

IMPERIAL

Superposition of Doppler-shifting magnetopause Kelvin-Helmholtz modes

through Dynamic Mode Decomposition of a global MHD simulation

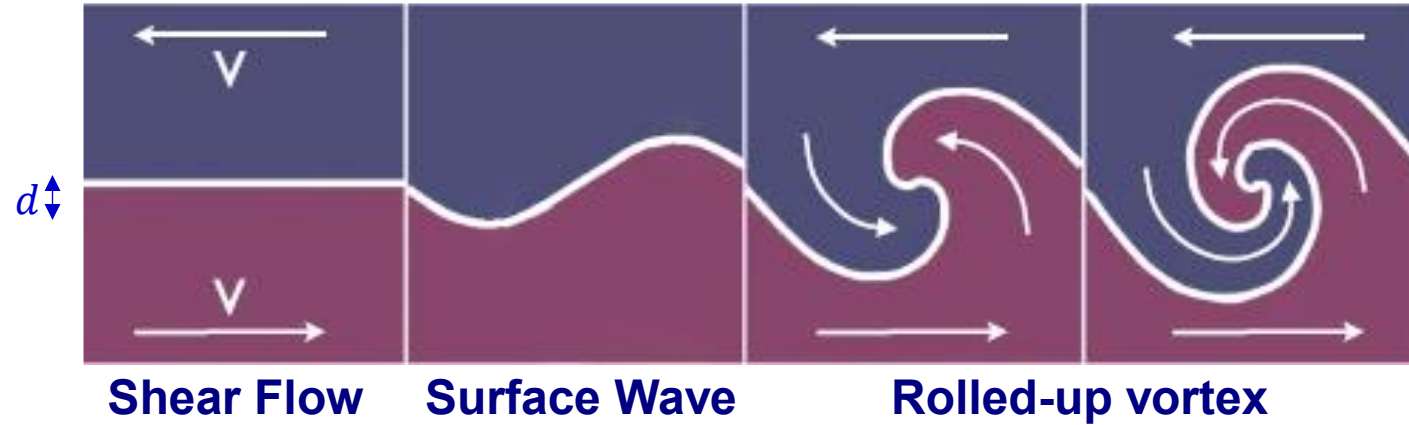
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Department of Physics, Imperial College London, UK

Kelvin-Helmholtz Instability (KHI) at the magnetopause

Johnson+ (2014, Space Sci Rev)

Fluid instability driven by shear flows, e.g. across Earth's magnetopause

If shear flow overcomes magnetic tension (weakest for northward IMF) surface waves grow into nonlinear rolled-up vortices



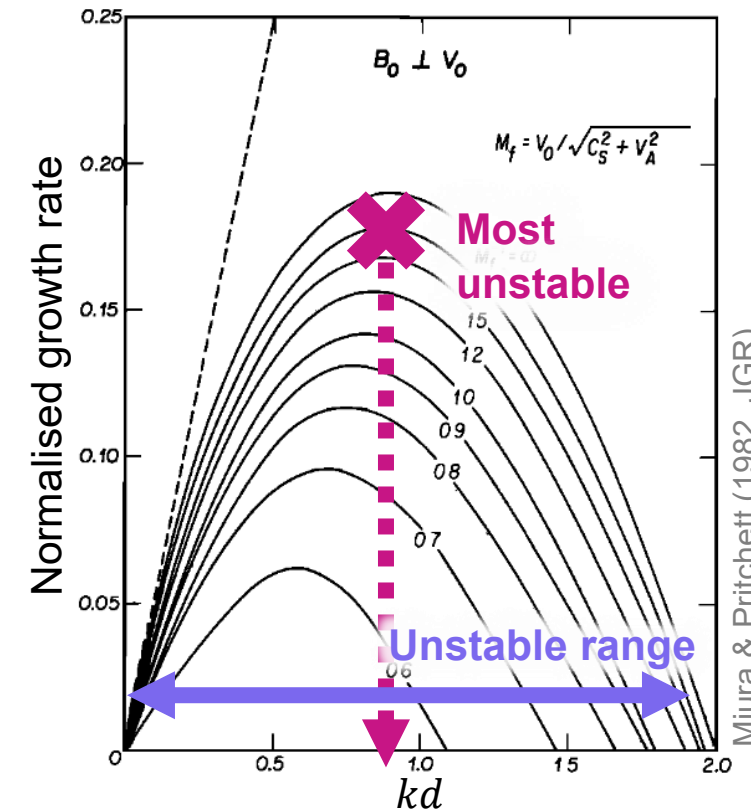
Local linear theory predicts fastest-growing KH wave when $kd \sim 0.5 - 1$

- Often assumed to explain observations (Hasegawa+, 2004, Nature; Lin+, 2014, JGR)
- Cannot account for longer wavelengths observed at lunar distance (Zhou+, 2022, GRL)

KHI is broad instability able to amplify perturbations across $0 < kd \leq 2$

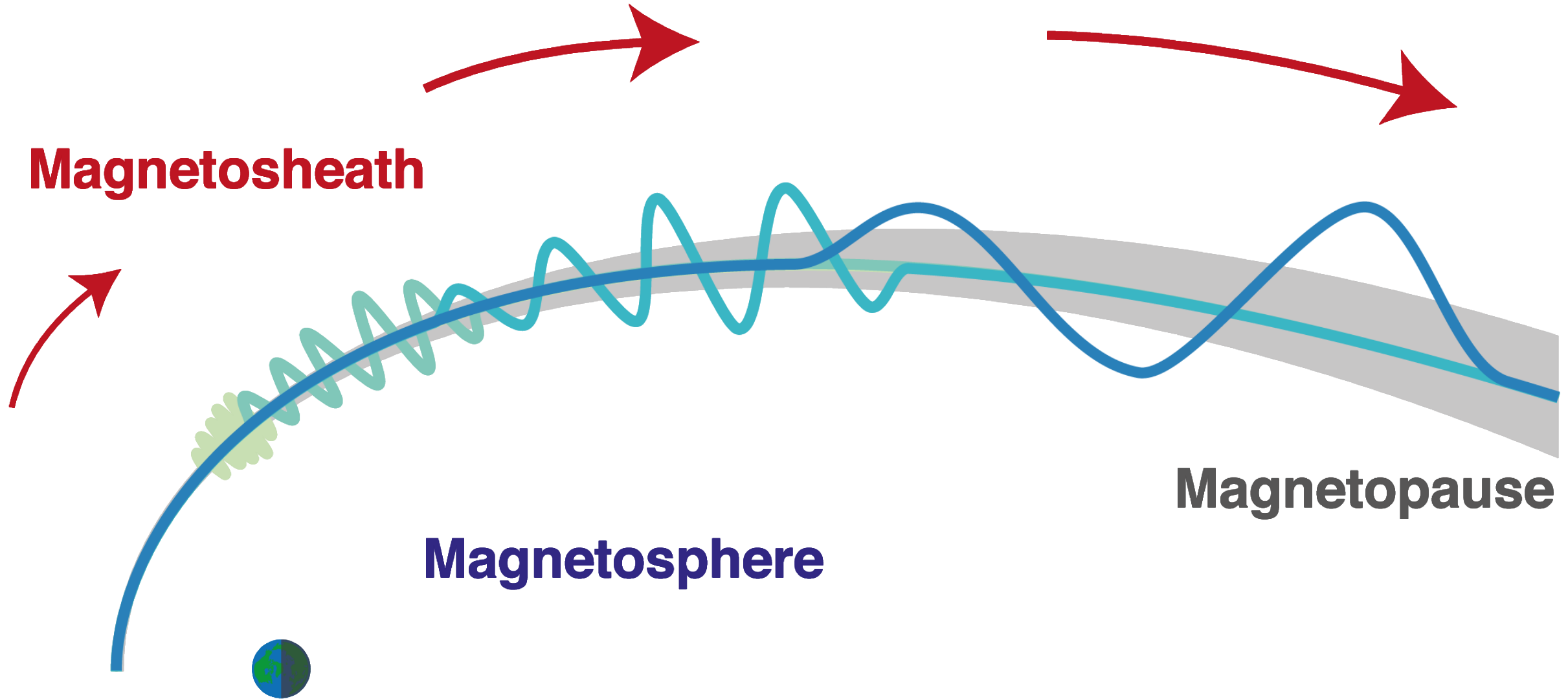
- Pre-existing seed fluctuations may grow further due to KHI (Nykyri+, 2017, JGR; Nakamura+, 2020, JGR)
- Challenges assumption of locally fastest-growing

KHI is a convective instability, advecting waves into new regions, so purely local treatment inadequate



