	6416	2888	-2176	-4883	-1553	-644	-287.6	-161.1	3.9
45	1955	-111.9	11	64	217.6	3089	5845	-4267	-2829	-1225	-416	-251.6	-30.5	8.0
46	1956	79.1	-16.8	89.8	718	6996	7793	-9048	-4087	-1511	-604	-236	-256.8	-7.0
47	1957	-179.9	98.6	198.4	46.7	5110	6859	-5911	-3804	-1257	-552	-394	-237.9	-1.9
48	1958	-170.9	1											
49	1959	-155.7	-											
50	1960	-82.3	-											
51	1961	-63.3	-											
52	1962	-151.3	1											
53	1963	-346.8	1											
54	1964	-195.3	-1											
55	1965	-80.1	-											
56	1966	-274.2	-1											
57	1967	46.9	1											
58	1968	-165.9	1											
59	1969	-139	-											
60	1970	-59	-											
61	1971	-46	-											
62	1972	-76.6	-											
63	1973	-97	-											
64	1974	-298.1	-											
65	1975	-196.8	-											
66	1976	-62	-											
67	1977	-178.3	-											
68	1978	-82	-											
69	1979	-211.3	-1											
70	1980	-102.2	-											
71	1981	-157.6	-											
72	1982	-187.1	-											
73	1983	-83	-											
74	1984	-193	-											
75	1985	-74	-											
76	1986	-11.4	1											
77	1987	-63.4	-											
78	1988	-141.7	-											
79	1989	-111.7	-											
80	1990	-60.2	-											
81	1991	-164.2	-											
82	1992	-108.2	-											
83	1993	-145.3	-64.1	102.4	658	9456	790	-4813	-3015	-1726	-565	-426	-155	8.1
84	1994	-86.7	-79.1	163.2	1442.6	6137	-2139	-3375	-1327	-557	-70	-172.8	-92.6	-13.0
85	1995	-120.7	92.6	223.8	379.7	4133	7688	-4424	-4711	-1789	-583	-327	-87	39.6
86	1996	-94	-53	40	831	6071	13106	-11575	-5587	-1793	-621	-162	-299	-11.3
87	1997	278	-21	36	556	9738	10870	-12716	-5000	-1914	-752	-500	-320	21.3
88	1998	-93	-39	-2	554	6190	2292	-2243	-4616	-1281	-555	-213	-222	-19.0
89	1999	9	-20	79	473	5957	7322	-6320	-4656	-1612	-858	-328	-174.5	-10.7
90	2000	-28.6	12.9	21.9	1288.3	6192	1732	-6121	-2063	-697	-126	-294.3	-184	-22.2
91	2001	-36.7	-3.3	178.6	565.7	5403	-1526	-2466	-1266	-497	-196.3	-46.1	-185.9	-6.5
92	2002	-6.3	36.1	96	564.5	4946	5067	-6427	-2605	-755	-376	-229	-115.1	16.3
93	2003	-54.1	15.7	57.1	881.4	5427	3700	-6195	-2411	-835	-339	-213.3	-94.4	-5.0
94	2004	-25.6	6.4	132.5	1202.4	3010	2209	-3030	-1951	-347	-293	-372	-286	21.3
06	2016	-64.5	71.3	37.4	1604.6	4020	645	-3818	-1666	-528	445	203	-567	31.9
07	2017	-269.5	50.2	769.3	1067	7017	5787	-7795	-4222	-1205	-439	-271	-305	15.3
08	2018	-170	-107	59	1014	12741	1500	-9194	-3983	-1409	-461	-230	-180.6	-35.1
09	2019	-158.1	-5.3	224.8	1014.2	4218	6254	-5762	-3812	-1021	-256	-401	-182	9.5
10	2020	-118.7	-44.3	106.3	604.7	7275	4171	-6896	-3239	-1196	-401	-203	-250.1	-15.9
11	2021	-106	30.6	119.9	598.6	4973	1176	-4673	-1177	-638	-144	-42.9	-274	-13.1
12	2022	-171.9	-17.7	254.6	290.9	3324	11282	-8871	-3759	-1228	-385	-327	-244.9	12.2
13	2023	19.8	-129.1	-40.6	695.8	8199	1030	-5459	-2449	-813	-184	-188	-385	24.7
14	2024	-156.1	-20	103.1	1002	2977	4972	-5724	-2088	-618	-406	-265.3	-179	-33.5

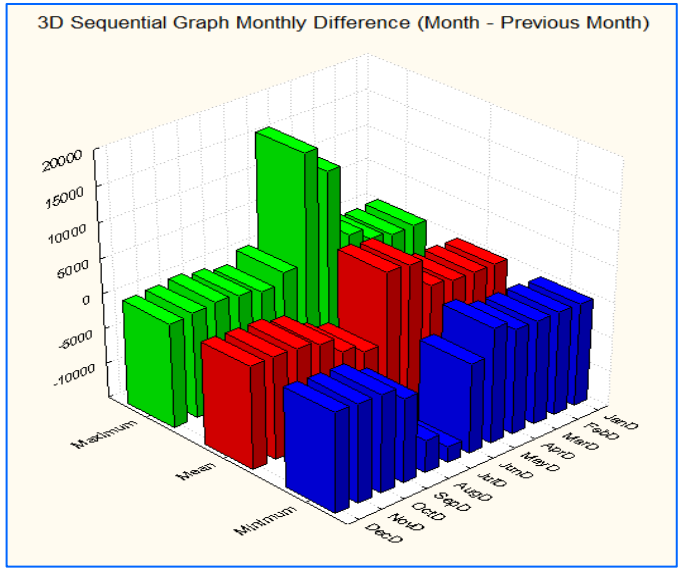
USGS 06191500 Yellowstone River at Corwin Springs MT. Descriptive statistics & discharge monthly difference



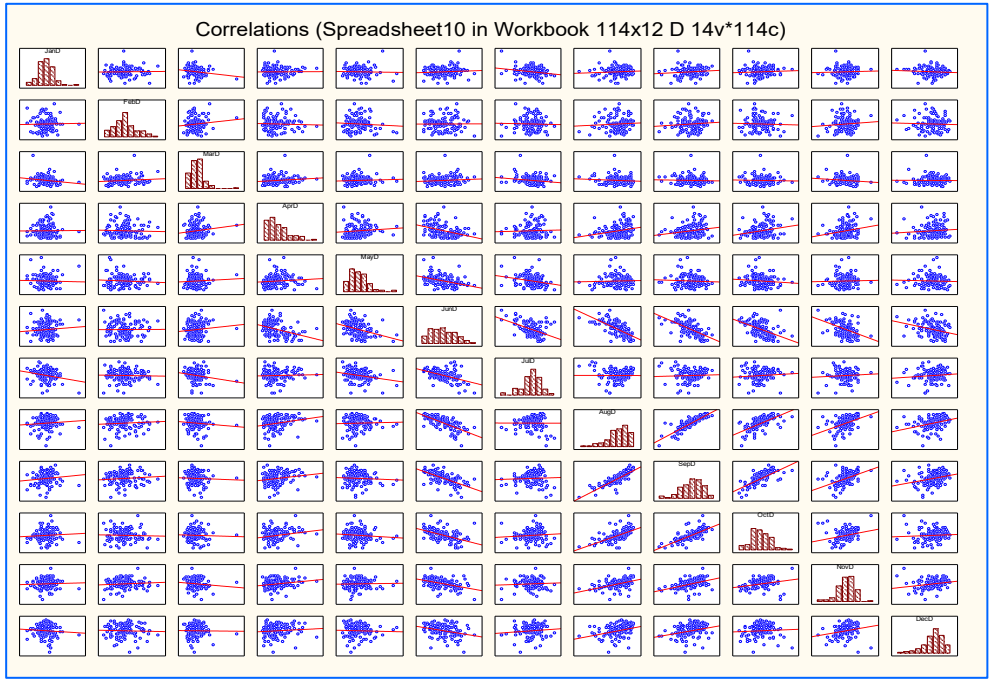
Variable	Descriptive Statistics (Spreadsheet10)					
	Valid N	Mean	Minimum	Maximum	Variance	Std.Dev.
JanD	114	-114.3	-346.8	278.0	7671	87.6
FebD	114	-13.1	-144.0	194.9	4680	68.4
MarD	114	99.2	-94.3	769.3	12273	110.8
AprD	114	689.3	9.0	2605.5	262989	512.8
MayD	114	4805.7	1531.0	12741.0	4089241	2022.2
JunD	114	5117.5	-2208.0	16443.0	16789728	4097.5
JulD	114	-4917.7	-12716.0	860.0	5971367	2443.6
AugD	114	-3537.3	-10360.0	-706.0	3423648	1850.3
SepD	114	-1186.2	-2945.0	-5.0	319970	565.7
OctD	114	-411.1	-996.0	445.0	65587	256.1
NovD	114	-302.9	-856.0	203.0	25145	158.6
DecD	114	-231.4	-655.0	-30.5	13151	114.7



USGS 06191500 Yellowstone River at Corwin Springs MT. Correlation



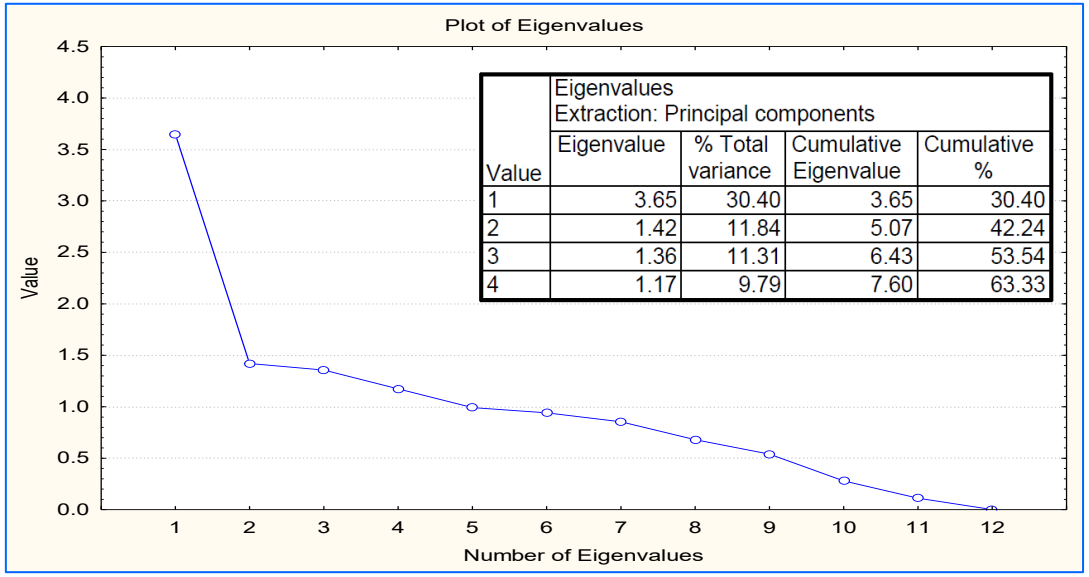
Variable	Correlations Marked correlations are significant at p < .05000 N=114 (Casewise deletion of missing data) JanD	FebD	MarD	AprD	MayD	JunD	JulD	AugD	SepD	OctD	NovD	DecD
JanD	1.00	0.02	-0.18	0.01	-0.03	0.09	-0.22	0.08	0.12	0.07	0.04	-0.08
FebD	0.02	1.00	0.12	-0.04	-0.08	0.01	-0.03	0.08	0.10	-0.04	0.09	-0.04
MarD	-0.18	0.12	1.00	0.14	0.06	0.11	-0.19	-0.10	-0.04	-0.04	-0.12	-0.01
AprD	0.01	-0.04	0.14	1.00	0.10	-0.38	0.04	0.25	0.20	0.23	0.22	0.09
MayD	-0.03	-0.08	0.06	0.10	1.00	-0.37	-0.25	0.04	-0.06	-0.10	0.03	-0.04
JunD	0.09	0.01	0.11	-0.38	-0.37	1.00	-0.49	-0.70	-0.64	-0.50	-0.43	-0.31
JulD	-0.22	-0.03	-0.19	0.04	-0.25	-0.49	1.00	-0.00	0.05	0.09	0.07	0.11
AugD	0.08	0.08	-0.10	0.25	0.04	-0.70	-0.00	1.00	0.87	0.64	0.46	0.33
SepD	0.12	0.10	-0.04	0.20	-0.06	-0.64	0.05	0.87	1.00	0.72	0.46	0.30
OctD	0.07	-0.04	-0.04	0.23	-0.10	-0.50	0.09	0.64	0.72	1.00	0.27	0.06
NovD	0.04	0.09	-0.12	0.22	0.03	-0.43	0.07	0.46	0.46	0.27	1.00	0.23
DecD	-0.08	-0.04	-0.01	0.09	-0.04	-0.31	0.11	0.33	0.30	0.06	0.23	1.00



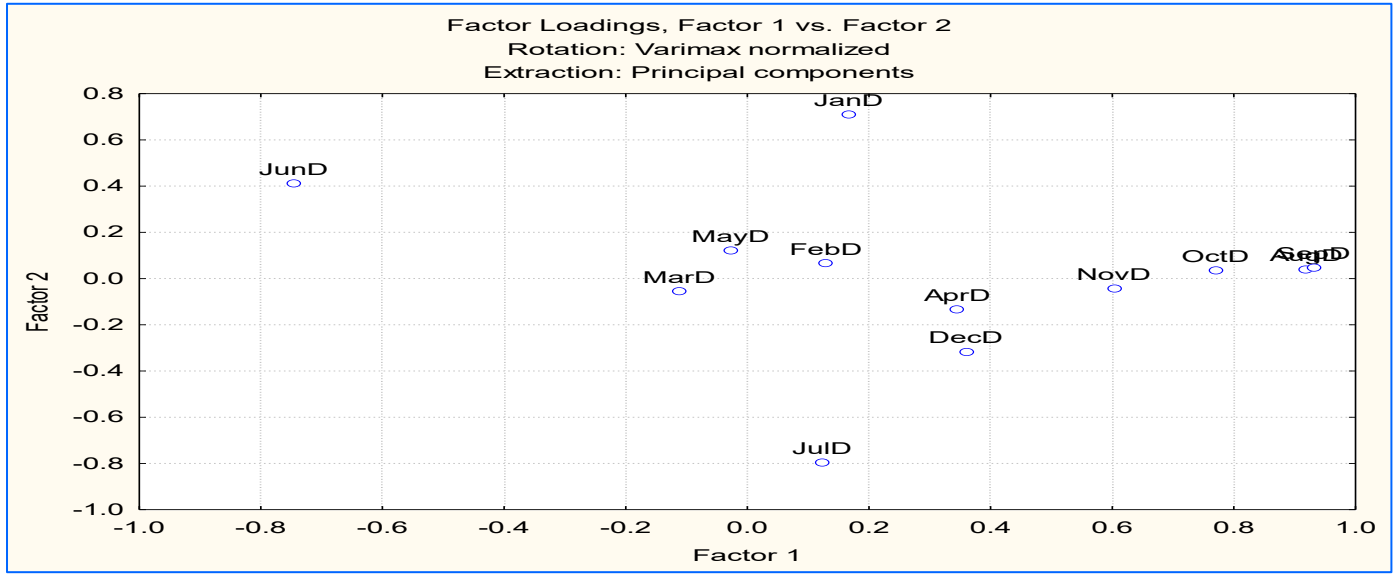
	1 Year	2 JanD	3 FebD	4 MarD	5 AprD	6 MayD	7 JunD	8 JulD	9 AugD	10 SepD	11 OctD	12 NovD	13 DecD	14 AverageD
1	1911	-94.5	0	149	352	2971	13908	-8999	-5489	-1629	-514	-318	-131	17.1
2	1912	14	-91	-37	83	1871	9260	-4506	-3666	-1722	-549	-469	-337	-12.4
3	1913	-101	-50	170	710	5709	10611	-8864	-3816	-2628	-996	-407	-239	8.3
4	1914	-50	-100	100	380	5178	3902	-4615	-3152	-721	205	-659	-368	8.3
5	1915	-150	-100	100	1511	1655	2039	-1364	-2091	-972	-256	-440	-57	-10.4
6	1916	-212.4	64.3	256.1	506	2391	9330	-1230	-7703	-2084	-585	-492	-166	6.3
7	1917	-100	-134.8	121.8	293	2998	9522	-700	-8445	-1905	-855	-528	-146	9.8
8	1918	-157	-144	180	424	3167	16443	-12694	-4960	-1339	111	-856	-321	-12.2
9	1919	-166	-36.1	72.1	716	4424	-1627	-2529	-706	-238	-97	-39.7	-112.7	-28.3
10	1920	-78.7	52.5	91	57.6	2975	8361	-3876	-4659	-1526	-561	-351	-283	16.9
11	1921	-89.2	29	104.2	137	5052	8133	-9218	-2694	-766	-540	-200	-44	0.3
12	1922	-118	-100	25	323	3037	6635	-5918	-2160	-943	-551	-216	-165	-12.6
13	1923	-52.3	-35.5	-5.8	179.6	3502	4965	-3424	-3023	-1107	-409	-314	-165.3	9.2
14	1924	4.3	-6.1	-18.4	98.5	4130	1546	-2363	-2069	-671	-70	-297	-182	8.5
15	1925	-120	137	46	973	7232	4062	-3581	-5752	-1486	-604	-407	-304	16.3
16	1926	-184	-10	1	1462	4991	-750	-3129	-1591	-554	-174	-216	-88	-20.2
17	1927	-67.1	22.1	70	486	3285	12420	-7950	-5104	-1504	-609	-45	-655	29.1
18	1928	-303	-100	153	331	12106	-2050	-2930	-4713	-1773	-547	-378	-217.7	-35.1
19	1929	-107.8	-73.5	142.6	133.4	2782	6512	-4467	-3342	-929	-347	-279	-41.5	-1.4
20	1930	-183.4	-23.8	80.9	1124.8	3439	2973	-3683	-1580	-1281	131	-753	-190	4.5
21	1931	-118.7	-28.5	-18.3	316.5	3284	2486	-4341	-1058	-455	-151.8	-83.2	-177.5	-26.8
22	1932	-27.9	-5.1	106.9	251.6	4598	5613	-3952	-4302	-1310	-478	-182.3	-256.8	4.6
23	1933	-53.7	-30.6	170.4	385	1872	8798	-6961	-2139	-1270	-450	-50	-100	14.3
24	1934	-95	8.9	30	1756.1	3405	-1760	-2144	-707	-378	-66.6	-80.9	-103.7	-11.3
25	1935	-128.8	-116	-24.4	462.9	3222.5	6999	-4971	-3493	-1079	-517	-254.7	-153.4	-4.4
26	1936	-69.6	5.1	76.5	1200.1	6767	481	-5073	-1811	-901	-432.3	-245	-158.9	-13.4
27	1937	-102.4	-37.8	0.9	164.4	4971.1	2103	-3210	-2311	-785	-236	-262.3	-126.8	14.0
28	1938	-135.5	-33.4	185.7	614.3	3949	8541	-6226	-4381	-1360	-272	-372	-261.7	18.9
29	1939	-62.4	-64	119.2	1083.9	6105	-342	-2904	-2501	-929	-269	-363	-122.9	-20.8
30	1940	-109	-39.9	181.3	421.5	5819	1683	-4973	-1935	-485	-251	-203.2	-127.5	-1.6
31	1941	-8.8	19.8	32.7	206.5	3977.5	1519	-2837	-1311	-5	-191	-503	-284	51.3
32	1942	-226	-112.5	-94.3	1086.8	2339	5756	-3289	-4039	-1180	-442	-149.8	-123.8	-39.7
33	1943	-67.6	-58.9	221.7	1729.4	3451	9398	-2960	-7728	-2253	-901	-365	-292	14.6
34	1944	-139	-5.9	-22.1	161	3401	4002	-2775	-3075	-911	-375	-229	-220.8	-15.7
35	1945	-149.9	-27.5	100.9	123.8	3025.5	6017	-1278	-4745	-1515	-486	-431	-360	22.9
36	1946	-33	-95.6	83.6	1949	3527	3249	-4888	-2561	-735	-195	-269	-176.2	-12.0
37	1947	-238.4	-32.8	188.2	320.2	6848	3612	-3837	-3831	-1735	-502	-454	-220	9.9
38	1948	-129.4	-22.4	36.1	307.7	6092	6585	-7981	-2875	-1219	-474	-304	-256	-20.0
39	1949	-220.7	-19.4	140	1235.1	6301	2683	-5293	-2912	-1029	-353	-280	-261.8	-2.5
40	1950	-253.3	53.1	-16	596	2538	9081	-2400	-6041	-1893	-362	-411	-337	46.2
41	1951	-341.5	-9.5	-7	593	6413	3056	-2104	-3984	-2294	-767	-491	-401.2	-28.1
42	1952	-187	1.2	116.7	1493.3	6929	3508	-6635	-3017	-1413	-585	-365.2	-142.8	-24.7
43	1953	59.9	-17.6	54	266.7	2106	8239	-4500	-3954	-1252	-496	-232	-187.3	7.2
44	1954	-146.5	-25.1	16.8	602.1	6416	2888	-2176	-4883	-1553	-644	-287.6	-161.1	3.9
45	1955	-111.9	11	64	217.6	3089	5845	-4267	-2829	-1225	-416	-251.6	-30.5	8.0
46	1956	79.1	-16.8	89.8	718	6996	7793	-9048	-4087	-1511	-604	-236	-256.8	-7.0
47	1957	-179.9	98.6	198.4	46.7	5110	6859	-5911	-3804	-1257	-552	-394	-237.9	-1.9
83	1993	-145.3	-64.1	102.4	658	9456	790	-4813	-3015	-1726	-565	-426	-155	8.1
84	1994	-86.7	-79.1	163.2	1442.6	6137	-2139	-3375	-1327	-557	-70	-172.8	-92.6	-13.0
85	1995	-120.7	92.6	223.8	379.7	4133	7688	-4424	-4711	-1789	-583	-327	-87	39.6
86	1996	-94	-53	40	831	6071	13106	-11575	-5587	-1793	-621	-162	-299	-11.3
87	1997	278	-21	36	556	9738	10870	-12716	-5000	-1914	-752	-500	-320	21.3
88	1998	-93	-39	-2	554	6190	2292	-2243	-4616	-1281	-555	-213	-222	-19.0
89	1999	9	-20	79	473	5957	7322	-6320	-4656	-1612	-858	-328	-174.5	-10.7
90	2000	-28.6	12.9	21.9	1288.3	6192	1732	-6121	-2063	-697	-126	-294.3	-184	-22.2
91	2001	-36.7	-3.3	178.6	565.7	5403	-1528	-2466	-1266	-497	-196.3	-46.1	-185.9	-6.5
92	2002	-6.3	36.1	96	564.5	4946	5067	-6427	-2605	-755	-376	-229	-115.1	16.3
93	2003	-54.1	15.7	57.1	881.4	5427	3700	-6195	-2411	-835	-339	-213.3	-94.4	-5.0
94	2004	-25.6	6.4	132.5	1202.4	3010	2209	-3030	-1951	-347	-293	-372	-286	21.3
06	2016	-64.5	71.3	37.4	1604.6	4020	645	-3818	-1666	-528	445	203	-567	31.9
07	2017	-269.5	50.2	769.3	1067	7017	5787	-7795	-4222	-1205	-439	-271	-305	15.3
08	2018	-170	-107	59	1014	12741	1500	-9194	-3983	-1409	-461	-230	-180.6	-35.1
09	2019	-158.1	-5.3	224.8	1014.2	4218	6254	-5762	-3812	-1021	-256	-401	-182	9.5
10	2020	-118.7	-44.3	106.3	604.7	7275	4171	-6896	-3239	-1196	-401	-203	-250.1	-15.9
11	2021	-106	30.6	119.9	598.6	4973	1176	-4673	-1177	-638	-144	-42.9	-274	-13.1
12	2022	-171.9	-17.7	254.6	290.9	3324	11282	-8871	-3759	-1228	-385	-327	-244.9	12.2
13	2023	19.8	-129.1	-40.6	695.8	8199	1030	-5459	-2449	-813	-184	-188	-385	24.7
14	2024	-156.1	-20	103.1	1002	2977	4972	-5724	-2088	-618	-406	-265.3	-179	-33.5

Monthly Hydrograph, visualization of changes of monthly discharge: Minimum, Mean & Maximum for 114 years

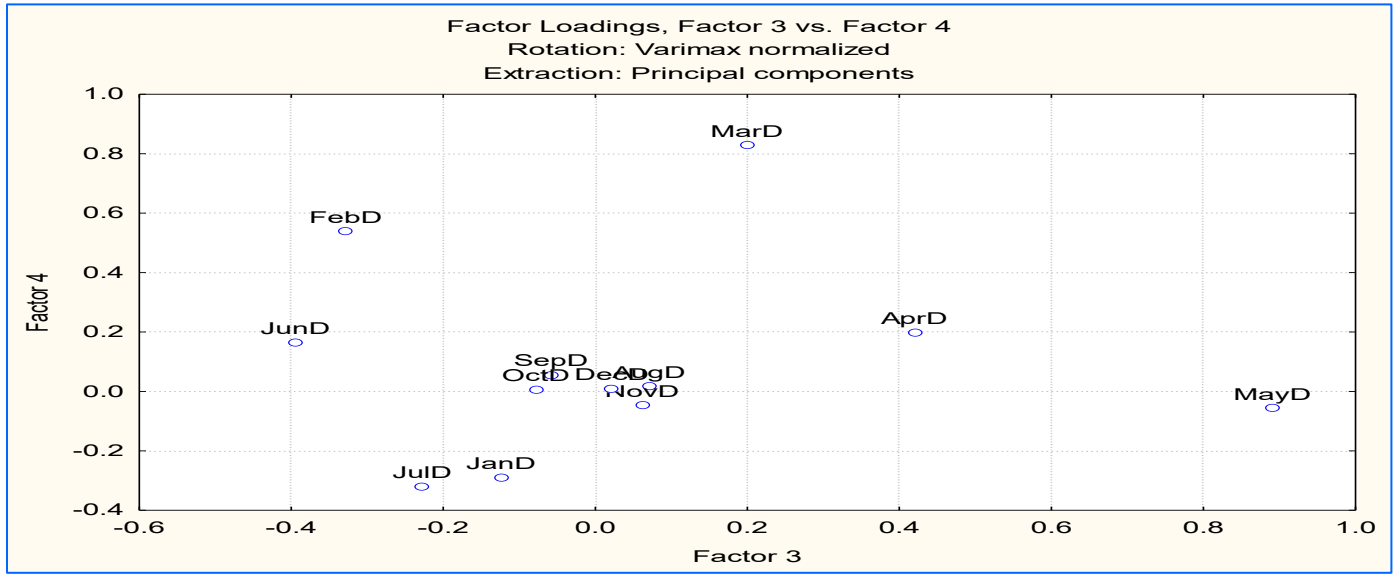
USGS 06191500 Yellowstone River at Corwin Springs MT. Eigenvalues, Factor Loadings



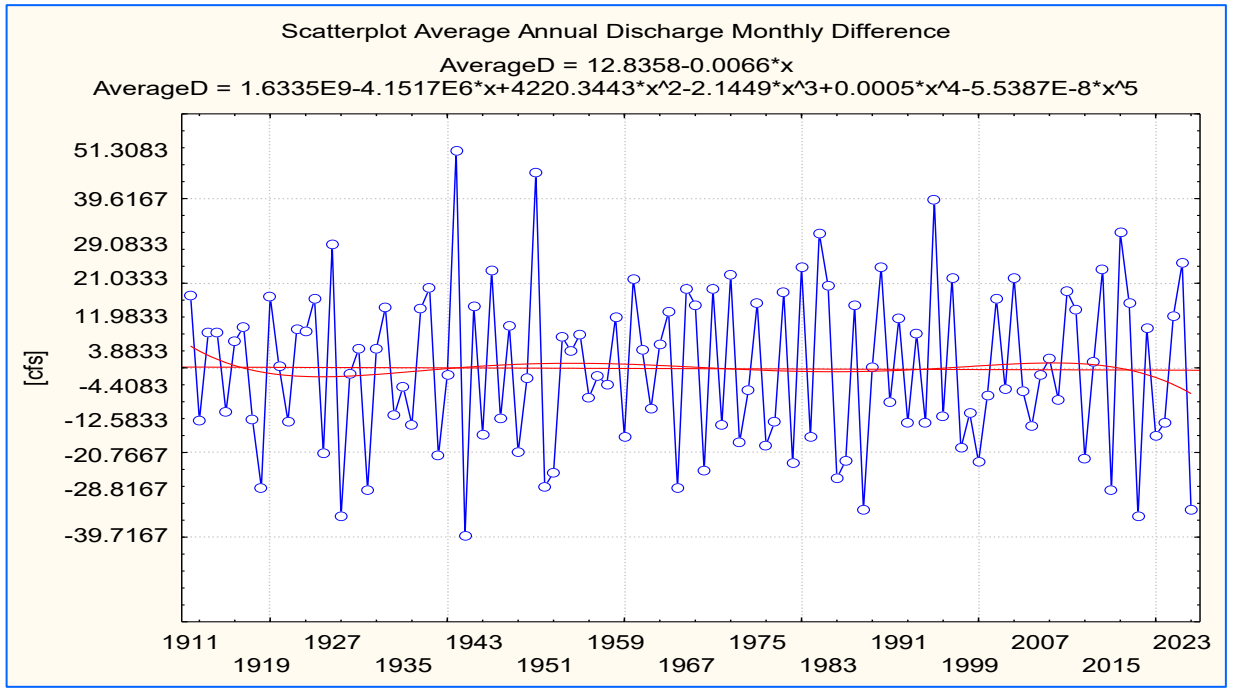
Value	Eigenvalues Extraction: Principal components			
	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.65	30.40	3.65	30.40
2	1.42	11.84	5.07	42.24
3	1.36	11.31	6.43	53.54
4	1.17	9.79	7.60	63.33



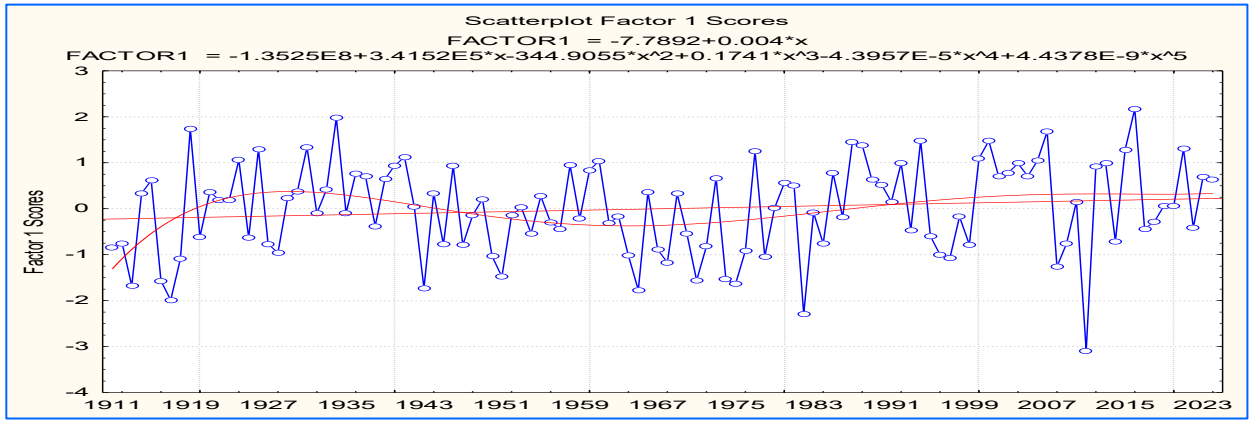
Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000)			
	Factor 1	Factor 2	Factor 3	Factor 4
JanD	0.17	*0.71*	-0.12	-0.29
FebD	0.13	0.07	-0.33	0.54
MarD	-0.11	-0.05	0.20	*0.83*
AprD	0.35	-0.13	0.42	0.20
MayD	-0.03	0.12	*0.89*	-0.06
JunD	*-0.75*	0.41	-0.39	0.17
JulD	0.12	*-0.80*	-0.23	-0.32
AugD	*0.92*	0.04	0.07	0.02
SepD	*0.93*	0.05	-0.06	0.06
OctD	*0.77*	0.04	-0.08	0.01
NovD	0.60	-0.04	0.06	-0.05
DecD	0.36	-0.32	0.02	0.01
Expl.Var	3.55	1.46	1.36	1.24
Prp.Totl	0.30	0.12	0.11	0.10

