

The climate model hierarchy

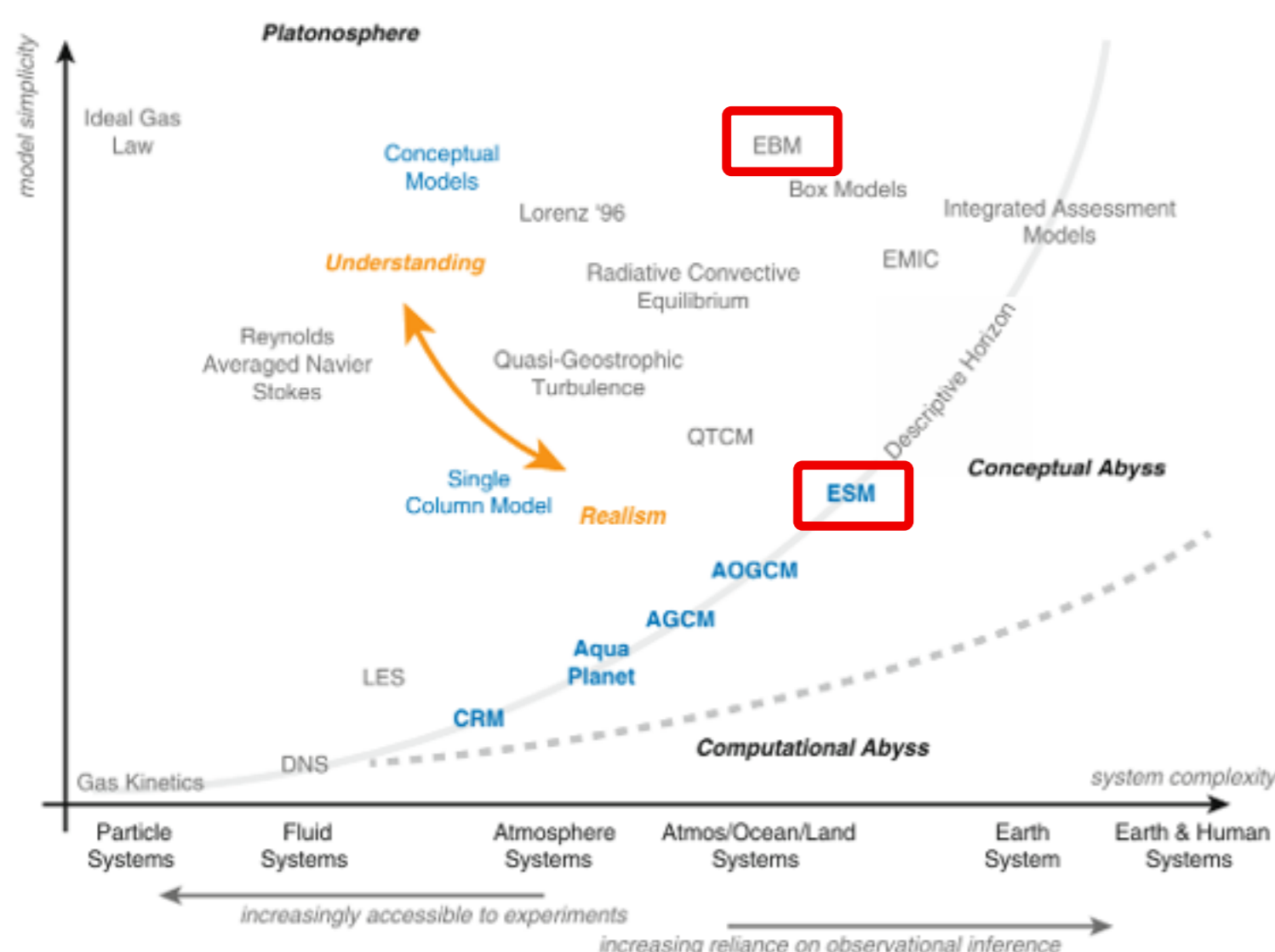


Figure 1. Sketch of climate model space. EBM: energy balance model, ESM: earth system model. Source: Bony et al. (2013)