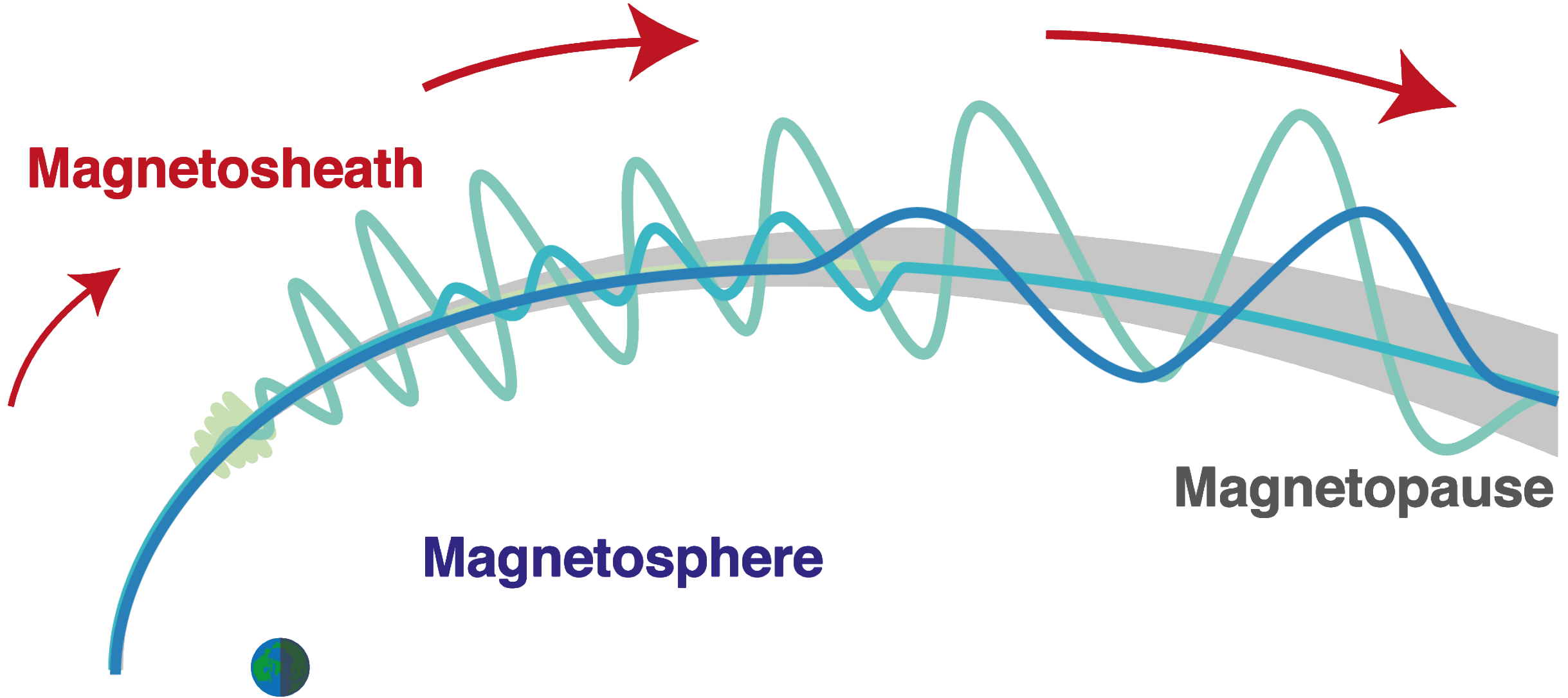
Magnetopause KHI globally

Paradigm 1: Locally most-unstable



Magnetopause KHI globally

Paradigm 2: Amplified upstream waves



Gorgon global MHD simulation

Gorgon MHD code at Imperial

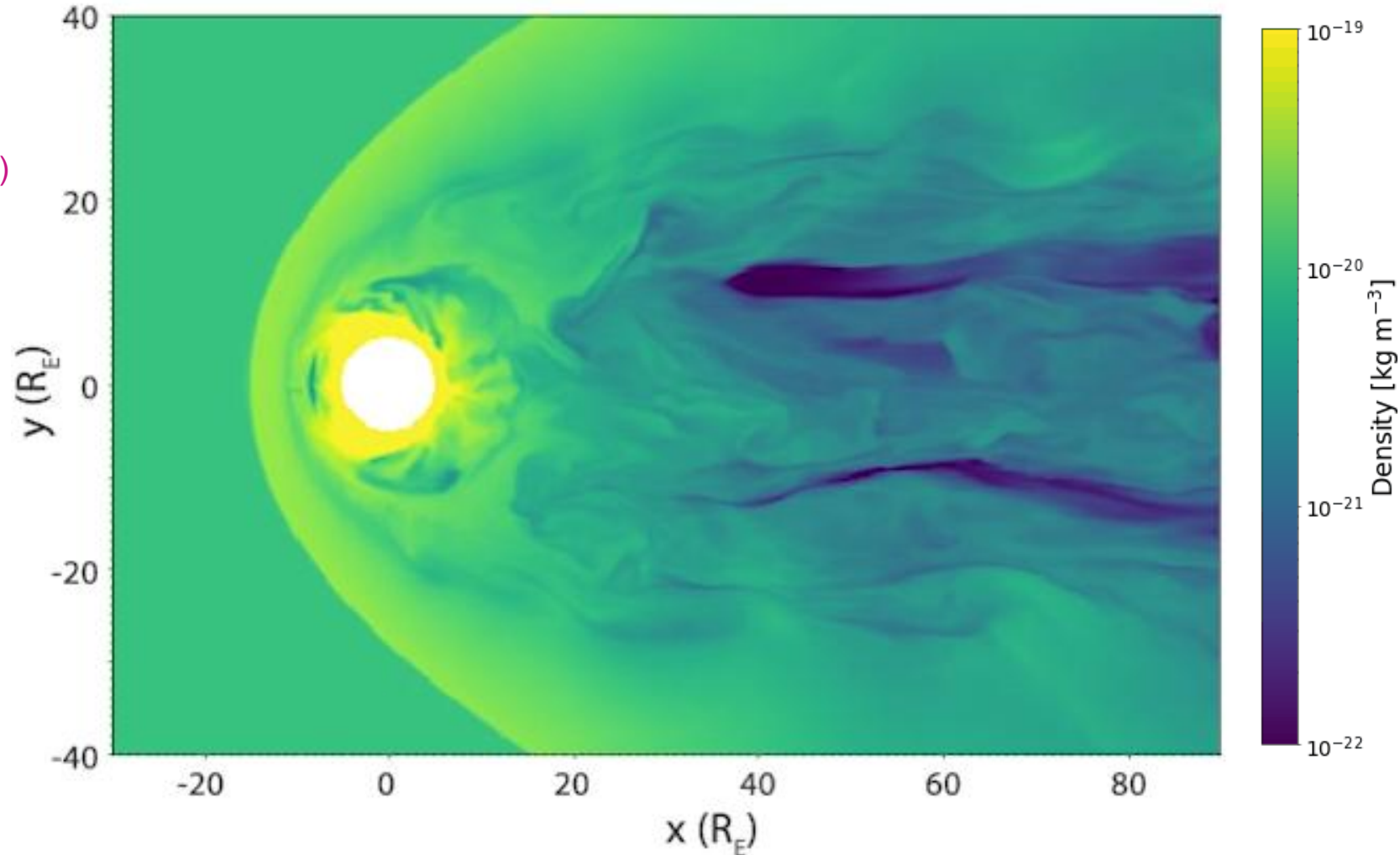
- Originally developed for lab plasmas
- Adapted to Earth's magnetosphere
(see LaMoury+ & Burne+ on Friday, ST4.2)

Constant northward IMF run

- Initialised until steady-state reached, use 110 snapshots of 15s cadence
- Non-linear waves along flanks, boundary confirmed KH-unstable

Efficiently analyse periodic behaviour across the simulation with focus on **relative variations** and their **physical mechanisms**

- Need to reduce data volume (Fourier methods do not)
- Grid resolution affects wave properties' absolute values, but physics is self-consistent



