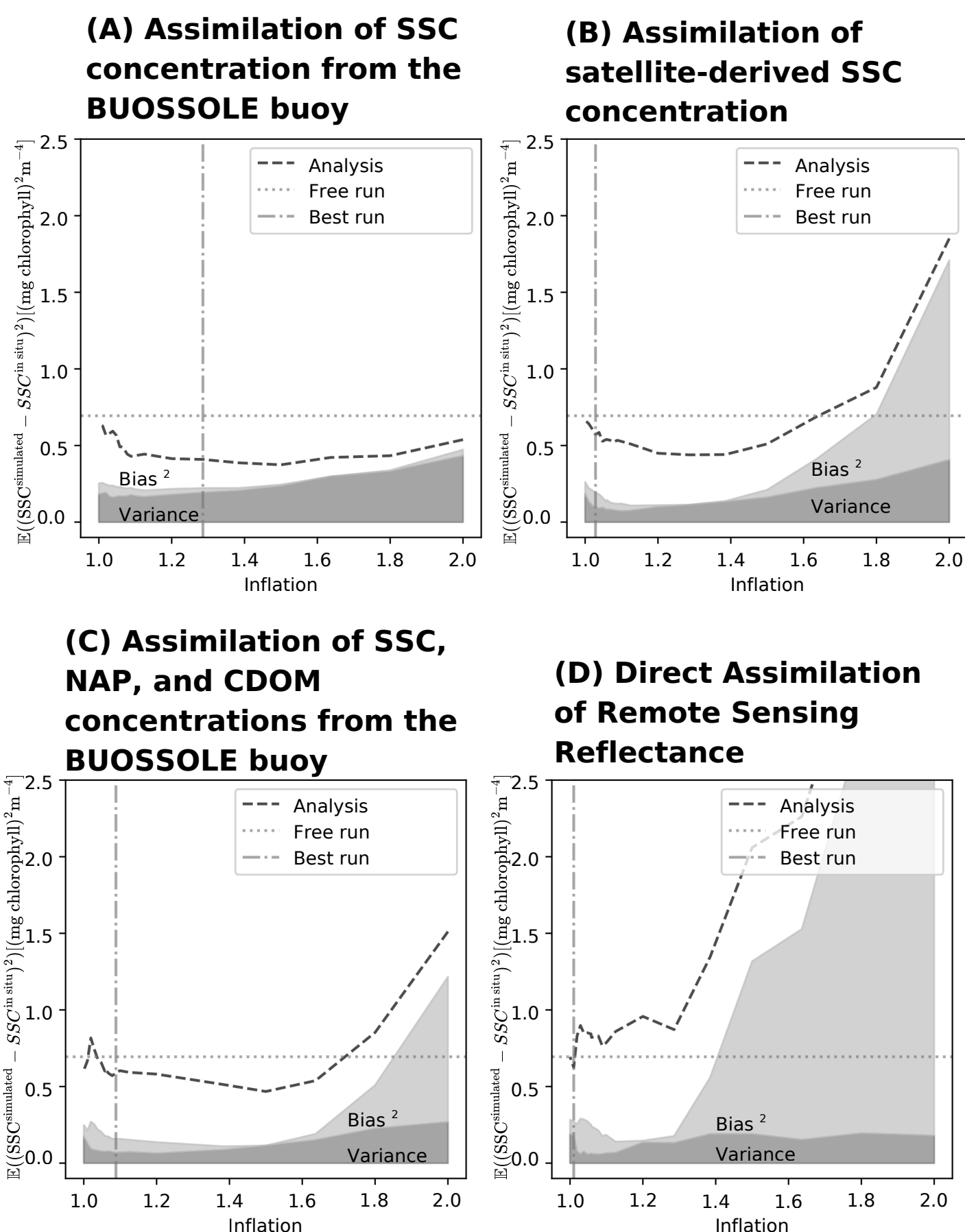


## ANALYSIS AS A FUNCTION OF INFLATION:

The performance of each different approach was tested with the Error Subspace Transform Kalman Filter (ESTKF) as a function of the inflation factor by comparing the analyzed model with in situ data collected in different cruises from 2001 to 2010.



**Figure 1:** Error decomposition for the variable SSC, for the different sets of experiments.

## ADAPTIVE INFLATION SCHEME:

The approach proposed focuses on an accurate description of the error.

### LINEARIZATION BY TIME-SCALE SEPARATION:

By approximating the slow process with an Ornstein-Uhlenbeck process, where the drift, diffusion, and limiting behavior change slowly in time,