Background: Energy balance models

Radiative energy balance models (EBMs) describe the planetary climate based on a balance between absorbed solar radiation and infrared emission, setting the temperature T . In 1D, as a function of latitude θ ,

$$C(\theta) \frac{\partial T(\theta, t)}{\partial t} = \text{net flux} = F_0 S(\theta) - \epsilon \sigma T(\theta, t)^4, \quad (1)$$

where $C(\theta)$ (in $JK^{-1}m^{-2}$) is the heat capacity, ϵ is the emissivity, σ the Stefan-Boltzmann constant and the mean annual profile

$$S(\theta) = \left(1 - \frac{3}{5} \sin^2 \theta\right), \quad F_0 = \frac{5S_\odot \alpha_c}{4},$$

where $S_\odot = 1340 Wm^{-2}$ is the solar constant and $\alpha_c = (1 - \alpha)$ is the co-albedo. See North & Kim (2017). Figure 2 shows a schematic of the model in OD.

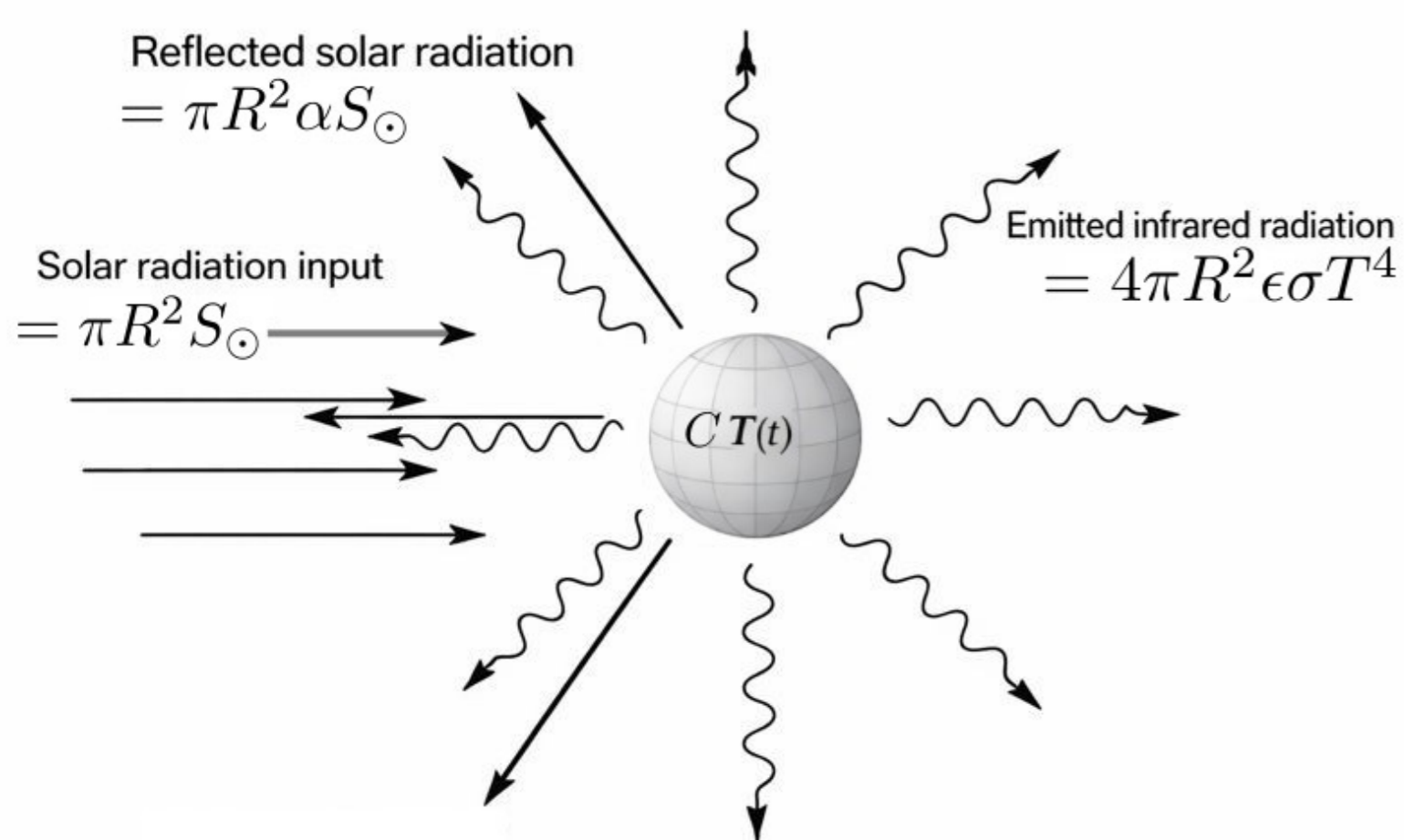


Figure 2. Schematic of a zero-dimensional EBM.