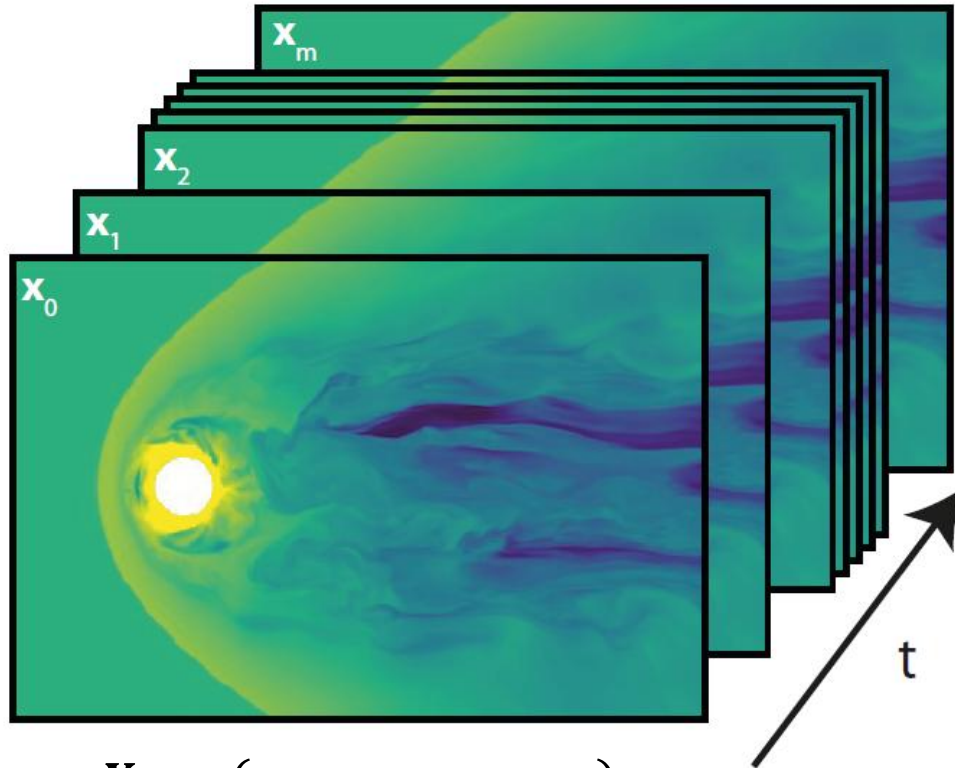
Introducing Dynamic Mode Decomposition

Dynamic Mode Decomposition (DMD)

Dimensionality-reduction Machine Learning technique

$$\mathbf{x}_{k+1} \approx \mathbf{A}\mathbf{x}_k$$

Identifies coherent spatio-temporal structures via least-squares regression and factorisation
 Each DMD mode has constant data-driven oscillation frequency and decay/growth rate

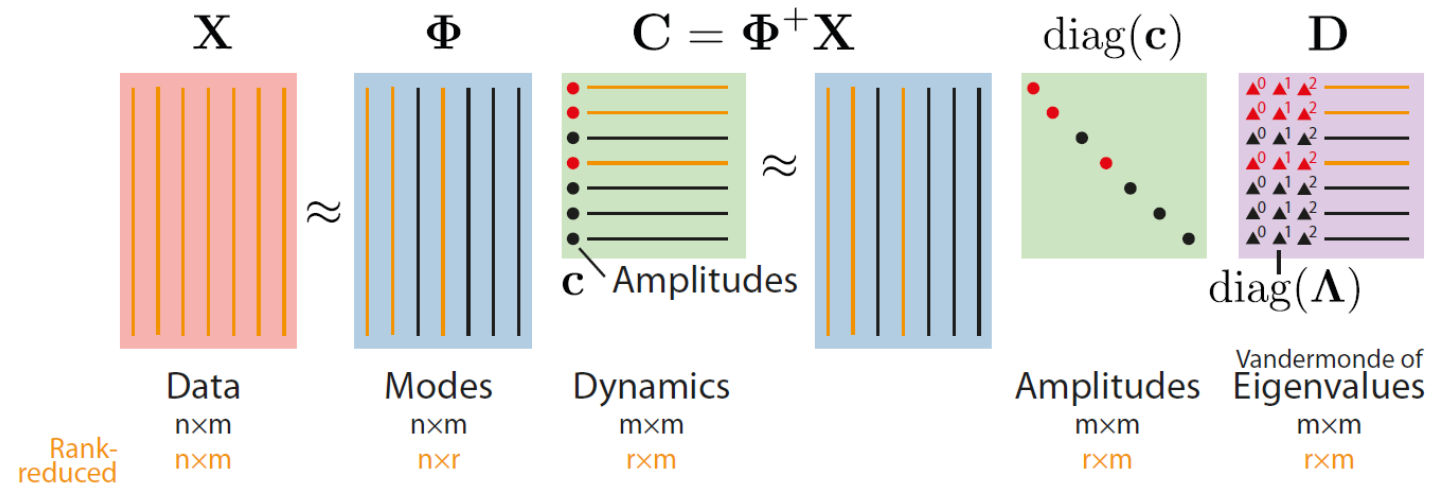


$$\mathbf{X}_1 = (\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{m-1})$$

$$\mathbf{X}_2 = (\mathbf{x}_1, \mathbf{x}_1, \dots, \mathbf{x}_m)$$

1) Regression: $\mathbf{A} = \mathbf{X}_2\mathbf{X}_1^+$

2) Factorisation: $\mathbf{A}\Phi = \Phi\Lambda$



Original data is product of

- **Spatial patterns** (eigenvectors)
- **Fluctuation magnitudes** (amplitudes)
- **Temporal evolution** (eigenvalues)

DMD of a global MHD simulation

DMD typically applied to single physical quantity, but in MHD both plasma & magnetic field important.

Normalise fluctuations from mean so variances are MHD wave energy densities:

$$\delta\bar{v} = \frac{1}{2} \sqrt{\rho_0} \delta v$$

√Kinetic

$$\delta\bar{B} = \frac{1}{2\sqrt{\mu_0}} \delta B$$

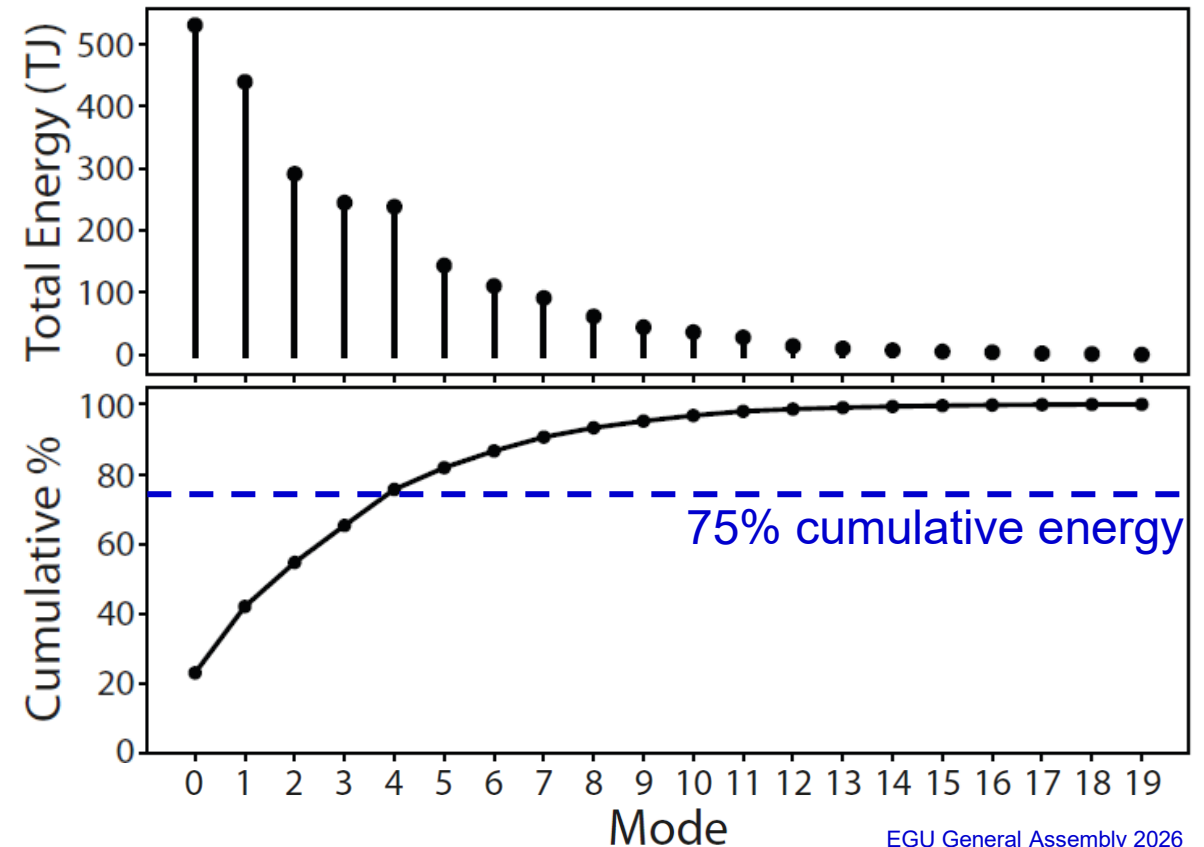
√Magnetic

$$\delta\bar{p} = \frac{1}{2\sqrt{\gamma p_0}} \delta p$$

√Internal

Extract 20 independent DMD modes from normalised simulation data

- Total energy quickly decays for higher mode numbers / frequencies
- 5 modes explain >75% total fluctuation energy, so restrict analysis to these (frequencies 0.5 – 2.7 mHz)