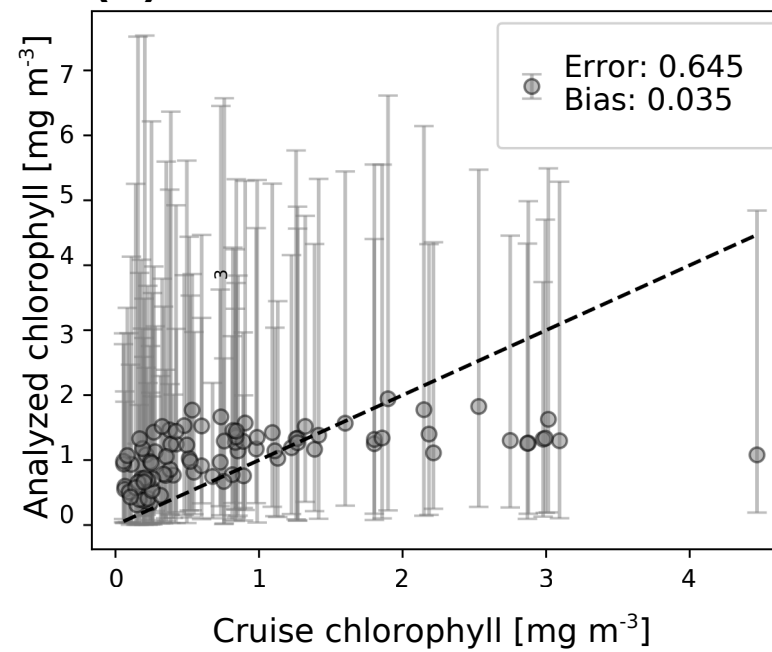
$$\Delta y^{(t)} = \beta^{(t)}(y^{(t-1)} - \mu^{(t)})\Delta t + \sigma_T^{(t)} dw$$

We proceeded to build an estimator of the inflation factor that depends on the expectation of the squared innovations,

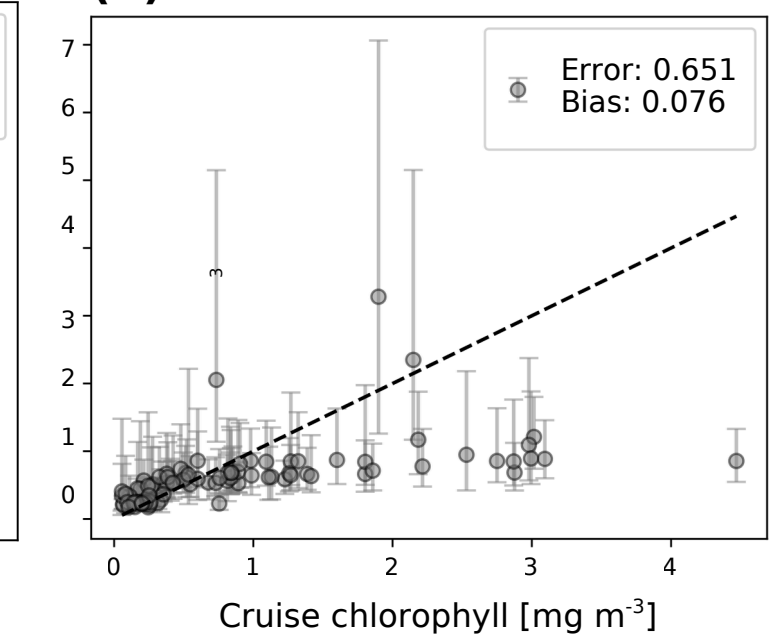
$$\hat{\rho}^{(t)} - 1 = \frac{\sum_i \Delta t \hat{\beta}_i^{(t)} \left( \mathbb{E}_t[(\vec{t}_i^{(t)} - H(\vec{x}^{(t)})_i]^2) - \mathbf{R}_{i,i}^{(t)}]}{\sum_i \hat{\mathbf{P}}_{i,i}}$$

Where the summation is over the observations "y", H() is the observation operator on a state vector "x", R is the observation error covariance, P is the ensemble covariance and a hat stands for an estimator of the true value.

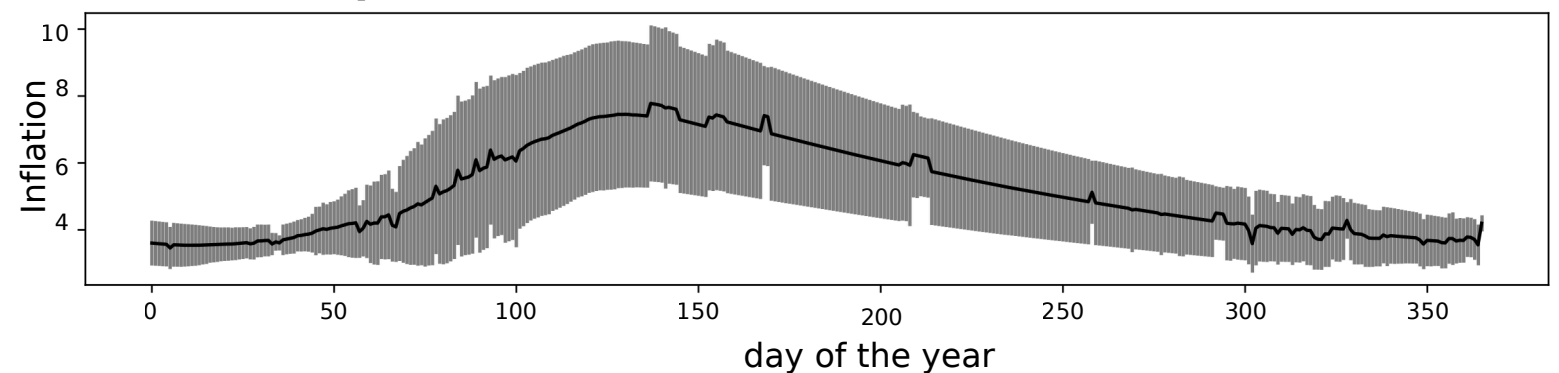
### (A) Direct Assimilation



### (B) Indirect assimilation



### (C) Time dependence of inflation for the direct assimilation



**Figure 2:** Scatter plot for SSC, using data points from summer and spring (A)(B), and time dependence of the inflation for the direct assimilation (C).

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