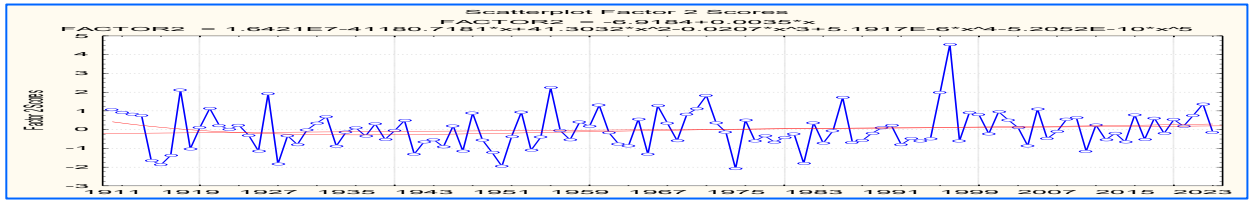


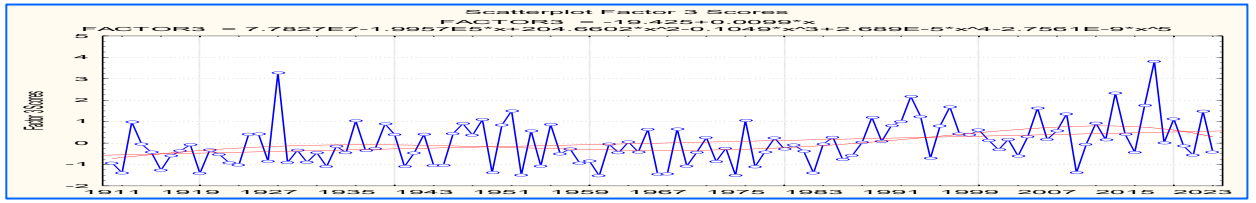
USGS 06191500 Yellowstone River at Corwin Springs MT. Factor Loadings & Scores

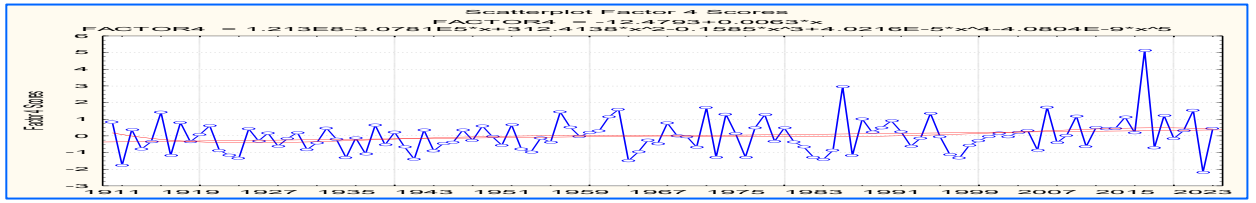


Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	3.65	30.40	3.65	30.40
2	1.42	11.84	5.07	42.24
3	1.36	11.31	6.43	53.54
4	1.17	9.79	7.60	63.33



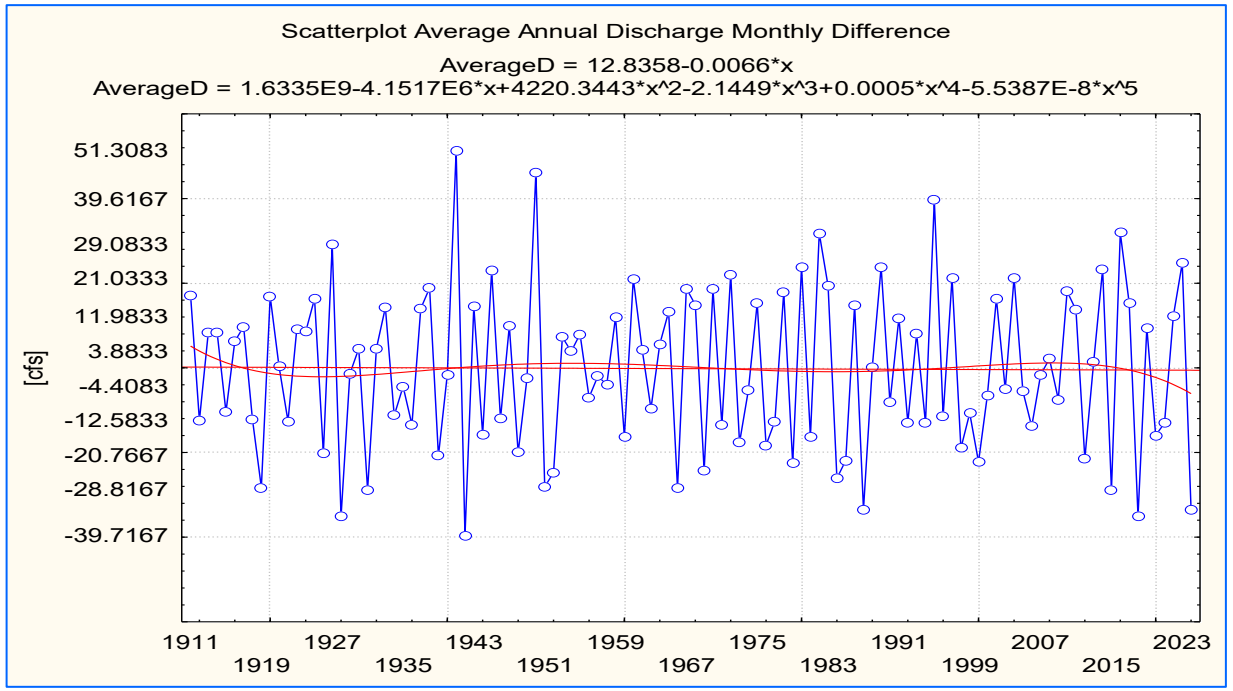


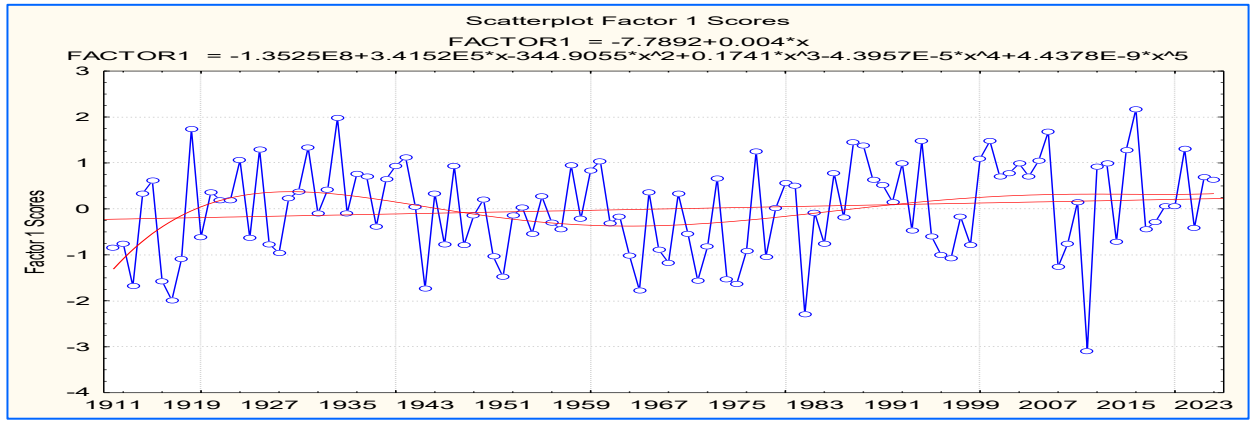


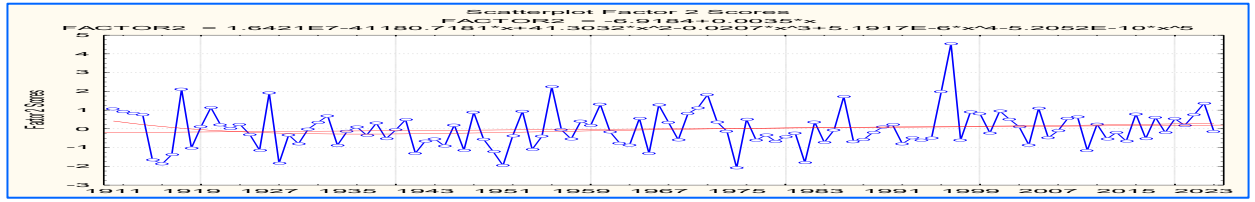


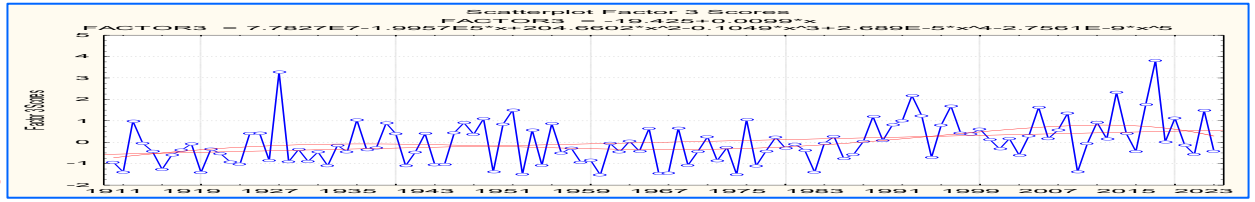
Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
JanD	0.17	*0.71*	-0.12	-0.29
FebD	0.13	0.07	-0.33	0.54
MarD	-0.11	-0.05	0.20	*0.83*
AprD	0.35	-0.13	0.42	0.20
MayD	-0.03	0.12	*0.89*	-0.06
JunD	*-0.75*	0.41	-0.39	0.17
JulD	0.12	*-0.80*	-0.23	-0.32
AugD	*0.92*	0.04	0.07	0.02
SepD	*0.93*	0.05	-0.06	0.06
OctD	*0.77*	0.04	-0.08	0.01
NovD	0.60	-0.04	0.06	-0.05
DecD	0.36	-0.32	0.02	0.01
Expl.Var	3.55	1.46	1.36	1.24
Prp.Totl	0.30	0.12	0.11	0.10

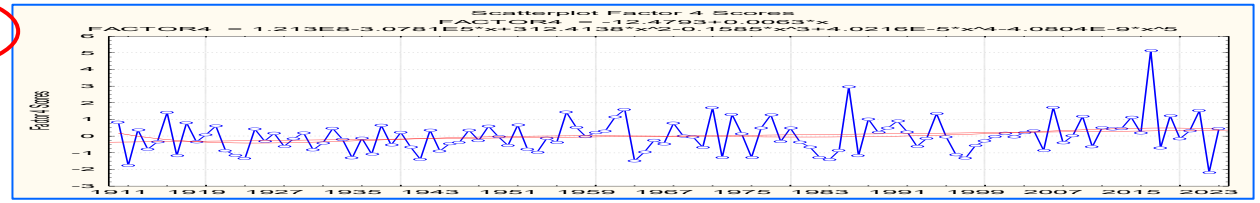
USGS 06191500 Yellowstone River at Corwin Springs MT. Factor Loadings, Scores & Uncertainty











Variable	Factor Loadings (Varimax normalized) Extraction: Principal components (Marked loadings are >.700000) Factor 1	Factor 2	Factor 3	Factor 4
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JulD	0.12	*-0.80*	-0.23	-0.32
AugD	*0.92*	0.04	0.07	0.02
SepD	*0.93*	0.05	-0.06	0.06
OctD	*0.77*	0.04	-0.08	0.01
NovD	0.60	-0.04	0.06	-0.05
DecD	0.36	-0.32	0.02	0.01
Expl.Var	3.55	1.46	1.36	1.24
Prp.Totl	0.30	0.12	0.11	0.10

Value	Eigenvalues Extraction: Principal components Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
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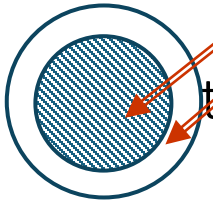
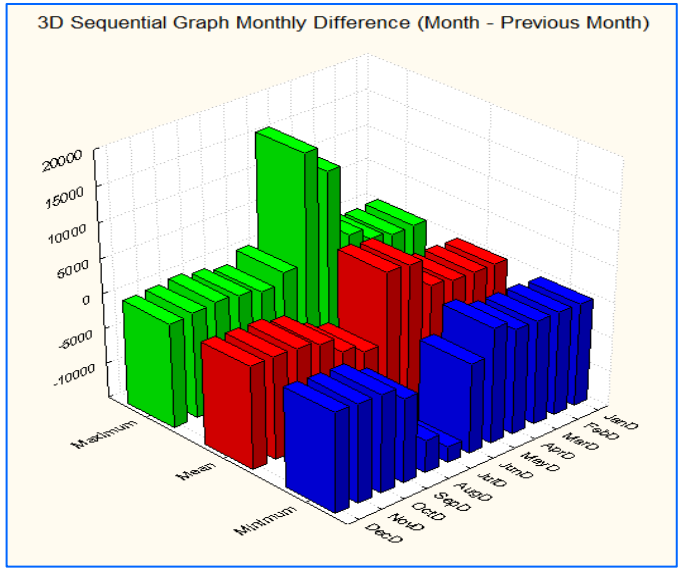
2	1.42	11.84	5.07	42.24
3	1.36	11.31	6.43	53.54
4	1.17	9.79	7.60	63.33

Uncertainty is 36.67%, model does not reflect this part of the variability of the data



USGS 06191500 Yellowstone River at Corwin Springs MT. Transposed Matrix (12Dx114)



Monthly Hydrograph, visualization of changes of monthly discharge: Minimum, Mean & Maximum for the 114 years.

Calculated Matrix $Q_{12Dx114}$ (1911-2024) of monthly discharge changes (parts)

Descriptive statistical characteristics

Variable	Valid N	Mean	Median	Minimum	Maximum	Variance	Std.Dev.
Y1911	12	17.1	-112.8	-8999	13908	28778006	5364.5
Y1912	12	-12.4	-214.0	-4506	9260	11509721	3392.6
Y1913	12	8.3	-170.0	-8864	10611	22453128	4738.5
Y1914	12	8.3	-75.0	-4615	5178	6778932	2603.6
Y1915	12	-10.4	-125.0	-2091	2039	1514619	1230.7
Y1916	12	6.3	-189.2	-7703	9330	14449109	3801.2
Y1917	12	9.8	-140.4	-8445	9522	16024330	4003.0
Y1918	12	-12.2	-150.5	-12694	16443	42339834	6529.9
Y1919	12	-28.3	-104.8	-2529	4424	2702784	1644.0
Y1920	12	16.9	-180.9	-4659	8361	10759130	3280.1
Y1921	12	0.3	-66.6	-9218	8133	16757043	4093.5
Y1922	12	-12.6	-141.5	-5918	6635	8575330	2928.4
Y1923	12	9.2	-108.8	-3424	4965	5393768	2322.4
Y1924	12	8.5	-44.2	-2363	4130	2717931	1648.6
Y1925	12	16.3	-212.0	-5752	7232	10774602	3282.5
Y1926	12	-20.2	-179.0	-3129	4991	3668420	1915.3
Y1927	12	29.1	-56.1	-7950	12420	23418257	4839.2
Y1928	12	-35.1	-340.5	-4713	12106	16855271	4105.5
Y1929	12	-1.4	-90.6	-4467	6512	7489715	2736.7
Y1930	12	4.5	-103.6	-3683	3439	3663022	1913.9
Y1931	12	-28.8	-101.0	-4341	3284	3391132	1841.5
Y1932	12	4.6	-105.1	-4302	5613	8081084	2842.7
Y1933	12	14.3	-51.8	-6961	8798	12358779	3515.5
Y1934	12	-11.3	-87.9	-2144	3405	2095018	1447.4
Y1935	12	-4.4	-141.1	-4971	6999	8913365	2985.5
Y1936	12	-13.4	-114.3	-5073	6767	7051951	2655.6
Y1937	12	14.0	-114.6	-3210	4971	4142599	2035.4
Y1938	12	18.9	-203.8	-6228	8541	13553891	3681.6
Y1939	12	-20.8	-195.9	-2904	6105	4940940	2222.8
Y1940	12	-1.6	-118.3	-4973	5819	5977087	2444.8
Y1941	12	51.3	-6.9	-2837	3978	2570720	1603.3
Y1942	12	-39.7	-136.8	-4039	5756	6235988	2497.2
Y1943	12	14.5	-179.8	-7728	9398	16169742	4021.2
Y1944	12	-15.7	-179.9	-3075	4002	4168508	2041.7
Y1945	12	22.9	-255.0	-4745	6017	6581406	2565.4
Y1946	12	-12.0	-135.9	-4888	3527	5267522	2295.1
Y1947	12	9.9	-229.1	-3837	6848	8459292	2908.5
Y1948	12	-20.0	-192.7	-7981	6585	14037608	3746.7
Y1949	12	-2.5	-250.4	-5293	6301	7848366	2801.5
Y1950	12	46.2	-295.2	-6041	9081	12323043	3510.4
Y1951	12	28.1	-371.4	-3984	6413	7043316	2653.9
Y1952	12	-24.7	-164.9	-6535	6929	10746334	3278.1
Y1953	12	7.2	-102.4	-4500	8239	10016383	3164.9
Y1954	12	3.9	-153.8	-4883	6416	7400381	2720.4
Y1955	12	8.0	-71.2	-4267	5845	6519792	2553.4
Y1956	12	-7.0	-126.4	-9048	7793	19231260	4385.3

Variable	Valid N	Mean	Median	Minimum	Maximum	Variance	Std.Dev.
Y1957	12	-1.9	-208.9	-5911	6859	11340772	3367.6
Y1958	12	-4.0	-106.6	-4322	6043	5437846	2331.9
Y1959	12	12.9	-192.0	-6939	10230	15852464	3981.5
Y1960	12	-16.2	-120.4	-5134	4748	5387895	2321.2
Y1961	12	21.0	-66.2	-6089	5365	7426788	2725.2
Y1962	12	4.4	-190.7	-5573	6078	10523388	3244.0
Y1963	12	-9.6	-247.9	-6512	6385	11503479	3391.7
Y1964	12	5.6	-189.7	-5502	7801	11725130	3424.2
Y1965	12	13.3	-252.6	-6327	11214	19346913	4398.5
Y1966	12	-28.4	-214.0	-3721	5233	4449620	2109.4
Y1967	12	18.8	-134.0	-6806	10540	18821714	4338.4
Y1968	12	14.9	-306.9	-5701	9587	13474556	3670.8
Y1969	12	-24.3	-173.8	-3782	7385	7600175	2756.8
Y1970	12	18.5	-119.0	-6365	9611	16345987	4043.0
Y1971	12	-13.4	-244.9	-6393	10300	19673318	4435.5
Y1972	12	22.0	-195.8	-9279	10575	21864951	4676.0
Y1973	12	-17.7	-88.5	-4707	4505	5842934	2417.2
Y1974	12	-5.3	-241.9	-8990	14350	32175837	5672.4
Y1975	12	15.2	-120.4	-8099	9035	14100121	3755.0
Y1976	12	-18.3	-163.4	-5690	7412	13160695	3627.8
Y1977	12	-12.6	-156.3	-4065	2953	2890559	1700.2
Y1978	12	17.8	-166.5	-5275	7906	11046410	3323.6
Y1979	12	-22.6	-187.8	-5103	5066	7177341	2679.1
Y1980	12	23.9	-147.6	-3144	5162	4191381	2047.3
Y1981	12	-16.2	-142.4	-5355	4638	7097267	2664.1
Y1982	12	31.8	-97.4	-7572	7832	13147063	3625.9
Y1983	12	19.3	-78.5	-3908	6684	6833912	2614.2
Y1984	12	-26.1	-244.5	-4073	5662	7624044	2761.2
Y1985	12	-21.9	-138.8	-3695	5628	4456842	2111.1
Y1986	12	14.9	-166.7	-8968	9817	18720576	4326.7
Y1987	12	-33.5	-111.1	-1763	3409	1675997	1294.6
Y1988	12	0.2	-76.8	-4362	4542	3944162	1986.0
Y1989	12	23.8	-164.8	-4958	5565	6458724	2541.4
Y1990	12	-8.2	-151.6	-4626	5199	6254482	2500.9
Y1991	12	11.8	-171.3	-7279	6608	11871883	3445.6
Y1992	12	-12.9	-256.0	-2411	5721	4600030	2144.8
Y1993	12	8.1	-150.1	-4813	9456	11476811	3388.0
Y1994	12	-13.0	-89.7	-3375	6137	5260245	2293.5
Y1995	12	39.6	-103.9	-4711	7688	11073255	3327.7
Y1996	12	-11.3	-128.0	-11575	13106	34385261	5863.9
Y1997	12	21.3	-170.5	-12716	10870	36786022	6065.1
Y1998	12	-19.0	-153.0	-4616	6199	6565479	2563.1
Y1999	12	-10.7	-97.3	-6320	7322	14038190	3746.8
Y2000	12	-22.2	-77.3	-6121	6192	7758246	2785.4
Y2001	12	-6.5	-116.0	-2466	5403	3626009	1904.2
Y2002	12	16.3	-60.7	-6427	5067	9030260	3005.0

Variable	Valid N	Mean	Median	Minimum	Maximum	Variance	Std.Dev.
Y2003	12	-5.1	-74.3	-6195	5427	8089348	2844.2
Y2004	12	21.3	-155.8	-3030	3010	2619281	1618.4
Y2005	12	-5.5	-173.3	-4186	5488	5165749	2272.8
Y2006	12	-13.8	-65.5	-6105	8321	10350648	3217.2
Y2007	12	-1.6	-151.4	-2748	5451	4091383	2022.7
Y2008	12	2.1	-192.1	-6871	8235	16167541	4020.9
Y2009	12	-7.4	-201.0	-6388	7304	13686118	3699.5
Y2010	12	18.1	-68.8	-6861	9842	14982136	3870.7
Y2011	12	13.7	-236.6	-10360	11899	25333109	5033.2
Y2012	12	-21.4	-72.9	-6169	4668	7417798	2723.6
Y2013	12	1.4	-108.4	-3714	6256	5221085	2285.0
Y2014	12	23.3	-218.6	-6237	8562	13029453	3609.6
Y2015	12	-28.8	-151.5	-4129	4416	3759004	1938.8
Y2016	12	31.9	54.3	-3818	4020	3394708	1842.5
Y2017	12	15.3	-270.3	-7795	7017	14993529	3872.1
Y2018	12	-35.0	-175.3	-9194	12741	24392510	4938.9
Y2019	12	9.5	-170.1	-5762	6254	9731001	3119.5
Y2020	12	-15.9	-160.9	-6896	7275	11859432	3443.8
Y2021	12	-13.1	-74.4	-4673	4973	4565771	2136.8
Y2022	12	12.3	-208.4	-8871	11282	21196167	4603.9
Y2023	12	24.7	-156.6	-5459	8199	9586971	3096.3
Y2024	12	-33.5	-167.6	-5724	4972	6580212	2565.2

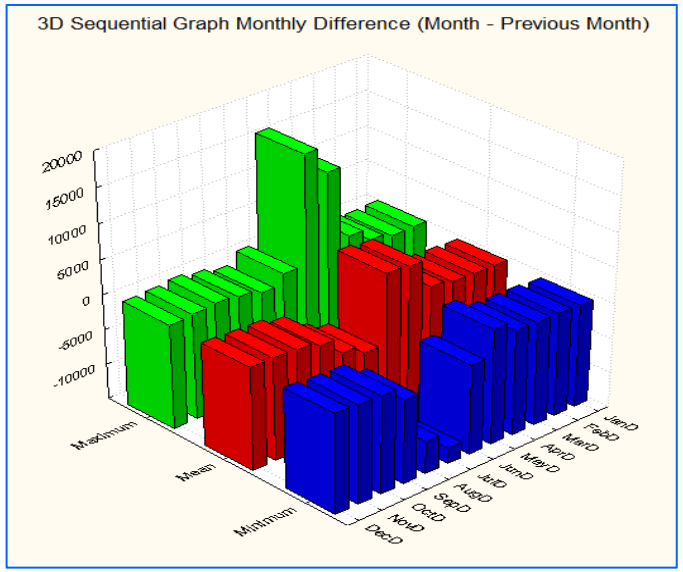
	1 Year	2 Y1911	3 Y1912	4 Y1913	5 Y1914	6 Y1915	7 Y1916	8 Y1917
1	JanD	-94.5	14	-101	-50	-150	-212.4	-100
2	FebD	0	-91	-50	-100	-100	64.3	-134.8
3	MarD	149	-37	170	100	100	256.1	121.8
4	AprD	352	83	710	380	1511	506	293
5	MayD	2971	1871	5709	5178	1655	2391	2998
6	JunD	13908	9260	10611	3902	2039	9330	9522
7	JulD	-8999	-4506	-8864	-4615	-1364	-1230	-700
8	AugD	-5489	-3666	-3816	-3152	-2091	-7703	-8445
9	SepD	-1629	-1722	-2628	-721	-972	-2084	-1909
10	OctD	-514	-549	-996	205	-256	-585	-855
11	NovD	-318	-469	-407	-659	-440	-492	-528
12	DecD	-131	-337	-239	-368	-57	-166	-146
13	AverageD	17.125	-12.4167	8.25	8.333333	-10.4167	6.25	9.75

	9 Y1918	10 Y1919	11 Y1920	12 Y1921	13 Y1922	14 Y1923	15 Y1924	16 Y1925
1	-157	-166	-78.7	-89.2	-118	-52.3	4.3	-120
2	-144	-36.1	52.5	29	-100	-35.5	-6.1	137
3	180	72.1	91	104.2	25	-5.8	-18.4	46
4	424	716	57.6	137	323	179.6	98.5	973
5	3167	4424	2975	5052	3037	3502	4130	7232
6	16443	-1627	8361	8133	6635	4965	1546	4062
7	-12694	-2529	-3876	-9218	-5918	-3424	-2363	-3581
8	-4960	-706	-4659	-2594	-2160	-3023	-2069	-5752
9	-1339	-238	-1526	-766	-943	-1107	-671	-1486
10	111	-97	-561	-540	-551	-409	-70	-604
11	-856	-39.7	-351	-200	-216	-314	-297	-407
12	-321	-112.7	-283	-44	-165	-165.3	-182	-304
13	-12.1667	-28.2833	16.86667	0.333333	-12.5833	9.225	8.525	16.33333

	17 Y1926	18 Y1927	19 Y1928	20 Y1929	21 Y1930	22 Y1931	23 Y1932	24 Y1933
1	-184	-67.1	-303	-107.8	-183.4	-118.7	-27.9	-53.7
2	-10	22.1	-100	-73.5	-23.8	-28.5	-5.1	-30.6
3	1	70	153	142.6	80.9	-18.3	106.9	170.4
4	1462	486	331	133.4	1124.8	316.5	251.6	385
5	4991	3285	12106	2782	3439	3284	4598	1872
6	-750	12420	-2050	6512	2973	2486	5613	8798
7	-3129	-7950	-2930	-4467	-3683	-4341	-3952	-6961
8	-1591	-5104	-4713	-3342	-1580	-1058	-4302	-2139
9	-554	-1504	-1773	-929	-1281	-455	-1310	-1270
10	-174	-609	-547	-347	131	-151.8	-478	-450
11	-216	-45	-378	-279	-753	-83.2	-182.3	-50
12	-88	-655	-217.7	-41.5	-190	-177.5	-256.8	-100
13	-20.1667	29.08333	-35.1417	-1.4	4.541667	-28.7917	4.616667	14.25833

	105 Y2014	106 Y2015	107 Y2016	108 Y2017	109 Y2018	110 Y2019	111 Y2020	112 Y2021
1	-174.1	-154.4	-64.5	-269.5	-170	-158.1	-118.7	-106
2	-117.5	16.6	71.3	50.2	-107	-5.3	-44.3	30.6
3	267.1	201.8	37.4	769.3	59	224.8	106.3	119.9
4	920.3	1548	1604.6	1067	1014	1014.2	604.7	598.6
5	8562	4416	4020	7017	12741	4218	7275	4973
6	3550	249	645	5787	1500	6254	4171	1176
7	-6237	-4129	-3818	-7795	-9194	-5762	-6896	-4673
8	-3935	-1345	-1666	-4222	-3983	-3812	-3239	-1177
9	-1160	-618	-528	-1205	-1409	-1021	-1196	-638
10	-658	-184	445	-439	-461	-256	-401	-144
11	-475	-148.6	203	-271	-230	-401	-203	-42.9
12	-263	-198.2	-567	-305	-180.6	-182	-250.1	-274
13	23.31667	-28.8167	31.9	15.33333	-35.05	9.466667	-15.925	-13.0667

USGS 06191500 Yellowstone River at Corwin Springs MT. Transposed Matrix (12Dx114)



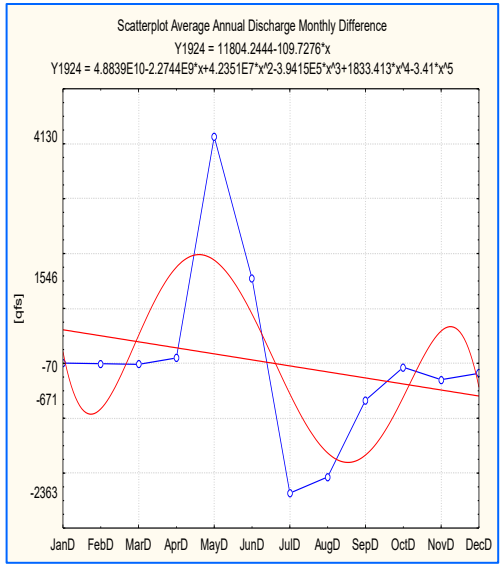
Monthly Hydrograph, visualization of changes of monthly discharge: Minimum, Mean & Maximum for the 114 years.