DMD Mode energy

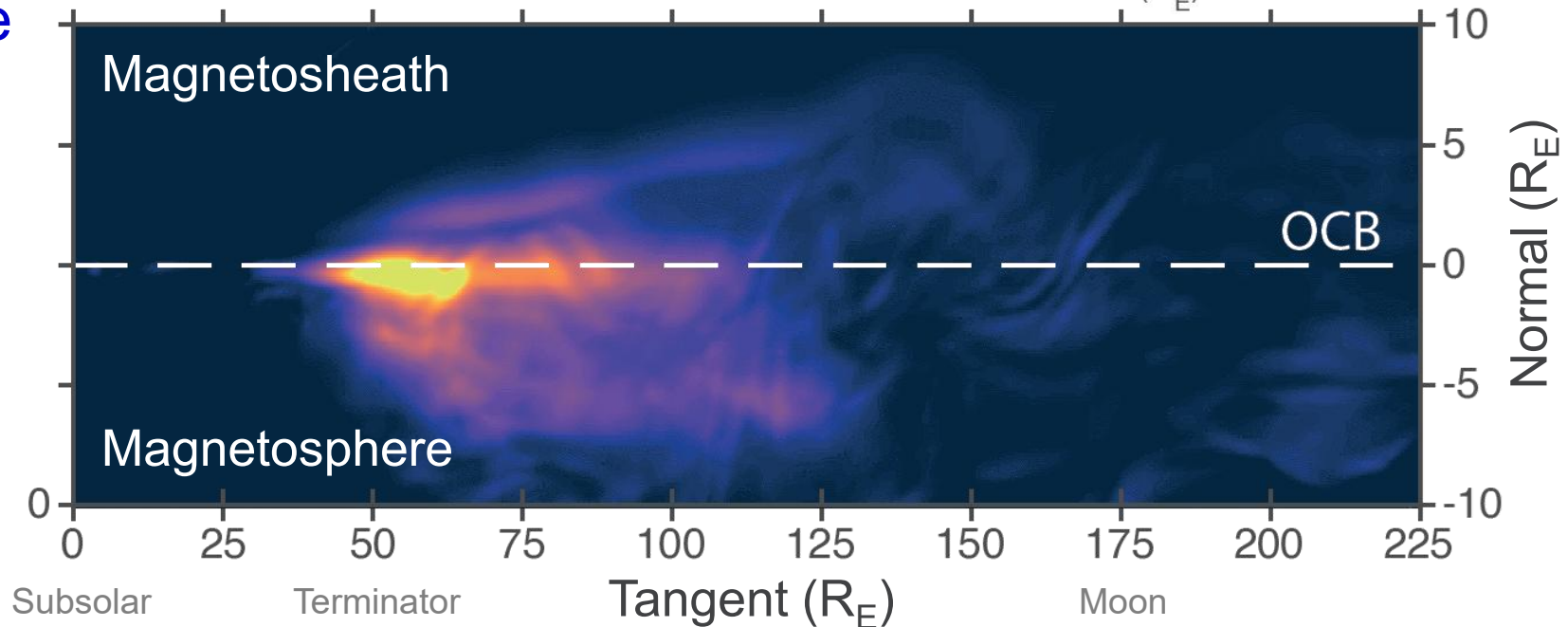
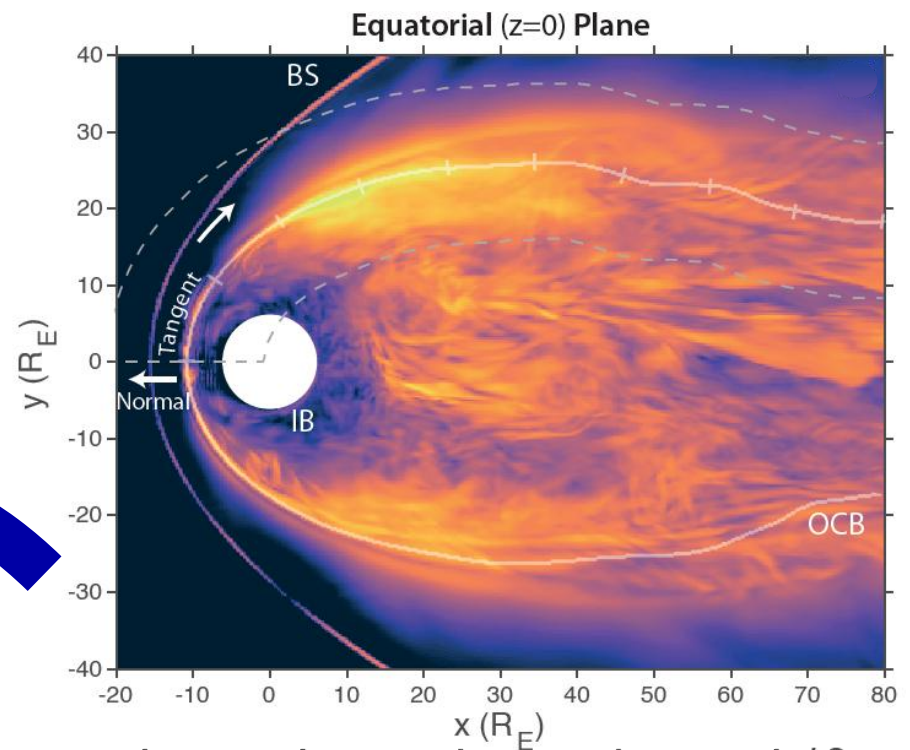
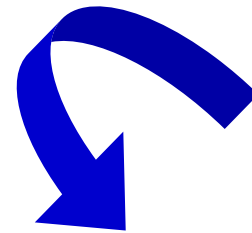
Example Mode 2 (1.6 mHz)

Total energy density in equatorial plane peaks at equilibrium magnetopause (open-closed boundary of B_0)

Coordinates that “unwrap” the boundary
Normal – Tangent plane (dawn flank)

Energy at the magnetopause shows instability behaviour

- Initially negligible
- Rapidly increases at $40 R_E$ (near terminator)
- Remains large $\sim 25 R_E$
- Slowly decays over $\sim 75 R_E$



DMD Mode wavelength

Example Mode 2 (1.6 mHz)

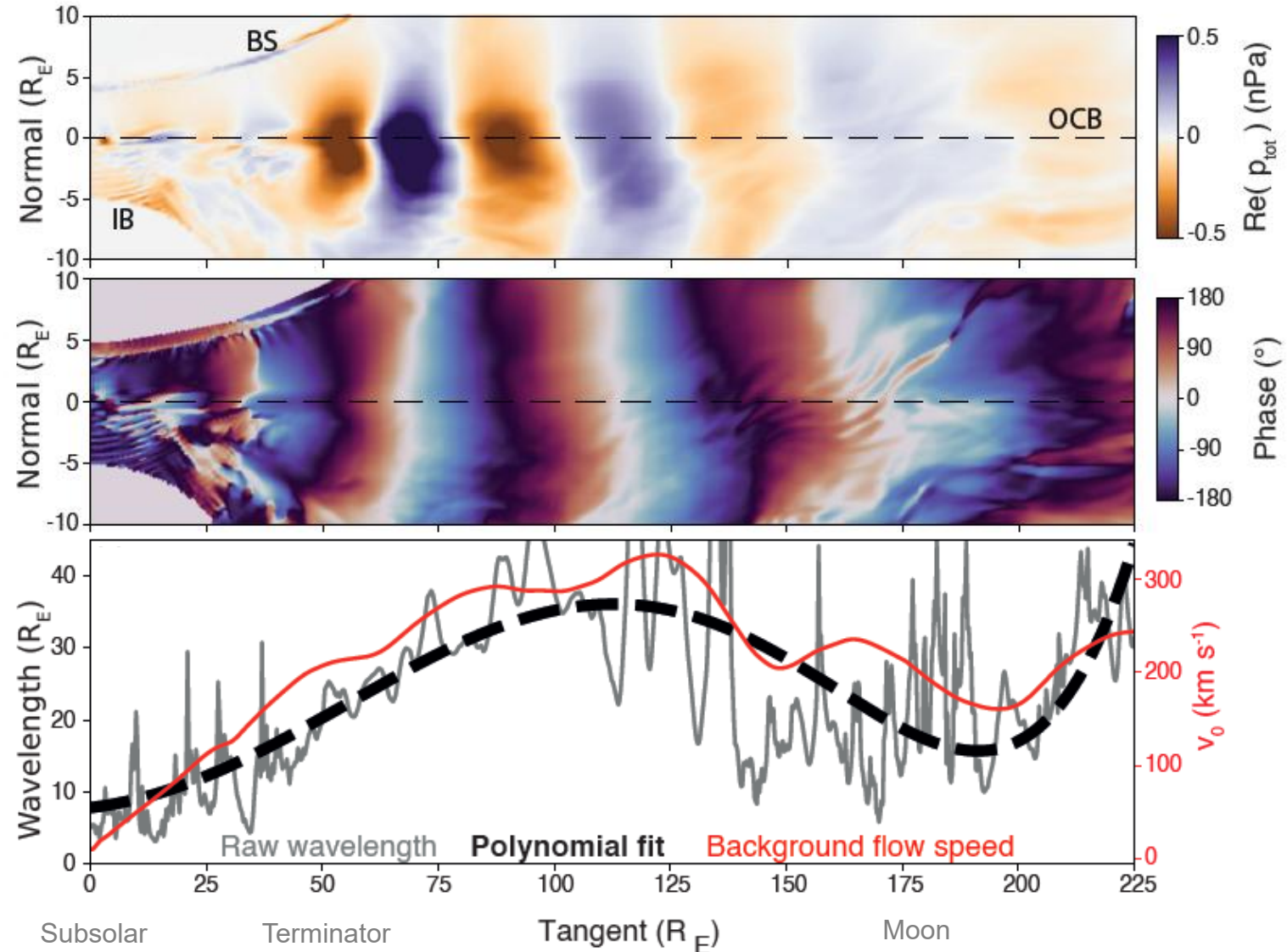
Linear total pressure perturbation

$\delta p_{tot} = \delta p + \mathbf{B}_0 \cdot \delta \mathbf{B} / \mu_0$ continuous across magnetopause