We consider Earth-like values for all relevant parameters, including $\alpha_c = 0.7$ and $\epsilon = 0.567$. For simplicity, we assume a uniform heat capacity $C \approx 2 \times 10^8 JK^{-1}m^{-2}$ representative of the ocean mixed layer. Nondimensionalizing Eq. (1) in terms of the equatorial radiative equilibrium temperature

$$T_0 = \left(\frac{F_0}{\epsilon \sigma}\right)^{1/4},$$

one finds by dimensional analysis that the time scale of temperature changes due to a radiative imbalance is given by

$$\tau_{rad} = CT_0/F_0.$$

The linear response time to small perturbation from radiative equilibrium is $\tau_{rad}/4$. For Earth-like parameters, the radiative time is comparable to $\tau_{rad} \approx 1$ yr.

An advective-convective 2D EBM

We parametrize macroturbulence by batropic dynamics, and tropical convection via a temperature threshold $T_c \approx 301 K$, see e.g. Hu et al. (2023), to which the temperature is rapidly relaxed when exceeding the threshold, and stochastic momentum forcing injecting the released thermal energy as kinetic energy.

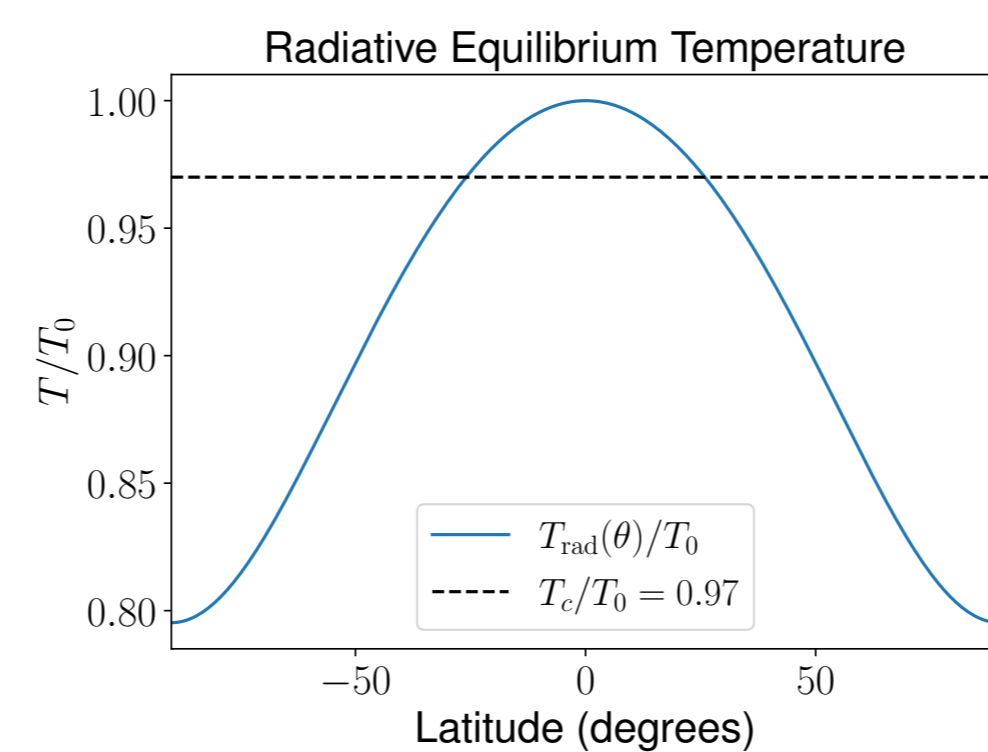


Figure 3. Radiative equilibrium temperature profile (blue solid curve) and sample temperature threshold (black dashed line).

The convection parameterization defines a tropical region of extent $[-\theta_c, \theta_c]$.