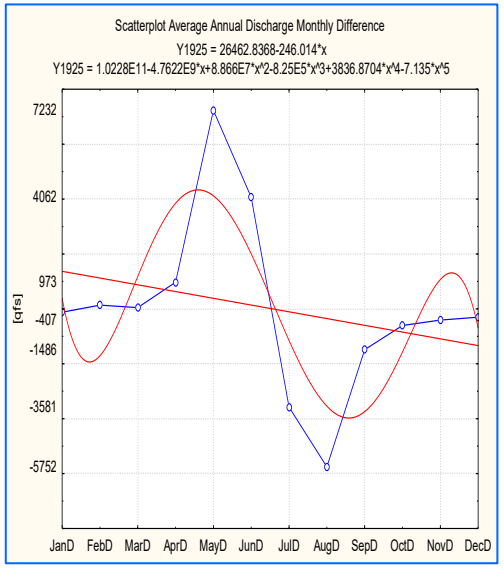
Calculated Matrix $Q_{12Dx114}$ (1911-2024) of monthly discharge changes (parts)

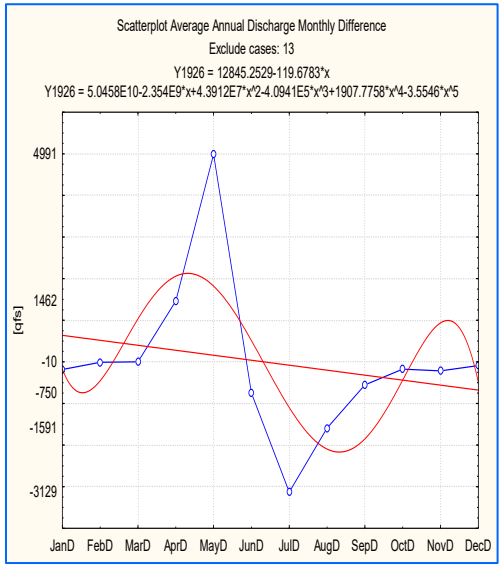
Discharge monthly changes for years: 1924, 1925, 1926,2023 & 2024

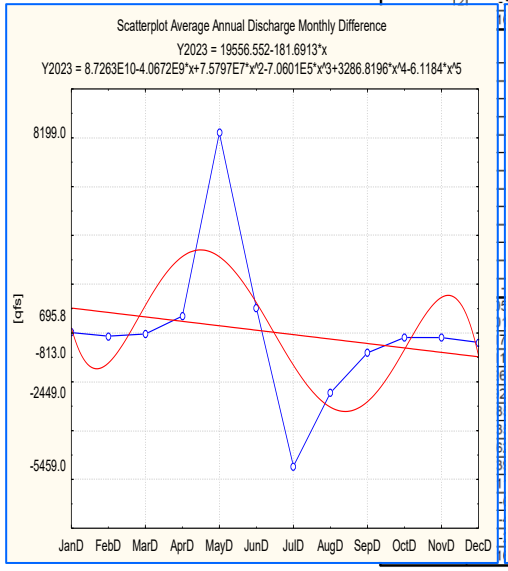
	1 Year	2 Y1911	3 Y1912	4 Y1913	5 Y1914	6 Y1915	7 Y1916	8 Y1917
1	JanD	-94.5	14	-101	-50	-150	-212.4	-100
2	FebD	0	-91	-50	-100	-100	64.3	-134.8
3	MarD	149	-37	170	100	100	256.1	121.8
4	AprD	352	83	710	380	1511	506	293
5	MayD	2971	1871	5709	5178	1655	2391	2998
6	JunD	13908	9260	10611	3902	2039	9330	9522
7	JulD	-8999	-4506	-8864	-4615	-1364	-1230	-700
8	AugD	-5489	-3666	-3816	-3152	-2091	-7703	-8445
9	SepD	-1629	-1722	-2628	-721	-972	-2084	-1909
10	OctD	-514	-549	-996	205	-256	-585	-855
11	NovD	-318	-469	-407	-659	-440	-492	-528
12	DecD	-131	-337	-239	-368	-57	-166	-146
13	AverageD	17.125	-12.4167	8.25	8.333333	-10.4167	6.25	9.75

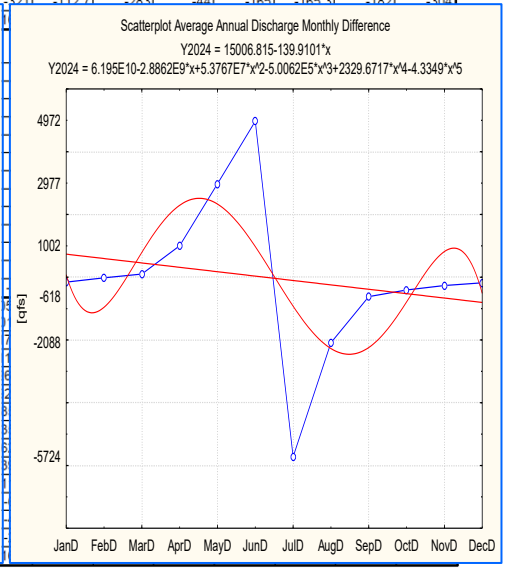
	9 Y1918	10 Y1919	11 Y1920	12 Y1921	13 Y1922	14 Y1923	15 Y1924	16 Y1925
1	-157	-166	-78.7	-89.2	-118	-52.3	4.3	-120
2	-144	-36.1	52.5	29	-100	-35.5	-6.1	137
3	180	72.1	91	104.2	25	-5.8	-18.4	46
4	424	716	57.6	137	323	179.6	98.5	973
5	3167	4424	2975	5052	3037	3502	4130	7232
6	16443	-1627	8361	8133	6635	4965	1546	4062
7	-12694	-2529	-3876	-9218	-5918	-3424	-2363	-3581
8	-4960	-706	-4659	-2594	-2160	-3023	-2069	-5752
9	-1339	-238	-1526	-766	-943	-1107	-671	-1486
10	111	-97	-561	-540	-551	-409	-70	-604
11	-856	-39.7	-351	-200	-216	-314	-297	-407
12	-321	-112.7	-283	-44	-165	-165.3	-182	-304











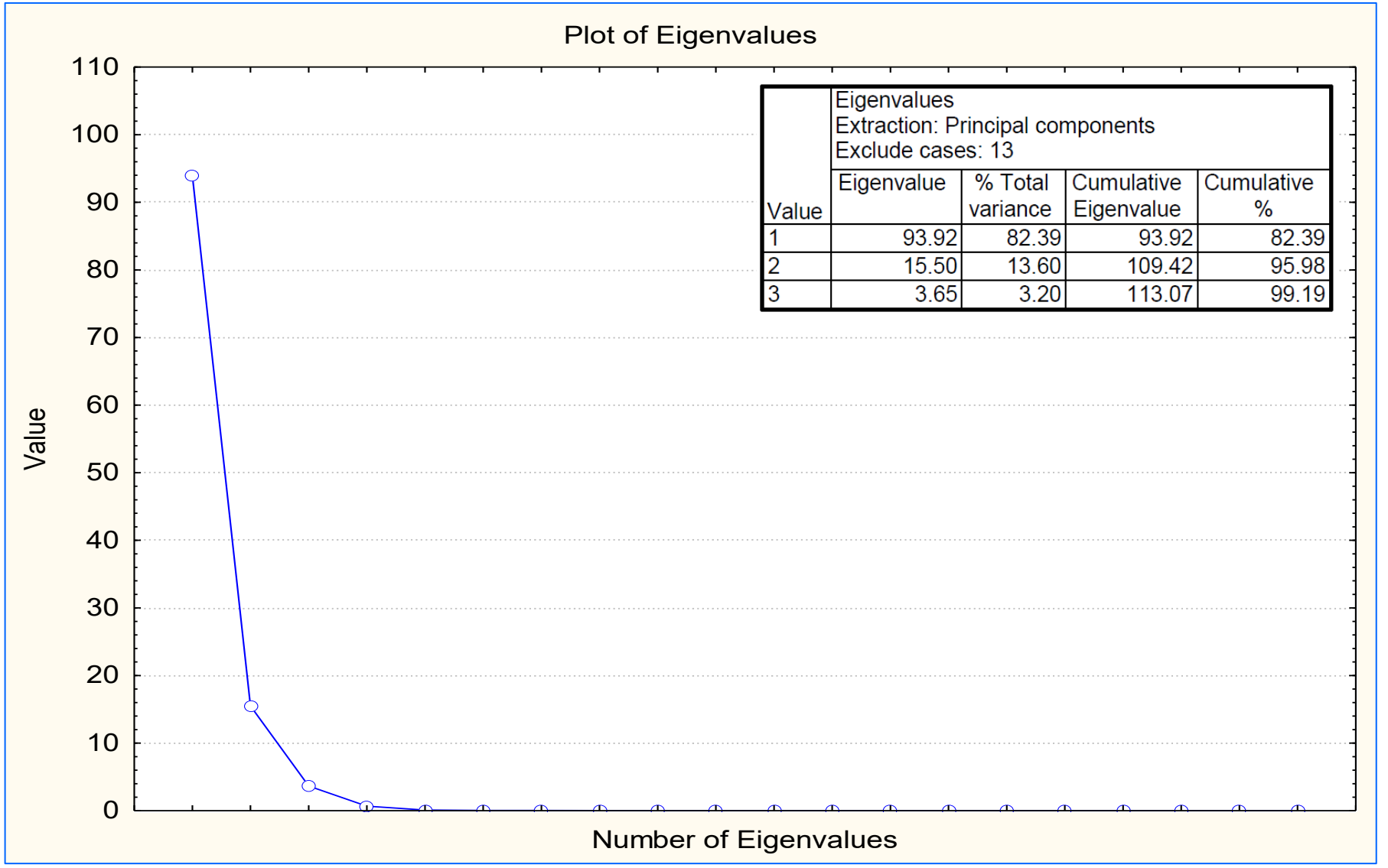
USGS 06191500 Yellowstone River at Corwin Springs MT. Transposed Matrix, correlation (p. 1 & p. 30) & Eigenvalues

Correlations
Marked correlations are significant at p < .05000
N=12 (Casewise deletion of missing data)
Exclude cases: 13

Variable	Y1911	Y1912	Y1913	Y1914	Y1915	Y1916	Y1917	Y1918	Y1919	Y1920	Y1921	Y1922
Y1911	1.00	0.99	0.97	0.85	0.82	0.87	0.84	0.99	0.18	0.97	0.94	0.97
Y1912	0.99	1.00	0.95	0.82	0.83	0.91	0.88	0.97	0.13	0.99	0.90	0.95
Y1913	0.97	0.95	1.00	0.93	0.86	0.81	0.78	0.96	0.40	0.95	0.98	0.99
Y1914	0.85	0.82	0.93	1.00	0.88	0.74	0.74	0.84	0.66	0.87	0.93	0.91
Y1915	0.82	0.83	0.86	0.88	1.00	0.85	0.84	0.78	0.46	0.88	0.80	0.83
Y1916	0.87	0.91	0.81	0.74	0.85	1.00	1.00	0.80	0.07	0.94	0.71	0.78
Y1917	0.84	0.88	0.78	0.74	0.84	1.00	1.00	0.77	0.09	0.93	0.68	0.75
Y1918	0.99	0.97	0.96	0.84	0.78	0.80	0.77	1.00	0.20	0.94	0.96	0.98
Y1919	0.18	0.13	0.40	0.66	0.46	0.07	0.09	0.20	1.00	0.22	0.47	0.37
Y1920	0.97	0.99	0.95	0.87	0.88	0.94	0.93	0.94	0.22	1.00	0.90	0.94
Y1921	0.94											0.99
Y1922	0.97											1.00
Y1923	0.94											0.96
Y1924	0.70											0.79
Y1925	0.75											0.80
Y1926	0.38											0.54
Y1927	1.00											0.98
Y1928	0.26											0.40
Y1929	0.99											0.98
Y1930	0.85											0.93
Y1931	0.83											0.93
Y1932	0.91											0.93
Y1933	0.99											0.98
Y1934	0.12											0.30
Y1935	0.98											0.99
Y1936	0.54											0.69
Y1937	0.73											0.82
Y1938	0.98											0.98
Y1939	0.42											0.56
Y1940	0.67											0.80
Y1941	0.70											0.82
Y1942	0.96											0.94
Y1943	0.91											0.85
Y1944	0.91											0.93
Y1945	0.88											0.83
Y1946	0.86											0.92
Y1947	0.75											0.83
Y1948	0.90											0.97
Y1949	0.74											0.84
Y1950	0.93											0.86
Y1951	0.68											0.75
Y1952	0.77											0.87
Y1953	0.99											0.96
Y1954	0.68											0.73
Y1955	0.97	0.97	0.99	0.94	0.88	0.86	0.84	0.96	0.38	0.98	0.97	0.99

Correlations
Marked correlations are significant at p < .05000
N=12 (Casewise deletion of missing data)
Exclude cases: 13

Variable	Y2019	Y2020	Y2021	Y2022	Y2023	Y2024
Y2001	0.51	0.76	0.87	0.29	0.93	0.49
Y2002	0.98	0.98	0.91	0.93	0.86	0.98
Y2003	0.95	0.99	0.96	0.87	0.92	0.96
Y2004	0.96	0.97	0.92	0.86	0.90	0.94
Y2005	0.88	0.98	0.98	0.74	0.99	0.86
Y2006	0.75	0.92	0.98	0.58	0.99	0.74
Y2007	0.45	0.71	0.84	0.22	0.89	0.43
Y2008	0.97	0.89	0.76	0.90	0.75	0.90
Y2009	0.97	0.99	0.92	0.88	0.90	0.94
Y2010	0.95	0.81	0.66	1.00	0.57	0.95
Y2011	0.83	0.67	0.47	0.79	0.48	0.73
Y2012	0.94	0.97	0.95	0.85	0.90	0.95
Y2013	0.74	0.92	0.96	0.56	0.99	0.71
Y2014	0.90	0.99	0.97	0.77	0.97	0.88
Y2015	0.77	0.92	0.98	0.62	0.97	0.78
Y2016	0.81	0.93	0.97	0.66	0.96	0.81
Y2017	0.98	0.99	0.93	0.90	0.90	0.96
Y2018	0.79	0.95	0.99	0.63	1.00	0.78
Y2019	1.00	0.94	0.83	0.96	0.79	0.98
Y2020	0.94	1.00	0.97	0.84	0.95	0.93
Y2021	0.83	0.97	1.00	0.71	0.98	0.85
Y2022	0.96	0.84	0.71	1.00	0.62	0.97
Y2023	0.79	0.95	0.98	0.62	1.00	0.77
Y2024	0.98	0.93	0.85	0.97	0.77	1.00



Eigenvalues
Extraction: Principal components
Exclude cases: 13

Value	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	93.92	82.39	93.92	82.39
2	15.50	13.60	109.42	95.98
3	3.65	3.20	113.07	99.19

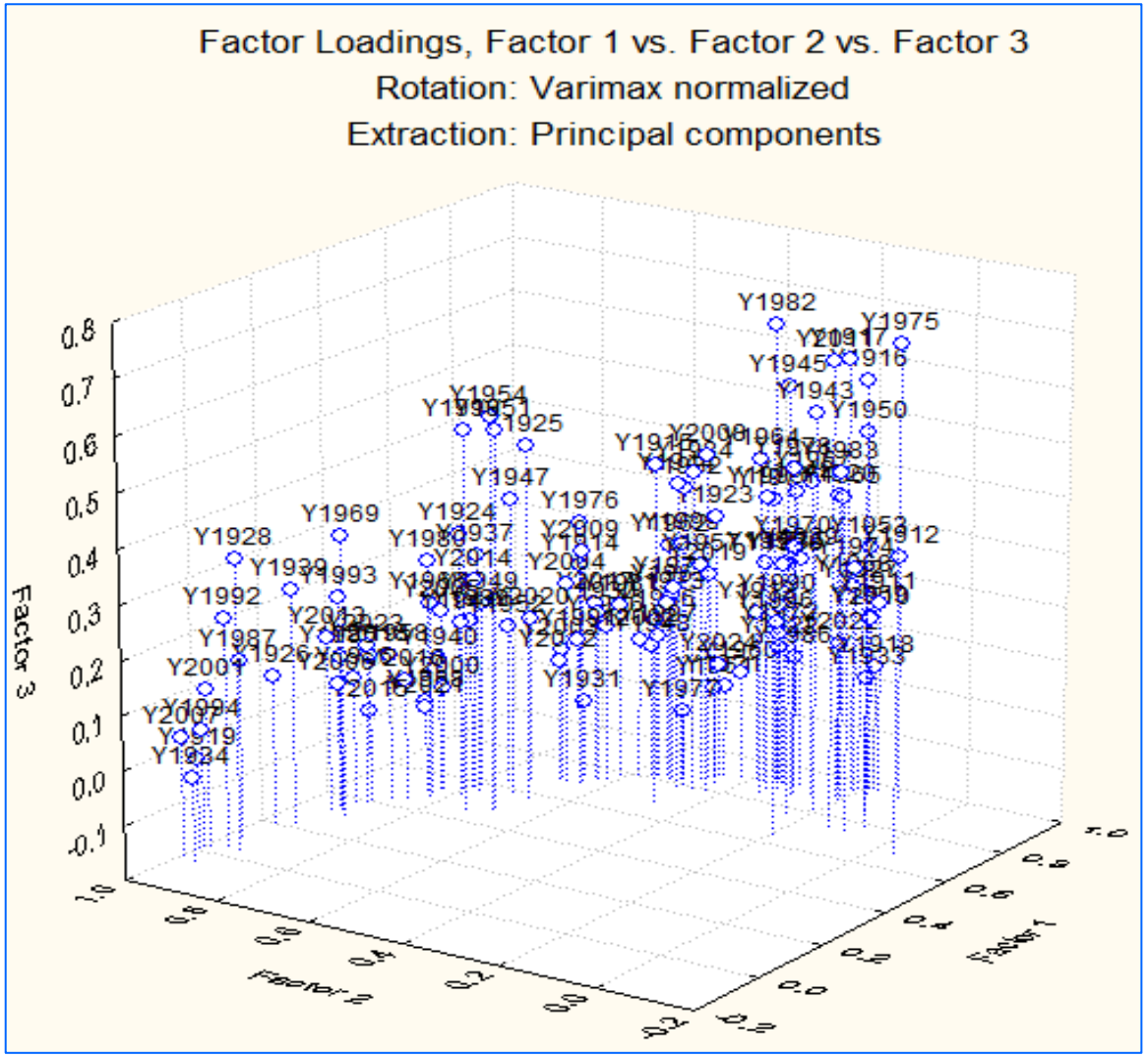
USGS 06191500 Yellowstone River at Corwin Springs MT. Transposed Matrix, Factor Loadings

Variable	Factor Loadings (Varim. norm.) Extraction: Principal compon. (Marked loadings are >.700000) Exclude cases: 13 Factor 1	Factor 2	Factor 3
Y1911	0.97	0.18	0.15
Y1912	0.96	0.12	0.25
Y1913	0.91	0.40	0.11
Y1914	0.72	0.66	0.21
Y1915	0.69	0.47	0.43
Y1916	0.78	0.08	0.62
Y1917	0.74	0.10	0.66
Y1918	0.98	0.19	0.03
Y1919	0.01	1.00	-0.04
Y1920	0.91	0.22	0.36
Y1921	0.89	0.45	-0.03
Y1922	0.93	0.36	0.05
Y1923	0.84	0.45	0.30
Y1924	0.53	0.79	0.29
Y1925	0.58	0.68	0.46
Y1926	0.20	0.97	0.07
Y1927	0.96	0.21	0.17
Y1928	0.05	0.93	0.34
Y1929	0.92	0.31	0.22
Y1930	0.74	0.63	0.11
Y1931	0.74	0.67	-0.06
Y1932	0.80	0.48	0.36
Y1933	0.98	0.20	0.00
Y1934	-0.05	0.97	-0.05
Y1935	0.92	0.34	0.21
Y1936	0.38	0.92	0.03
Y1937	0.57	0.78	0.24
Y1938	0.91	0.35	0.21
Y1939	0.23	0.95	0.23
Y1940	0.53	0.84	0.04
Y1941	0.56	0.81	0.11
Y1942	0.88	0.30	0.36
Y1943	0.81	0.21	0.53
Y1944	0.79	0.49	0.36
Y1945	0.77	0.24	0.59
Y1946	0.75	0.62	0.09
Y1947	0.59	0.72	0.34
Y1948	0.82	0.57	0.04
Y1949	0.59	0.79	0.14
Y1950	0.85	0.13	0.50
Y1951	0.50	0.70	0.50
Y1952	0.64	0.76	0.09
Y1953	0.95	0.18	0.26
Y1954	0.49	0.69	0.53

Variable	Factor Loadings (Varim. norm.) Extraction: Principal compon. (Marked loadings are >.700000) Exclude cases: 13 Factor 1	Factor 2	Factor 3
Y1955	0.90	0.37	0.21
Y1956	0.82	0.56	0.09
Y1957	0.85	0.48	0.20
Y1958	0.44	0.89	0.07
Y1959	0.97	0.19	0.12
Y1960	0.90	0.43	-0.01
Y1961	0.88	0.47	-0.03
Y1962	0.81	0.53	0.24
Y1963	0.83	0.53	0.14
Y1964	0.84	0.34	0.42
Y1965	0.91	0.22	0.36
Y1966	0.51	0.84	0.15
Y1967	0.88	0.26	0.38
Y1968	0.95	0.22	0.18
Y1969	0.31	0.90	0.32
Y1970	0.91	0.32	0.24
Y1971	0.88	0.34	0.34
Y1972	0.93	0.36	0.07
Y1973	0.83	0.54	0.13
Y1974	0.95	0.21	0.21
Y1975	0.69	-0.04	0.72
Y1976	0.70	0.65	0.29
Y1977	0.85	0.52	-0.08
Y1978	0.86	0.29	0.41
Y1979	0.83	0.53	0.15
Y1980	0.49	0.83	0.24
Y1981	0.78	0.61	0.12
Y1982	0.66	0.20	0.73
Y1983	0.89	0.21	0.41
Y1984	0.80	0.46	0.39
Y1985	0.35	0.93	0.08
Y1986	0.94	0.34	0.03
Y1987	0.11	0.97	0.13
Y1988	0.51	0.86	-0.03
Y1989	0.58	0.80	0.11
Y1990	0.88	0.33	0.14
Y1991	0.74	0.67	0.06
Y1992	0.05	0.96	0.22
Y1993	0.33	0.92	0.20
Y1994	0.03	1.00	0.02
Y1995	0.87	0.35	0.34
Y1996	0.92	0.37	0.09
Y1997	0.83	0.56	0.06
Y1998	0.46	0.73	0.50

Variable	Factor Loadings (Varim. norm.) Extraction: Principal compon. (Marked loadings are >.700000) Exclude cases: 13 Factor 1	Factor 2	Factor 3
Y1999	0.83	0.51	0.24
Y2000	0.54	0.84	-0.01
Y2001	0.04	0.99	0.09
Y2002	0.81	0.59	0.05
Y2003	0.72	0.70	0.04
Y2004	0.70	0.68	0.17
Y2005	0.53	0.83	0.13
Y2006	0.35	0.93	0.03
Y2007	-0.03	1.00	0.02
Y2008	0.79	0.43	0.43
Y2009	0.72	0.65	0.23
Y2010	0.97	0.19	0.12
Y2011	0.75	0.13	0.65
Y2012	0.70	0.69	0.02
Y2013	0.32	0.93	0.12
Y2014	0.58	0.79	0.19
Y2015	0.41	0.90	-0.03
Y2016	0.46	0.86	0.02
Y2017	0.75	0.64	0.13
Y2018	0.41	0.91	0.07
Y2019	0.86	0.47	0.18
Y2020	0.67	0.73	0.10
Y2021	0.51	0.86	-0.04
Y2022	0.96	0.25	0.06
Y2023	0.40	0.91	0.09
Y2024	0.88	0.46	0.01
Expl.Var	60.36	44.40	8.32
Prp.Totl	0.53	0.39	0.07

Value	Eigenvalues Extraction: Principal components Exclude cases: 13 Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	93.92	82.39	93.92	82.39
2	15.50	13.60	109.42	95.98
3	3.65	3.20	113.07	99.19



USGS 06191500 Yellowstone River at Corwin Springs MT. Transposed Matrix, Factor Analysis Results

Factor Loadings (Varim. norm.)
Extraction: Principal compon.
(Marked loadings are >.700000)
Exclude cases: 13

Variable	Factor 1	Factor 2	Factor 3
Y1911	0.97	0.18	0.15
Y1912	0.96	0.12	0.25
Y1913	0.91		
Y1914	0.72		
Y1915	0.69		
Y1916	0.78		
Y1917	0.74		
Y1918	0.98		
Y1919	0.01		
Y1920	0.91		
Y1921	0.89		
Y1922	0.93		
Y1923	0.84		
Y1924	0.53		
Y1925	0.58		
Y1926	0.20		
Y1927	0.96		
Y1928	0.05		
Y1929	0.92		
Y1930	0.74		
Y1931	0.74		
Y1932	0.80		
Y1933	0.98		
Y1934	-0.05		
Y1935	0.92		
Y1936	0.38		
Y1937	0.57		
Y1938	0.91		
Y1939	0.23		
Y1940	0.53		
Y1941	0.56		
Y1942	0.88		
Y1943	0.81		
Y1944	0.79		
Y1945	0.77		
Y1946	0.75		
Y1947	0.59		
Y1948	0.82		
Y1949	0.59		
Y1950	0.85		
Y1951	0.50		
Y1952	0.64		
Y1953	0.95		
Y1954	0.49		

Factor Loadings (Varim. norm.)
Extraction: Principal compon.
(Marked loadings are >.700000)
Exclude cases: 13

Variable	Factor 1	Factor 2	Factor 3
Y1955	0.90	0.37	0.21
Y1956	0.82	0.56	0.09
Y1957	0.85	0.48	0.20
Y1958	0.44	0.89	0.07
Y1959	0.97	0.19	0.12
Y1960	0.90	0.43	-0.01
Y1961	0.88	0.47	-0.03
Y1962	0.81	0.53	0.24
Y1963	0.83	0.53	0.14
Y1964	0.84	0.34	0.42
Y1965	0.91	0.22	0.36
Y1966	0.51	0.84	0.15
Y1967	0.88	0.26	0.98
Y1968	0.95		
Y1969	0.31		
Y1970	0.91		
Y1971	0.88		
Y1972	0.93		
Y1973	0.83		
Y1974	0.95		
Y1975	0.69		
Y1976	0.70		
Y1977	0.85		
Y1978	0.86		
Y1979	0.83		
Y1980	0.49		
Y1981	0.78		
Y1982	0.66		
Y1983	0.89		
Y1984	0.80		
Y1985	0.35		
Y1986	0.94		
Y1987	0.11		
Y1988	0.51		
Y1989	0.58		
Y1990	0.88		
Y1991	0.74		
Y1992	0.05		
Y1993	0.33		
Y1994	0.03		
Y1995	0.87		
Y1996	0.92		
Y1997	0.83		
Y1998	0.46		

Factor Loadings (Varim. norm.)
Extraction: Principal compon.
(Marked loadings are >.700000)
Exclude cases: 13

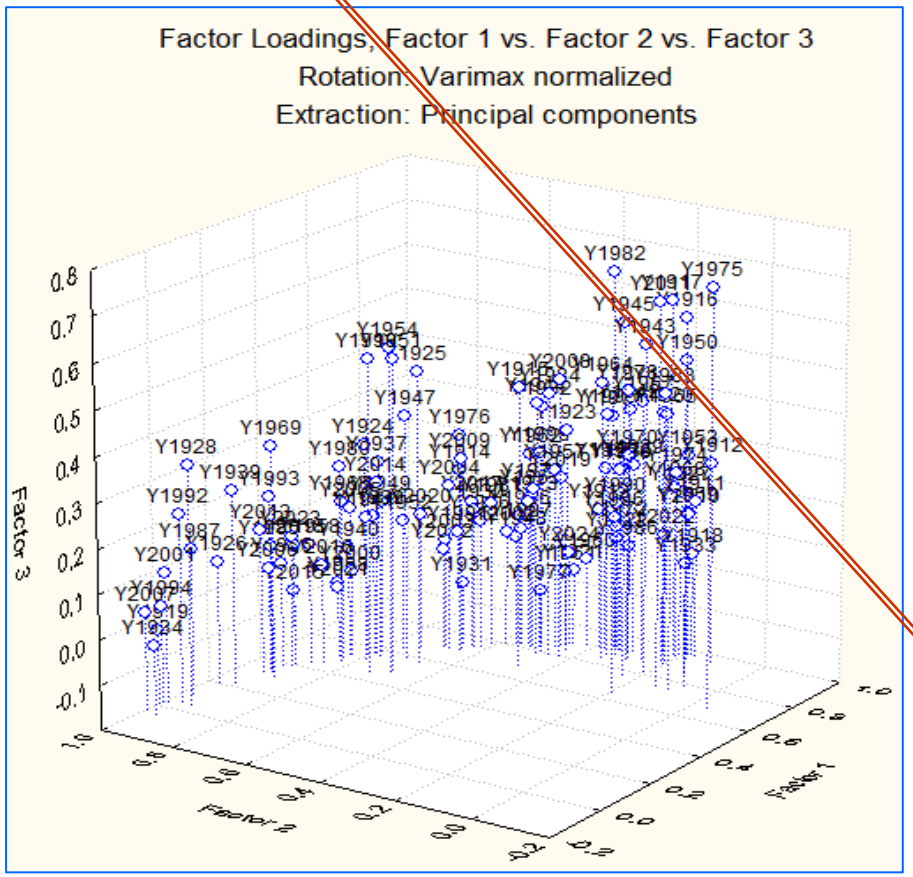
Variable	Factor 1	Factor 2	Factor 3
Y1999	0.83	0.51	0.24
Y2000	0.54	0.84	-0.01
Y2001	0.04	0.99	0.09
Y2002	0.81	0.59	0.05
Y2003	0.72	0.70	0.04
Y2004	0.70	0.68	0.17
Y2005	0.53	0.83	0.13
Y2006	0.35	0.93	0.03
Y2007	-0.03	1.00	0.02
Y2008	0.79	0.43	0.43
Y2009	0.72	0.65	0.23
Y2010	0.97	0.19	0.12
Y2011	0.75	0.13	0.65
Y2012	0.70	0.69	0.02
Y2013	0.32	0.93	0.12
Y2014	0.58	0.79	0.19
Y2015	0.41	0.90	-0.03
Y2016	0.46	0.86	0.02
Y2017	0.75	0.64	0.13
Y2018	0.41	0.91	0.07
Y2019	0.86	0.47	0.18
Y2020	0.67	0.73	0.10
Y2021	0.51	0.86	-0.04
Y2022	0.96	0.25	0.06
Y2023	0.40	0.91	0.09
Y2024	0.88	0.46	0.01
Expl.Var	60.36	44.40	8.32
Prp.Totl	0.53	0.39	0.07

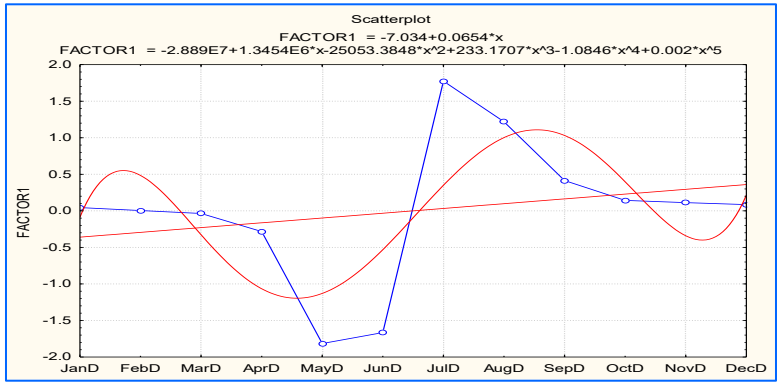
Eigenvalues
Extraction: Principal components
Exclude cases: 13

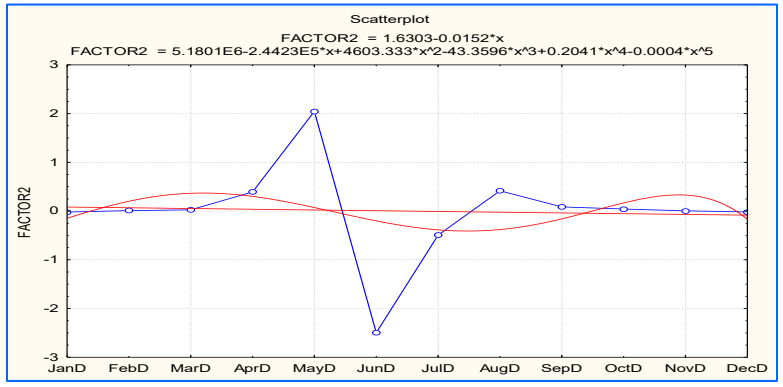
Value	Eigenvalue	% Total variance	Cumulative Eigenvalue	Cumulative %
1	93.92	82.39	93.92	82.39
2	15.50	13.60	109.42	95.98
3	3.65	3.20	113.07	99.19

Factor scores for 3

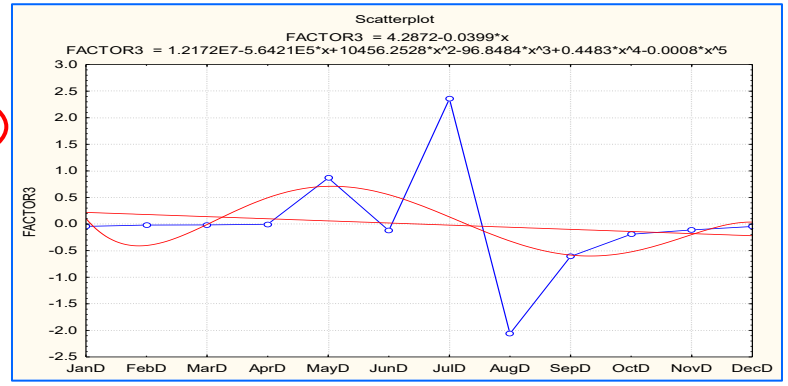
	1 Year	2 FACTOR1	3 FACTOR2	4 FACTOR3
1	JanD	0.04	-0.02	-0.04
2	FebD	0.00	0.01	-0.02
3	MarD	-0.04	0.03	-0.02
4	AprD	-0.28	0.39	-0.00
5	MayD	-1.81	2.05	0.87
6	JunD	-1.67	-2.50	-0.12
7	JulD	1.78	-0.49	2.36
8	AugD	1.22	0.42	-2.06
9	SepD	0.41	0.08	-0.61
10	OctD	0.14	0.04	-0.19
11	NovD	0.11	0.01	-0.11
12	DecD	0.09	-0.02	-0.05