Reconstruction of δp_{tot} shows wavelength varies for fixed frequency

Local wavelength (gradient of phase) highly correlated to **background flow** in boundary (proxy for vortex speed)

Frequency of advected waves are fixed in Earth frame...
... but accelerating flow speed causes wave **Doppler shift**
lengthening wavelength downtail



Superposition of Kelvin-Helmholtz modes

Comparison of DMD modes along magnetopause

Integrated total energy density ($\pm 5 R_E$ in n, $\pm 6.5 R_E$ in z)

Mode 2

- Quickly grows at $40 R_E$ (instability)
- Becomes dominant mode
- Then slowly decays

Mode 1

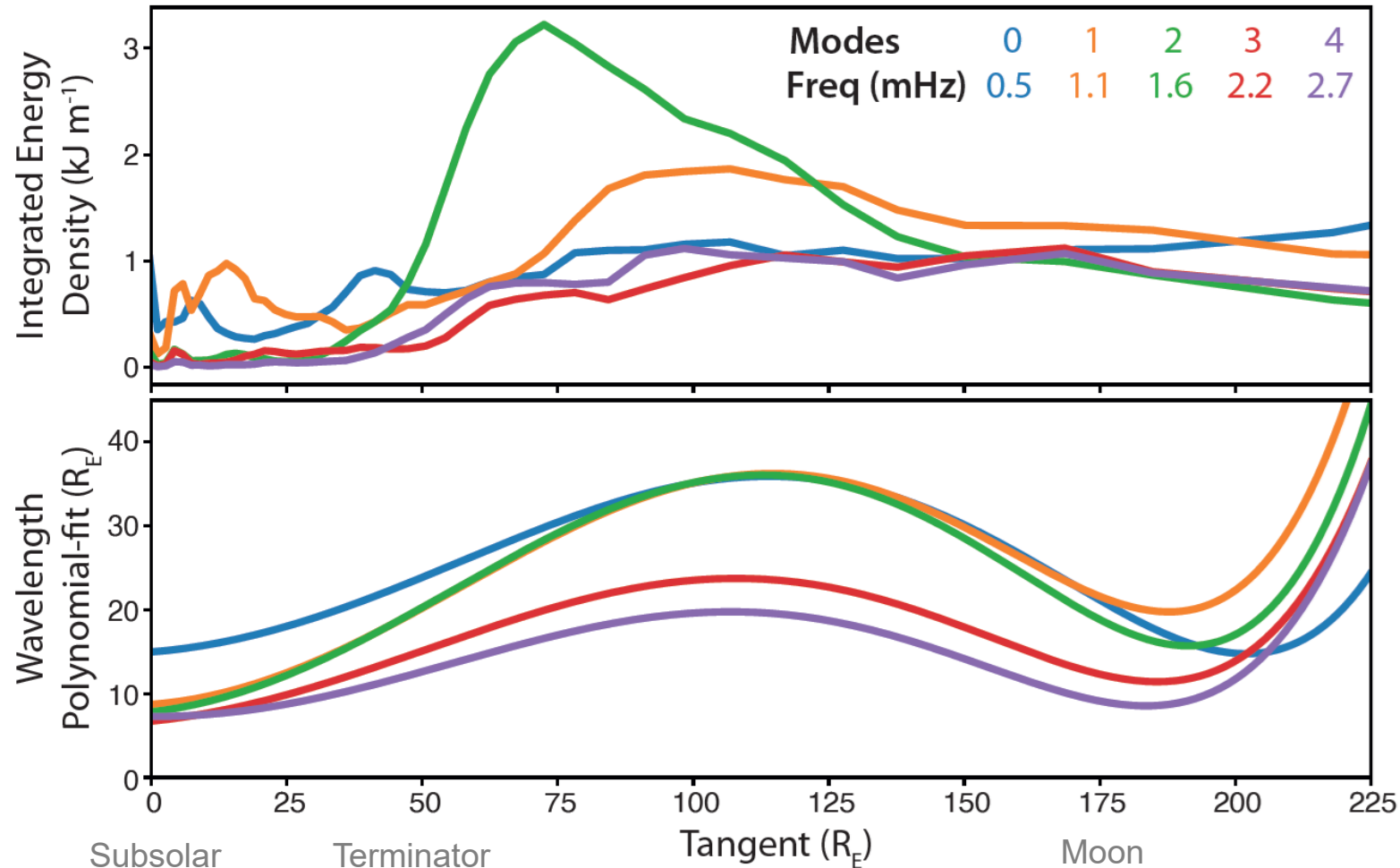
- Quickly grows earlier at $15 R_E$ (instability)
- Becomes dominant mode
- Starts decaying
- Energy slowly increases again throughout Mode 2 growth phase
- Becomes dominant far downtail

Mode 0

- Overall slow growth throughout
- Briefly becomes dominant

Modes 3 & 4

- Slow growth throughout
- Never dominant



Superposition of Kelvin-Helmholtz modes

Comparison of DMD modes along magnetopause

Polynomial fits to local wavelength

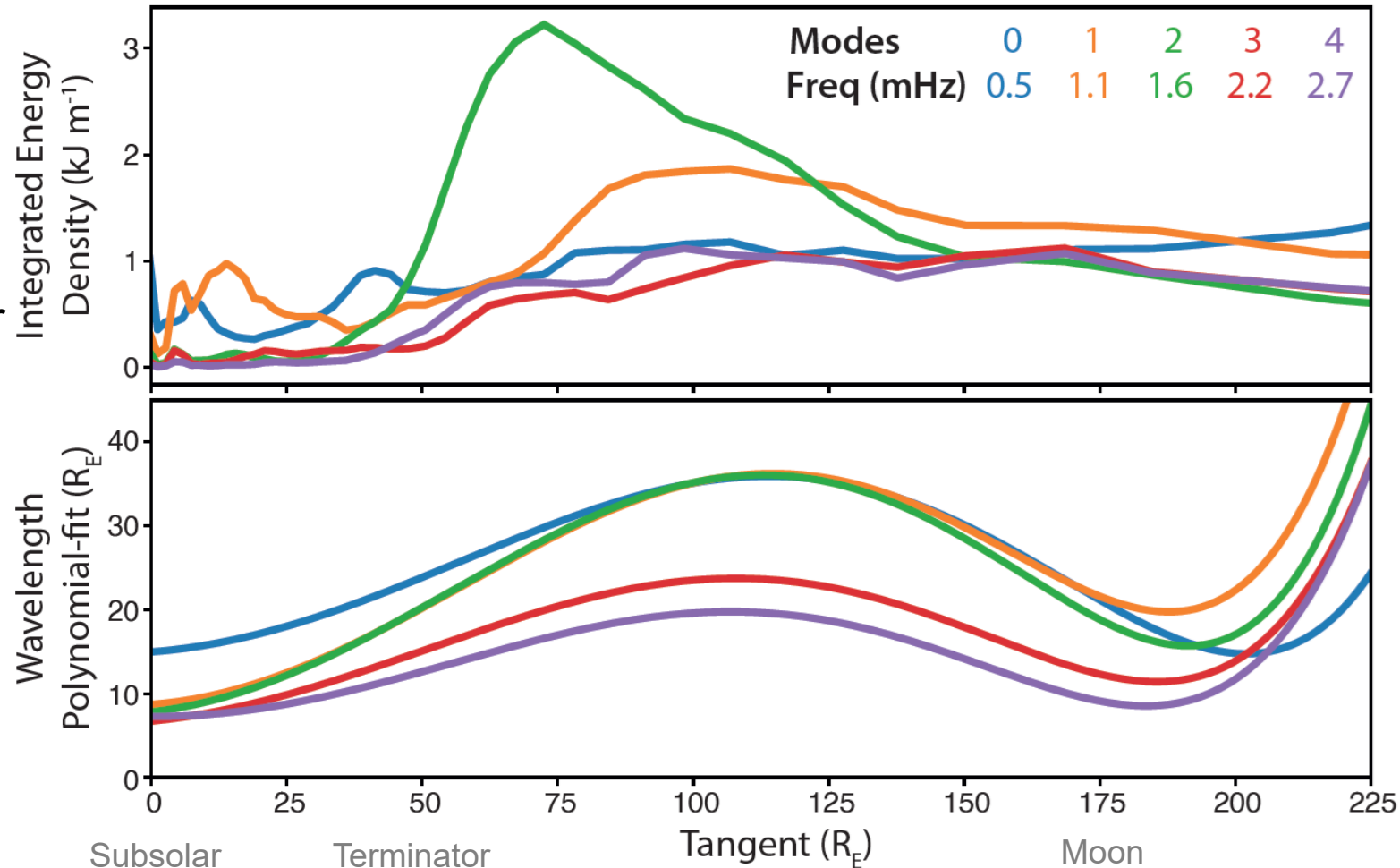
All modes show similar pattern of significant wavelength evolution (up to 30% over single cycle)