We study the following highly parameterized model

$$D_t \mathbf{u} = -\nabla p / \rho - \nu_R \mathbf{u} - \nu_E^{(n)} (-\nabla^2)^n \mathbf{u} + \mathcal{F} - 2\Omega \times \mathbf{u},$$

$$\nabla \cdot \mathbf{u} = 0,$$

$$CD_t T = F_0 S(\theta) - \epsilon \sigma T^4 - \kappa_E^{(n)} (-\nabla^2)^n (CT) - \text{convective cooling} + \rho \mathbf{u} \cdot \left[\nu_R \mathbf{u} + \nu_E^{(n)} (-\nabla^2)^n \mathbf{u} \right],$$

with material derivative $D_t = \partial_t + \mathbf{u} \cdot \nabla$, Rayleigh damping coefficient ν_R , hyperviscosity $\nu_E^{(n)}$, convective forcing \mathcal{F} effecting the conversion of thermal into kinetic energy, planetary rotation rate Ω about axis, $\Omega = \Omega \hat{\mathbf{z}}$, and a temperature threshold T_c , above which temperature locally relaxes to T_c on a timescale τ_c .

Figure 4 shows a schematic of the energy cycle in the model, which ignores available potential energy.

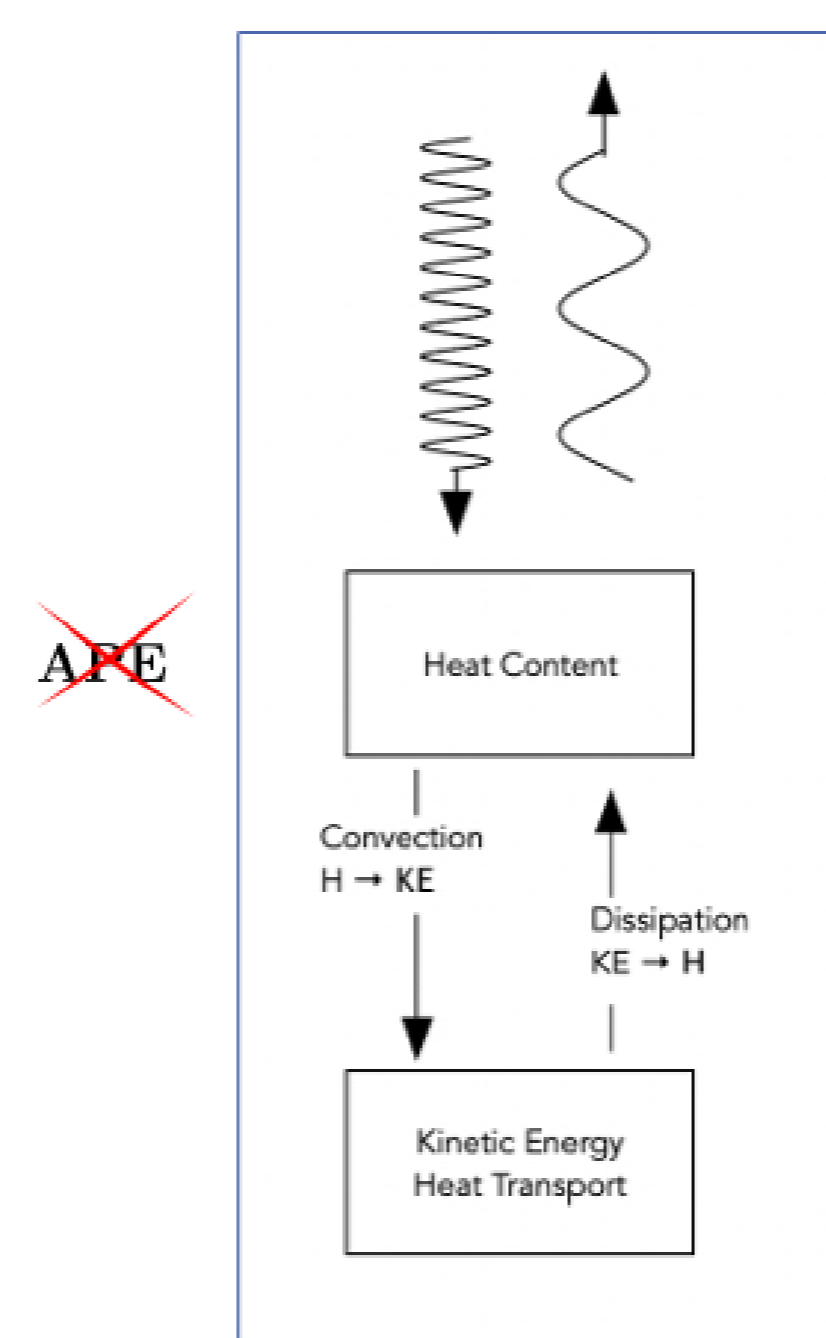


Figure 4. Schematic of the energy cycle in the model.