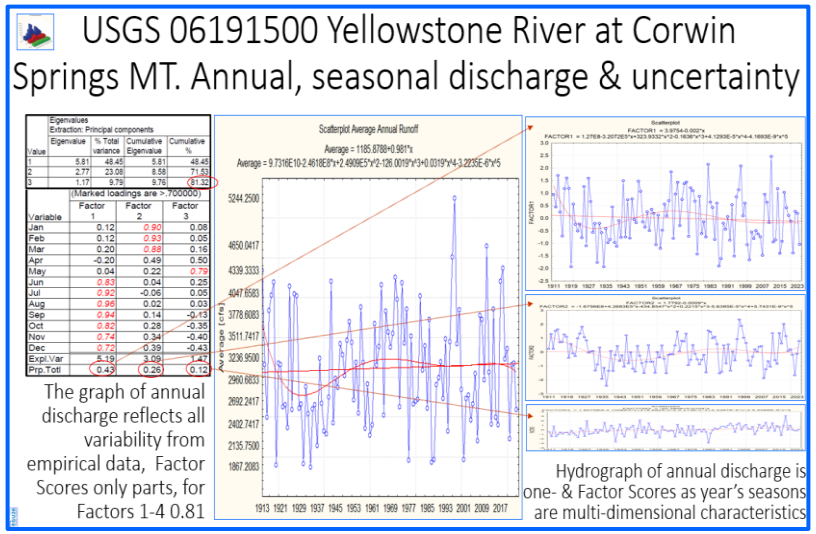




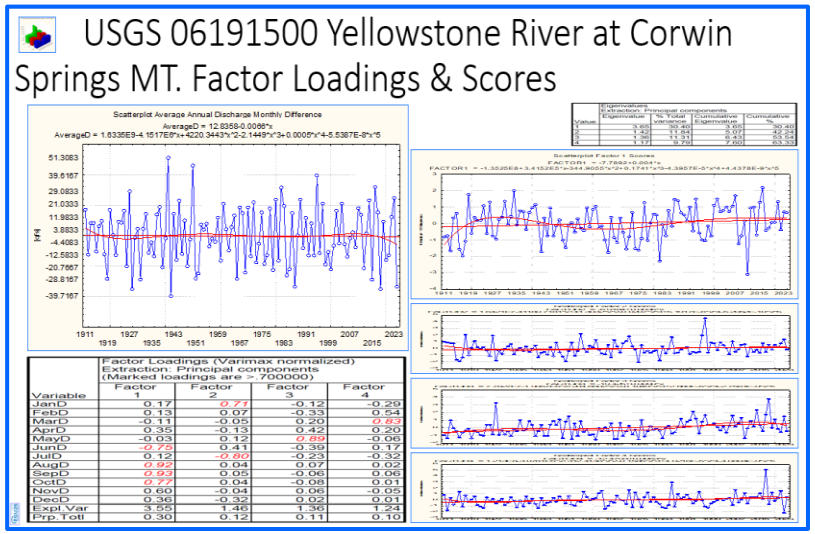
Uncertainty is 0.81%



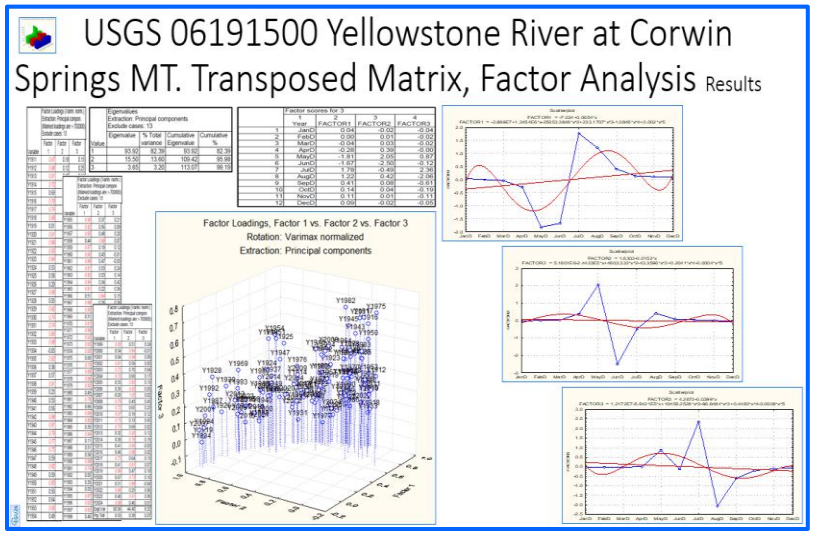
USGS 06191500 Yellowstone River at Corwin Springs MT. Structures of River Discharge from empirical (114x12), calculated (114x12D) & transposed (12Dx114) matrixes



Uncertainty is 18.68%



Uncertainty is 36.6%

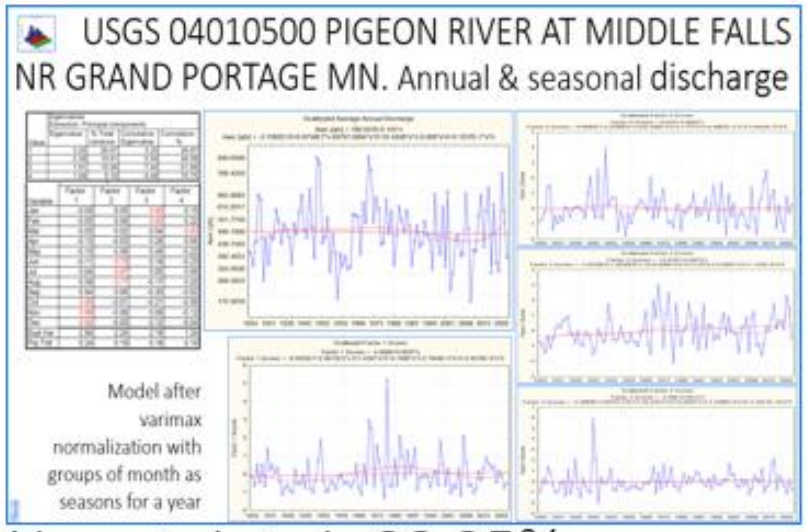


Uncertainty is 0.87%

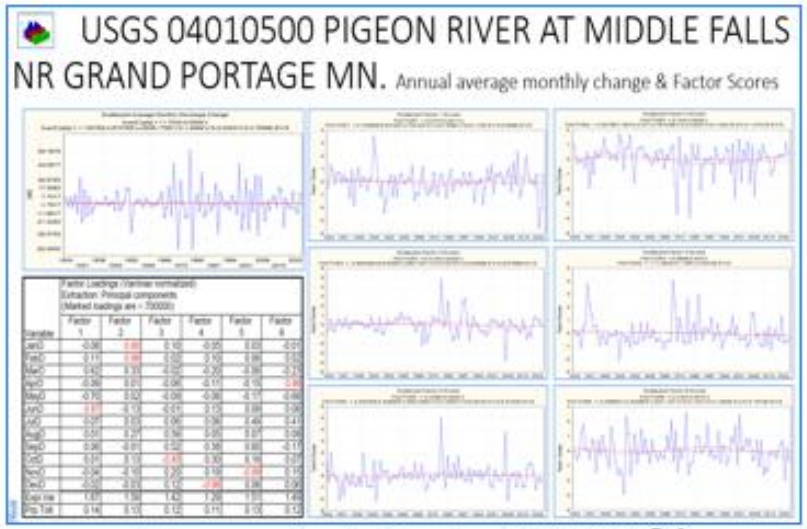
The graphics for models of annual discharge & monthly change of discharge reflects different information from empirical data & calculated data. The monthly discharge actual shows difference of seasons through years, monthly changes – monthly dynamics inside years. The third model shows influence months in distribution of discharge in a year & extract 3 Factors (patterns).

Structures for two watersheds from six matrixes

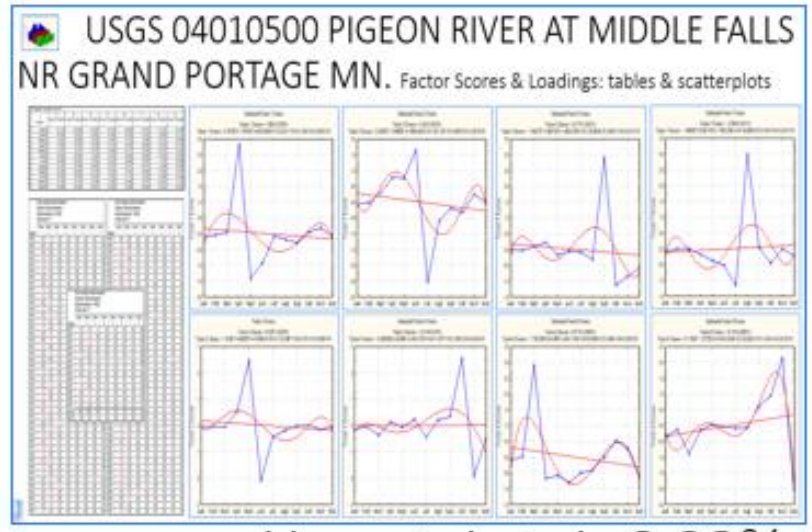
[101x12], [101x12D], [12Dx101] & [114x12], [114x12D], [12Dx114]



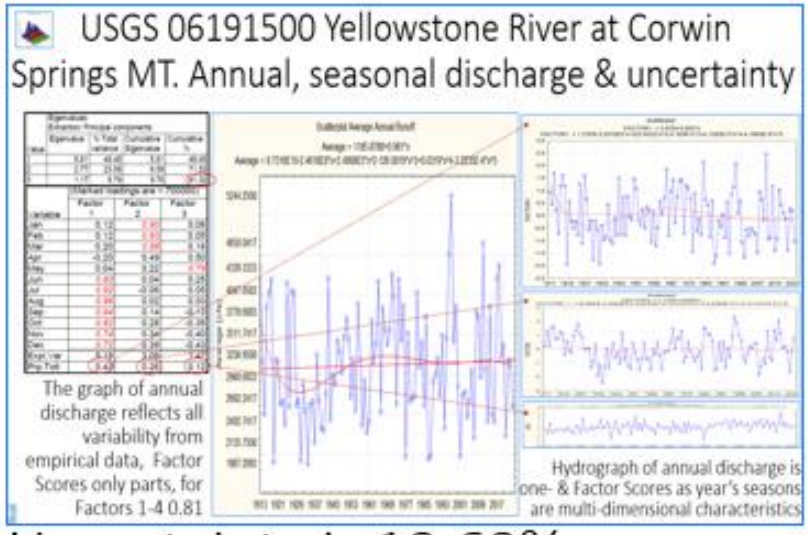
Uncertainty is 29.25%



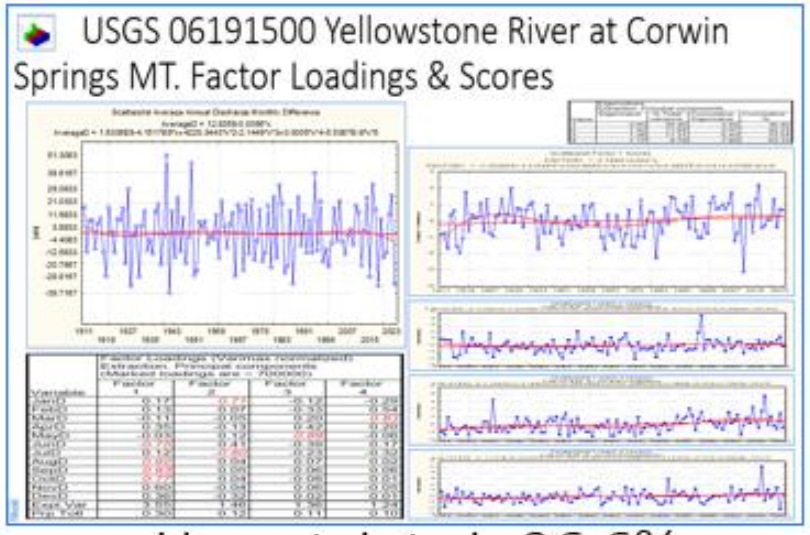
Uncertainty is 25.22%



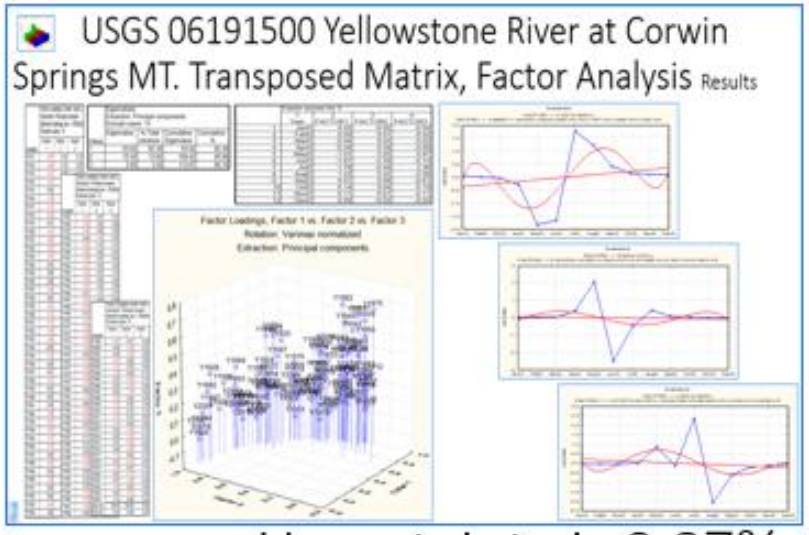
Uncertainty is 0.23%



Uncertainty is 18.68%



Uncertainty is 36.6%



Uncertainty is 0.87%

The concept of hydrograph analysis by R. F. Horton (1935)

HORTON HYDROLOGICAL LABORATORY
VOORHEESVILLE, N. Y.

PUBLICATION 101 FEBRUARY 1, 1935

SURFACE RUNOFF PHENOMENA

PART I — ANALYSIS OF THE HYDROGRAPH

By

ROBERT E. HORTON

UNIVERSITY OF MINNESOTA LIBRARY

EDWARDS BROTHERS, INC.
ANN ARBOR, MICHIGAN
1935

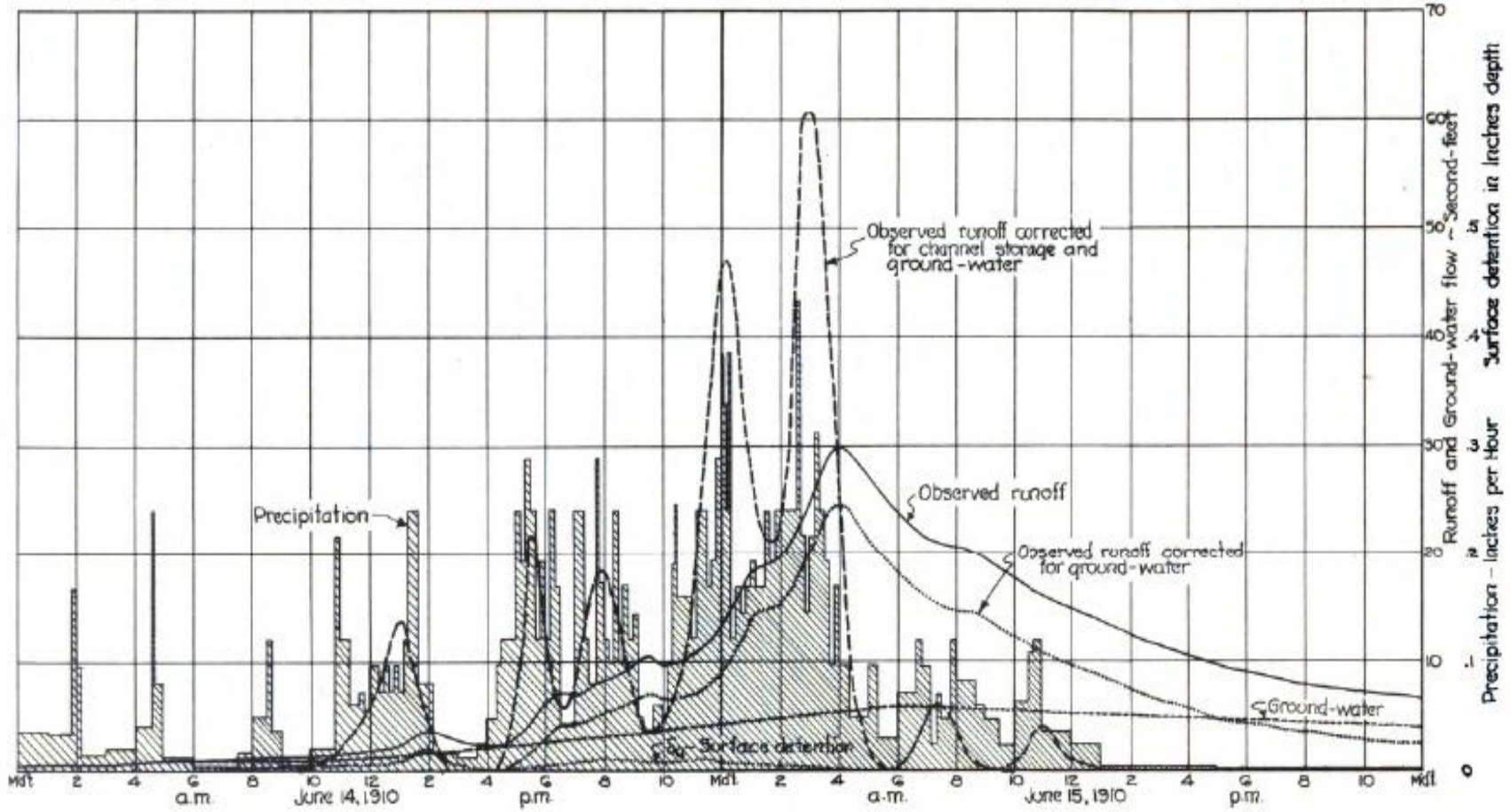
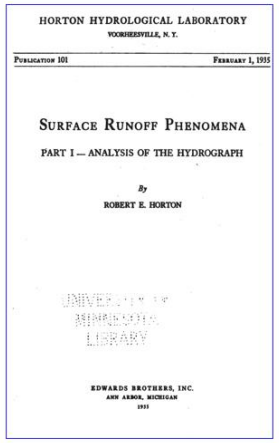


FIG. 16, ANALYSIS OF RUNOFF, RAPPEN BASIN, SWITZERLAND, JUNE, 1910.

Characteristics of the two watersheds by Gemini (Horton's concept) & in Concept of Systemic Analysis

Approach with time difference of 91 year (2026-1935)

The concept of hydrograph analysis by R. F. Horton (1935)



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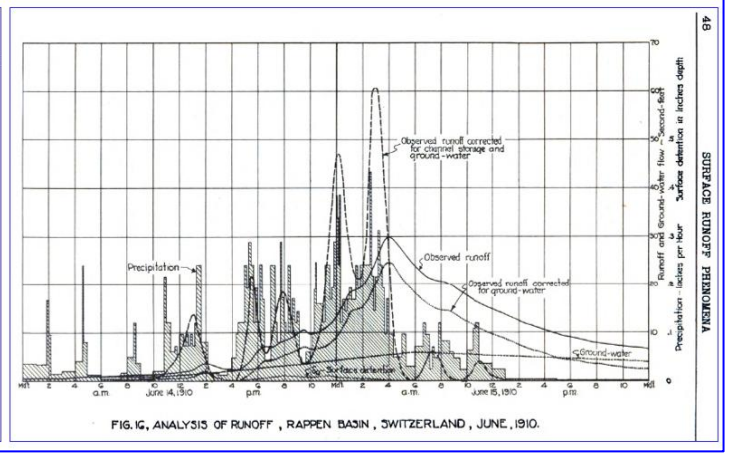
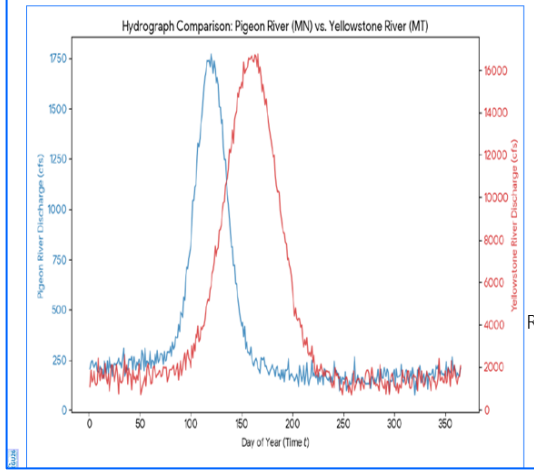


FIG. IG, ANALYSIS OF RUNOFF, RAPPEN BASIN, SWITZERLAND, JUNE, 1910.

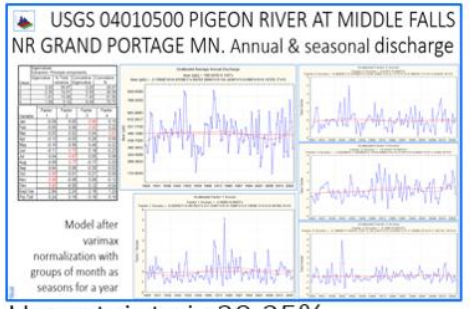
Characteristics of the two watersheds by Gemini



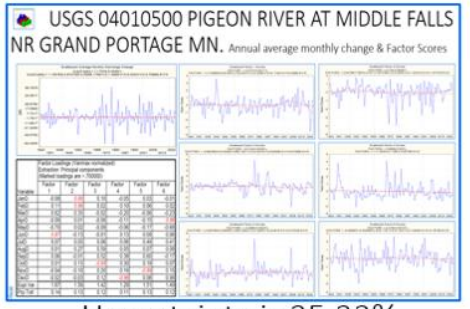
The following data points reflect the empirical representation of these two distinct subsystems: **Pigeon River** at Middle Falls, MN (USGS 04010500): Drainage Area: 609 square miles. Hydrologic Unit Code (HUC): 04010101. Context: Located in the Northern Lakes and Forests ecoregion, characterized by humid continental climates and significant snowmelt runoff in early spring. **Yellowstone River** at Corwin Springs, MT (USGS 06191500): Drainage Area: 2,615 square miles. Hydrologic Unit Code (HUC): 10070002. Annual Runoff Ratio: Approximately 54.6%. Dynamics: Exhibits a high-volume snow-driven hydrograph with peaks typically occurring in June. Recent extreme events, such as the 2022 flood (peaking at 43,300 cfs), illustrate the "fuzzy" nature of these systems where record-breaking variability challenges traditional static models.

Structures for two watersheds from six matrixes

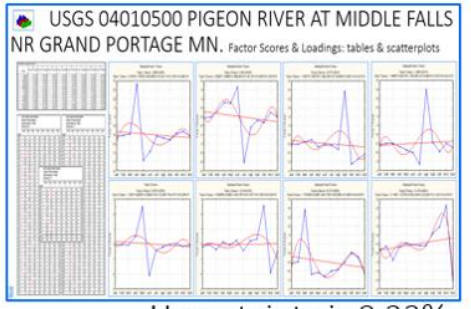
[101x12], [101x12D], [12Dx101] & [114x12], [114x12D], [12Dx114]



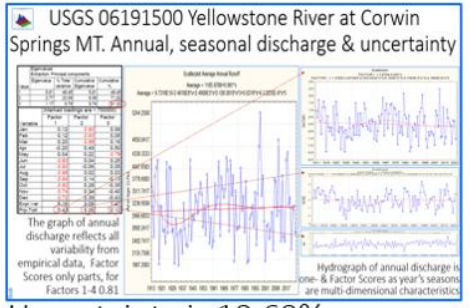
Uncertainty is 29.25%



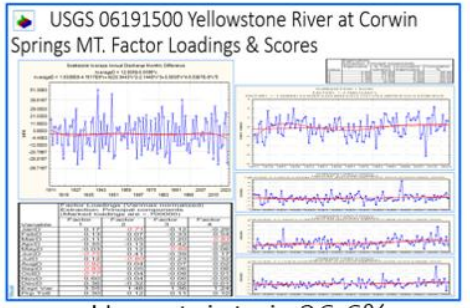
Uncertainty is 25.22%



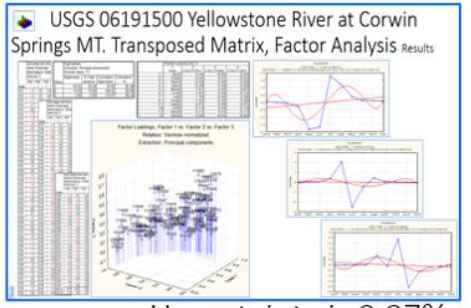
Uncertainty is 0.23%



Uncertainty is 18.68%



Uncertainty is 36.6%

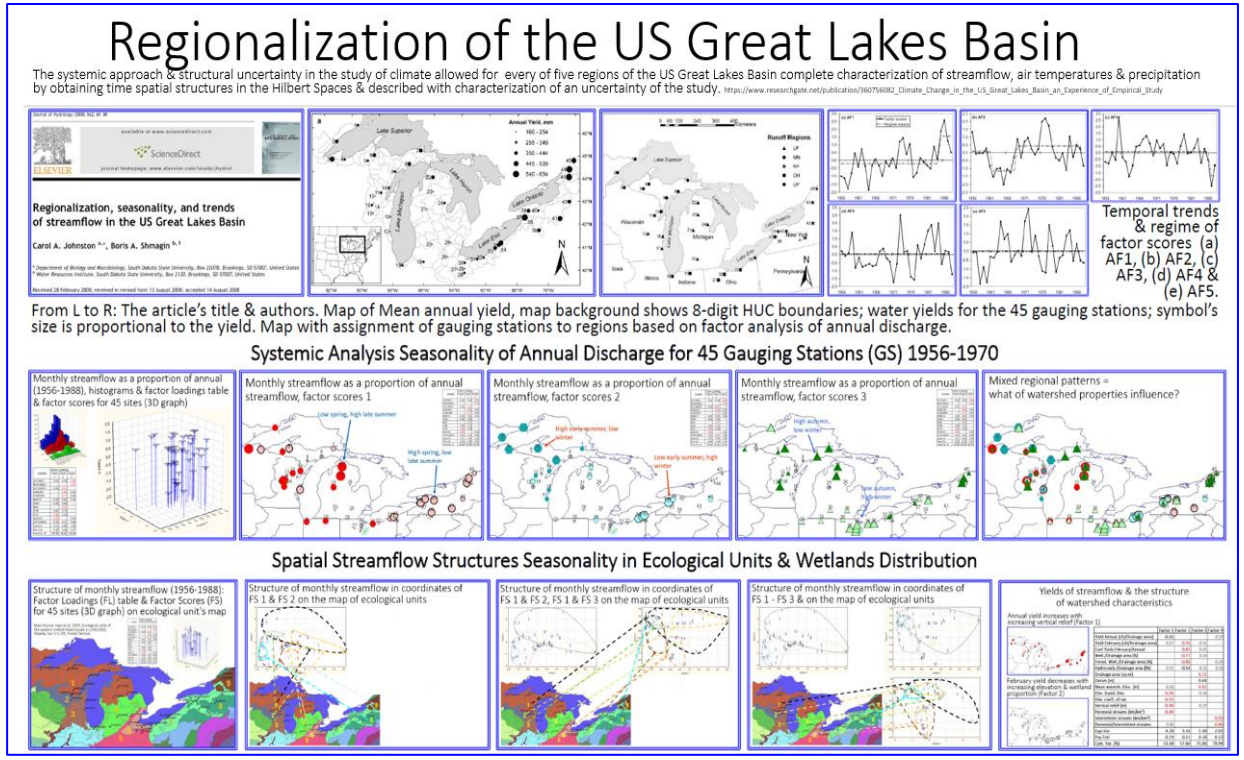


Uncertainty is 0.87%

Analysis Data in Systemic Approach shows & proves a new concept which allows us to begin the recognition of the science of Structural Hydrology & from there begins the engineering analysis of risk

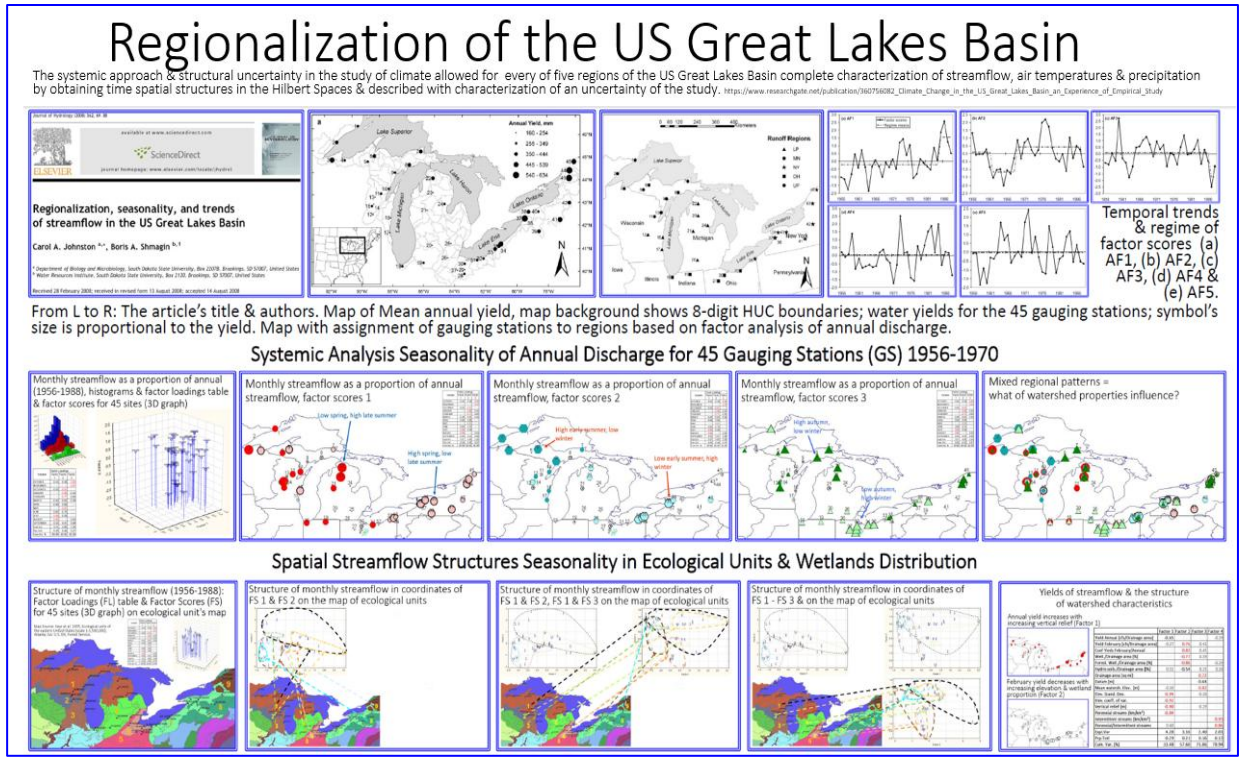
8 – Great Lakes US Basin Example

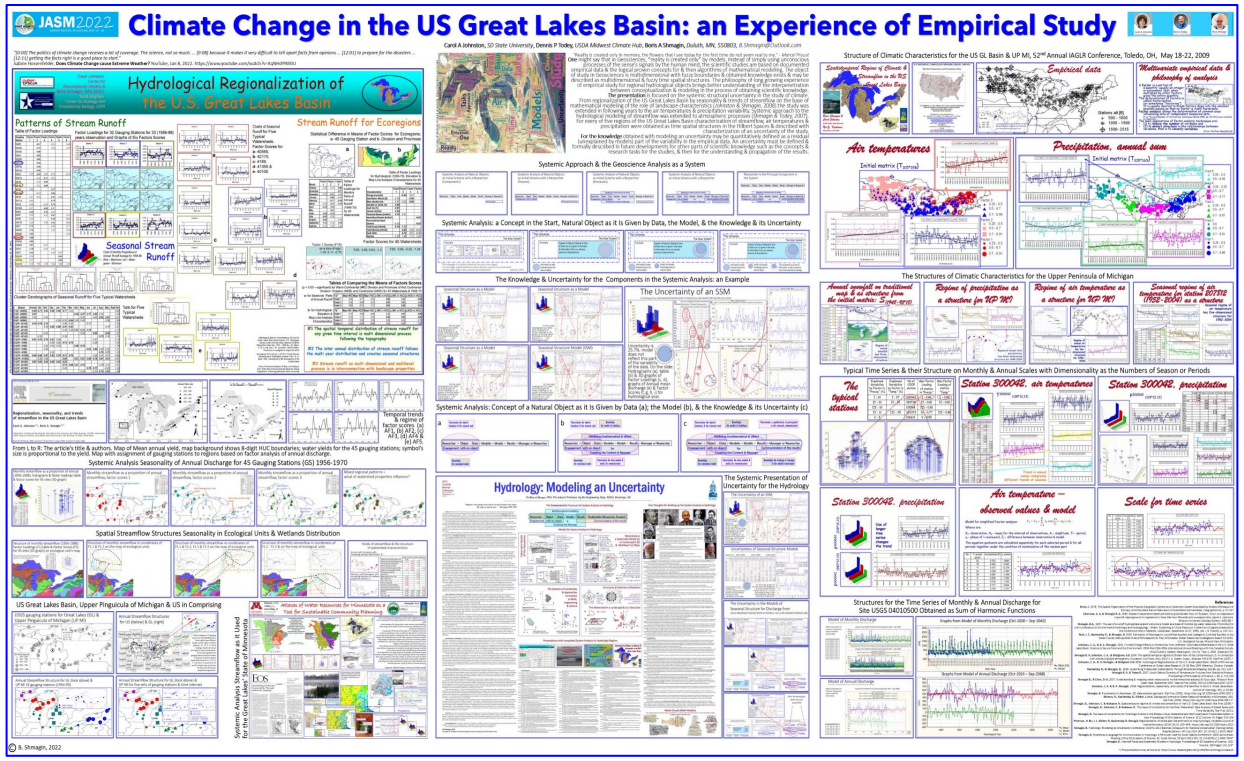
- Hydrological maps constructed from learned time-spatial structures
- Demonstrates scalability from mesoscale to regional systems



8 – Great Lakes US Basin Example

- Hydrological maps constructed from learned time-spatial structures
- Demonstrates scalability from mesoscale to regional systems
- Shows how fuzzy structures reveal hidden interactions





Regionalization of the US Great Lakes Basin

The systemic approach & structural uncertainty in the study of climate allowed for every of five regions of the US Great Lakes Basin complete characterization of streamflow, air temperatures & precipitation by obtaining time spatial structures in the Hilbert Spaces & described with characterization of an uncertainty of the study. https://www.researchgate.net/publication/360756082_Climate_Change_in_the_US_Great_Lakes_Basin_an_Experience_of_Empirical_Study

Journal of Hydrology (2008) 362, 69–88

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Regionalization, seasonality, and trends of streamflow in the US Great Lakes Basin

Carol A. Johnston [a,*], Boris A. Shmagin [b,1]

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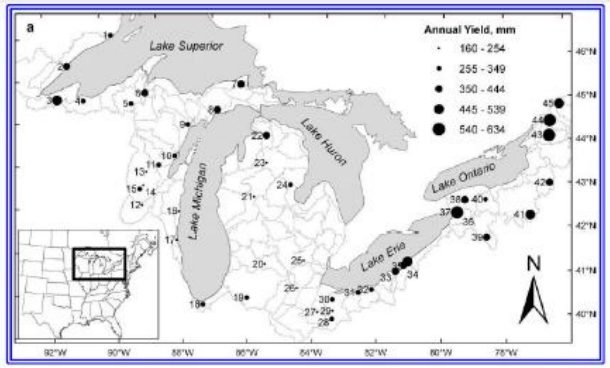
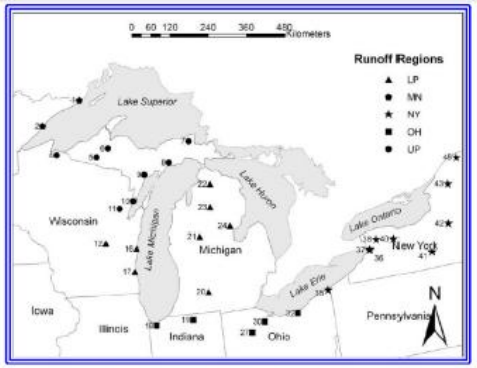
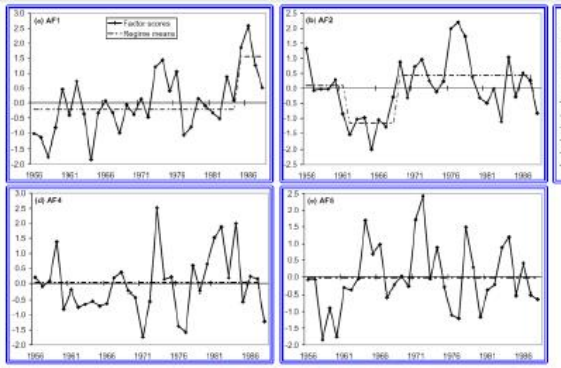
[b] Water Resources Institute, South Dakota State University, Box 2120, Brookings, SD 57007, United States

Received 28 February 2008; received in revised form 13 August 2008; accepted 14 August 2008





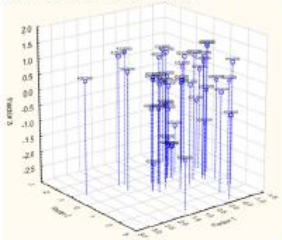


Temporal trends & regime of factor scores (a) AF1, (b) AF2, (c) AF3, (d) AF4 & (e) AF5.