Not explained by locally most-unstable KH-mode for thickening boundary layer

- Frequencies of DMD modes fixed
- Occurs throughout wave growth & decay phases

Consistent with Doppler shift due to accelerating flow downtail

Phase speed varies between modes due to location in boundary layer



Evolution of Kelvin-Helmholtz modes along magnetopause

Mode 1 Mode 2

Magnetosheath

Both modes' wavelengths increase due to fixed frequency but accelerating flow

Mode 2 now locally most unstable, dominating superposition

Dominant mode in superposition changes back to Mode 1 due to its amplification

Surviving Mode 1 seed fluctuations slowly amplified

Mode 1 grows as locally most unstable KH mode

Mode 1 dissipates but survives downtail

Magnetopause

Magnetosphere



Conclusions on Kelvin-Helmholtz evolution at magnetopause

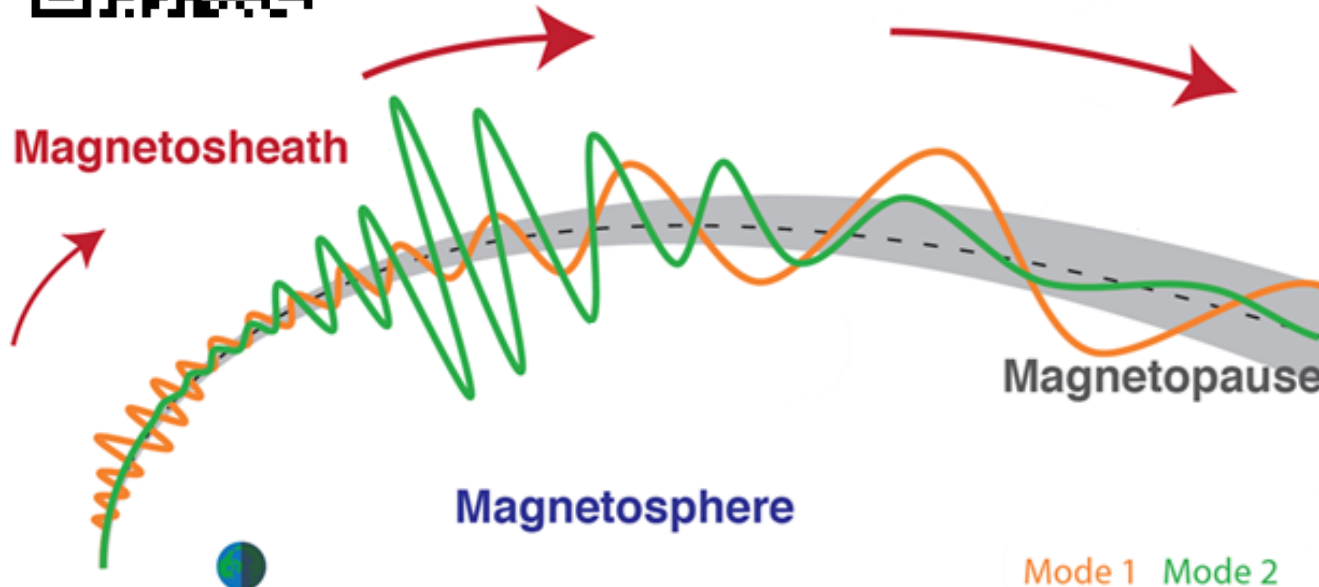
Dynamic Mode Decomposition of a global MHD simulation has revealed

- A superposition of KH-modes exists along the magnetopause
- KHI continues to amplify surviving waves far downtail
- **Dominant wave need not be the locally most-unstable KH-mode**
- Wavelengths lengthen downtail via Doppler shift by accelerating flow
- Physical mechanisms may explain KH observations at lunar distance



Soon to be published in GRL

DMD could prove a powerful tool in other areas of space physics



More Gorgon MHD modelling @ EGU

Fri 8 May Session ST4.2

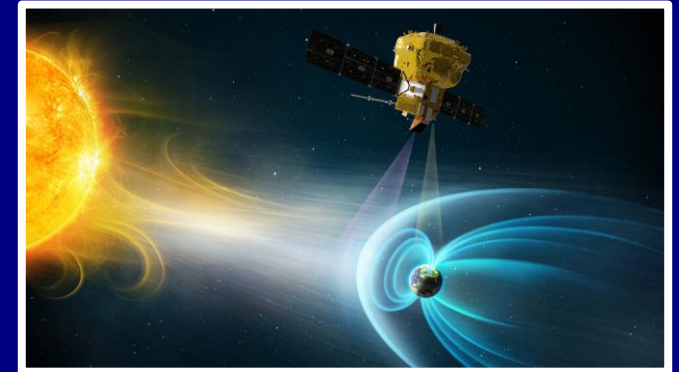
09:15 Adrian LaMoury
Real-time operational space weather forecasting

12:00 Sofia Burne
LEO magnetic field due to field-aligned currents

Research Associate in Magnetospheric Physics

Imperial College London (2yr)

How does Earth's magnetopause dynamically respond to driving?