This problem involves four dimensions: length, time, temperature and mass. Nondimensionalizing via

$$\mathbf{x} = R\hat{\mathbf{x}}; \quad \nabla = R^{-1}\hat{\nabla} \quad t = t_{rad}\hat{t}; \quad u_0 \equiv R/\tau_{rad};$$

$$\mathbf{u} = u_0 \hat{\mathbf{u}}; \quad p/\rho = u_0^2 \hat{p}$$

$$T = T_0 \hat{T}; \quad T_0 \equiv \left(\frac{F_0}{\epsilon \sigma}\right)^{1/4}, \quad \mathcal{F} = \frac{R}{\tau_{rad}^2} \hat{\mathcal{F}},$$

yields the nondimensional groups

$$N_1 = \tau_{rad} \nu_R, \quad N_2 = 2\Omega \tau_{rad}, \quad N_3 = \frac{\rho u_0^2}{CT_0}, \quad N_4 = \frac{T - T_c}{T_c}.$$

$$N_5 = \frac{\tau_{rad}}{\tau_c} \gg 1, \quad N_6 = \frac{\ell_c}{R} \ll 1, \quad N_7 = \frac{\ell_g}{R}, \quad Re = \kappa_E^{(n)} \frac{\tau_{rad}}{R^2 n}.$$

Alternatively, group as ratios of time scales, ordered as

$$\tau_{eddy} \gg \tau_{rad} \gg 1/\nu_R \gtrsim 1/\Omega \gg \tau_c.$$

Results of numerical simulations

We solve these equations on a sphere using Dedalus (Burns et al. 2020), integrating for several radiative time units (decades) at low numerical cost, resolving macroturbulence over a wide range of scales (most testing done at T255, i.e. grid spacing of $\lesssim 100$ km). For simplicity, we ignore rotation.