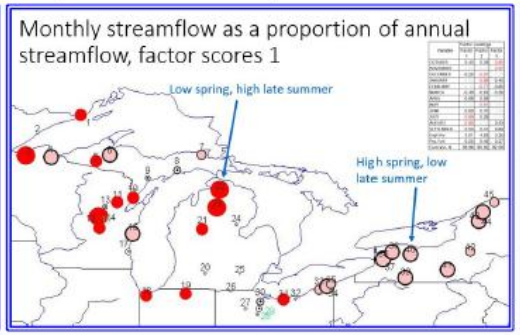
From L to R: The article's title & authors. Map of Mean annual yield, map background shows 8-digit HUC boundaries; water yields for the 45 gauging stations; symbol's size is proportional to the yield. Map with assignment of gauging stations to regions based on factor analysis of annual discharge.

Systemic Analysis Seasonality of Annual Discharge for 45 Gauging Stations (GS) 1956-1970

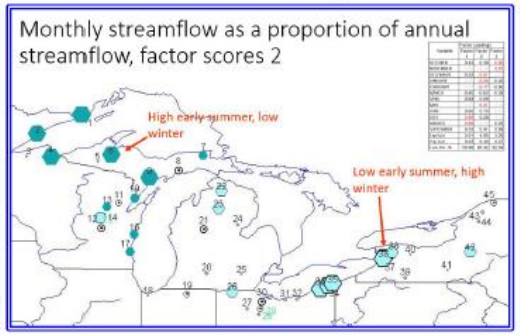
Monthly streamflow as a proportion of annual (1956-1988), histograms & factor loadings table & factor scores for 45 sites (3D graph)



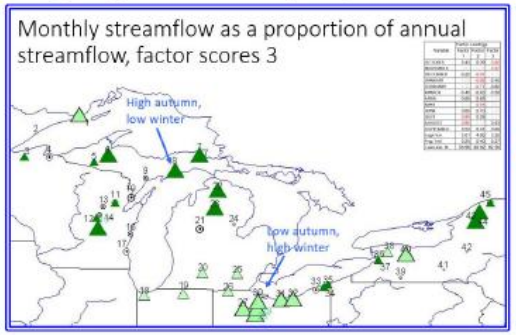
Monthly streamflow as a proportion of annual streamflow, factor scores 1



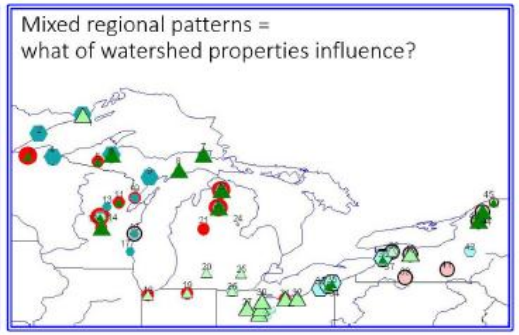
Monthly streamflow as a proportion of annual streamflow, factor scores 2



Monthly streamflow as a proportion of annual streamflow, factor scores 3

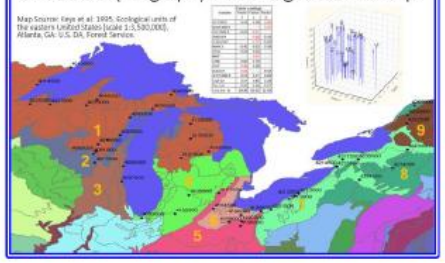


Mixed regional patterns = what of watershed properties influence?

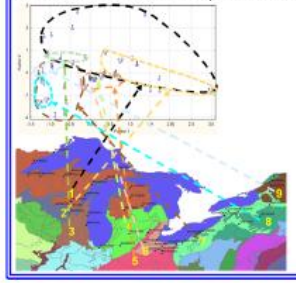


Spatial Streamflow Structures Seasonality in Ecological Units & Wetlands Distribution

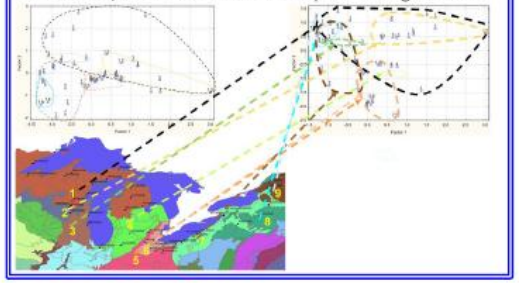
Structure of monthly streamflow (1956-1988): Factor Loadings (FL) table & Factor Scores (FS) for 45 sites (3D graph) on ecological unit's map



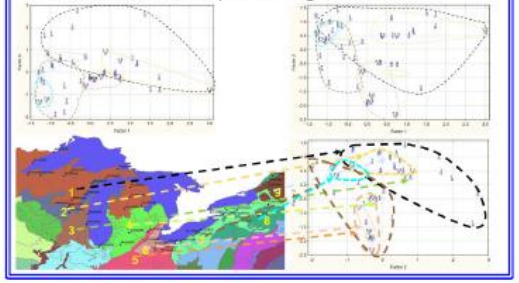
Structure of monthly streamflow in coordinates of FS 1 & FS 2 on the map of ecological units



Structure of monthly streamflow in coordinates of FS 1 & FS 2, FS 1 & FS 3 on the map of ecological units



Structure of monthly streamflow in coordinates of FS 1 - FS 3 & on the map of ecological units



Yields of streamflow & the structure of watershed characteristics

Annual yield increases with increasing vertical relief (Factor 1)

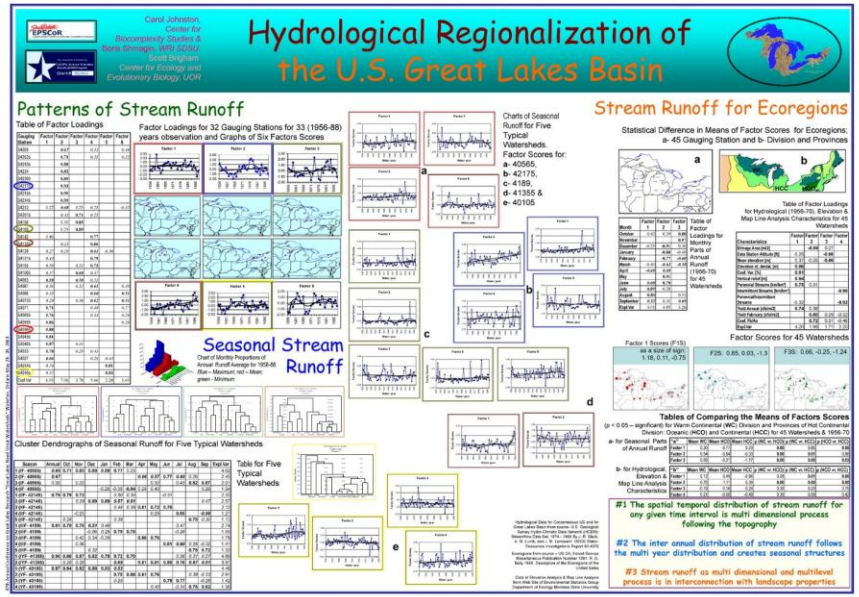
February yield decreases with increasing elevation & wetland proportion (Factor 2)

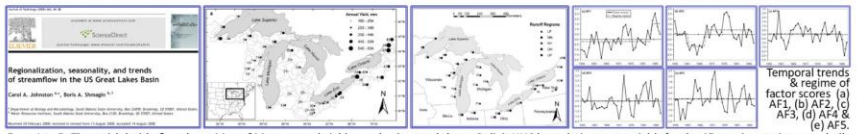
	Factor 1	Factor 2	Factor 3	Factor 4
Yield Annual [cfs/Drainage area]	-0.65			-0.29
Yield February [cfs/Drainage area]	-0.37	0.76	0.41	
Coef Yields February/Annual		0.83	0.41	
Wetl./Drainage area [%]		-0.77	0.19	
Forest, Wetl./Drainage area [%]		-0.86		-0.29
Hydrolsoils./Drainage area [%]	0.31	-0.54	0.25	0.26
Drainage area [sq mi]			0.72	
Datum [m]			-0.64	
Mean watersh. Elev. [m]	-0.30		-0.82	
Elev. Stand. Dev.	-0.94		-0.26	
Elev. coeff. of var.	-0.92			
Vertical relief [m]	-0.90		-0.29	
Perennial streams [km/km²]	-0.84			
Intermittent streams [km/km²]				0.93
Perennial/Intermittent streams	0.40			0.86
Expl.Var	4.28	3.16	2.40	2.03
Prp.Tot	0.29	0.21	0.16	0.13
Cum. Var. [%]	33.48	57.60	71.86	78.94

Climate Change in the US Great Lakes Basin: an Experience of Empirical Study

Carol A Johnston, *SD State University*, Dennis P Todey, *USDA Midwest Climate Hub*, Boris A Shmagin, *Duluth, MN, 550803, B.Shmagin@Outlook.com*

"[0:00] The politics of climate change receives a lot of coverage. The science, not so much. ... [0:08] because it makes it very difficult to tell apart facts from opinions ... [12:01] to prepare for the disasters ... [12:11] getting the facts right is a good place to start."
Sabine Hossenfelder, **Does Climate Change cause Extreme Weather?** YouTube, Jan 8, 2022. https://www.youtube.com/watch?v=KqNHdY90StU

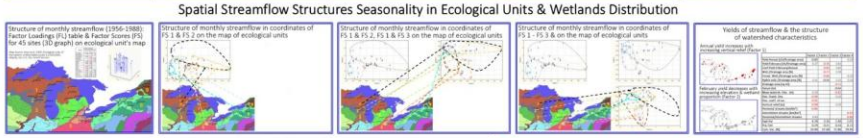




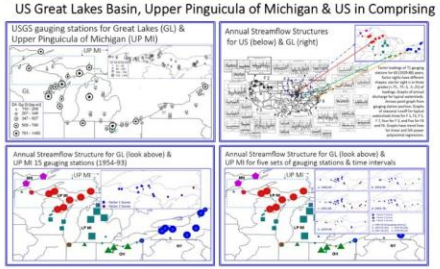
From L to R: The article's title & authors. Map of Mean annual yield, map background shows 8-digit HUC boundaries; water yields for the 45 gauging stations; symbol's size is proportional to the yield. Map with assignment of gauging stations to regions based on factor analysis of annual discharge.

Systematic Analysis Seasonality of Annual Discharge for 45 Gauging Stations (GS) 1956-1970

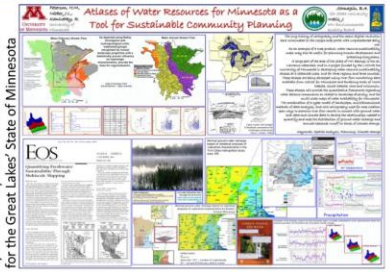
Spatial Streamflow Structures Seasonality in Ecological Units & Wetlands Distribution



US Great Lakes Basin, Upper Pinguicula of Michigan & US in Comprising



Systemic Analysis Streamflow as it Used for the Great Lakes' State of Minnesota



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"Reality is created only in memory, the flowers that I see today for the first time do not seem real to me." - *Marcel Proust*

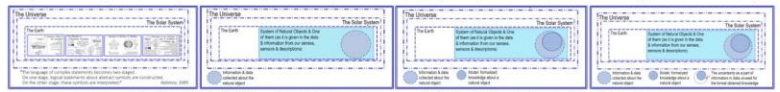
One might say that in Geosciences, "reality is created only" by models. Instead of simply using unconscious processes of the sense's signals by the human mind, the scientific studies are based on documented empirical data & the logical proven concepts for & then algorithms of mathematical modeling. The object of study in Geosciences is multidimensional with fuzzy boundaries & obtained knowledge exists & may be described as multidimensional & fuzzy time spatial structures. The philosophy of long growing experience of empirical study for regional hydrological objects brings better understanding of the interpretation between conceptualization & modeling in the process of obtaining scientific knowledge.

The presentation is focused on the systemic structural uncertainty in the study of climate. From regionalization of the US Great Lakes Basin by seasonality & trends of streamflow on the base of mathematical modeling of the role of landscape characteristics (*Johnston & Shmagin*, 2008) the study was extended in following years to the air temperatures & precipitation variability. Systemic approach to the hydrological modeling of streamflow was extended to atmospheric processes (*Shmagin & Todey*, 2007). For every of five regions of the US Great Lakes Basin characterization of streamflow, air temperatures & precipitation were obtained as time spatial structures in the Hilbert Spaces & described with characterization of an uncertainty of the study.

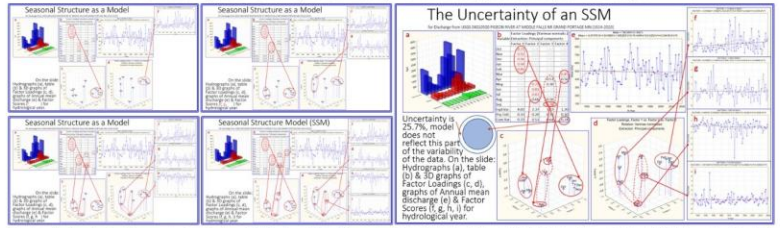
For the knowledge obtained with modeling an uncertainty may be quantitatively defined as a residual (unexplained by models) part of the variability in the empirical data. An uncertainty must be defined & formally described in future developments for other parts of scientific knowledge such as the concepts & research tasks for the studies, & then for the understanding & propagation of the results.

Systemic Approach & the Geoscience Analysis as a System

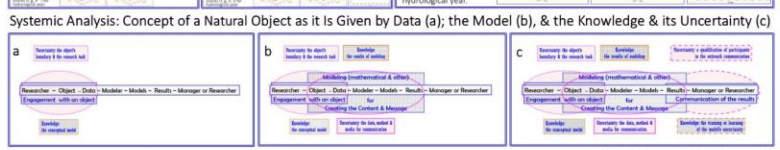
Systemic Analysis: a Concept in the Start, Natural Object as it Is Given by Data, the Model, & the Knowledge & its Uncertainty



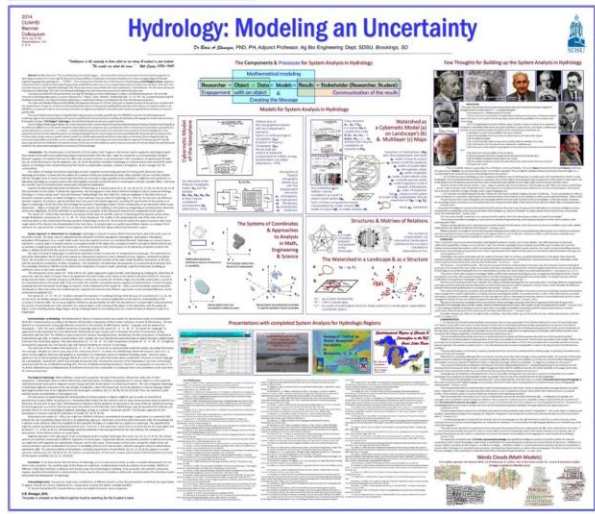
The Knowledge & Uncertainty for the Components in the Systemic Analysis: an Example



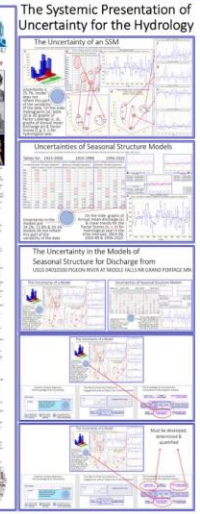
Systemic Analysis: Concept of a Natural Object as it Is Given by Data (a); the Model (b), & the Knowledge & its Uncertainty (c)



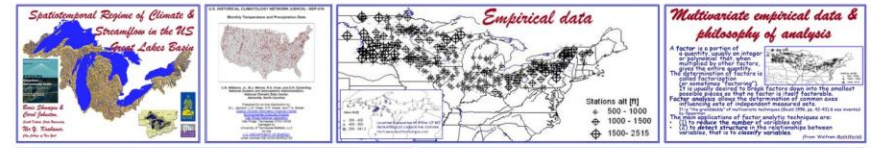
Hydrology: Modeling an Uncertainty

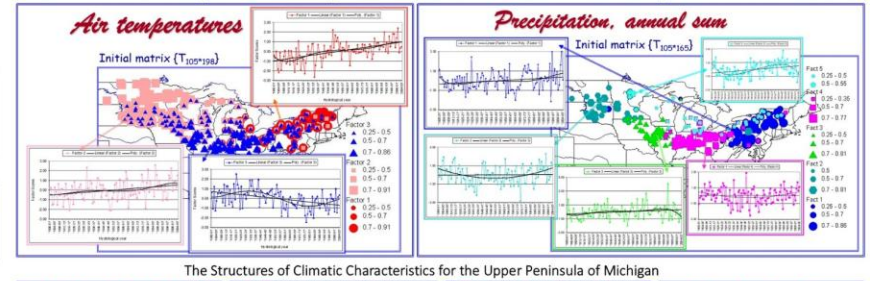


The Systemic Presentation of Uncertainty for the Hydrology

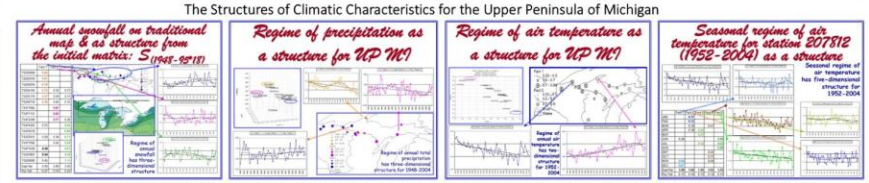


Structure of Climatic Characteristics for the US GL Basin & UP MI, 52nd Annual IAGLR Conference, Toledo, OH, May 18-22, 2009

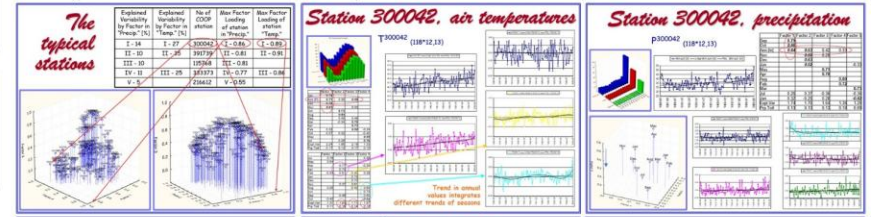


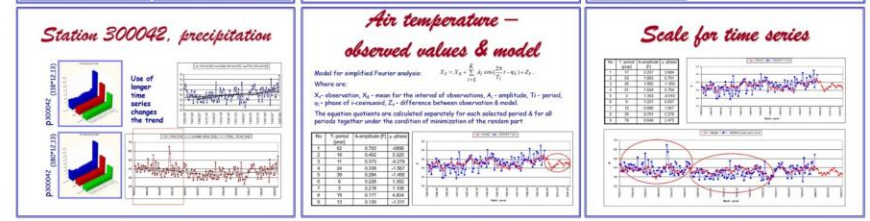


The Structures of Climatic Characteristics for the Upper Peninsula of Michigan

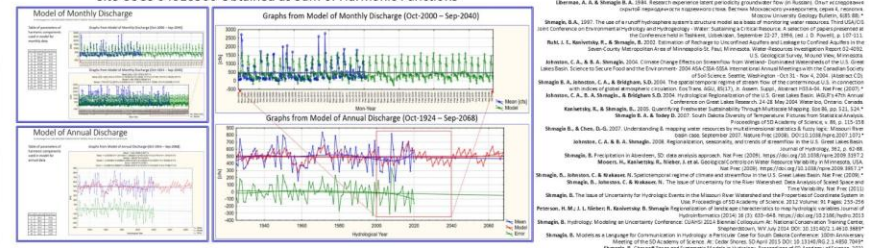


Typical Time Series & their Structure on Monthly & Annual Scales with Dimensionality as the Numbers of Season or Periods





Structures for the Time Series of Monthly & Annual Discharge for Site USGS 04010500 Obtained as Sum of Harmonic Functions



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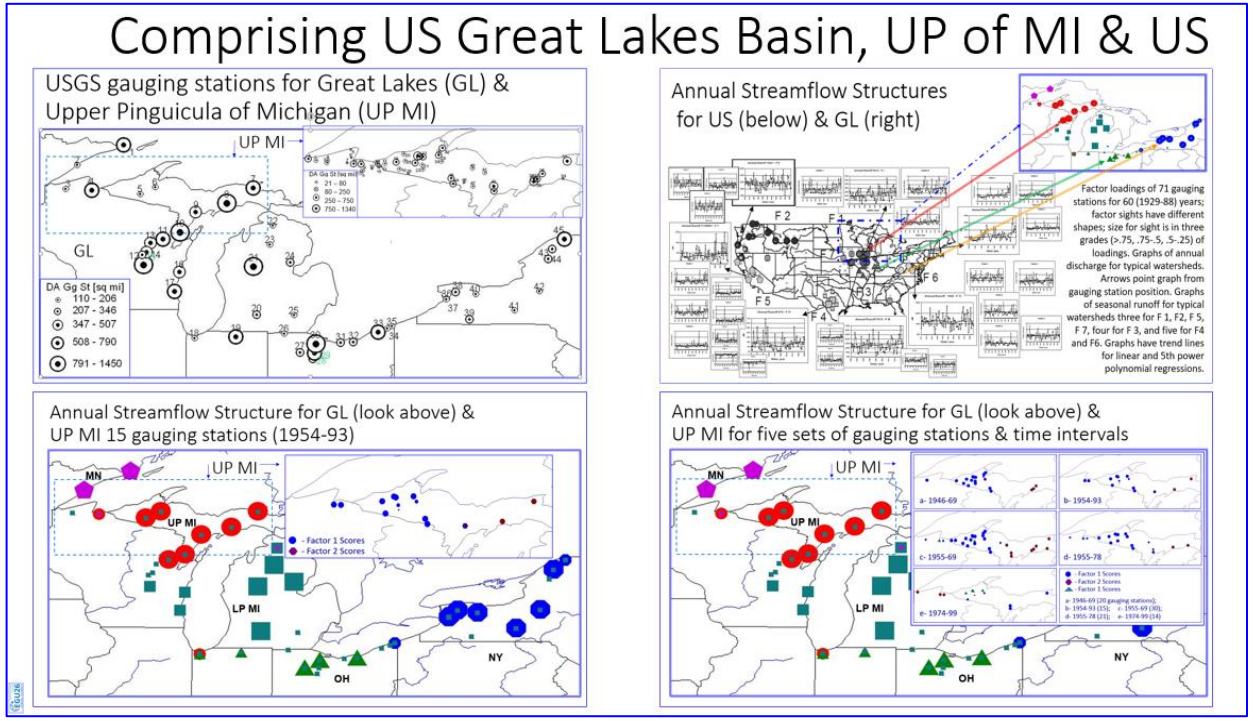
*The publications may be found at: https://www.researchgate.net/profile/Boris-Shmagin/research

9 – Hydrology: Scientific Hydrology

- Rooted in multiscale cartography

Scientific Hydrology & Stochastic Hydrology are two traditions.
These traditions evolved in parallel, but not together.



9 – Hydrology: Scientific Hydrology

- Rooted in multiscale cartography
- Surface–groundwater interactions in the Systemic Approach

Scientific Hydrology & Stochastic Hydrology are two traditions.
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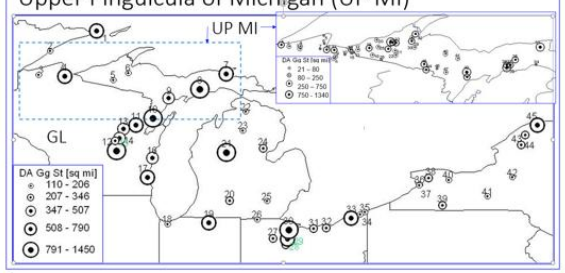
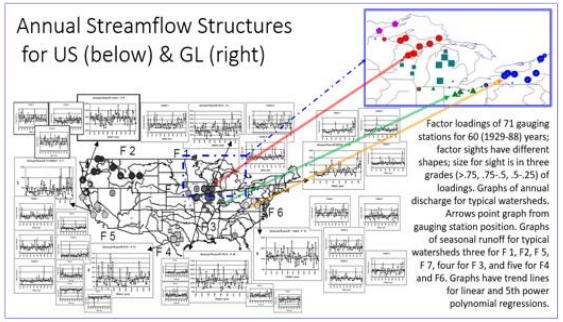
Comprising US Great Lakes Basin, UP of MI & US

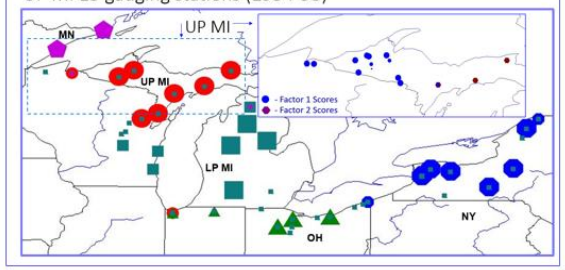
USGS gauging stations for Great Lakes (GL) & Upper Pinguicula of Michigan (UP MI)



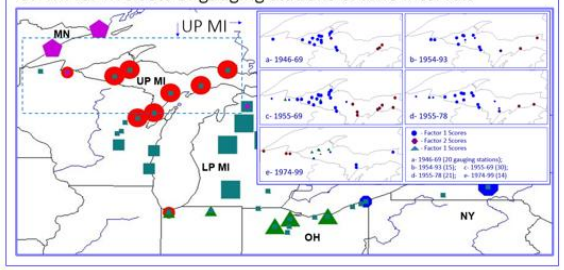
Annual Streamflow Structures for US (below) & GL (right)



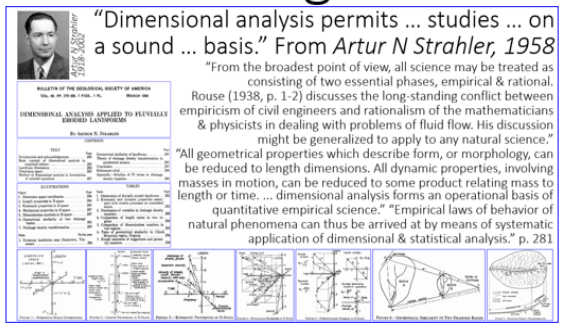
Annual Streamflow Structure for GL (look above) & UP MI 15 gauging stations (1954-93)



Annual Streamflow Structure for GL (look above) & UP MI for five sets of gauging stations & time intervals



Surface–groundwater in Systemic Approach

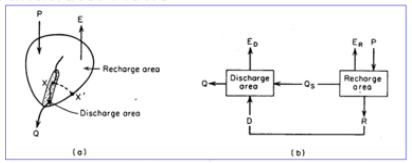


"Dimensional analysis permits … studies … on a sound … basis." From *Artur N Strahler, 1958*

"From the broadest point of view, all science may be treated as consisting of two essential phases, empirical & rational. Rouse (1938, p. 1-2) discusses the long-standing conflict between empiricism of civil engineers and rationalism of the mathematicians & physicists in dealing with problems of fluid flow. His discussion might be generalized to apply to any natural science."