Harnessing SMILE soft X-ray imaging + advanced statistics / machine learning

Available from October 2026
Deadline 20 May



Backup slide: DMD eigenvalues

DMD eigenvalues are complex numbers and can take any continuous value

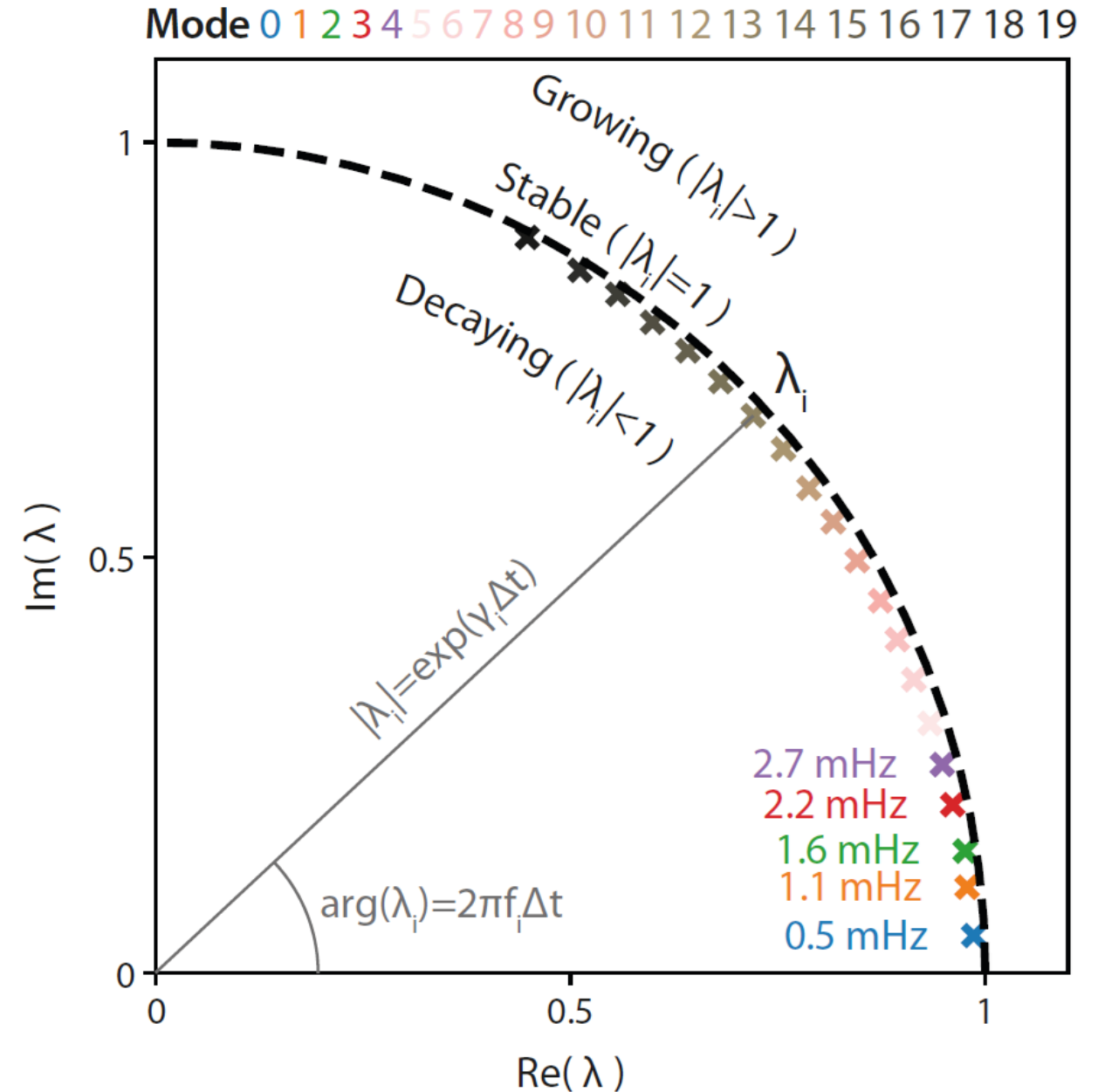
Typically shown in Argand diagram

Eigenvalue magnitudes indicate decay/growth

- Almost 1 indicating quasi-steady state

Frequencies related to eigenvalue argument

- Mode numbers by decreasing amplitude
- Also corresponds to increasing frequencies



Backup slide: Mode hierarchy

Energy partition across simulation for each mode

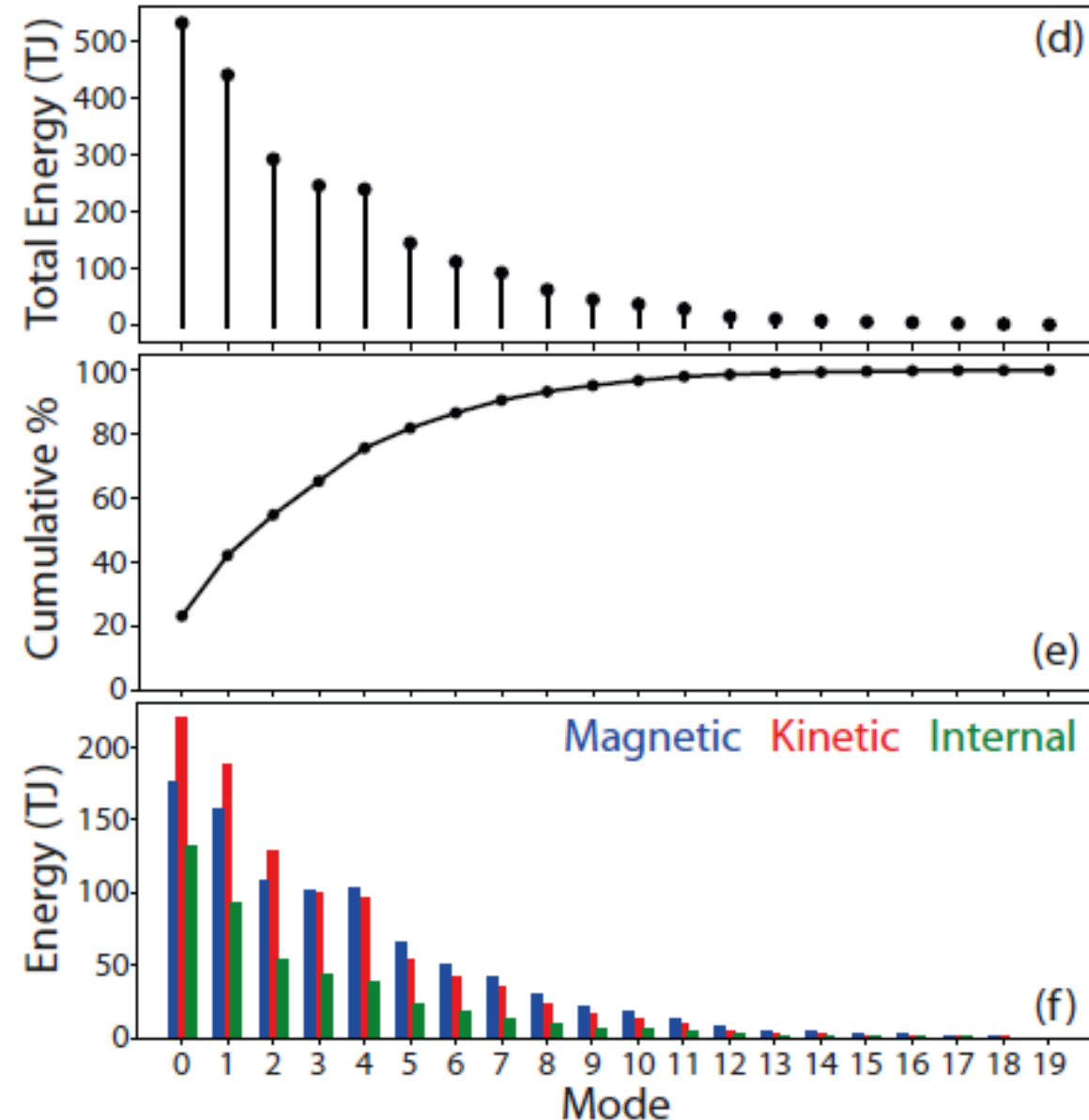
Kinetic & magnetic energies dominate over internal for all modes

Slight changes in balance between kinetic & magnetic with mode number (frequency)

In MHD theory energy partition depends on:

- Wave mode
- Propagation angle
- Plasma beta

Differences likely due to different physical processes and/or different regions of geospace

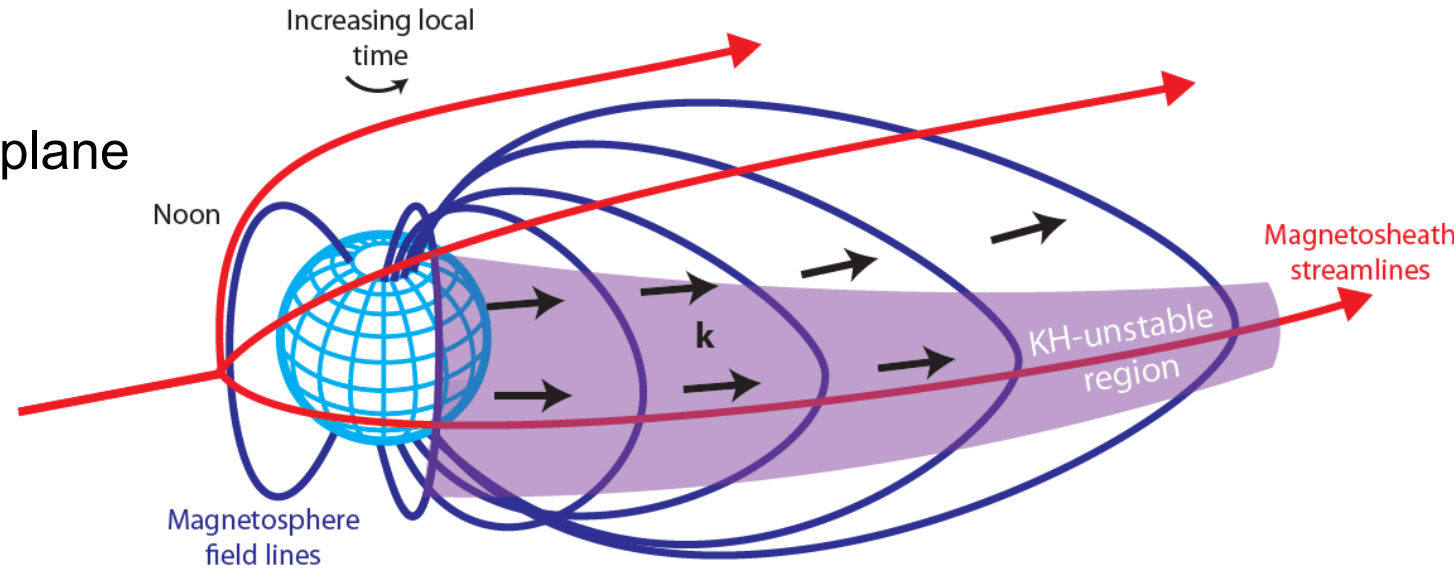


Backup slide: Meridional variation

Mode 2 (1.6 mHz)