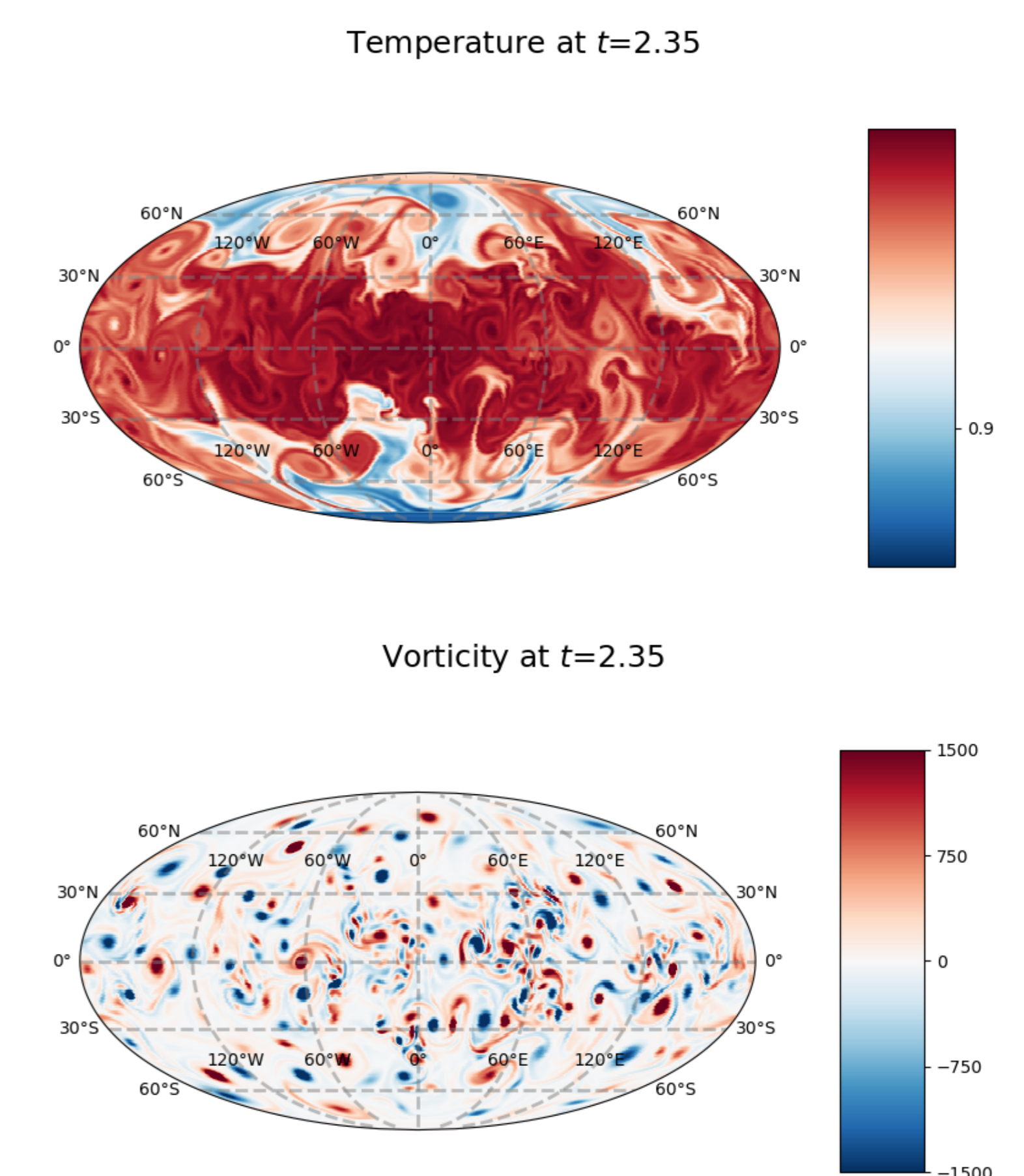


Figure 7. Snapshots from a non-rotating model simulation.