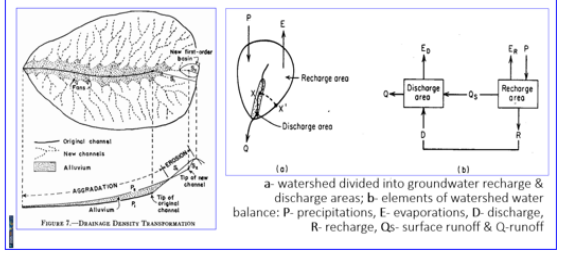
"All geometrical properties which describe form, or morphology, can be reduced to length dimensions. All dynamic properties, involving masses in motion, can be reduced to some product relating mass to length or time. … dimensional analysis forms an operational basis of quantitative empirical science." "Empirical laws of behavior of natural phenomena can thus be arrived at by means of systematic application of dimensional & statistical analysis." p. 281

Components of Watershed Water Balance with Groundwater Flows



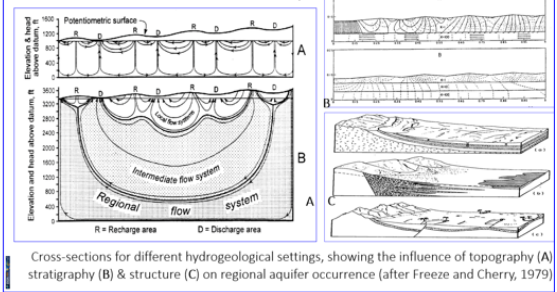
Watershed divided into recharge & discharge areas; elements of watershed water balance: P- precipitation, E- evapotranspiration, Q- runoff, Q_S- the surface water component of average annual runoff, E_R- the average annual evapotranspiration from recharge area, E_D- the average annual evapotranspiration from discharge area, R- the average annual ground water recharge, D- the average annual ground water discharge; X-X'- cross-section from shown in (b) - quantitative flow net & recharge-discharge profile in a two-dimensional section across the heterogeneous groundwater basin. *R. Allan Freeze, John A. Cherry , 1979. Groundwater.*

Similarity of Models: Geomorphologic (left) & Hydrogeologic



a- watershed divided into groundwater recharge & discharge areas; b- elements of watershed water balance: P- precipitations, E- evaporations, D- discharge, R- recharge, Q_S- surface runoff & Q-runoff

Watershed in a Landscape



Cross-sections for different hydrogeological settings, showing the influence of topography (A) stratigraphy (B) & structure (C) on regional aquifer occurrence (after Freeze and Cherry, 1979)

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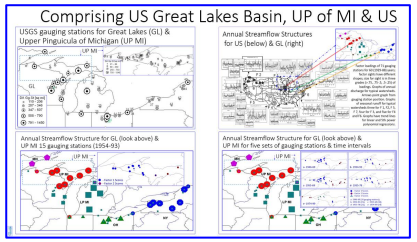
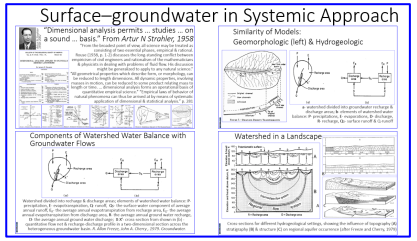
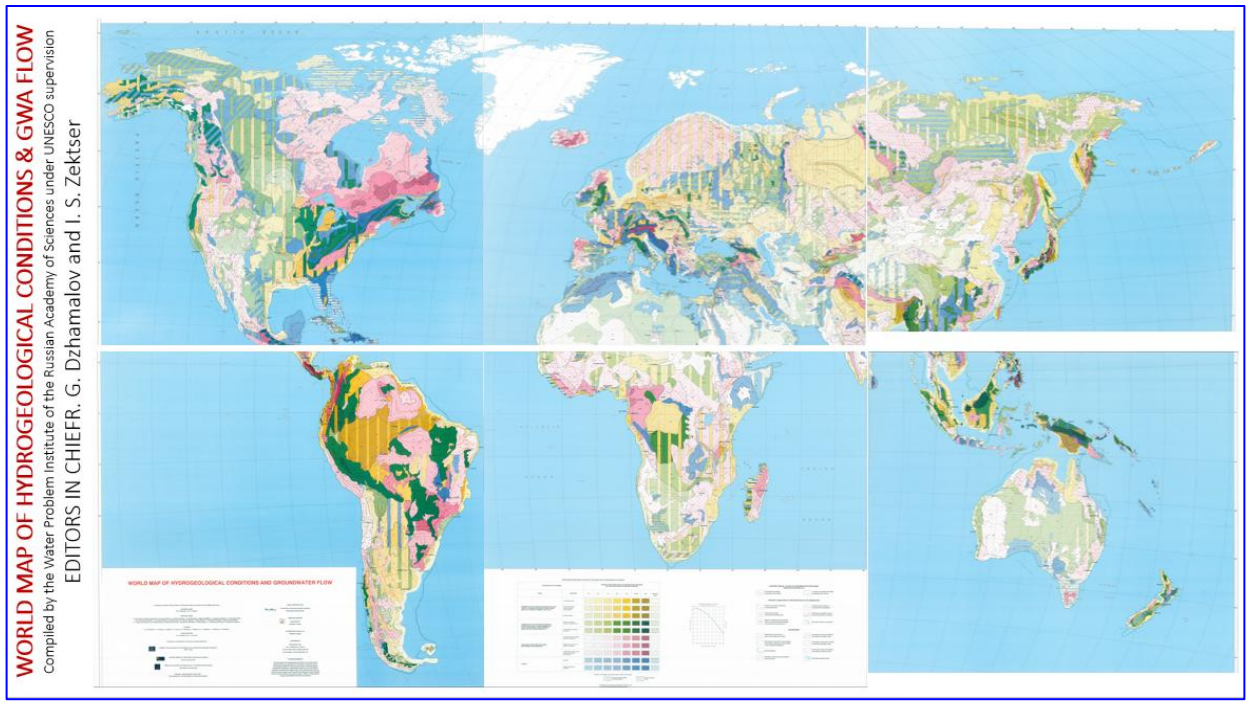
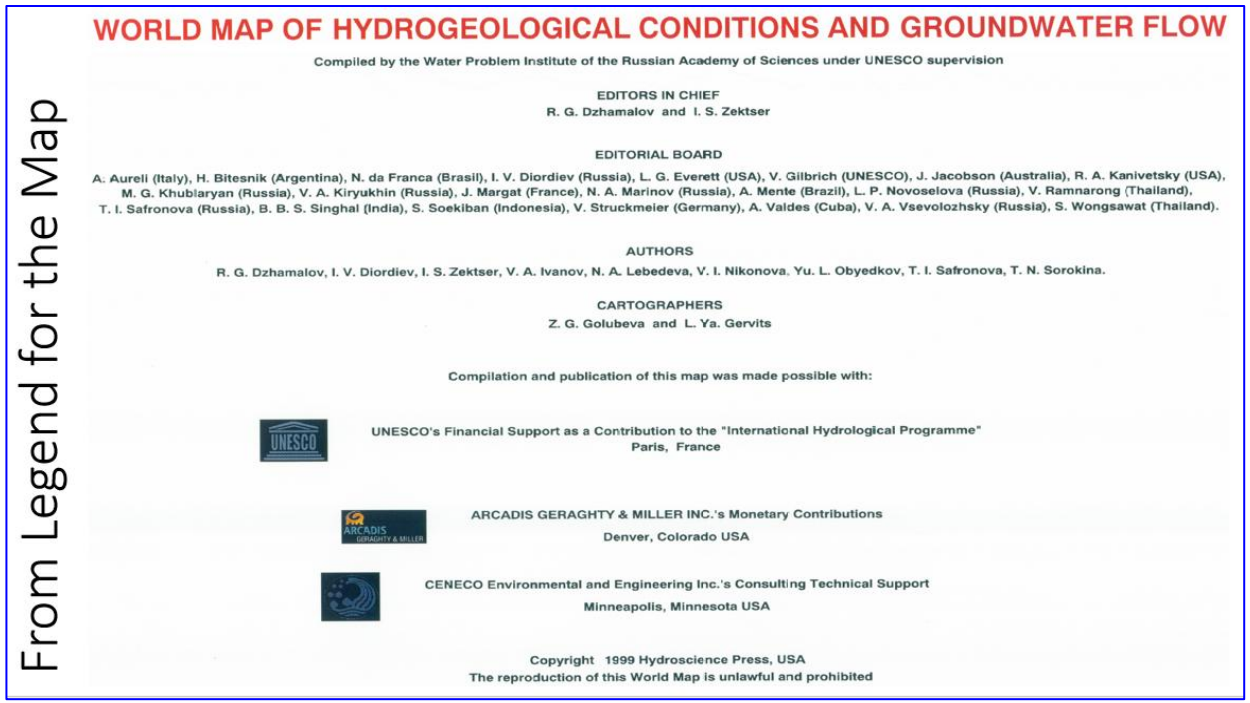
- Rooted in multiscale cartography
- Surface–groundwater interactions in the Systemic Approach
- Regional/global water-resource assessment









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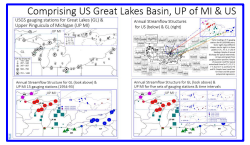

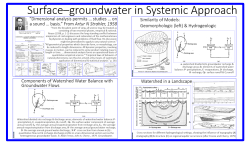

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- Report 50 Years of Hydro(geo)logical Cartography, Gilbrich & Struckmeier, 2014

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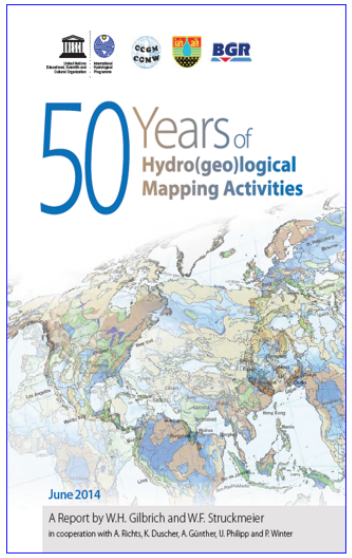
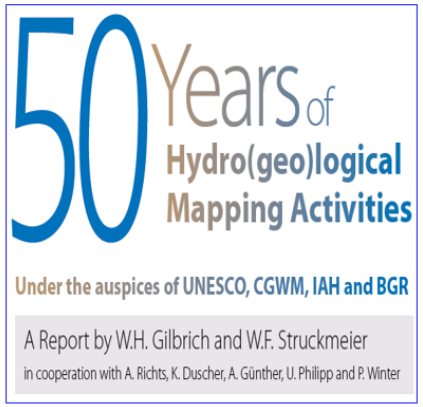
Report 50 Years of Hydro(geo)logical Cartography, Gilbrich & Struckmeier, 2014

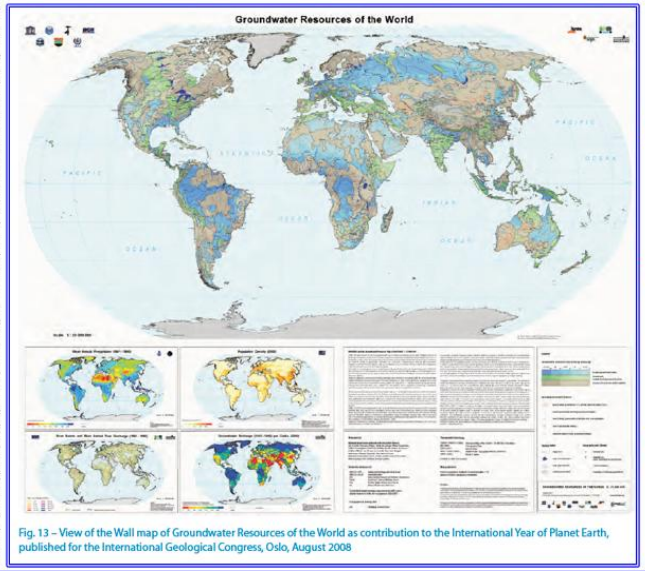


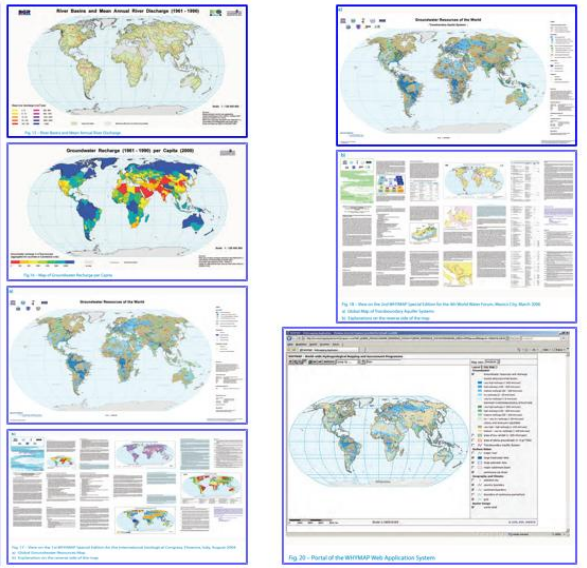


From the Report: "People world-wide are using hydrological maps, often much simplified, for the transfer of knowledge from scientists to politicians & to the general public, because maps deliver an idea of hydrological processes & of trends much better than only numerical presentations."

"Only true information is politically correct & in this spirit UNESCO & the cooperating organizations & associations have tried to deliver the tools needed. UNESCO & its Partners are looking back on a fifty years work & they know that this work never can & will end."

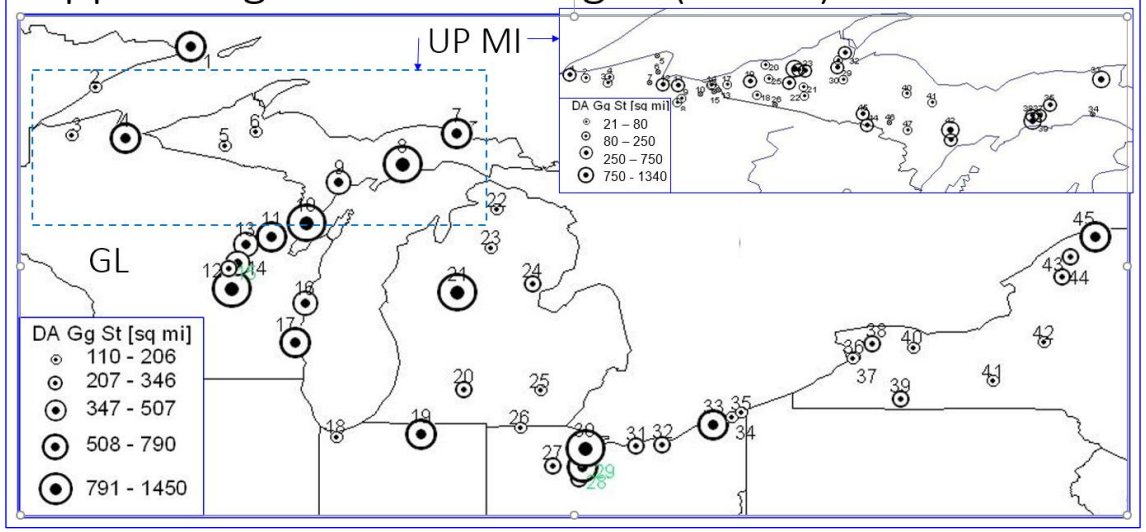
The subsequent maps & diagrams presented in the Report served as generalizations & an information system



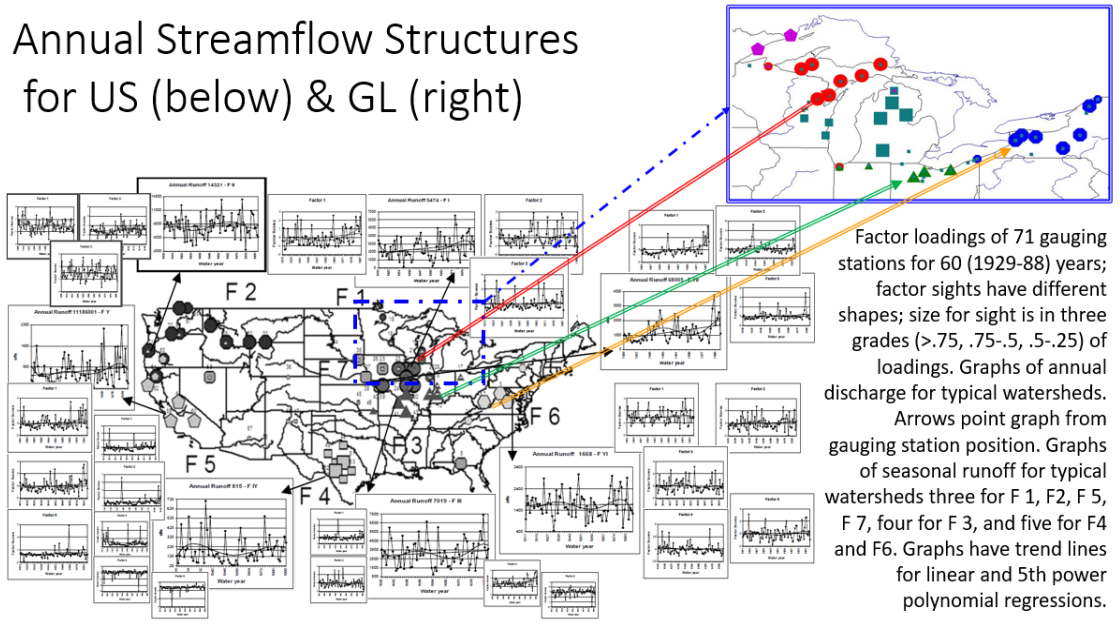


Comprising US Great Lakes Basin, UP of MI & US

USGS gauging stations for Great Lakes (GL) & Upper Pinguicula of Michigan (UP MI)

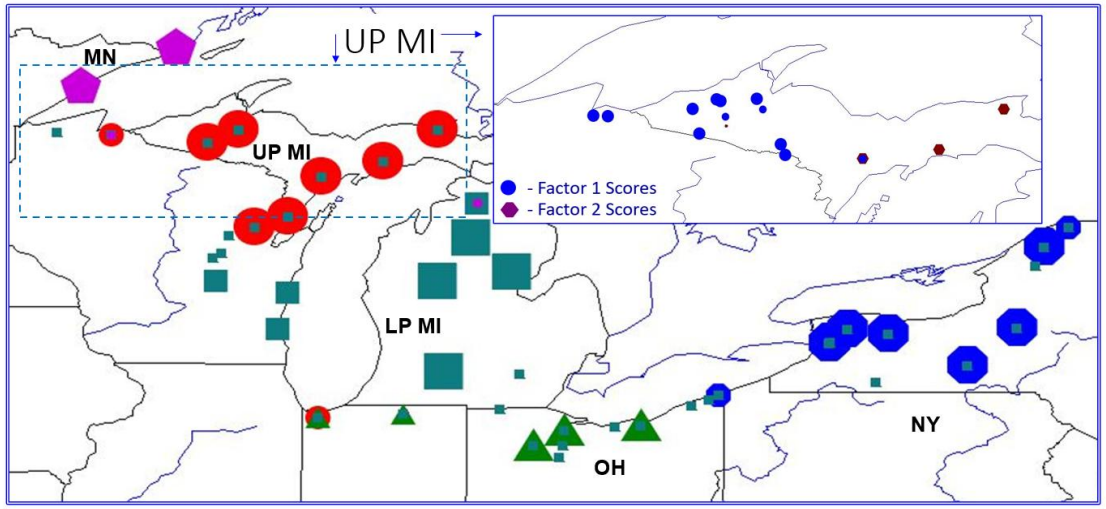


Annual Streamflow Structures for US (below) & GL (right)

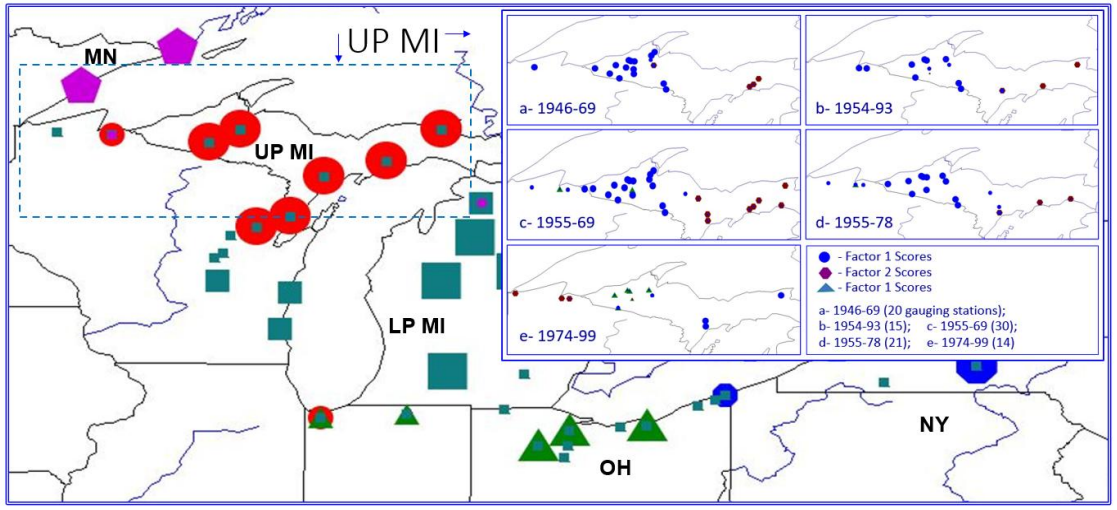


Factor loadings of 71 gauging stations for 60 (1929-88) years; factor sights have different shapes; size for sight is in three grades (>.75, .75-.5, .5-.25) of loadings. Graphs of annual discharge for typical watersheds. Arrows point graph from gauging station position. Graphs of seasonal runoff for typical watersheds three for F 1, F2, F 5, F 7, four for F 3, and five for F4 and F6. Graphs have trend lines for linear and 5th power polynomial regressions.

Annual Streamflow Structure for GL (look above) & UP MI 15 gauging stations (1954-93)



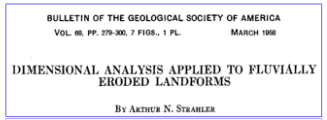
Annual Streamflow Structure for GL (look above) & UP MI for five sets of gauging stations & time intervals



Surface–groundwater in Systemic Approach

Artur N Strahler
1918-2002

"Dimensional analysis permits … studies … on a sound … basis." From *Artur N Strahler, 1958*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DIMENSIONAL ANALYSIS APPLIED TO FLUVIALLY ERODED LANDFORMS

By Arthur N. Strahler

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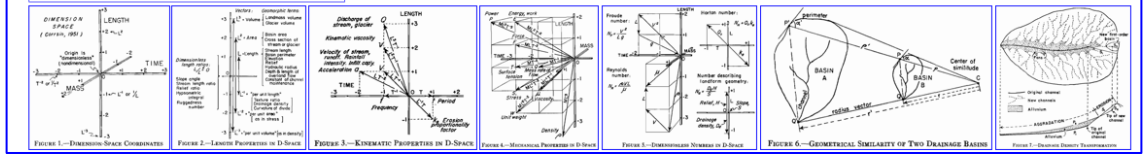


FIGURE 1.—DIMENSION-SPACE COORDINATES. FIGURE 2.—LENGTH PROPERTIES IN D-SPACE. FIGURE 3.—KINEMATIC PROPERTIES IN D-SPACE. FIGURE 4.—MECHANICAL PROPERTIES IN D-SPACE. FIGURE 5.—DIMENSIONLESS NUMBERS IN D-SPACE. FIGURE 6.—GEOMETRICAL SIMILARITY OF TWO DRAINAGE BASINS. FIGURE 7.—DRAINAGE DENSITY TRANSFORMATION.

Similarity of Models: Geomorphologic (left) & Hydrogeologic

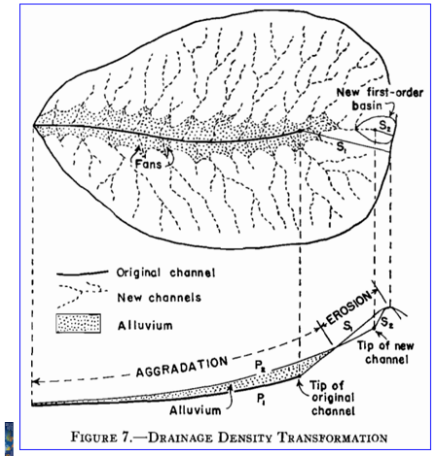
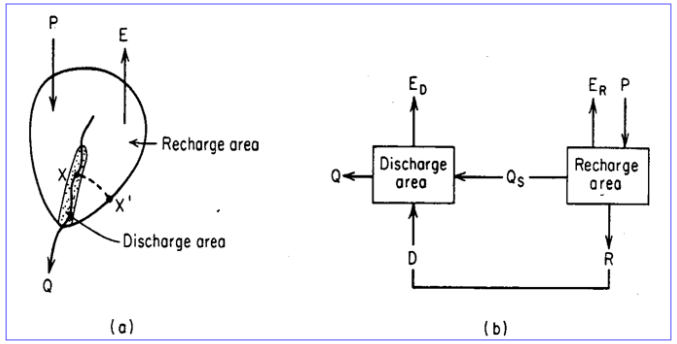
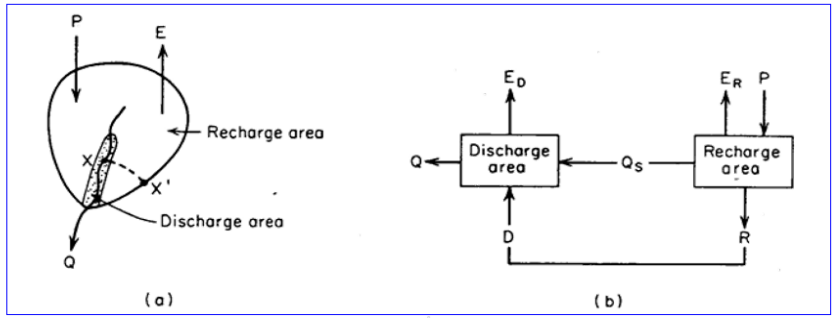


FIGURE 7.—DRAINAGE DENSITY TRANSFORMATION



a- watershed divided into groundwater recharge & discharge areas; **b**- elements of watershed water balance: P- precipitations, E- evaporations, D- discharge, R- recharge, Qs- surface runoff & Q-runoff

Components of Watershed Water Balance with Groundwater Flows



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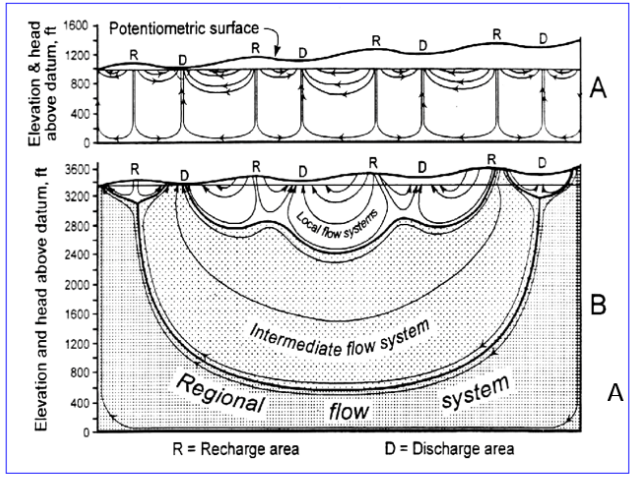
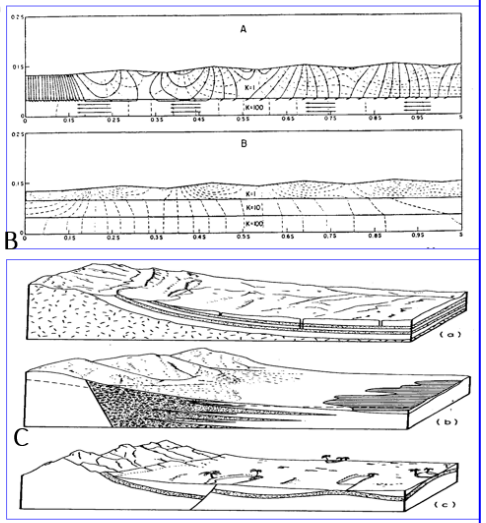
Watershed in a Landscape





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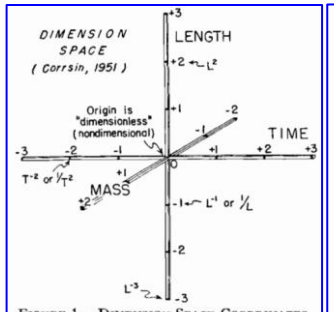

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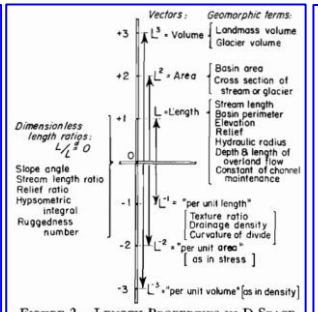
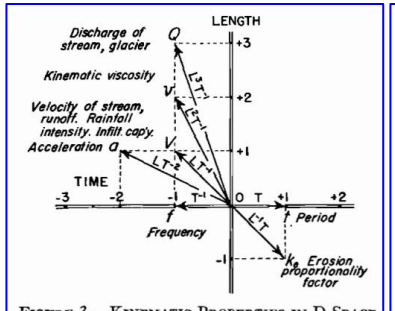


FIGURE 2.—LENGTH PROPERTIES IN D-SPACE



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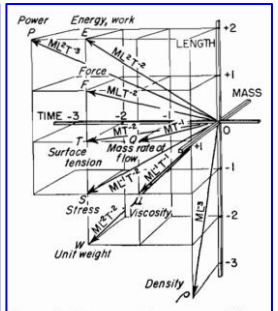

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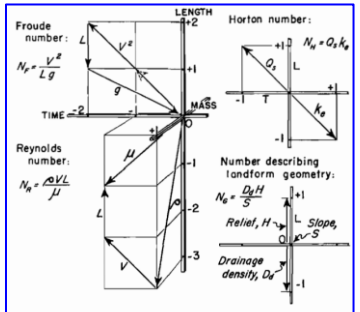


FIGURE 5.—DIMENSIONLESS NUMBERS IN D-SPACE

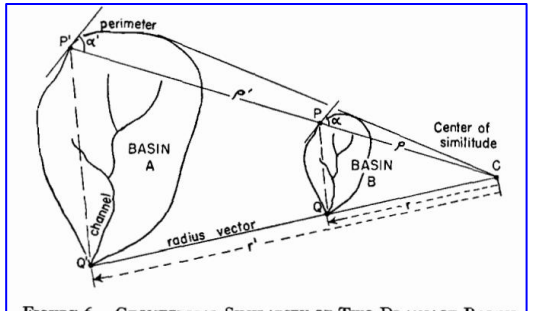
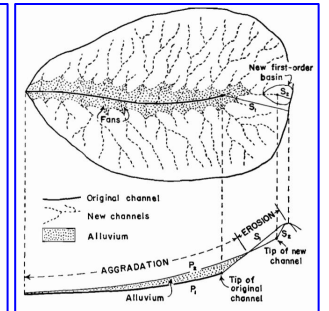


FIGURE 6.—GEOMETRICAL SIMILARITY OF TWO DRAINAGE BASINS



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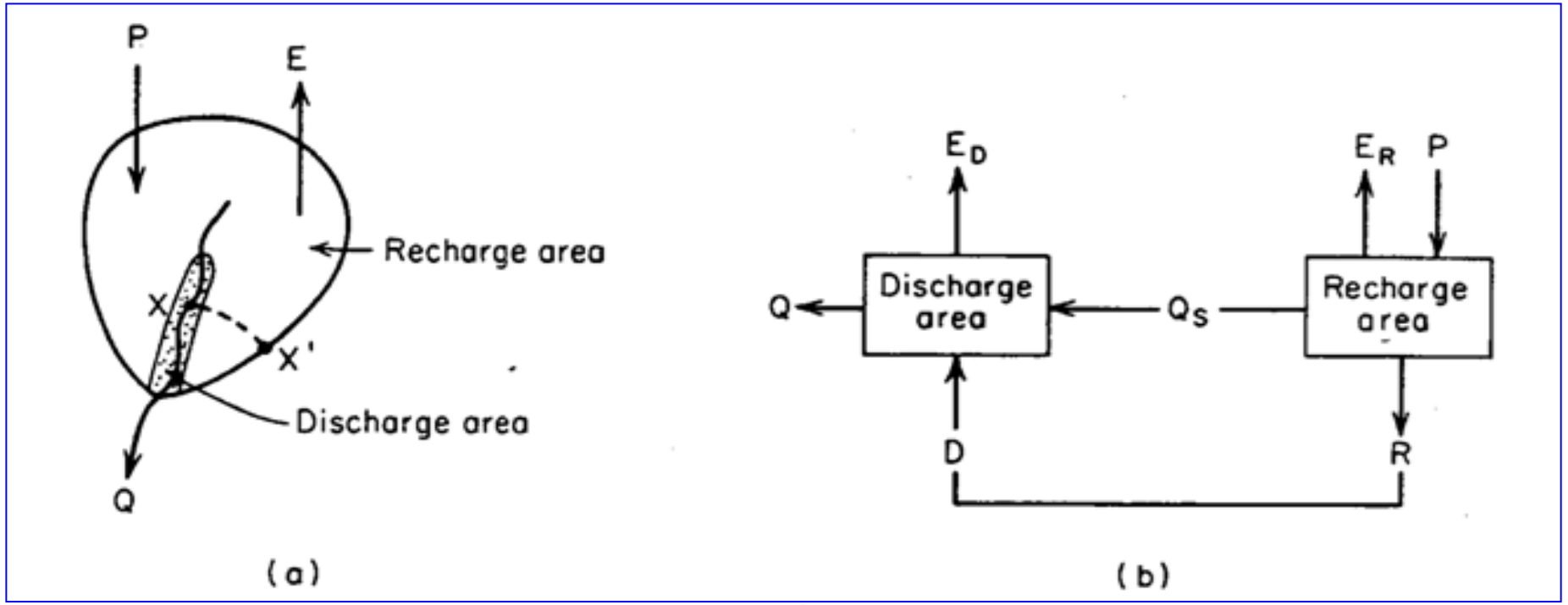

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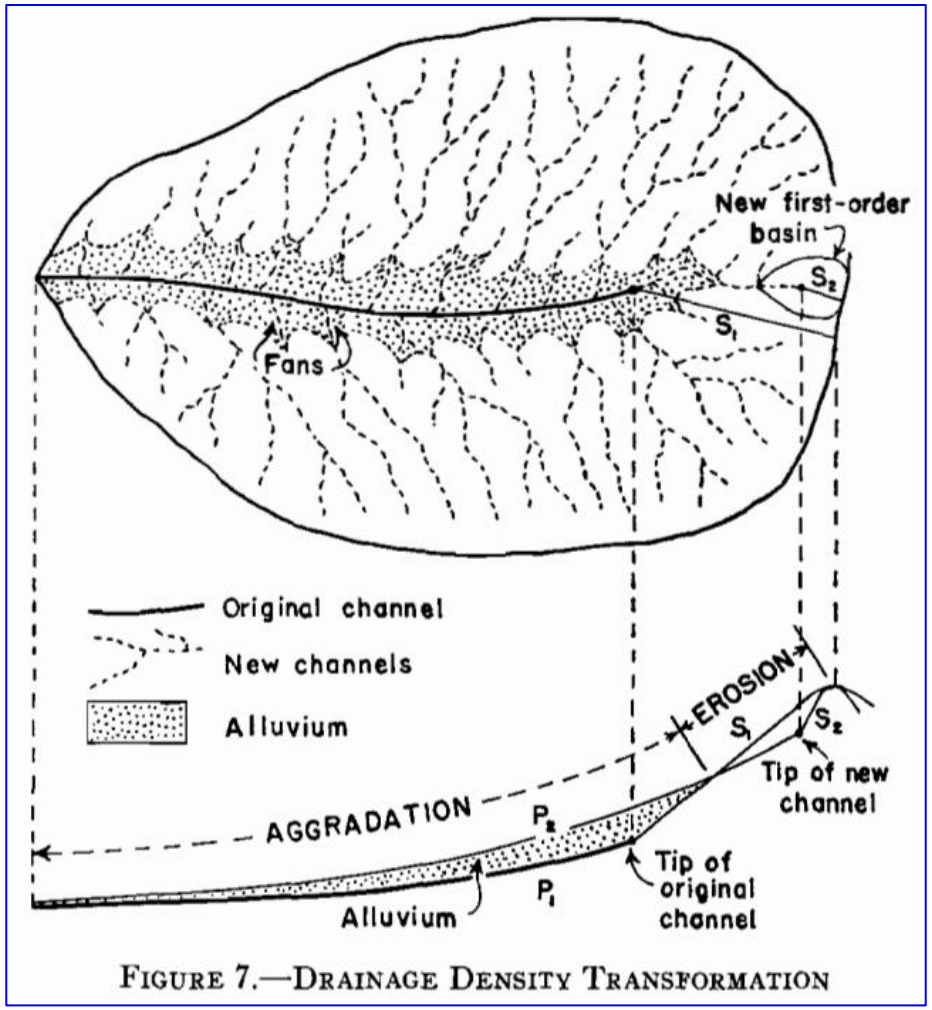