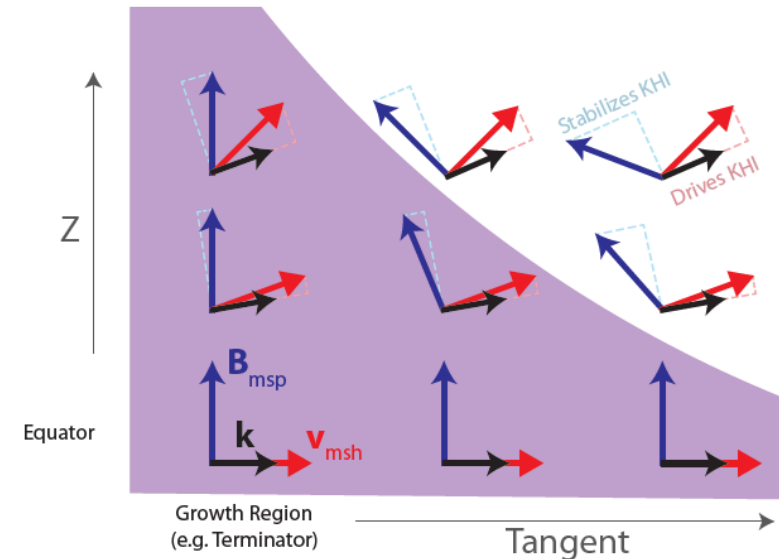
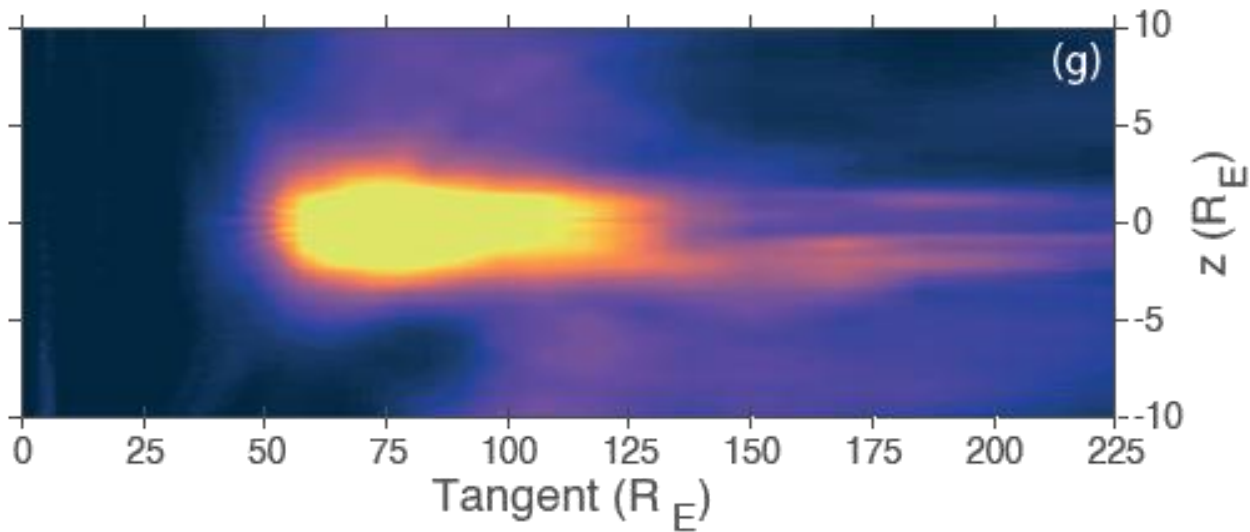
KH-mode energies confined near equatorial plane and extent reducing with local time:

- Reduction in shear flow driver
- Increase in magnetic tension stabiliser



Meridional-Tangent plane
Normal-integrated ($\pm 5 R_E$) Energy Densities

mJ m^{-2}



Backup slide: Energy partition

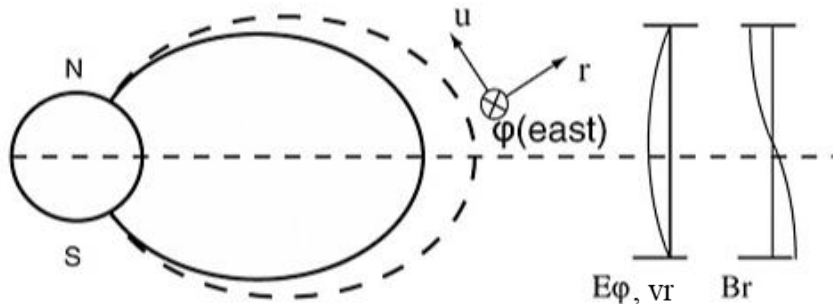
Mode 2 (1.6 mHz)

Energy primarily kinetic overall

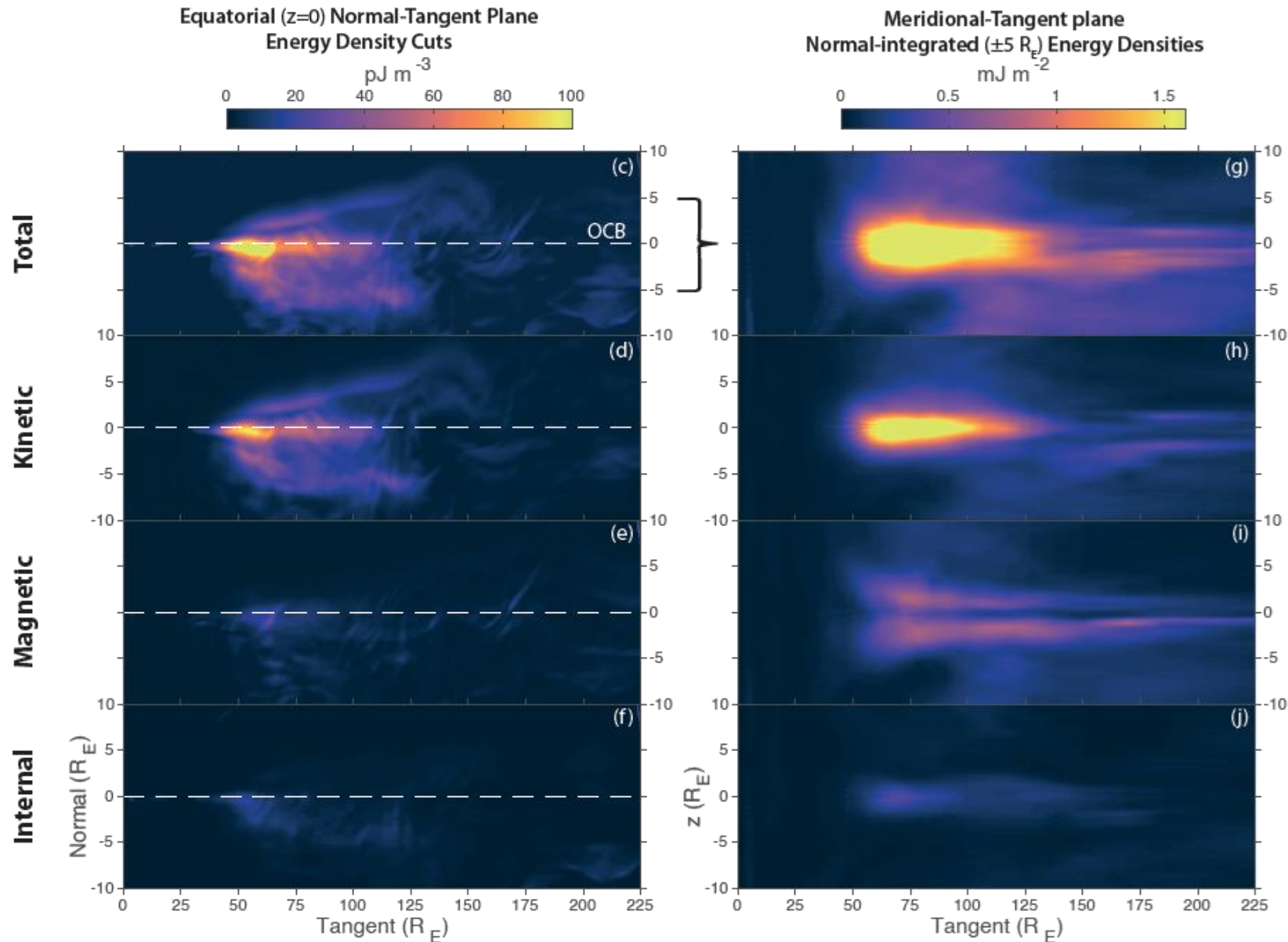
- Peaks in equatorial plane

Small magnetic energy contribution

- Minimum at equator
- Peaks several RE out of plane
- Consistent with wave north-south symmetry (fundamental mode)



Internal energy negligible



Backup slide: Linear theory

Good agreement between simulation and linear theory:

- Most-unstable mode at KH growth locations
- Regions the KHI can amplify existing modes