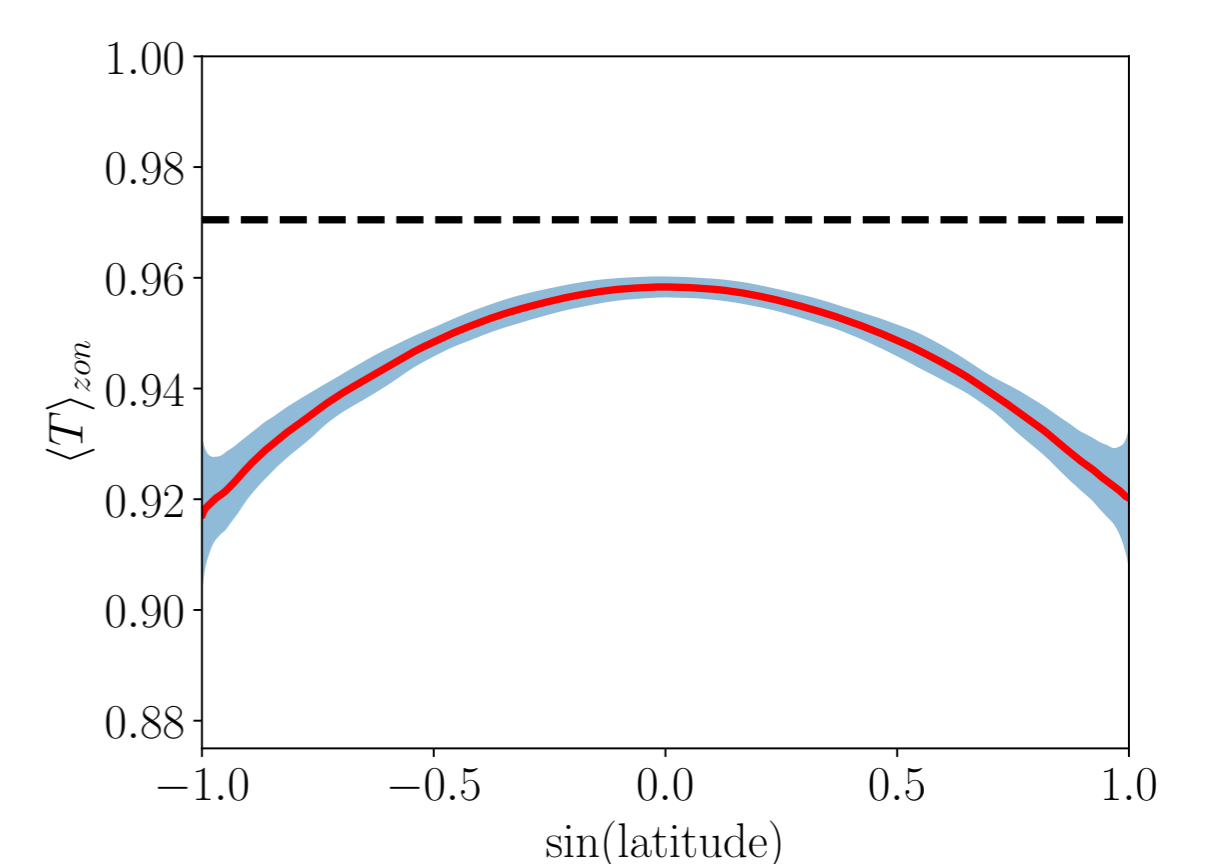


Figure 8. Zonally averaged temperature profile and fluctuations (larger at the pole due to reduced area). Cold fluid from high altitudes entrained into equatorial region reduces zonal mean.

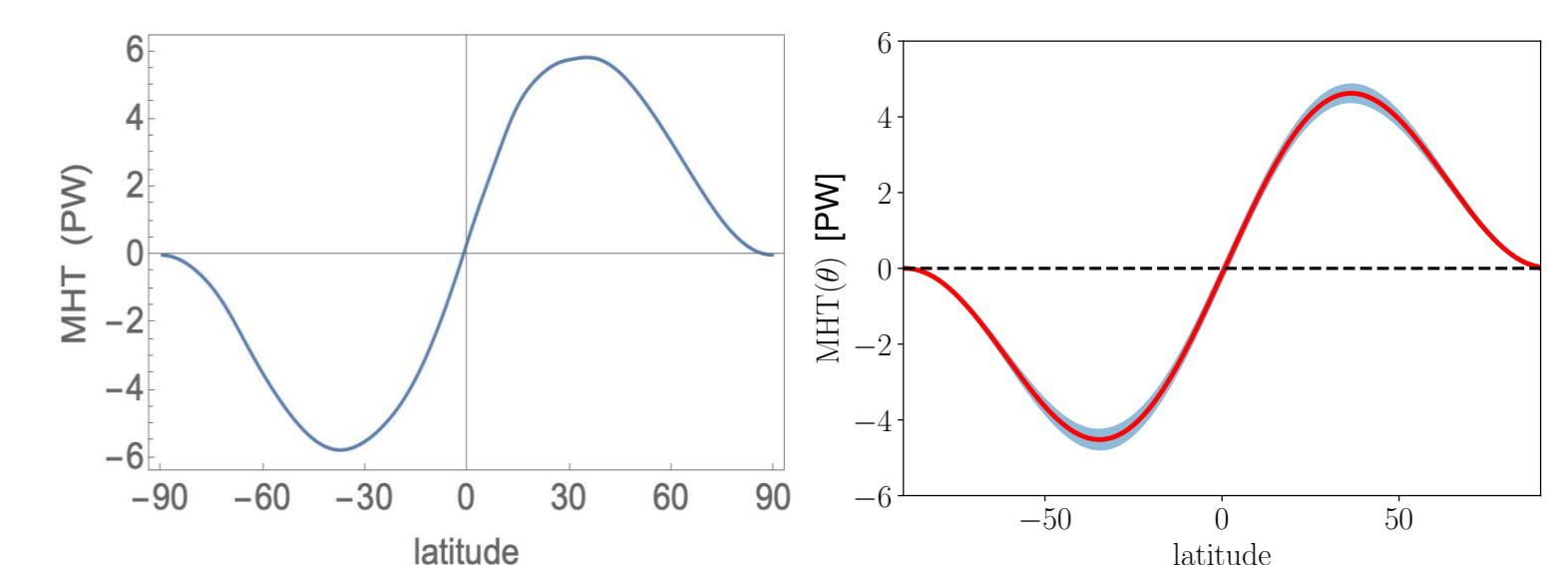


Figure 9. Meridional heat transport (defined e.g. in Donohoe and Battisti (2012), from CERES (left), EBM (right) resemble each other, a.k.a. Bjerknes compensation, see Stone (1978).

References

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