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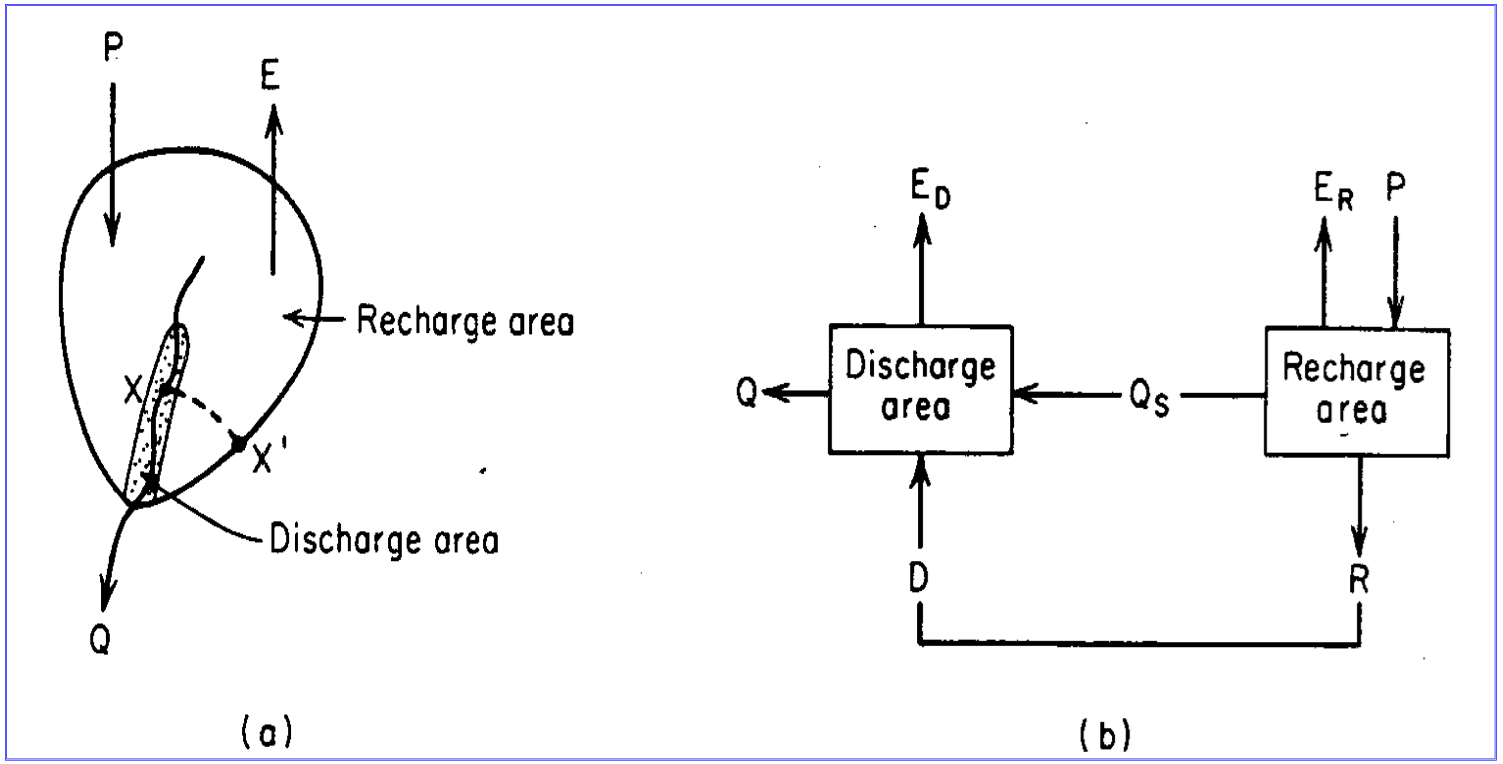

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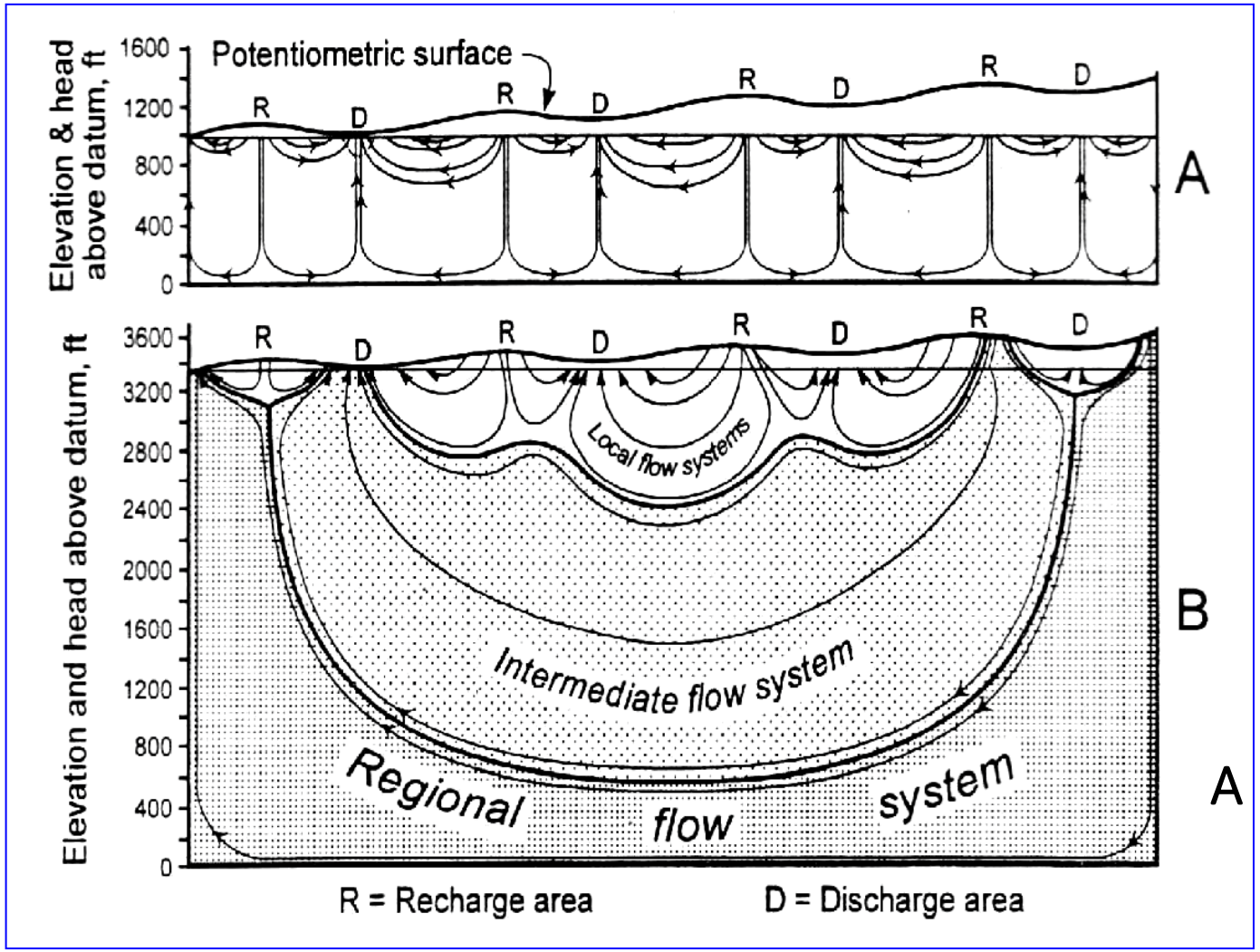

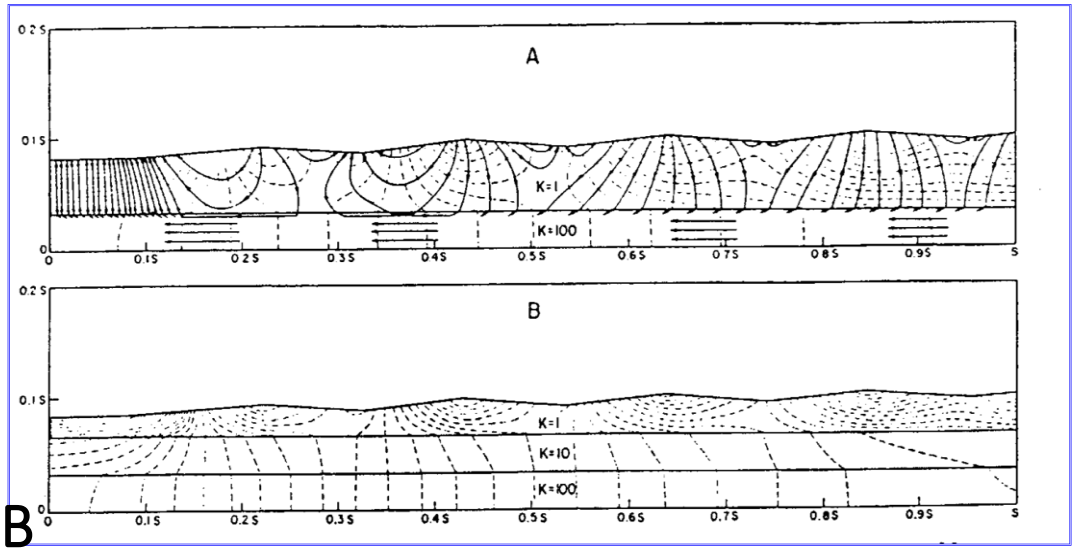

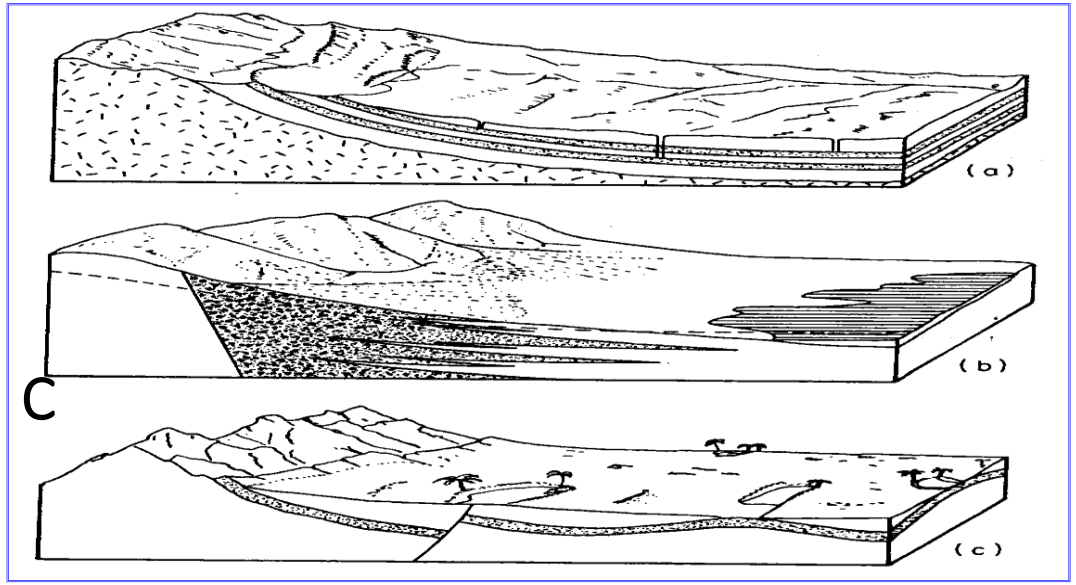

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WORLD MAP OF HYDROGEOLOGICAL CONDITIONS & GWA FLOW

Compiled by the Water Problem Institute of the Russian Academy of Sciences under UNESCO supervision

EDITORS IN CHIEFR. G. Dzhamalov and I. S. Zektser

From Legend for the Map

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Compiled by the Water Problem Institute of the Russian Academy of Sciences under UNESCO supervision

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CARTOGRAPHERS

Z. G. Golubeva and L. Ya. Gervits

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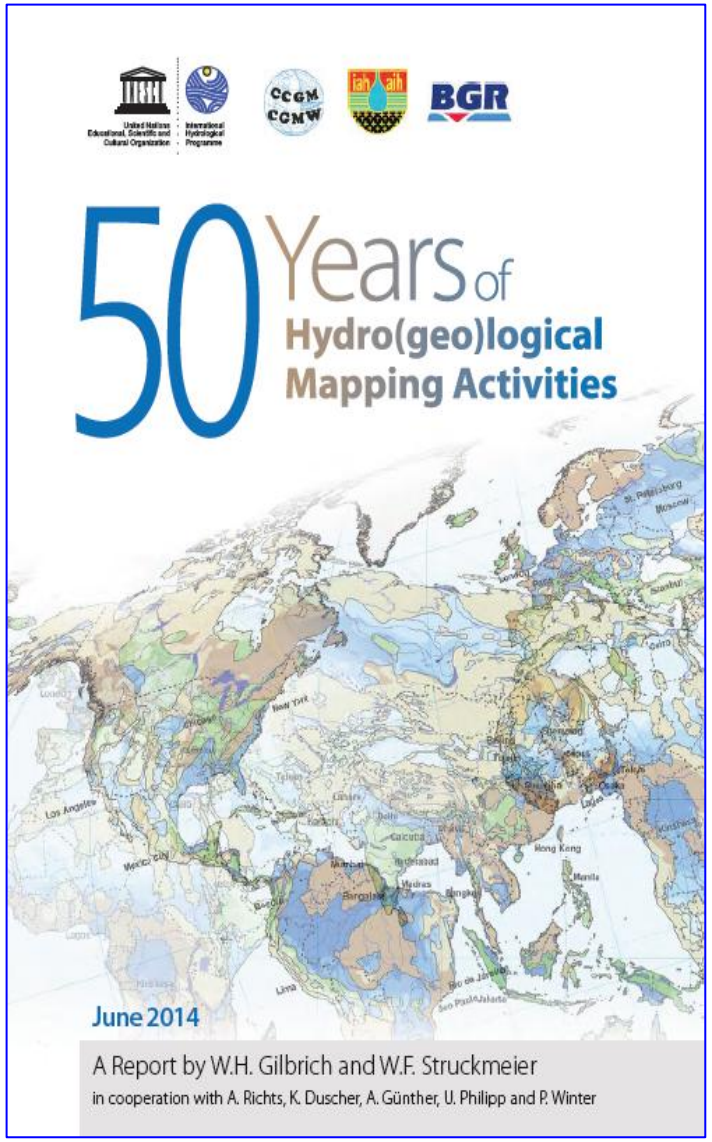
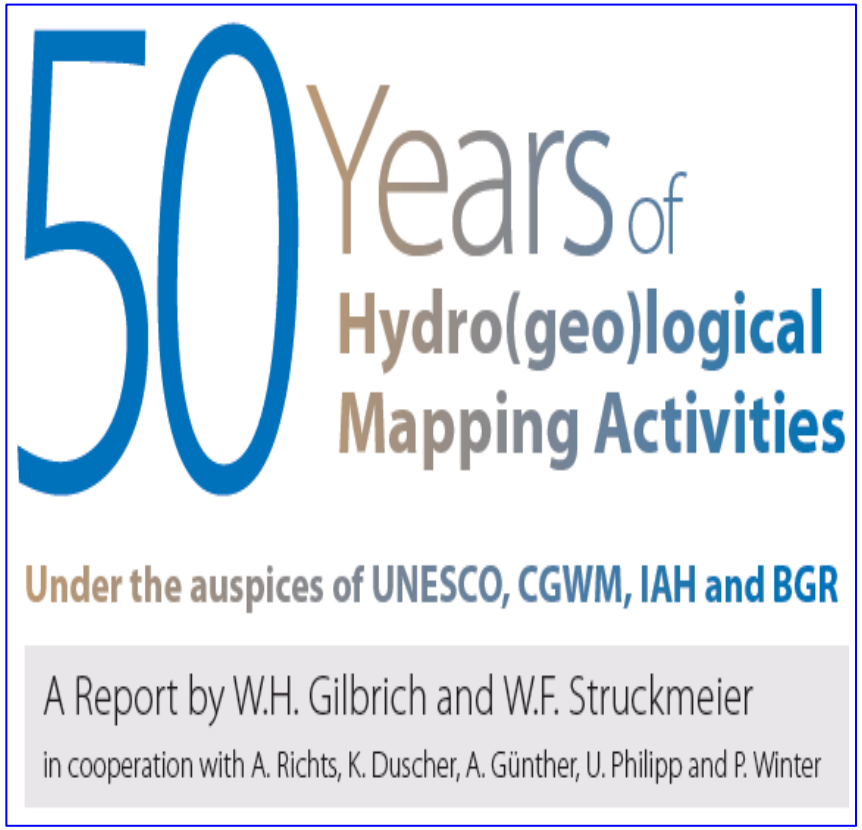
Report 50 Years of Hydro(geo)logical Cartography, Gilbrich & Struckmeier, 2014



From the Report: "People world-wide are using hydrological maps, often much simplified, for the transfer of knowledge from scientists to politicians & to the general public, because maps deliver an idea of hydrological processes & of trends much better than only numerical presentations."



“Only true information is politically correct & in this spirit UNESCO & the cooperating organizations & associations have tried to deliver the tools needed. UNESCO & its Partners are looking back on a fifty years work & they know that this work never can & will end.”

STAHY'2023 13th International Workshop Statistical Hydrology, 8th - 10th November 2023, Institute for Experiential AI, Northeastern University, Boston, Massachusetts (USA) Thematic Cartography in Hydrology: Ontology & Semantics in the Use of Statistical Learning

Шнягин Б. А. Карты подземного стока: предмет из истории кафедры гидрологии МГУ прошлого века или инструмент познания комплекса наук о Земле. Всерос. Конф. «Ломоносовские чтения» 2024. Секция «Геология». Подсекция Гидрогеология. Главное здание, сектор «А», ауд. 42 а 27 марта, 16:45-18:00

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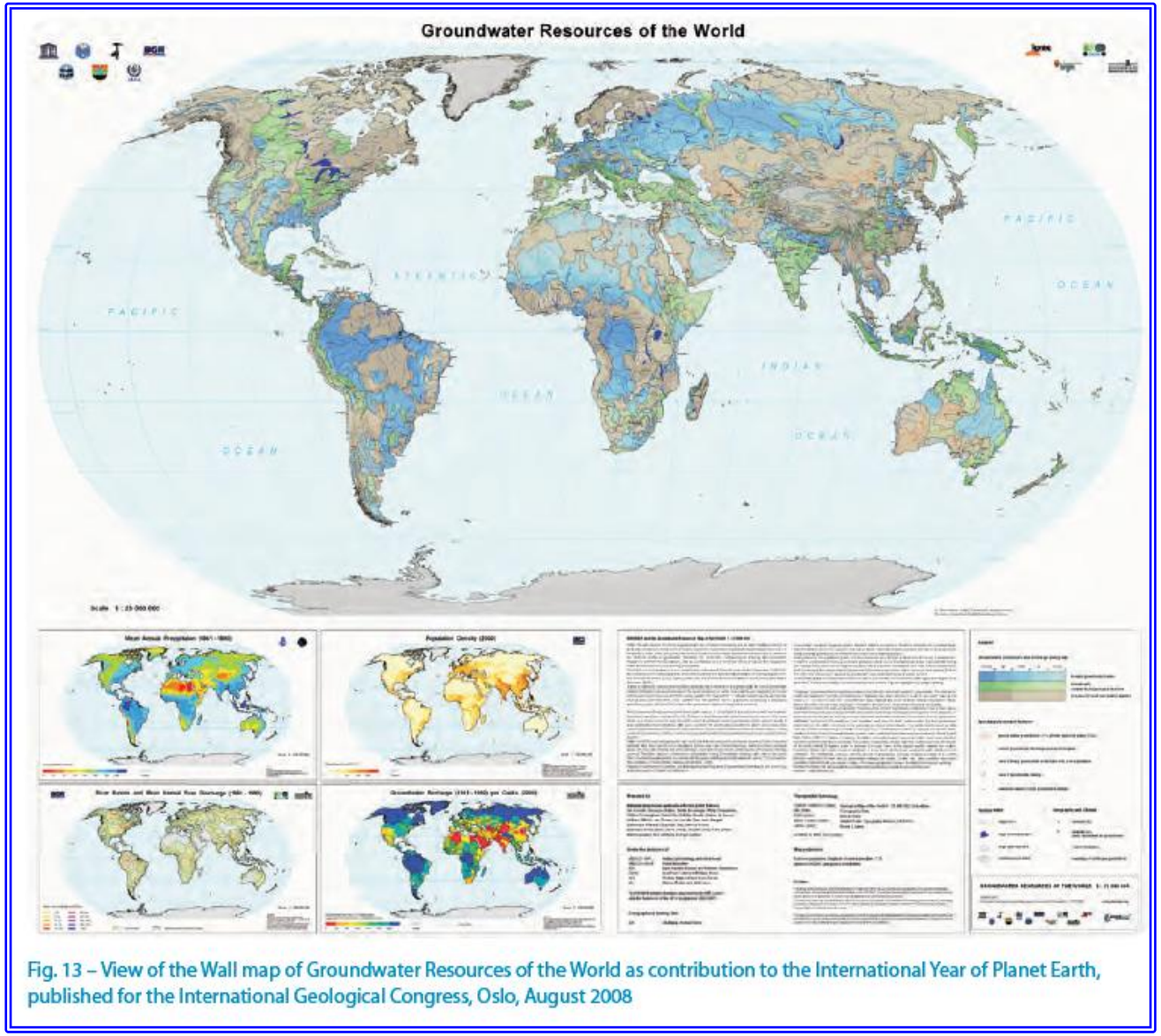


Fig. 13 – View of the Wall map of Groundwater Resources of the World as contribution to the International Year of Planet Earth, published for the International Geological Congress, Oslo, August 2008

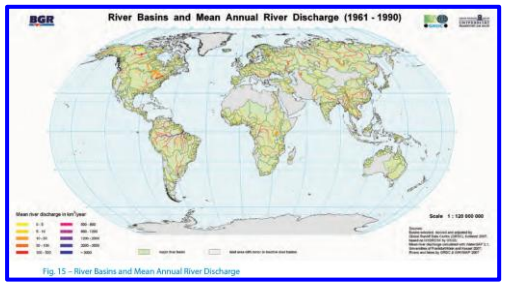


Fig. 15 – River Basins and Mean Annual River Discharge

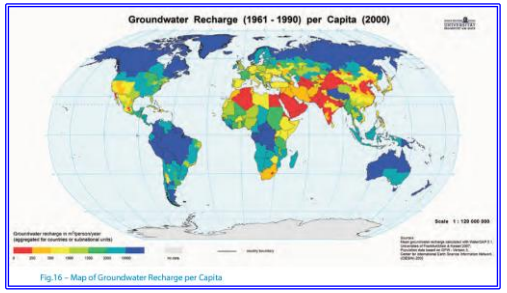
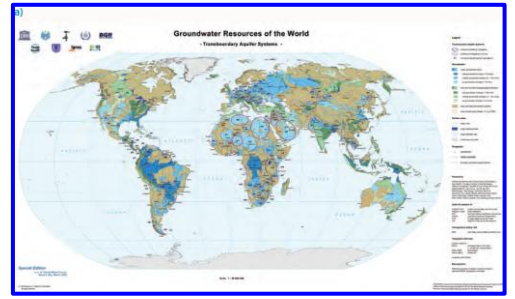


Fig.16 – Map of Groundwater Recharge per Capita

Fig. 17 – View on the 1st WHYMAP Special Edition for the International Geological Congress, Florence, Italy, August 2004
a) Global Groundwater Resources Map
b) Explanation on the reverse side of the map



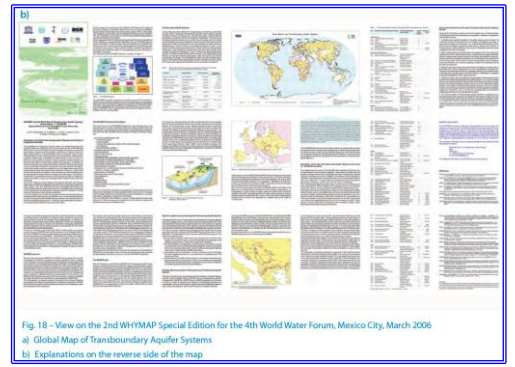


Fig. 18 – View on the 2nd WHYMAP Special Edition for the 4th World Water Forum, Mexico City, March 2006
a) Global Map of Transboundary Aquifer Systems
b) Explanations on the reverse side of the map

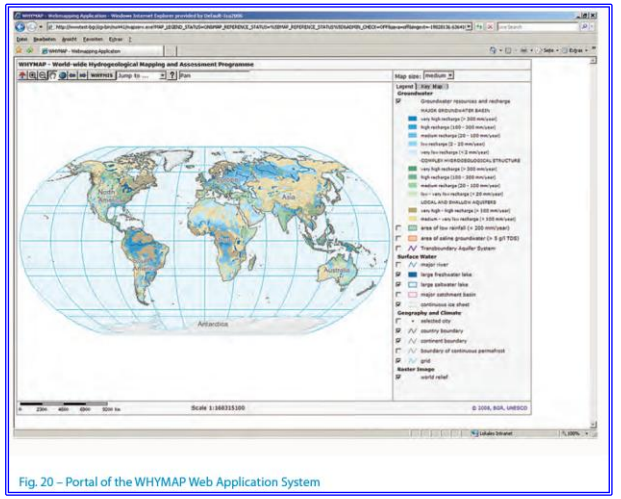


Fig. 20 – Portal of the WHYMAP Web Application System

10 – Hydrology: Stochastic Hydrology

- Variability, randomness, long-memory processes
- (Klemeš; Koutsoyiannis)

Scientific Hydrology & Stochastic Hydrology are two traditions.

These traditions evolved in parallel, but not together.

Knowledge & Uncertainty in Hydrology

Vit Klemeš (1932-2010)

"… one must begin with the fact that while the basis of all our knowledge is, in the last analysis, empirical, the knowledge itself is not merely a sum of empirical facts but emerges from the ability of the human mind to discover relationships among these facts." p. 95

"Because of the extreme complexity of the environment in which hydrologic processes evolve & the fragmentary (both time & space-wise) information that hydrologic data normally contain, empirical relationships based on these data usually can give only approximate results." p 98

Klemeš, V. (1982) Empirical & causal models in hydrology. In: Scientific Basis of Water Resource Management. NAS, Washington, DC.

"The study of hydrological uncertainties, & eventually the development of the discipline referred to as stochastic hydrology, has been stimulated by water management's need to make decisions involving stochastic hydrological variables. It is pointed out that, in spite of this initial stimulus, stochastic hydrology is a discipline essentially independent of water management, & that a hydrological uncertainty need not necessarily be important for a water management decision & may carry a different degree of importance in different contexts." p. 175

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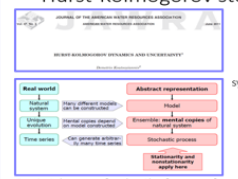
A (epistolary) discussion regarding two distinct

approaches to hydrology: a natural-scientific description of the dynamics of the runoff shell within the hydrosphere (runoff maps), versus an approach tailored to the anthroposphere (the design of structures & their protection)—including, in both cases, the associated time series.

Model by Prof D. Koutsoyiannis:
Hurst-Kolmogorov stochastic dynamics



"The nonstatic, ever changing hydroclimatic processes are often described as nonstationary. … the notions of stationarity and nonstationarity, defined within stochastics, suggests that claims of nonstationarity cannot stand… … change is not synonymous with nonstationarity, & even prominent change at a multitude of time scales, small & large, can be described satisfactorily by a stochastic approach admitting stationarity." "The stationary description with Hurst-Kolmogorov stochastic dynamics demonstrates that nonstationary & classical stationary descriptions underestimate the uncertainty." "… using examples of hydro-meteorological time series … show … the consistency of the Hurst-Kolmogorov approach with reality."

Fig. 2. Schematic for the Clarification of the Notions of Stationarity & Nonstationarity.

Hurst-Kolmogorov stochastic dynamics.
Reality in Engineer's View, from Privet Discussion

Presenter: You refer to reality as a proof of your concept. How do you define "the reality", there is a discussion about it in philosophy for centuries?

Professor Demetris Koutsoyiannis: The reality issue is a big question in philosophy. I use a rather simple approach based on the axioms
(a) there is a real world,
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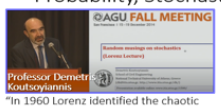
The Models for Hydrologic Process Are Based on Probability, Stochastic Processes & Statistics



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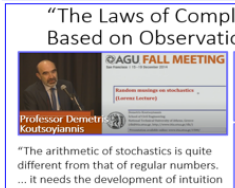
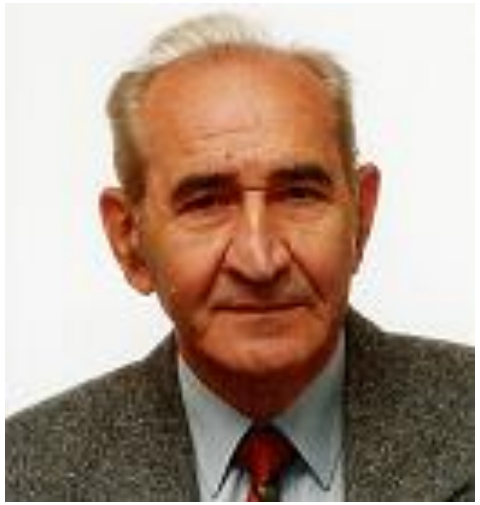
"The Laws of Complex Systems …
Based on Observational Data & Using Statistics"



"The arithmetic of stochastics is quite different from that of regular numbers. … it needs the development of intuition & interpretations which differ from those built upon deterministic considerations. Using stochastic tools in a deterministic context may result in mistaken conclusions. "

Knowledge & Uncertainty in Hydrology



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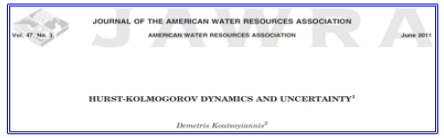
JOURNAL OF THE AMERICAN WATER RESOURCES ASSOCIATION
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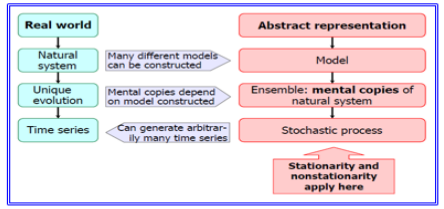
HURST-KOLMOGOROV DYNAMICS AND UNCERTAINTY¹
Demetris Koutsoyiannis²



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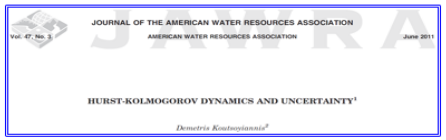
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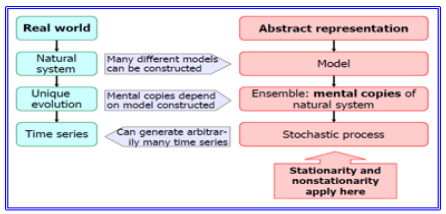

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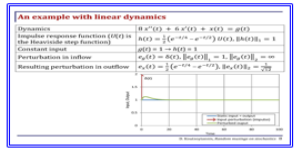

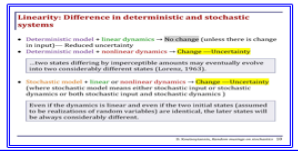

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посвященном 100-летию со дня рождения профессора Даниила Яковлевича РАТКОВИЧА
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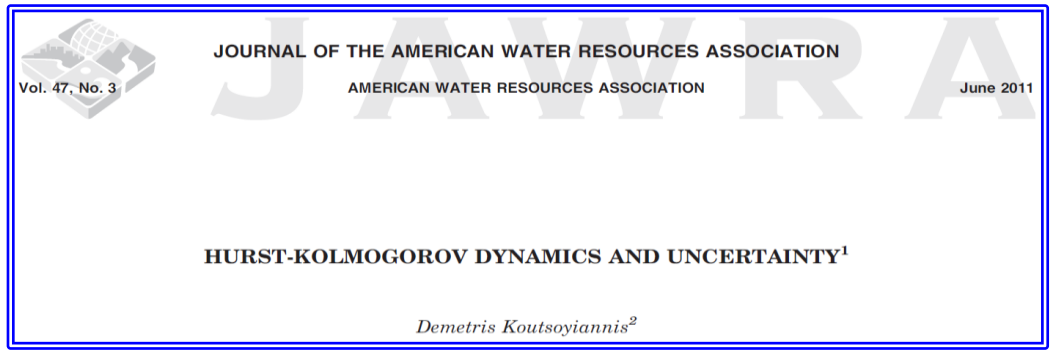

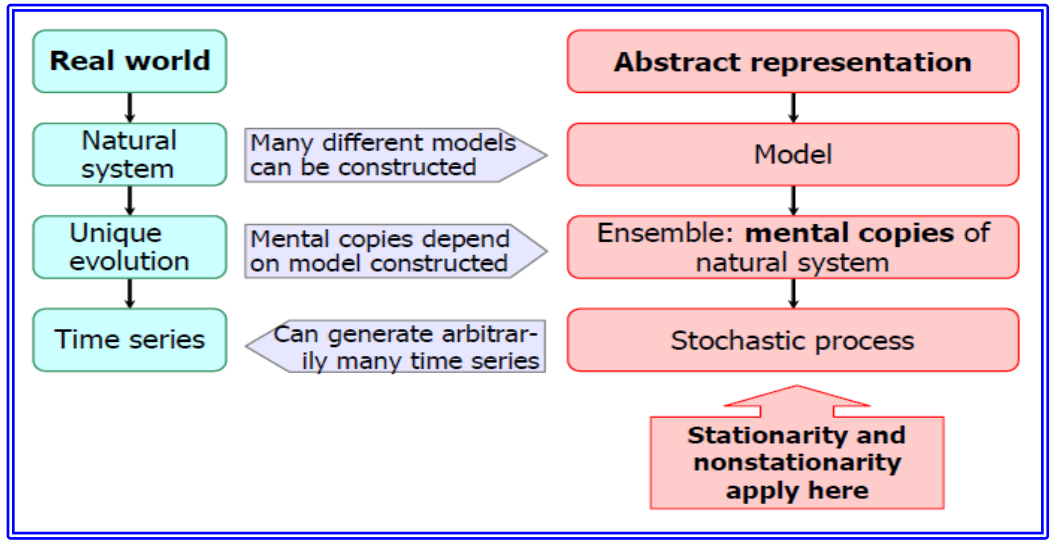

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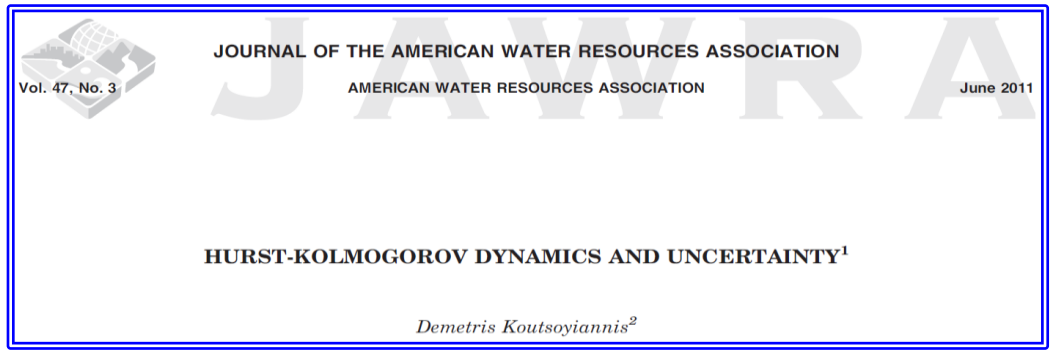

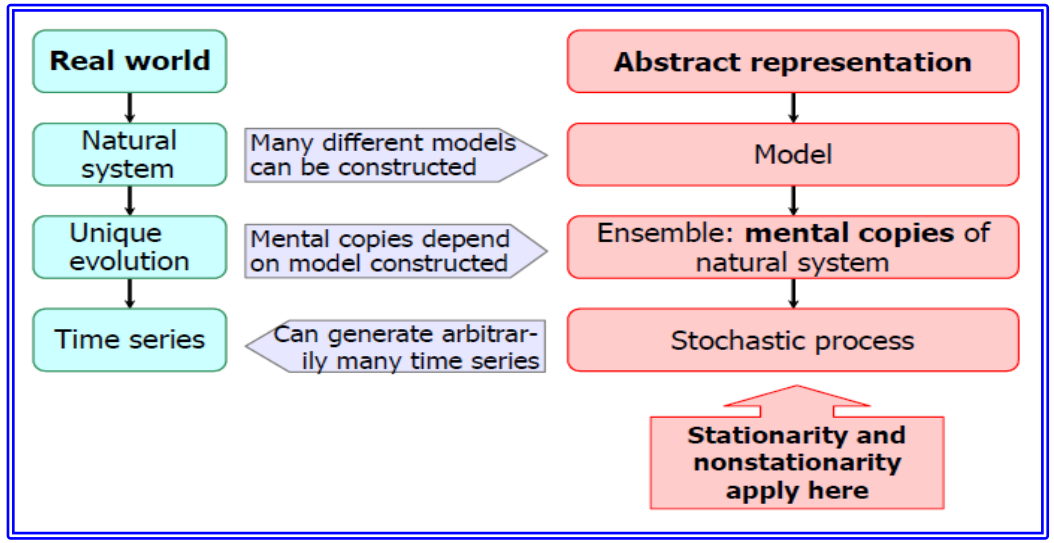

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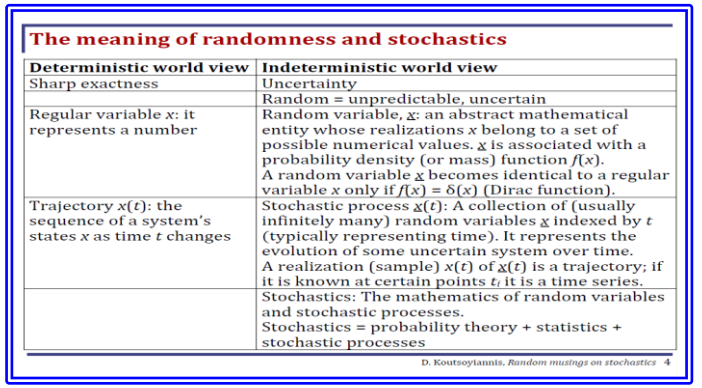
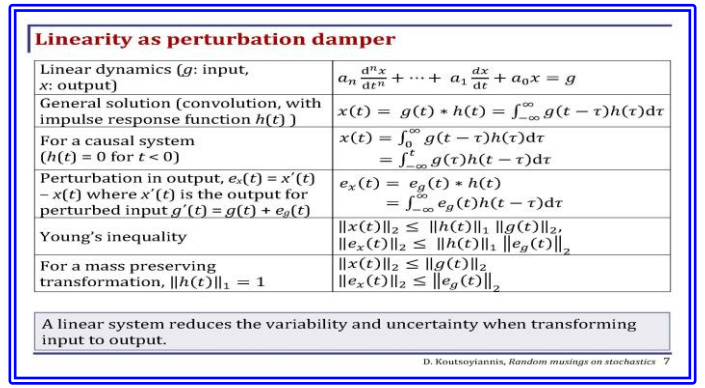
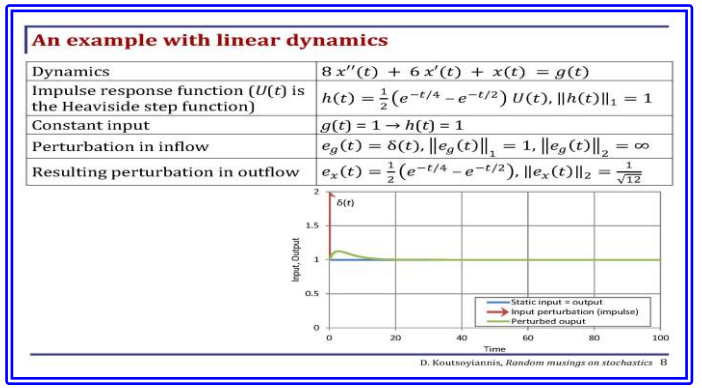
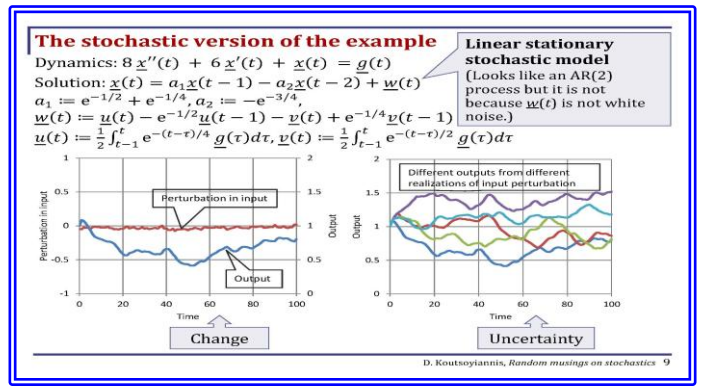
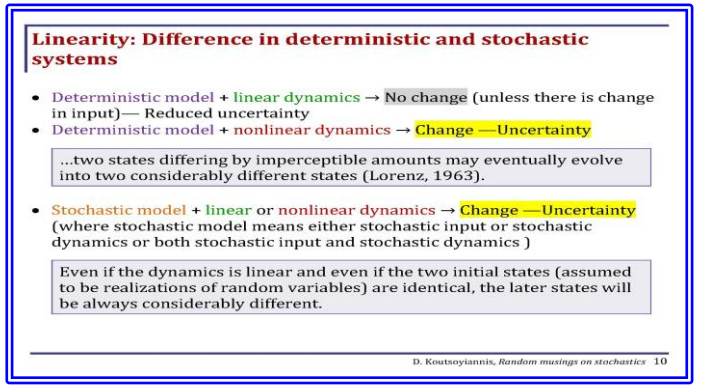
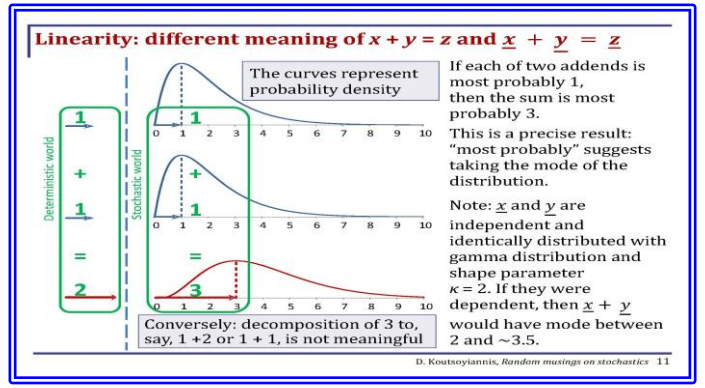








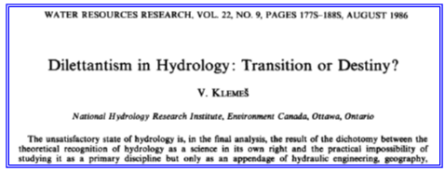




11 – Toward a Unified Structural Hydrology

- In Search for Hydrology as a Natural Science

In Search for Hydrology as a Natural Science



WATER RESOURCES RESEARCH, VOL. 22, NO. 9, PAGES 177S–188S, AUGUST 1986

Dilettantism in Hydrology: Transition or Destiny?

V. KLEMEŠ

National Hydrology Research Institute, Environment Canada, Ottawa, Ontario

The unsatisfactory state of hydrology is, in the final analysis, the result of the dichotomy between the theoretical recognition of hydrology as a science in its own right and the practical impossibility of studying it as a primary discipline but only as an appendage of hydraulic engineering, geography,

"The **unsatisfactory state of hydrology** is … **the result of the dichotomy between the theoretical recognition of hydrology as a science in its own right & the practical impossibility of studying it as a primary Discipline** but only as an appendage of hydraulic engineering, geography, geology, etc. … This state of affairs often paralyzes hydrologists' ability to differentiate between hydrology & water management, hydrology & statistics, facts & assumptions, science & convenience etc., with consequent dangers both to scientific development of hydrology & to its practical utility. The danger increases with the proliferation of computerized "hydrologic" models, whose cheaply arranged ability to fit data."

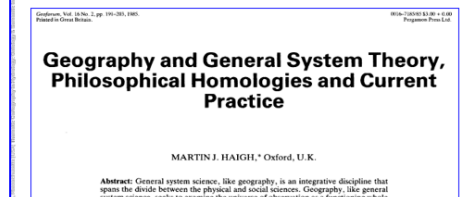
"These points are illustrated, among other things, by discussion of flood frequency analysis."

"However, most important of all is the realization that calling something hydrology does not necessarily make it hydrology. Without recognizing this, hydrology cannot be cured of its present dilettantism of which misconceptions are only the symptoms. Clarity in these matters will make the difference between institutionalizing dilettantism in hydrology & overcoming it."

11 – Toward a Unified Structural Hydrology

- In Search for Hydrology as a Natural Science
- The Theory in Hydrology & General System Theory

The Geography & General System Theory (GST)



Geography and General System Theory, Philosophical Homologies and Current Practice

MARTIN J. HAIGH,* Oxford, U.K.

Abstract: General system science, like geography, is an integrative discipline that spans the divide between the physical and social sciences. Geography, like general system science, seeks to examine the universe of observation as a functioning whole and attempts to study together the things other disciplines study separately. However, within both geography and the system sciences at large, there are many different philosophies and methodologies. A special attribute of the general system approach is an explicit adoption of an organismic rather than a mechanistic world view. This has caused general system practitioners to develop theory in more bioscientific areas of concern such as growth, hierarchical organisation and the theory of evolution. To date, much of the systems science in geography has preferred the static, mechanistic ethos of systems analysis and systems engineering. However, general system science's concern with historic processes and the dynamic self-determined relationships between systems structure, functioning and self-creation may be more appropriate to geographical research. A search is made for general system methodologies in current geographical research and for particular applications of aspects of the new general systems theory of evolution and theory of systems attractors as defined by Ilya Prigogine and Erich Jantsch to geography.

Today, we seem to be at the tip of an iceberg of scientific change . . . Every discipline is in the midst of a revolution . . . What is exciting about this theoretical chaos is not that each discipline will emerge with a new paradigm to guide future investigations but that a new grand paradigm may be forming, one that will integrate all structure and processes from the farthest reaches of the Universe to the reasonances of subatomic particles. The Grand Paradigm is somewhere in the future and we may live to see it THEISEN, (1981, p. 758).

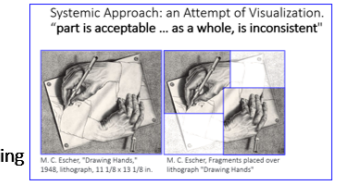
Introduction

Two centuries ago the whole of science could be invested in a single individual, a 'natural philosopher', who could take a holistic view. The proliferation of knowledge and the development of distinct academic disciplines and subdisciplines have fragmented science into hundreds of tiny independent subjects, each working in isolation. Given the fact that the world's scientific literature is increasing at an inexorable 11 13% per year, specialisation has become inevitable. But as the early philosophers of geography, such as HUMBOLDT (1845), were well aware, fragmentation of knowledge limits our understanding of the 'harmonious unity of the cosmos'. In Russia during the 1870s, Dokuchayev battled against further fragmentation, arguing that although the science of the nineteenth century had achieved success in the study of individual objects and phenomena, it had, as a result, lost view of the interrelationships of nature, the study of which was "the best and highest beauty of natural science" (ISACHENKO, 1973, p. 25). Dokuchayev's legacy includes much of modern Soviet thinking in ecology and the distinctive Russian School of landscape science, but his work did little to staunch the general trend of scientific fragmentation.

*Oxford Polytechnic, Oxford, U.K.

To BERTALANFFY (1950, 1969, 1975), founder

Kenneth E Boulding
1910 -1993



Martin J Haigh
1950 -2022

On Knowledge & Science in GST

" Knowledge ... is a function of human organisms & of social organization. Knowledge, that is to say, is always what somebody knows: the most perfect transcript of knowledge in writing is not knowledge if nobody knows it. Knowledge how ever grows by the receipt of meaningful information-that is, by the intake of messages by a knower which are capable of reorganizing his knowledge."

"We will quietly duck the question as to what reorganizations constitute "growth" of knowledge by defining "semantic growth" of knowledge as those reorganizations which can profitably be talked about, in writing or speech, by the Right People."

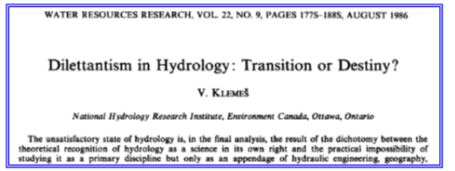
"Science, that is to say, is what can be talked about profitably by scientists in their role as scientists."

Kenneth E. Boulding "General Systems Theory - The Skeleton Of Science" Management Science, 2(3) 1956, pp. 198-199

The Theory in Hydrology & General System Theory

From an Abstract of the Presentation:
Author [Kenneth E Boulding] examines in the book a social function of knowledge, which is created, communicated and accepted individually. It will be challenging to consider for our time of an Informational Age the state of science & practical applications (engineering) for Hydrology. I regard rephrasing the subtitle [of the book Image] as "hydrological knowledge from nature & in society". The task of my presentation is to take the memorable year of 2011 for Missouri River & compound the theory & practice in hydrology in respect of this case of flooding as a natural & manmade event.

In Search for Hydrology as a Natural Science



WATER RESOURCES RESEARCH, VOL. 22, NO. 9, PAGES 177S–188S, AUGUST 1986

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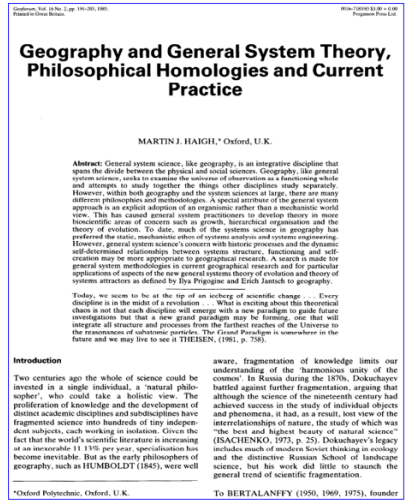
11 – Toward a Unified Structural Hydrology

- In Search for Hydrology as a Natural Science
- The Theory in Hydrology & General System Theory
- A call to unify Science

The Geography & General System Theory (GST)



Geography and General System Theory, Philosophical Homologies and Current Practice

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Abstract: General system science, like geography, is an integrative discipline that spans the divide between the physical and social sciences. Geography, like general system science, seeks to examine the universe of observation as a functioning whole and attempts to study together the things other disciplines study separately. However, within both geography and the system sciences at large, there are many different philosophies and methodologies. A special attribute of the general system approach is an explicit adoption of an organismic rather than a mechanistic world view. This has caused general system practitioners to develop theory in more biocientific areas of concern such as growth, hierarchical organisation and the theory of evolution. To date, much of the systems science in geography has preferred the static, mechanistic ethos of systems analysis and systems engineering. However, general system science's concern with historic processes and the dynamic self-determined relationships between systems structure, functioning and self-creation may be more appropriate to geographical research. A search is made for general system methodologies in current geographical research and for particular applications of aspects of the new general systems theory of evolution and theory of systems attractors as defined by Ilya Prigogine and Erich Jantsch to geography.

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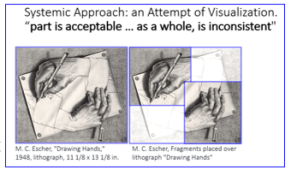

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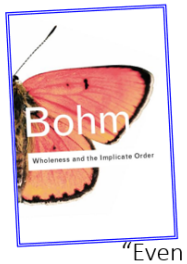

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Wikipedia, 2017

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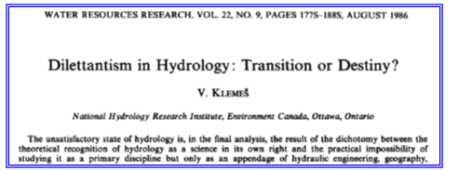

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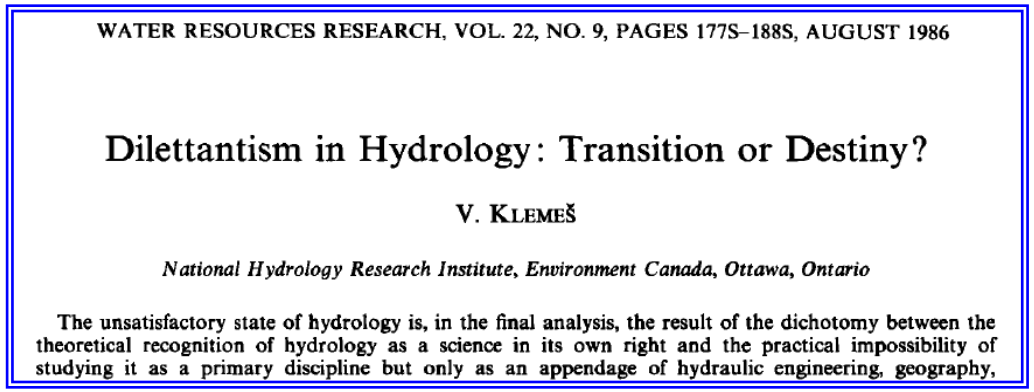

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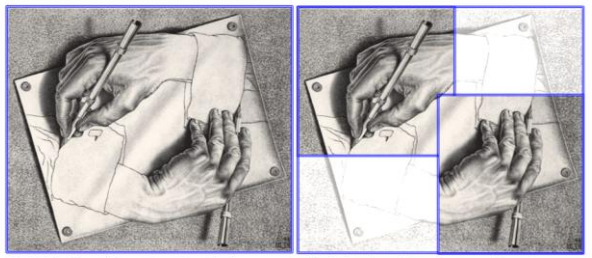

M. C. Escher, "Drawing Hands," 1948, lithograph, 11 1/8 x 13 1/8 in.

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STAHY'2023 13th International Workshop Statistical Hydrology, 8th - 10th November 2023, Institute for Experiential AI, Northeastern University, Boston, Massachusetts (USA) Thematic Cartography in Hydrology: Ontology & Semantic in the Use of Statistical Learning

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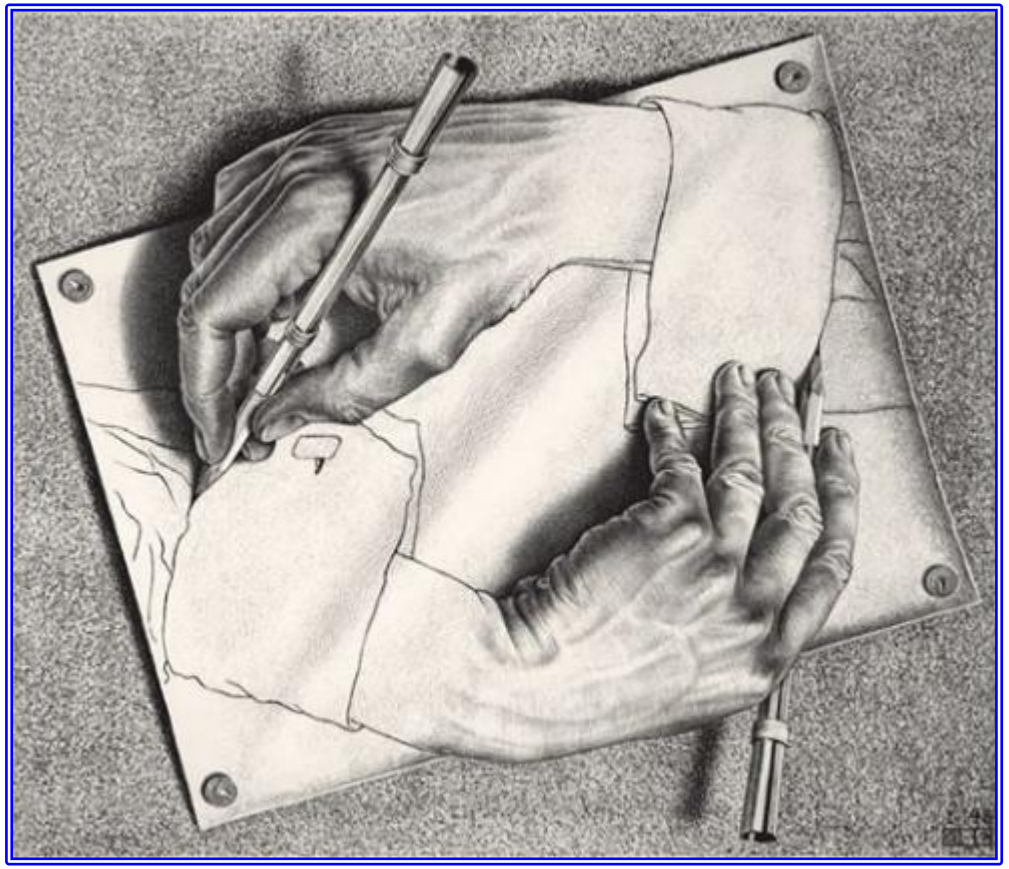

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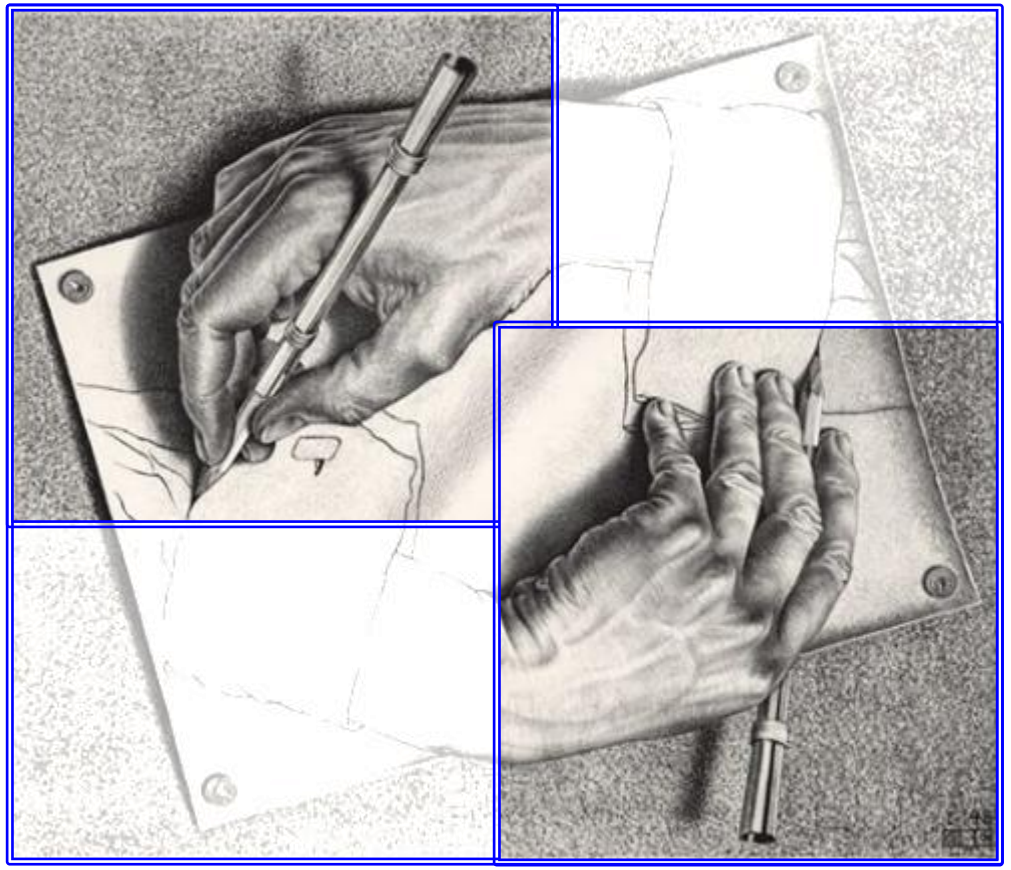

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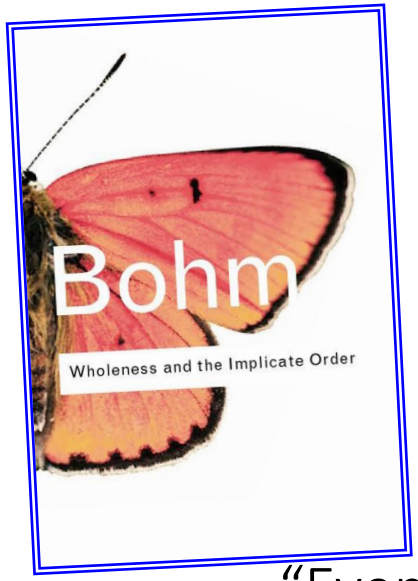

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Wikipedia, 2017

12 – Conclusion

- Presented part of modern natural science with name of Structural Hydrology
- Statistical learning provides a systemic, cybernetic view of the Geosphere
- Fuzzy time-spatial structures capture the essence of variability
- Watersheds serve as powerful units for modeling Geospheric interactions
- This approach opens a path toward unified hydrological science of

Structural Hydrology

Acknowledgments

References are provided in the full version of the slides. Citation practices have undergone significant changes in recent times, largely due to the advent of search engines.
All my published & completed works are available on ResearchGate/Boris Shmagin.

As I bring to a close the resolution of the research challenges that first began to emerge in 1967—during & immediately following my initial fieldwork over the winter holidays (in the settlement of Kusye-Alexandrovsky, Perm Krai)— & which have occupied me continuously ever since, I owe the opportunity to present at this session to a great many individuals: mentors, teachers, friends, colleagues, collaborators, & students—both those still with us & those who have passed away—to whom I extend my sincere gratitude or whom I remember with deep appreciation.

Very glad for the occasion to thank of two European scientists: Demetris Koutsoyiannis & Alberto Montanari, whose willingness & possibility communication with whom the topic will not be completed.

USA. Minnesota, Duluth, Thursday, January 22, 2026, 1:41 PM, -15F

USA. Minnesota, Pequaywan Lake, 30 mi NE from Duluth, Thursday, January 24, 2026, 3:01 PM, -21F

USA, South Dakota, Brookings, Wednesday, January 22, 2014, 5:48 PM, -32F

The Substance of Uncertainty in Systemic Approach: Statistical Learning for Time Series of Geospheres: Natural Scientist's Point of View

Questions?

Vienna, Austria & Online
3–8 May 2026
EGU26
GENERAL ASSEMBLY

Boris A. Shmagin, Ph. D.
Member of the Am. Inst. of Hydrology,
b.shmagin@outlook.com (left)
Professor Nir Krakauer, Ph. D.
at City College of New York NY, New York, USA

On campus of CCNY, NY, NY,
in Dr. David B. Steinman Hall, Aug 18, 2025

An Appendix

A Slide

A slide is introduced as a visualization & a carrier of a "unit of knowledge"*, & regarded as the elementary medium for textual & visual information concerning an object or process.

In a series of slides, each successive slide adds information to the one preceding it; this incremental information conveys additional knowledge to the recipient (whether a colleague, a listener, or a reader).Slides are widely utilized in education, science, &other fields—typically without explicit consideration of their specific role or characteristics.

* / "2.1. We picture facts to ourselves. … 2.12. The picture is a model of reality. … 2.221. What the picture depicts is its sense. … 2.223. In order to tell whether a picture is true or false we must compare it with reality. … 3. A logical picture of facts is a thought."— Ludwig Wittgenstein, The *Tractatus*

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«Time is the measurement by humans...» by Gemini 3

***Time is the measurement by humans**, at a certain level of cultural development, of the sequence of events. These metrics are determined by the cycles of rotation of the Earth around its axis, the Sun in the Solar System, and the Moon around the Earth. The force that determines the characteristics of cyclical motion is gravity. Analytically, gravity is indefinable for a set of bodies greater than three.*

AI Overview

Based on scientific, historical, and physical principles, the statements provided reflect both human-centric timekeeping and fundamental physics challenges.

Time as a Human Measurement of Sequence:

Time is understood as a measure of consistent, non-stop change in our surroundings, used to order events. It is often described as a human-defined concept used to quantify the duration between, and the sequence of, events.

Metrics Determined by Celestial Cycles:

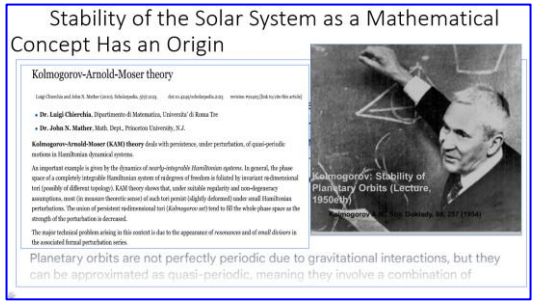
Historically, the fundamental units of time—days, months, and years—are derived from the rotation of the Earth on its axis, the revolution of the Moon around the Earth, and the revolution of the Earth around the Sun.

Gravity as the Driver of Cycles:

Gravity is the force that acts as the centripetal agent, keeping planets in orbit around the Sun and moons around planets, which creates the consistent cyclical motion used to measure time.

The Three-Body Problem:

In physics, while two-body systems (like the Earth and Sun) can be accurately modeled, the "three-body problem" refers to the difficulty of creating an analytical, closed-form solution to predict the motion of three or more celestial bodies interacting gravitationally. Such systems are often chaotic, meaning their long-term behavior cannot be predicted with absolute precision, making a perfect analytical definition for "greater than three" impossible.



In summary, time is measured by cyclic, gravitationally driven celestial events, but predicting these interactions in systems of more than two bodies is complex and often analytically unsolvable.

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